XII

PHYSICS

Volume 1 Study Material

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COLOMBIA PUBLICATION PROUDLY PRESENTS || 12th PHYSICS VOLUME 1 COMPLETE GUIDE ||

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INDEX

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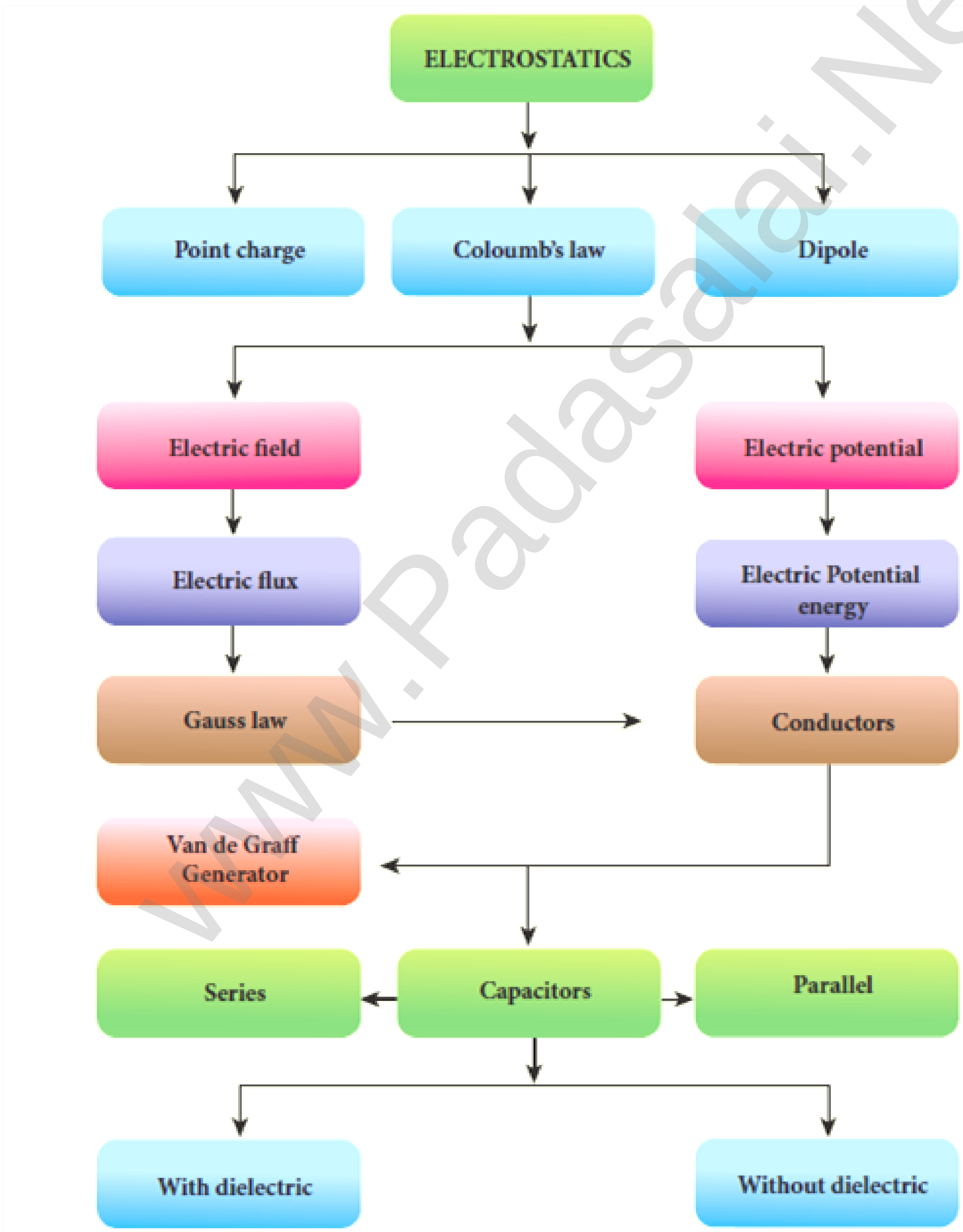
UNIT 1

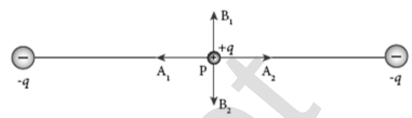
ELECTROSTATICS



PART – A (1 MARKS)

1. Two identical point charges of magnitude –q are fixed as shown in the figure below. A third charge +q is placed midway between the two charges at the point P. Suppose this charge +q is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will +q be stable with respect to the displacement?

(a) A_1 and A_2 (b) B_1 and B_2
(c) both directions (d) No stable



Answer :- (b) B_1 and B_2

Angle between B_1 and B_2 is $90°$

$$V = \frac{1}{4\pi\varepsilon o}\frac{P\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon o}\frac{P\cos 90°}{r^2} = 0$$

$W = V.q = 0$

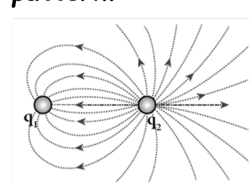
$+q$ be stable along B_1 and B_2.

If the displacement of the charge is in equatorial plane , then it will be stable .

2. Which charge configuration produces a uniform electric field?
(a) point charge (b) uniformly charged infinite line
(c) uniformly charged infinite plane (d) uniformly charged spherical shell

Answer :- (c) uniformly charged infinite plane

3. What is the ratio of the charges $\left|\frac{q_1}{q_2}\right|$ for the following electric field line pattern?



(a) $^1/_5$ (b) $^{25}/_{11}$ (c) 5 (d) $^{11}/_{25}$

Answer :- (d) $^{11}/_{25}$

$$\frac{No.\ of\ lines\ entering\ (q_1)}{No.\ of\ lines\ entering\ (q_2)} = \left|\frac{q_1}{q_2}\right| = {}^{11}/_{25}$$

4. An electric dipole is placed at an alignment angle of $30°$ with an electric field of 2×10^5 N C^{-1}. It experiences a torque equal to 8 N m. The charge on the dipole if the dipole length is 1 cm is
(a) 4 mC (b) 8 mC (c) 5 mC (d) 7 mC

Answer :- (b) 8 mC

τ =pEsin =(2qa)E sinθ

2a = 1 cm = 1 x 10^{-2} m

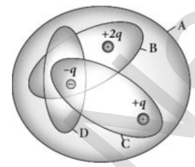
$q = \frac{\tau}{2aesin\theta} = \frac{8}{1 X 10^{-2} X 2 X 10^{5} X sin30°}$

$sin30° = \frac{1}{2}$

$q = \frac{8 x 2}{2 X 10^{3} X sin30°}$ =8x10^{-3} C =8mc

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.

(a) D < C < B < A (b) A < B = C < D
(c) C < A = B < D (d) D > C > B > A



Answer :- (a) D < C < B < A

Net charge A=3q-q=2q,

Net charge B=2q-q=q

Net charge C=-q+q=0,

Net charge D=-q

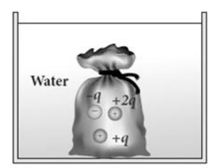
$\emptyset_A = \frac{2q}{\varepsilon_0}$; $\emptyset_B = \frac{q}{\varepsilon_0}$

$\emptyset_C = 0$; $\emptyset_D = -\frac{q}{\varepsilon_0}$

$\emptyset \propto q$

6. The total electric flux for the following closed surface which is kept inside water

(a) $\frac{80q}{\varepsilon_0}$ **(b)** $\frac{q}{40\varepsilon_0}$ **(c)** $\frac{q}{40\varepsilon_0}$



Answer :- (b) $\frac{q}{40\varepsilon_0}$

Net charge Q=-q+q+2q=2q

$\emptyset = \frac{Q}{\varepsilon} = \frac{Q}{\varepsilon_0\varepsilon_r} = \frac{2q}{80\varepsilon_0} = \frac{q}{40\varepsilon_0}$

7. Two identical conducting balls having positive charges q_1 and q_2 are separated by a centre to centre distance r. If they are made to touch each other and then separated to the same distance, the force between them will be
(a) less than before (b) same as before (c) more than before (d) zero

Answer :- (c) more than before

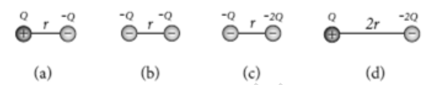
Initially F $\propto q_1 q_2$
After they are made to touch each other, the charge on each ball will be equal.
$q = \frac{q_1 + q_2}{2}$

Now new force $F^{l}=q \times q$ (or) $F^{l} \propto q^2$

$\left[\frac{q_1+q_2}{2}\right]^2 > q_1q_2$ (i.e) $q^2 > q_1q_2$ $\therefore F^{l} > F$

8. Rank the electrostatic potential energies for the given system of charges in increasing order.

(a) 1 = 4 < 2 < 3 **(b) 2 = 4 < 3 < 1**
(c) 2 = 3 < 1 < 4 **(d) 3 < 1 < 2 < 4**



Answer :- (a) 1 = 4 < 2 < 3

The electrostatic potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

a) $U_1 = k - \frac{Q^2}{r}$ b) $U_2 = k\frac{Q^2}{r}$ c) $U_3 = k\ \frac{2Q^2}{r}$ d) $U_4 = k - \frac{2Q^2}{2r} = k - \frac{Q^2}{r}$

9. An electric field $\vec{E} = 10x\hat{j}$ exists in a certain region of space. Then the potential difference V = $V_o - V_A$, where V_o is the potential at the origin and V_A is the potential at x = 2 m is:

(a) 10 V **(b) −20 V** **(c) +20 V** **(d) −10 V**

Answer :- (c) +20 V

$E = -\frac{dv}{dx}$; $dv = -E.dx$

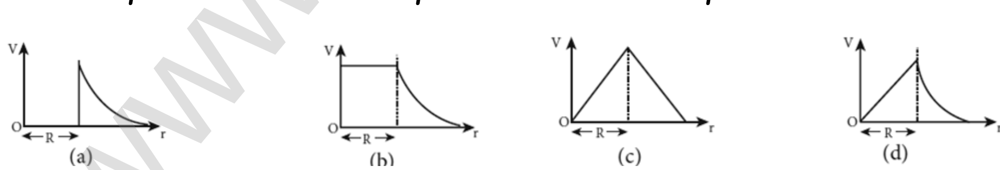
$dv = -10x.dx$

$\int_{V_B}^{V_A} dv = - \int_0^2 10x.dx = -10\left[\frac{x^2}{2}\right]_0^2 = -10 \times \frac{2^2}{2}$;

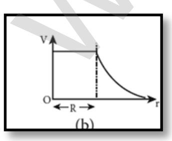
$V_A - V_B = -5 \times (2)^2 = -20$ J

$V_o - V_A = 20$ J

10. A thin conducting spherical shell of radius R has a charge Q which is uniformly distributed on its surface. The correct plot for electrostatic potential due to this spherical shell is



Answer :-



1. Potential is constant inside spherical shell V=constant
2. Potential decreased outside spherical shell as distance increase $V \propto 1/r$

11. Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is

(a) 8.80×10^{-17} J (b) -8.80×10^{-17} J (c) 4.40×10^{-17} J (d) 5.80×10^{-17} J

Answer **:-** *(a) 8.80×10^{-17} J*

$Q = 50e = 50 \times (-1.6 \times 10^{-19}) = -80 \times 10^{-19}$
$\Delta V = V_A - V_B = -4 - 7 = -11$ V
Workdown $= Q \times \Delta V = -80 \times 10^{-19} \times (-11)$
$= 8.8 \times 10^{-17}$ J

12. If voltage applied on a capacitor is increased from V to 2V, choose the correct conclusion.

(a) Q remains the same, C is doubled (b) Q is doubled, C doubled
(c) C remains same, Q doubled (d) Both Q and C remain same

Answer **:-** *c) C remains same, Q doubled*

1) $Q \propto V$, Charge depends on potential, If V is doubled Q also doubled

2) $C = \frac{\varepsilon_0 A}{d}$,Capacitance is independent of C and V , Capacitance remain same if voltage is doubled

13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?

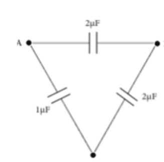
(a) Capacitance (b) Charge (c) Voltage (d) Energy density

Answer **:-** *(d) Energy density*

$A \rightarrow 2A$, $d \rightarrow 2d$

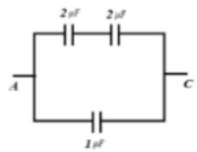
$C=\frac{\varepsilon_0 A}{d}$	$Q \propto V$	$V=\frac{Q}{C}=\frac{Qd}{\varepsilon_0 A}$	$U_E=\frac{1}{2}\epsilon_0 E^2 Ad$
$C'=\frac{\varepsilon_0 2A}{2d}=\frac{\varepsilon_0 A}{d}$	V-doesn't change	$V'=\frac{Q2d}{\varepsilon_0 2A}=\frac{Qd}{\varepsilon_0 A}$	$U'_E=\frac{1}{2}\epsilon_0 E^2 2A2d$ $=4\frac{1}{2}\epsilon_0 E^2 Ad$
C=C'	Q-same	V=V'	$U'_E = 4U_E$

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the pointsA and C is



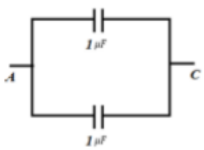
(a) 1μF (b) 2 μF (c) 3 μF (d)$\frac{1}{4}$ μF

Answer **:-** *(b) 2 µF*



C_1 and C_2 are in series

$$\frac{1}{c_s} = \frac{1}{c_1} + \frac{1}{c_2} = \frac{1}{2} + \frac{1}{2} = 1\ \mu F$$



C_1 and C_s are in parallel

$$C_p = C_3 + C_s = 1 + 1 = 2\ \mu F$$

15. ***Two metallic spheres of radii 1 cm and 3 cm are given charges of -1×10^{-2} C and 5×10^{-2} C respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is***

(a) 3×10^{-2} C (b) 4×10^{-2} C (c) 1×10^{-2} C (d) 2×10^{-2} C

Answer **:-** *(a) 3×10^{-2} C*

$q_1 = -1 \times 10^{-2}\ C$; $q_2 = 5 \times 10^{-2}\ C$

$Q = q_1 + q_2 = (-1+5) \times 10^{-2} = 4 \times 10^{-2}\ C$

Potentials equal,

$$V_1 = V_2$$

$$K\frac{q_1}{r_1} = K\frac{q_2}{r_2}$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \rightarrow \frac{q_1}{1\ cm} = \frac{q_2}{3\ cm}$$

$$3q_1 = q_2$$

$$q_1 + q_2 = 4 \times 10^{-2}\ C$$

$$q_1 + 3q_1 = 4 \times 10^{-2}\ C$$

$$4q_1 = 4 \times 10^{-2}\ C$$

$q_1 = 1 \times 10^{-2}\ C$; $q_2 = 3q_1 = 3 \times 10^{-2}\ C$

PART – B (2 MARKS)

1. ***CHARGING BY RUBBING***

Some materials are found to be charged by rubbing with suitable materials.

EX

1. *Amber rod is charged by rubbing with animal fur.*
2. *Glass rod is charged by rubbing with silk cloth.*

2. ***CONCLUSION***

- *Like charges repel force.*
- *Unlike charges attract force.*

Two kinds of charges

1. *Positive charge (+ve)*
2. *Negative charge (-ve)*

3. ***NEUTRAL CHARGES***

If the net charge is zero in the object it is said to be electricity neutral.

CHARGE OF AN ATOM

All material are made up of atoms which are electrically neutral. But its constituent particles posses charges.

i. *Protons* → *+ve*
ii. *Electrons* → *-ve*
iii. *Neutrons* → *zero*

4. ***TRIBOELECTRIC CHARGING***

*When an object is rubbed with another object [**EX**: Rubber with silk cloth], some amount of charges is transferred from one object to the other due to friction between them. The object is said to be **'Electrically charged'.** This method of charging the objects through rubbing is called **'Triboelectric charging'.***

5. BASIC PROPERTIES OF CHARGES

I. ELECTRIC CHARGE

Electric charge is inherent and fundamental property of particles.

Charge is the physical property of particles is matter that experiences is directly Proportional to the force when placed in an electromagnetic field.

S.I unit → Coulomb [c]

II. CONSERVATION OF CHARGES

The total electric charge in the universe is constant and charge can be neither created nor destroyed . In any physical process, the net charge is always zero.

III. QUANTISATION OF CHARGES

The charge of any object is equal to an integral multiple of the fundamental unit of charge e,

$$\boxed{q=ne}$$

n → *integer [0, ±1, ±2,..]*

e → *charge of an electron.*

The charge of electron is found to be '-e' → -1.6×10^{-19} C

The charge of proton is found to be '+e' → 1.6×10^{-19} C

Charge Quantization is applicable for microscopic level and not for macroscopic. When a glass rod is rubbed with silk cloth, approximately 10^{10} *charges are transferred and the charges are treated to be continuous.*

The smallest charge in nature :- *Charge of electron (-e) and charge of proton (+e)*

6. COULOMB'S LAW

The force of attraction or repulsion between two point charges [q_1 *and* q_2*] separated by a distance 'r' is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charge*

7. RELATIVE PERMITTIVITY [Dielectric constant, ε_r]

Force between two point charges in medium is always less than that in vacuum.

In Vacuum:

$$[\vec{F}_{21}]_{vac} = \frac{1}{4\pi\varepsilon o}\frac{q1q2}{r^2}\ \hat{r}_{12}$$

In Medium:

$$[\vec{F}_{21}]_{Med} = \frac{1}{4\pi\varepsilon}\frac{q1q2}{r^2}\ \hat{r}_{12}$$

$$\frac{\vec{F}\ 21\ vac}{\vec{F}\ 21\ \ Med} = \frac{\varepsilon}{\varepsilon o} > 1$$

Since $\varepsilon > \varepsilon o$

$$\vec{F}\ 21\ vac > \vec{F}\ 21\ Med$$

The ratio $\frac{\varepsilon}{\varepsilon o}$ *is called relative permittivity of the Medium.*

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon o}$$

In air (or) vacuum $\varepsilon_r = 1$

For all other medium $\varepsilon_r > 1$

8. LIMITATION OF COULOMB'S LAW

The coulomb force is the only for point charges and is applicable for charges whose size is smaller compared to the distance between them.

9. ACTION AT A DISTANCE

Consider a point charge kept at a point in space.If another charge is placed at some distance from the first charge , it experiences a attractive or repulsive force. This is called " action at a distance" [non- contract force]

10. ELECTRIC FIELD

The region of space around a charged particle which exerts a force on another charged particle brought near it.

$\vec{F}$ *is the force experienced by a charge* q_o.

Electric field, $\vec{E} = \frac{\vec{F}}{qo}$

S.I unit → NC^{-1}

Direction of $\vec{E}$ *is along the direction of force.*

11. UNIFORM ELECTRIC FIELD

If the electric field has the same direction and constant magnitude at all points in space , then the electric field is said to be uniform .

12. NON- UNIFORM ELECTRIC FIELD

If the electric field has different direction or different magnitude or both at different points in space, then the electric field is said to be non-uniform.

13. LINEAR CHARGE DENSITY (λ)

The charge (Q) per unit length (L).

$$\boxed{\lambda = \frac{Q}{L}}$$

S.I unit → cm^{-1}

14. SURFACE CHARGE DENSITY (σ)

The charge (Q) per unit surface area(A).

$$\boxed{\sigma = \frac{Q}{A}}$$

Unit → cm^{-2}

15. VOLUME CHARGE DENSITY(ρ)

Charge (Q) per unit volume (V)

$$\boxed{\rho = \frac{Q}{V}}$$

unit → cm^{-3}

16. ELECTRIC FIELD LINES

Electric field lines are the imaginary lines drawn to visualize the electric field in same region of space .

17. ELECTRIC DIOPLE

Two equal and opposite charges separated by a small distance is called an Electric dipole.

18. ELECTRIC DIPOLE MOMENT (P)

The product of charge and the distance between them.

For a dipole of '+q' and '-q' separated by a distance 'r'

$\vec{P} = q2\vec{a} = 2q\vec{a}$

$\boxed{\vec{P}=2q\vec{a}}$

It is vector quantity . *S.I unit → cm.*

19. ELECTICSTATIC POTENTIAL(V)

The workdone to bring a unit positive charge from infinity to a point in the region of external electric field.

20. POTENTIAL DIFFERENCE (ΔV)

The workdone by an external force to bring a unit positive charge from one point to another point in space.

Unit → Jc^{-1} (or) Volt

21. ELECTRIC FLUX ($\emptyset_E$)

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux.

Unit → Nm^2c^{-1}

$\emptyset_E$ → *Scalar and it may +ve or –ve.*

22. GAUSS LAW

If a charge Q is enclosed by an arbitrary closed surface , then the total electric flux $\emptyset_E$ over the closed surface is $\frac{1}{\varepsilon o}$ times the total charges Q enclosed by the surfaces.

$$\boxed{\emptyset_E = \oint \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon o}}$$

$'Q'_{encl}$ *represents the charges inside the closed surfaces.*

23. CONDUCTORS

Electrical are materials which has a large number if mobile charges [free electrons] which are free to move in the material .

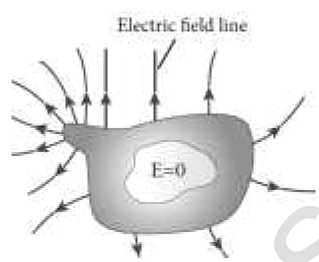
Ex : copper

24. ELECTROSTATIC EQUILIBRIUM:

*The free charges in a conductor are not bound and they are free to move in all directions , Hence there is no net motion of charges [namely electrons] along a particular direction and there is no net current in the conductor .This state is called **Electrostatic equilibrium.***

25. ELECTROSTATIC SHIELDING

Using Gauss law ,we proved that the electric field inside a charged spherical shell is zero. Further, we showed that the electric field inside both hollow and solid conductors is zero .It is very interesting property which has an important consequence.



Consider a cavity inside the conductor as shown in figure. whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside a cavity is zero. A sensitive electrical instrument which is to be productive from external electrical disturbance is kept inside the cavity this is called electrostatic shielding.

Faraday cage is an instrument used to demonstrate this effect .It is made up of metal bars. If an artificial lighting jolt is created outside, the person inside is not affected.

26. ELECTROSTATIC INDUCTION

*Charging of conductors without actual contact is called **electrostatic induction.***

27. DIELECTRICS OR INSULATORS

A dielectric is a non conducting material with no free electrons.

***Ex:** Ebonite, glass, mica. A dielectric is made up of either polar molecules or non polar molecules.*

28. NON – POLAR MOLECULES

Non-polar molecules are molecules which has no permanent dipole moments.

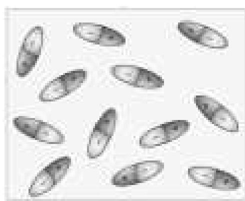
***Ex :** Hydrogen (H_2), oxygen (O_2), carbon dioxide(CO_2)*

29. POLAR MOLECULES

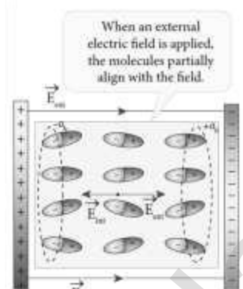
Polar molecules are molecules which have ***permanent dipole moments.***

Ex: *H_2O , N_2O , NH*

30. EFFECT OF ELECTRIC FIELD ON POLAR MOLECULES



(a)



(b)

In polar molecules ,the centre of positive and negative charges are separated even in the absence of an external electric field. This, they have a permanent dipole moment one to thermal motion ,the direction of each dipole moment is oriented randomly . Hence the dipole moment gets cancelled due to this random orientation as shown in figure . The net dipole moment is zero in the absence of an external electric field.

When an external electric field is applied, the dipoles align in the direction of the external electric field. This indicates a dipole moment and the dipole are polarized by the external electric field as shown in the figure. The external electric field indicates polarization in both the polar and non-polar molecule.

31. POLARI SATION $\vec{P}$

The total dipole moment per unit volume in a dielectric is called polarization ( )

For a linear dielectric, the polarization is directly proportional to the applied electric field.

$$\vec{P} \propto \vec{E}_{ext}$$

$$\boxed{\vec{P} = \chi_e \vec{E}_{ext}}$$

Where, X_e is constant called the electric susceptibility of the dielectric.

32. SUSCEPTIBILITY (X_e)

Susceptibility is defined as the case with which a dielectric is electricfield.

$$X_e = \frac{\vec{P}}{\vec{E}}$$

It is the polarisation induced in a dielectric per unit electric field.

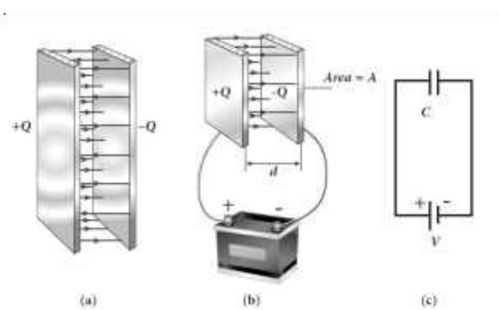
33. DIELECTRIC STRENGTH:

When the external electric field applied to a dielectric is very large ,it tears the atoms apart so that they bound charges become free charges. Then the dielectric starts to conduct electricity. This is called a dielectric breakdown. The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength .For example, that dielectric of air is 3 x 10^6 Vm^{-1}. If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in table below.

Substance	Dielectric strength (Vm^{-1})
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Air	3×10^6
Pyrex glass	14×10^6

34. CAPACITOR

Capacitor is a device used to store electric charge and electric energy.



CAPACITANCE OF A CAPACITOR

A capacitor consists of two parallel metal plates separated by a small distance as shown in figure (a). When the capacitor is connected to a battery of potential difference V the plate connected to the positive terminal of the battery acquires a charge of + Q and the plate connected to the negative terminal acquires charge of - Q .The potential difference between the plates is equal to the battery's terminal voltage as shown in figure (b).

As the battery voltage V is increased. The amount of charge stored in the plates also increase. Thus, the charge stored in the capacitor is proportional to the potential difference V. (i.e) $Q \propto V$

$$\boxed{Q = cV}$$

c is proportionality constant called the capacitance.

35. CAPACITANCE

The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on the conductor plates in the potential difference existing between the conductors .

$$\boxed{C = \frac{Q}{V}}$$

Unit of capacitance is COULOMB per volt or farad (F).

Farad is the larger unit of capacitance and capacitors are available in the range of

Microfarad [1μF $=10^{-6}$ F] to picofarad [1PF = 10^{-12}F]. The capacitor is represented by the symbol ⊣⊢ or ⊣(

36. TOTAL CHARGE STORED IN A CAPACITOR

The total charge stored in the capacitor is the sum of charges deposited on the two plates. The total charges stored in a capacitor is Q - Q =0 [because equal number of positive charge and negative charges are deposited on the plates which gets cancelled]

When a capacitor is said to store charges ,it actually represents the amount of charges stored in any one of the plates.

37. TYPES OF CAPACITOR

Available shapes of capacitors:

Cylindrical, disc.

***Available types of capacitors**:*

Tantalum,ceramic ,electrolytic

38. ENERGY DENSITY (U_E)

The energy stored per unit volume of space is defined as the energy density .

$$U_E = \frac{U}{Volume}$$

39. APPLICATIONS OF CAPACITORS

Capacitors are used in various electronic circuits. A few of the applications are ,

a) Most people are now familiar with the digital camera .The flash which comes from the camera when we take photographs is due to the energy released from the capacitor called the flash capacitor .

b) During cardiac arrest, device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of 175 μF charged to a high voltage of around 2000 V.

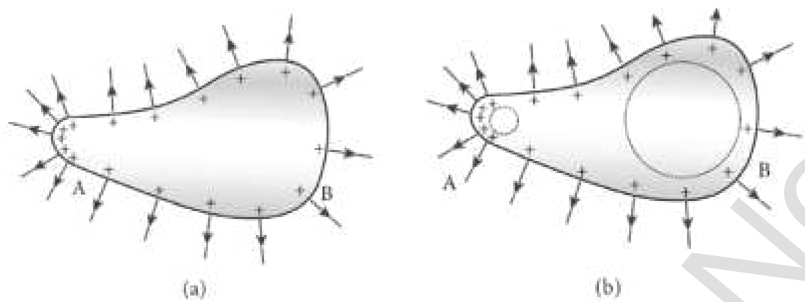
c) Capacitors are used in the ignition system of automobile energy to eliminate sparking.

d) Capacitors are used to reduce power fluctuations in power supplies and to increase efficiency of power transmission.

However, Capacitors have disadvantages as well. Even after the battery or power supply is removed the capacitor stores charges and energy for some time .For example, if the TV is switched off it is always advisable to not touch the back side of the TV panel .

40. ACTION OF POINTS (OR) CORONA DISCHARGE

Consider a charged conductor of irregular shaped as shown in the figure . In conductors, if the radius curvature is smaller ,the charge density around it will be larger, because of the large accumulator of charges over a small area.

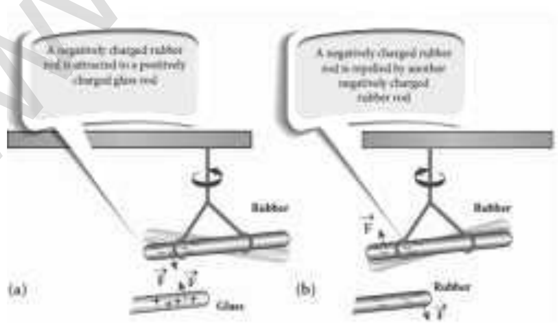


The electric field near the edge of smaller curvature is very high and it ionizes the surrounding air. (i.e) The positive ions are repelled are there sharp edge and negative ions are attracted towards the sharper edges . This causes neutralisation of charges and reduced the charge of the conductor near the sharp edge. This is called ***action at points (or) corona discharge.***

<u>PART – C (3 MARKS)</u>

41. EXPERIMENT

Consider a charged rubbed rod hanging from a thread. When another charged rubber rod is brought near it will be repelled instead if a charged glass rod is brought near rubber rod, it will be attracted.



(a) Unlike charges attract each other (b) Like charges repel each other

42. SIMILARITIES BETWEEN COULOMB'S LAW AND NEWTON'S LAW OF GRAVITATION

S.NO	COULOMB'S LAW	NEWTON'S LAW
1	Electrostatic force between charges q_1 and q_2.	Gravitational force between charges masses m_1 and m_2.
2	$F \propto q_1 q_1$	$F \propto m_1 m_2$
3	$F \propto \frac{1}{r^2}$	$F \propto \frac{1}{r^2}$
4	Attractive or Repulsive depending on the nature of the charge.	Always attractive.
5	Depends on the nature of the Medium in which the charges are kept.	Independent of the medium.
6	Varies when they are in motion.	Remains the same when the masses are at rest as in Motion.
7	$K = 9 X 109\ Nm^2C^{-2}$	$G = 6.67 X 10^{-11} Nm^2kg^{-2}$

K is much higher than G.

Electrostatic force is always greater [Magnitude] than gravitational force for smaller size object .

43. FORCE BETWEEN CHARGES [NEWTON'S III LAW

Prove that electrostatic force obeys Newton's third law:

Two point charges q_1 and q_2 ; distance ' r'.

Force on the point charge 'q_2' exerted by charging 'q_1'

$$\vec{F}_{21} = K \frac{q1q2}{r^2} \hat{r}_{12}$$

Force on the point charge 'q_1' exerted by charging 'q_2'

$$\vec{F}_{12} = K \frac{q1q2}{r^2} \hat{r}_{21}$$

But $\hat{r}_{21} = -\hat{r}_{12}$

Substituting,

$$\vec{F}_{12} = K\frac{q1q2}{r^2}[-\hat{r}_{12}] = -K\frac{q1q2}{r^2}[\hat{r}_{12}]$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Equal in Magnitude but opposite in direction.

44. MAGNITUDES OF ELECTROSTATIC AND GRAVITATIONAL FORCES

Why a charged comb attracts an uncharged piece of paper with greater force even through the paper is attracted by the gravitational force of the earth?

The electrostatic force between a proton and electron is found to be 10^{39} *times than that of the gravitational force between them.*

$$\boxed{F_e = 10^{39} F_G}$$

Thus, the gravitational force is negligible compared to the electrostatic force for small size objects in the atomic domain. This is the reason that a charged comb attracts an uncharged piece of paper with greater force even through the paper is attracted downwards by the gravitational force.

45. SUPERPOSITION PRINCIPLE

" The total force acting on a given charge is equal to the vector sum of forces Exerted on it by all the other charges "

Consider the system of 'n' charges, namely $q_1, q_2, q_3, \ldots q_n$.

The force on 'q_1' *exerted by charging* 'q_2'

$$\vec{F}_{12} = K\frac{q1q2}{r_{21}^2}\ \hat{r}_{21}$$

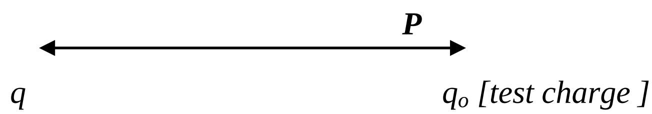
The force on 'q_1' *exerted by charging* 'q_3'

$$\vec{F}_{13} = K \frac{q1q2}{r_{13}^2} \hat{r}_{13}$$

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1n}$$

$$\vec{F}_1^{tot} = K \left[\frac{q1q2}{r_{21}^2} \hat{r}_{21} + \frac{q1q2}{r_{13}^2} \hat{r}_{13} + \ldots\ldots + \frac{q1q2}{r_{n1}^2} \hat{r}_{n1}\right]$$

46. ELECTRIC FIELD DUE TO POINT CHARGE



Consider a source point charge q placed at some point in space. Let another point charge q_o is placed at a point 'p' which is at a distance 'r' from q.

The force experienced by 'q_o' is $\vec{F} = K \frac{q\, qo}{r^2} \hat{r}$

Electric field $\vec{E} = \frac{\vec{F}}{qo}$ $\boxed{\vec{E} = \frac{1}{4\pi\varepsilon o} \frac{q}{r^2} \hat{r}}$

UNIT → *NC^{-1} ; Vector quantity.*

47. PROPERTIES OF ELECTRIC FIELD

1. *The electric field of a positive charge is directed away from the source charge and the electric field of a negative charge is directed towards from the source charge.*
2. ***Coulomb's law in terms of electric field:-***

   If the electric field at a point P is E,then the force experienced by q_o at p is

   $$\vec{F} = q_o\vec{E}$$

3. $\vec{E} = \frac{1}{4\pi\varepsilon o} \frac{q}{r^2} \hat{r}$. $\vec{E}$ *is independent of the test charge q_o and depends only on the*

source charge 'q'.

4. $\vec{E}$ is Vector quantity which posses both magnitude and direction . $E \propto \frac{1}{r^2}$

   $\vec{E}$ is decreases in magnitude as the distance increases and vice versa.

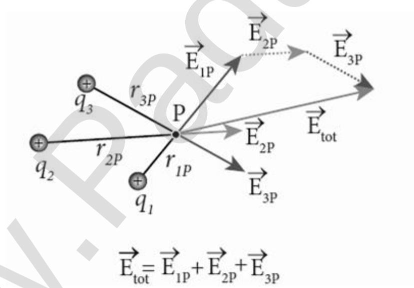
5. It is assumed that the test charge is sufficiently smaller such that brightly it near the source charge will not modify the electric field of the source charge.

6. $E = \frac{F}{q}$ is valid only for point charges , for continuous and finite charge distributions integration technique are used.

7. These are two kinds of electric fields, uniform and non uniform electric fields.

48. ELECTRIC FIELD DUE TO A SYSTEM OF POINT CHARGES

Electric field obeys the superposition principle ."The electric field due to a collector of charges at any point is equal to the vector sum of the individual charges"



Consider a collection of point charges q_1, q_2, q_3, ... q_n located at various points in space.

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots \vec{E}_n$$

$$\vec{E_1} = \frac{1}{4\pi\varepsilon o}\frac{q1}{r_{1P}^2}\ \hat{r}_{1P}\ ;\ \vec{E_2} = \frac{1}{4\pi\varepsilon o}\frac{q2}{r_{2P}^2}\ \hat{r}_{2P}\ ;\ \vec{E_n} = \frac{1}{4\pi\varepsilon o}\frac{qn}{r_{nP}^2}\ \hat{r}_{nP}\ .$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon o}\left[\frac{q1}{r_{1P}^2}\hat{r}_{1P} + \frac{q2}{r_{2P}^2}\hat{r}_{2P} + \ldots\ldots + \frac{qn}{r_{nP}^2}\hat{r}_{nP}\right]$$

$$\boxed{\vec{E}_{tot} = \frac{1}{4\pi\varepsilon o}\sum_{i=1}^{n}\frac{qi}{r_{iP}^2}\ \hat{r}_{ip}}$$

49. ELECTRIC FIELD DUE TO A CONTINUOUS CHARGE DISTRIBUTION

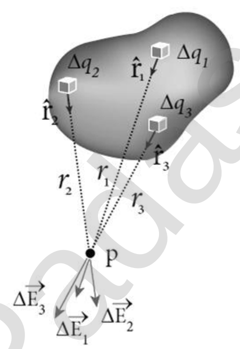
In order to find the electric due to charged wire , charged sphere etc.,

The charges are assumed to be distributed continuously, this is because the charges are closely spaced with lesser inter-particles distance. Hence, they are consider as continuously distributed and not discrete.

50. EXPRESSION FOR $\vec{E}$ OF A CONTINUOUS CHARGE DISTRIBUTION

Consider a charged irregular object. Divide the entire object into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3, \ldots \Delta q_n$ Each charge element Δq is taken as a point charge.

The electric field at a point P, due to the charged object is given by the sum of the field at 'P' due to all charge elements $\Delta q_1, \Delta q_2, \Delta q_3, \ldots \Delta q_n$



$$\vec{E} \approx \frac{1}{4\pi\varepsilon o}\left[\frac{q1}{r_{1P}^2}\hat{r}_{1P} + \frac{q2}{r_{2P}^2}\hat{r}_{2P} + \ldots\ldots + \frac{qn}{r_{nP}^2}\hat{r}_{nP}\right]$$

$$\vec{E} \approx \frac{1}{4\pi\varepsilon o}\sum_{i=1}^{n}\frac{qi}{r_{iP}^2}\hat{r}_{ip} \longrightarrow (1)$$

Δq_i is the i^{th} charge element , r_{ip} is the distance of the point P from the i^{th} charge element and $\hat{r}_{ip}$ unit vector.

To incorporate the continuous charge distribution take the limit as $\Delta q \rightarrow 0$

In this limit the summation in the equation (1) becomes an integral.

$$\vec{E} \approx \frac{1}{4\pi\varepsilon o}\int\frac{dq}{r^2}\hat{r}$$

$r \rightarrow$ distance of the point P from the infinite similar charge dq

51. Show that F=qE is applicable for continuous charge distribution also:

PROOF

The electric field for a continuous charge distribution is,

$$E = \frac{1}{4\pi\varepsilon o}\int \frac{dq}{r^2}$$

$$E = K\int \frac{dq}{r^2}$$

$$E = \frac{K}{r^2}\int dq$$

$$E = \frac{Kq}{r^2} \rightarrow (1)$$

Multiplying both sides of equation (1) by q,

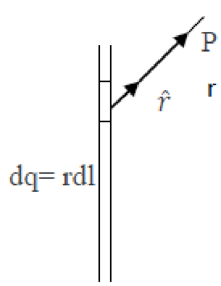
$$qE = \frac{Kq^2}{r^2}$$

$$F = \frac{Kq^2}{r^2}$$

F = qE is applicable for continuous charge distribution.

52. ELECTRIC FIELD DUE TO A LINEAR CHARGE DISTRIBUTION

Consider a linear distribution of charges over a length L of a wire. Let Q be the total change distributed over the length L. Consider an elementary length dl is dq.



The electric field ,

$$d\vec{E} = \frac{\lambda}{4\pi\varepsilon o}\frac{dq}{r2}\hat{r}$$

$$\lambda = \frac{dq}{dl} \quad \text{(or)} \quad dq = \lambda dl$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon o}\frac{\lambda dl}{r2}\hat{r}$$

$$\int dE = \int \frac{1}{4\pi\varepsilon o}\frac{\lambda dl}{r2}\hat{r}$$

$$\boxed{\vec{E} = \frac{\lambda}{4\pi\varepsilon o}\int \frac{dl}{r2}\hat{r}}$$

53. ELECTRIC FIELD DUE TO A SURFACE CHARGE DISTRIBUTION

Consider a charged surface .Let Q be the charges distributed over the surface of area A.Let an amount of charge dq is over dA

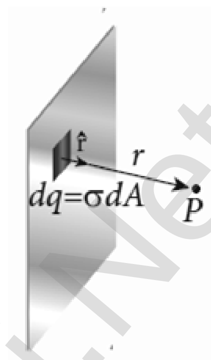
$$\sigma = \frac{dq}{dA} \quad (or) \quad dq = \sigma dA$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon o}\frac{dq}{r2}\hat{r}$$

$$\int d\vec{E} = \frac{1}{4\pi\varepsilon o}\frac{\sigma dA}{r2}\hat{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon o}\int\frac{\sigma dA}{r2}\hat{r}$$

$$\boxed{\vec{E} = \frac{\sigma}{4\pi\varepsilon o}\int\frac{dA}{r2}\hat{r}}$$



54. RULES FOR DRAWING ELECTRIC FIELD LINES FOR CHARGES

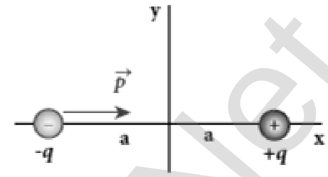
Electric field vectors are visualized by the concept of electric field lines . They form a set of continuous lines which represent electric field in some region of space visuality.

1. *The electric field lines start from a positive charge and end at negative Charge or at infinity . For a positive point charge the electric field lines point radially outward and negative point charge electric charge electric field lines point radially inward.*
2. *The electric field vectors at a point in space is tangential to the electric field lines at that point.*
3. *The electric field lines are denses [more close] in a region where the electric filed has large magnitude and less dense in a region where the electric field is of smaller magnitude . In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region.*
4. *No two electric field lines intersect each other. If two lines cross at a point, then these will be two different electric field vectors at the same point.*

5. *The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.*

55. EXPRESSION FOR ELECTRIC DIOPLE MOMENT $\vec{E}$

Consider two equal and opposite charges (+q,-q) separated by a distance '2a' along the x-axis



$$\vec{P} = (+q)\,\vec{r_+} + (-q)\,\vec{r_-}$$

$\vec{r_+}$ → *Position vector of +q from the origin .*

$\vec{r_-}$ → *Position vector of - q from the origin .*

Distance between the charges +q and –q is 2a.

$\vec{r_+} = a(\hat{\imath})$ and $\vec{r_-} = a(-\hat{\imath})$

$\vec{P} = qa(\hat{\imath}) - qa(-\hat{\imath})$

$\boxed{\vec{P} = 2qa\hat{\imath}}$

The electric dipole moment vector $\vec{P}$ lies along the line joining two charges and directed from –q and +q.

56. PROPERTIES OF ELECTRIC DIPOLE MOMENT

1. *$\vec{P}$ will point from –q to +q whatever be the direction of its placement.*

2. *The magnitude of electric dipole moment is equal to the product of magnitude of the charges and the distance between them.*

$$|\vec{P}| = 2qa$$

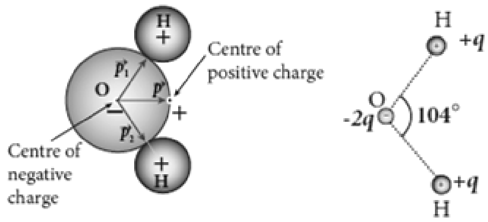
3. *For a collection of point charges, the electric dipole charges, the electric dipole moment is*

$$\boxed{\vec{P} = \sum_{i=1}^{n} q_i\,\vec{r}_i}$$

where $\vec{r}_i$ is the possibility vector of q_i , from the origin

57. DIPOLE MOMENT OF H_2O

The water molecules (H_2 O) has this charge configuration. The water molecules has three atoms [two H atom and one O atom]. 1 A°= 10^{-10} m



The centers of positive (H^+) and negative (O) charges of a water molecules lie at different points , hence it posses permanent dipole moment .

The O-H bond length is 0.958 A° due to which the electric dipole moment of water molecule has the magnitude $\boxed{P = 6.1 \times 10^{-30}\ \text{cm}}$ *The electric dipole moment $\vec{P}$ is directed from center of negative charge to the centre of positive charge .*

58. IMPORTANT INFERENCES

a) RELATION BETWEEN THE MAGNITUDE OF ELECTRIC FIELD DUE TO A DIPOLE AT LARGE DISTANCE ON THE DIPOLE AXIS AND EQUATORIAL PLANE

Comparing (a) and (b) magnitude of the electric field on the dipole axis is twice the magnitude of electric field on the dipole axis is twice the magnitude of electric field at points on the equatorial plane.

Direction $\vec{E}_{tot}$ and $\vec{P}$ are the same at a point at large distance on the axis but they are oppositely dissected at a point at large distance on the equatorial plane

b) COMPARISON BETWEEN ELCTRIC FIELD DUE TO A DIPOLE AND POINT CHARGE AT LARGE DISTANCE :

At large distance, the electric field due to a dipole [E_{tot}] varies as .

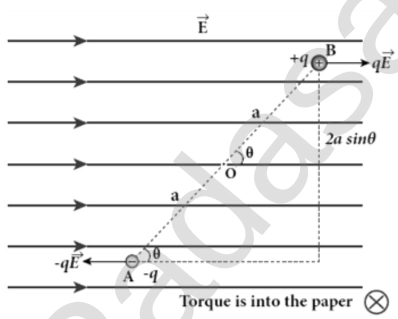
But for a point charge Electric field varies as . Thus at large distance , the electric field due to a dipole goes to zero faster than that of a point charge . Hence at very large distance the two charges of the dipole appear close and neutralize each other

a. ***POINT DIPOLE :***

If the distance 2a approaches zero and q approaches infinity such that the product $\vec{P}=2qa\hat{P}$ is finite . Then the dipoles is called a point dipole equations (a) and (b) hold true. Thus if the distance between the charge in a dipole is zero, then it is called a point dipole

59. TORQUE EXPERIENCE BY AN ELECTRIC DIPOLE IN UNIFOEM ELECTRIC FIELD:

Consider an electric dipole of dipole moment placed in a uniform Electric field



The charge +q in the dipole experience a force $q\vec{E}$ in the direction of the field and the charge –q experience force - $q\vec{E}$ in a direction opposite to the direction of $\vec{E}$ The forces acting at the two ends of the dipole are equal and opposite .Hence the total force acting on the dipole is zero because these two forces cancel each other . But the torque is not zero.

This torque makes the dipole to rotate through an angle θ. The total torque acting at the two ends of the dipole . OA and OB are the perpendicular distance and the force acting at the ends are + $q\vec{E}$ and - $q\vec{E}$

The total torque on the dipole about the point 'O'

$$\vec{\tau} = \overrightarrow{OA} \times (-q\vec{E}) + \overrightarrow{OB} \times (q\vec{E})$$

The total torque [from right – hand corkscrew rule] is perpendicular to the plane of paper and direction into it. Since θ is angle between $\overrightarrow{OA}$ and $-q\vec{E}$

$$\overrightarrow{OA} \times (-q\vec{E}) = |\overrightarrow{OA}||-q\vec{E}| \sin\theta\, \hat{n}$$

Similarly,

$$\overrightarrow{OB} \times (q\vec{E}) = |\overrightarrow{OB}||q\vec{E}| \sin\theta\, \hat{n}$$

$$\vec{\tau} = |\overrightarrow{OA}||-q\vec{E}| \sin\theta\, \hat{n} + |\overrightarrow{OB}||q\vec{E}| \sin\theta\, \hat{n}$$

$$|\overrightarrow{OA}| = |\overrightarrow{OB}| = a$$

$$\vec{\tau} = q\,aE \sin\theta\, \hat{n} + q\,aE \sin\theta\, \hat{n}$$

$$\vec{\tau} = 2\,q\,aE \sin\theta\, \hat{n}$$

$$|\vec{\tau}| = |2\,q\,aE \sin\theta\, \hat{n}|$$

$$\vec{\tau} = 2\,q\,aE \sin\theta\, |\hat{n}|$$

$$\vec{\tau} = 2\,q\,aE \sin\theta$$

Since $|\vec{P}| = 2aq$

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

$$\vec{\tau} = |\vec{P}||\vec{E}| \sin\theta$$

$$\vec{\tau} = |\vec{P}||\vec{E}| \sin\theta\, \hat{n}$$

Case (1) :

τ *is maximum ,* $\theta = 90°$

$$\tau = PE \sin 90°$$

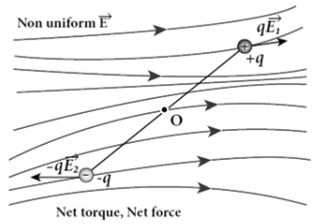
$$\boxed{\tau = PE}$$

This torque rotates the dipole and when it is align with the electric field $\vec{E}$.

$$\theta = 0 ;\ \tau = 0$$

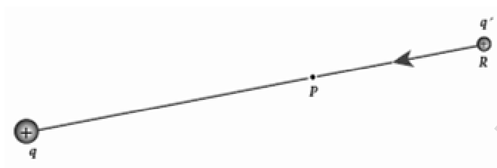
Case (2) :

If the electric field is non – uniform.



The force experienced by +q will be the same as experienced by –q. The net force is not zero. Hence in addition to the torque there will be a net force acting on the dipole .

60. EXPRESSION FOR POTENTIAL DIFFERNCE



Consider a positive charge q in space . It produces an electric field $\vec{E}$ which is pointed outwards around it. Let a +ve test charge q^l is brought from a point R to the point P. The charge q^l experience a electron static repulsive force due to +q. Work must be done by the charge q^l to overcome this repulsive force. This workdone is stored as potential energy. The test charge q^l is brought from R to P with a constant velocity so that the external force needed to bring the charge q^l from R to P must equal and opposite to the coulomb force of repulsion .

$$\vec{F}_{ext} = \vec{F}_{cou}$$

The workdone to move the charge from R to P

W=Force x distance

Let dr be the small distance moved by q^l and qw is the workdone.

$$dw = \vec{F}.\overrightarrow{dr}$$

Let the potential energy of q^l be U_P at P and U_R at R.

$$W = U_P - U_R \qquad U_P - U_R = \Delta U$$

$$W = \Delta U$$

$$\Delta U = \int_R^P \vec{F}_{ext} . \overrightarrow{dl}$$

$\vec{F}_{ext} = -q^l\,\vec{E}$

Negative sign → $\vec{F}_{ext}$ *is opposite to* $\vec{E}$.

$\Delta U = \int_R^P (-q^l\overrightarrow{E}).\overrightarrow{dr}$

$\boxed{\Delta U = q^l \int_R^P (-\overrightarrow{E}).\overrightarrow{dr}}$

The potential energy difference per unit charge

$\frac{\Delta U}{q^l} = \frac{q^l \int_R^P (-\overrightarrow{E}).\overrightarrow{dr}}{q^l} = -\int_R^P \vec{E}\,.\overrightarrow{dr}$

$\boxed{\frac{\Delta U}{q^l} = -\int_R^P \vec{E}\,.\overrightarrow{dr}}$

The above equation is independent of q^l .

The quantity $\frac{\Delta U}{q^l} = -\int_R^P \vec{E}\,.\overrightarrow{dr}$ *is called electro potential difference R and P.*

$V_P - V_R = \Delta V$

$V_P - V_R = \Delta V = -\int_R^P \vec{E}\,.\overrightarrow{dr}$

$\frac{\Delta U}{q^l} = -\int_R^P \vec{E}\,.\overrightarrow{dr}$

$\Delta U = -q^l \int_R^P \vec{E}\,.\overrightarrow{dr}$

$\Delta U = \Delta V q^l$ [*Since,* $\Delta V = -\int_R^P \vec{E}\,.\overrightarrow{dr}$]

If the point R is taken at infinity the potential is zero. [$V\infty = 0$ *because* $V = \frac{Kq}{r} = \frac{Kq}{r} = 0$]

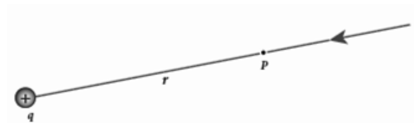
The electric potential at a point P is equal to the workdone by an external force to bring a unit positive charge with constant velocity from infinity to a point P in the velocity from infinity to a point P in the region external field which can be expressed mathematically as,

$\boxed{\Delta V = -\int_R^P \vec{E}\,.\overrightarrow{dr}}$

The electric potential at a point P depends only on the electric field due to the source charge q and not on the test charge q'

61. ELECTRIC POTENTIAL DUE TO A POINT CHARGE :

Consider a positive charge q at the origin . Let P be a point at a distance 'r' from the charge q.



Electric potential at the point 'P' is,

$$V = -\int_{\infty}^{r} \vec{E}\,.\,\overrightarrow{dr}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon o}\frac{q}{r^2}\hat{r}$$

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon o}\frac{q}{r^2}\hat{r}\,.\,\overrightarrow{dr}$$

Small displacement $\overrightarrow{dr}$ *is along the direction of* $\vec{r}$ *.Let* $\hat{r}$ *be the unit vector along* $\overrightarrow{dr}$*,*

$$\overrightarrow{dr} = dr.\hat{r}$$

$$V = -\frac{1}{4\pi\varepsilon o}\int_{\infty}^{r} \frac{q}{r^2}\hat{r}\,.\,dr.\hat{r}$$

$$= -\frac{1}{4\pi\varepsilon o}\int_{\infty}^{r} \frac{q}{r^2}\,dr(\hat{r}.\hat{r}) \qquad \hat{r}.\hat{r} = 1$$

$$= -\frac{1}{4\pi\varepsilon o}\int_{\infty}^{r} \frac{q}{r^2}\,dr$$

$$= -\frac{q}{4\pi\varepsilon o}\left[\frac{r^{-2+1}}{-2+1}\right]_{\infty}^{r}$$

$$= -\frac{q}{4\pi\varepsilon o}\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$= \frac{q}{4\pi\varepsilon o}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\boxed{V = \frac{q}{4\pi\varepsilon o r} \quad \text{(or)} \quad V = \frac{Kq}{r}}$$

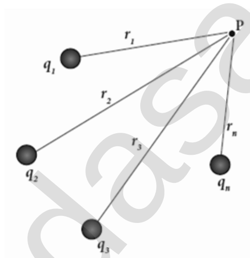
62. PROPERTIES OF POTENTIAL :

1. The potential V is positive for a positive source charge (V > 0) and V is negative if the source charge is negative (V < 0).

2. The motion of charges can be described easily in terms of potential that that of field.

3. V$\propto \frac{q}{r}$ Hence as the distance r increases the potential decreases . If the potential charge q is negative V$\propto -\frac{q}{r}$ the potential increases as the distance is increased . The electric potential is zero. [V = 0]

4. Super position principle :



Electric potential V obeys superposition principle. Consider a collection of charges $q_1, q_2, q_3, \ldots q_n$. The electric potential at a point 'P' due to these charges are the sum of electric potential due to individual charges.

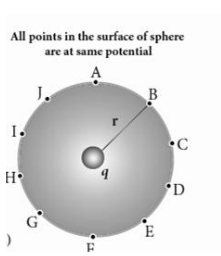
$$V_{tot} = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} + \frac{Kq_3}{r_3} + \ldots + \frac{Kq_n}{r_n}$$

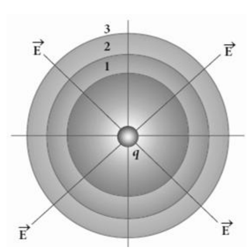
$$\boxed{V_{tot} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

Potential is always a scalar quantity .

63. EQUI-POTENTIAL SURFACE

An equipotential surface is a surface on which all the points are at the same potential





Consider a point charged 'q' in space. Assume an imaginary spherical surface of radius 'r' with the charge of its centre .

The potential of the point charge is $V=\frac{Kq}{r}$ *. As the radius r is the same at each and every point along the spherical surface. The potential V at all points on the surface of the sphere is also the same . Such a surface is called an equipotential surface.*

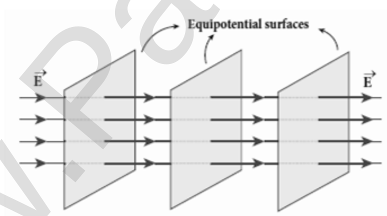
All spherical surface drawn concentric to the equipotential surface will also be equipotential surface . But the value of the potential will differ for different spherical surfaces [as r varies] . For a uniform field , the equipotential surfaces form a set of planes normal to the electric field $\vec{E}$

64. PROPERTIES OF EQUIPOTENTIAL SURFACES

1. *The workdone to move a charge q form one point to another on the same equipotential surface is zero.*

Example : *Consider two points A and B .The workdone to move a charge from A to B is* $W=q(V_B - V_A)$*. Since A and B have the same potential* $V_B - V_A = 0$ *(or)* $W=0$

2. *The electric field* $\vec{E}$ *is always normal to an equipotential surfaces.*



65. RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Consider a positive charge q kept fixed at the origin. To move a unit positive charge by a small distance dx in the electric field E, the work done is given by,

workdone= Force X distance

$W=F.x$

$F = qE$

$W=qE.x$

$dW= - qE.dx$

Given q is a unit point charge

$q = 1C$

$dW = - E.dx$

$E = -\frac{dv}{dx}$

The electric field is the negative gradient of the electric potential.

$\vec{E} = -[\frac{\partial v}{\partial x}\vec{i} + \frac{\partial v}{\partial x}\vec{j} + \frac{\partial v}{\partial x}\vec{k}]$

66. ELECTROSTATIC POTENTIAL ENERGY FOR COLLECTION OF POINT CHARGE WORKDONE TO MOVE A CHARGE

The electric potential at a point at a distance r from a point charge q_1 is

$V = \frac{1}{4\pi\varepsilon o}\frac{q_1}{r}$

In order to bring another charge q_2 from infinity to a point in the fielf of q_1 at a distance r from q_1 work has to be done .

$W = q_2 V$

Workdone is the product of charge and potential .This workdone is stored as the electrostatic potential energy (U) in a system of charges q_1 and q_2 is

$U = q_2 V = q_2 \frac{1}{4\pi\varepsilon o}\frac{q_1}{r}$

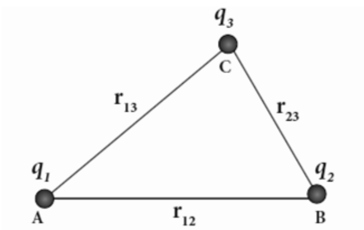
The electrostatic potential energy (U) in a system of charges q_1 and q_2 is

$$\boxed{U = \frac{1}{4\pi\varepsilon o}\frac{q_1 q_2}{r}}$$

Electrostatic potential energy depends only on the distance between the two charges q_1 and q_2 .This equation holds good for any collection of point charges .

67. EXPRESSION FOR ELECTROSTATIC POTENTIAL ENERGY OF A COLLECTION OF POINT CHARGES

Consider three point charges q_1 , q_2 and q_3. In order to assemble these three charges in a region of space work has to be done. Workdone is stored in the form of potential energy .Bring the three charges one by one to a configuration as shown in the figure.



Let the charge q_1 is brought from infinity to that point A. The charge q_1 does not experience any field no other charges was present in the present in the vicinity of q_1 . Hence, the workdone is zero .The potential due to the charge q_1 is zero. But the charge q_1 creates an electric field .Let another charge $q_2$is brought to a point B which is at a distance of r_{12} from q_1.Hence work must be done against the field .The workdone is bringing the charge q_1 to B is

$$W=q_1V_{1B}$$

Since, Workdone = charge X potential.

V_{1B} ⟶ *Electrostatic potential energy due to the charge q_1 at B.*

The electrostatic potential energy due to q_2 is,

$$U= \frac{1}{4\pi\varepsilon o}\frac{q_1q_2}{r_{12}}$$

Now the electric field is produced by the two charges q_1 and q_2 .Let another charge q_3 be brought to the point C which is at a distance of r_{13} from q_1 and r_{23} from q_2.

The workdone to bring the charge q_2 to C is,

$$W =q_3[V_{1C}+ V_{2C}]$$

$V_{1C} \longrightarrow$ *Electrostatic potential energy due to the charge* q_1 *at* C

$V_{2C} \longrightarrow$ *Electrostatic potential energy due to the charge* q_2 *at* C

The electrostatics potential energy due to the charge q_3 *is,*

$$U = \frac{1}{4\pi\varepsilon o}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

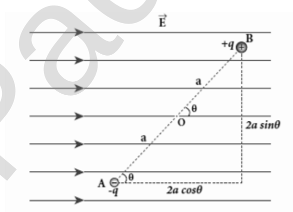
The total electrostatic potential energy for assembly q_1 , q_2 *and* q_3 *is* ,

$$\boxed{U = \frac{1}{4\pi\varepsilon o}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]}$$

The above expression is the same if the charges are brought to these positions in any other order. (i.e.) The electrostatic potential energy is independent of the manner in which the charges arrived.

68. ELECTROSTATIC POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM ELECTRIC FIELD

Consider a dipole placed in a uniform electric field $\vec{E}$ *as shown in the figure*



The electric field exerts a force on the dipole .As a result , the dipole experience a torque and aligns the dipole in the direction of electric field .To rotate the dipole at constant angular velocity from an initial angle 'θ' to another angle θ against the torque exerted by the electric field , an equal and opposite external torque must be applied on the dipole.

The workdone by the external torque to rotate the dipole from angle θ^1 *to* θ *at constant angular velocity is,*

$$W= \int_{\theta^1}^{\theta} \tau_{ext}.d\theta$$

Torque due to the field

$$\vec{\tau}_E = \vec{P} \times \vec{E}$$

$$|\tau_{ext}| = |\vec{\tau}_E| = |\vec{P} \times \vec{E}|$$

$$W= \int_{\theta^1}^{\theta} |\vec{P} \times \vec{E}|.d\theta$$

$$= \int_{\theta^1}^{\theta} |\vec{P}||\vec{E}| \sin\theta.d\theta$$

$$= PE\,[-\cos\theta]_{\theta^1}^{\theta}$$

$$= PE[-\cos\theta + \cos\theta^1]$$

$$W= PE\,[\cos\theta^1 - \cos\theta]$$

The above equation for workdone is equal to the potential energy difference between the angular position θ and θ^1.

$$U(\theta) - U(\theta^1) = \Delta U = -PE\cos\theta + PE\cos\theta^1$$

If θ^1=90° and is taken as reference point.

$$U(\theta^1) = PE\cos 90° = 0$$

$$U = -PE\cos\theta = -\vec{P}.\vec{E}$$

$$\boxed{U = -\vec{P}.\vec{E}}$$

The potential energy depends on $\vec{P}$ and $\vec{E}$, but also on the oriented θ of the dipole with respect to the electric field .

Special cases:

1. *If $\theta = \pi$, the dipole is aligned antiparallel to the electric field and potential energy is maximum.*
2. *If $\theta = 0$, the dipole is aligned parallel to the electric field and potential energy is Minimum.*

69. ELECTRIC FLUX FOR UNIFROM ELECTRIC FIELD

Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown in Figure . The electric flux for this case is

$$\boxed{\phi_E = EA}$$

Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pierce through the area A , The electric flux for this case is zero.

$$\boxed{\phi_E = 0}$$

If the area is inclined at an angle θ with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux.The electric flux,

$$\boxed{\phi_E = (E\cos\theta)\,A}$$

Further, θ is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is

$$\boxed{\phi_E = \vec{E}.\vec{A} = E\,A\cos\theta}$$

Here, note that $\vec{A}$ is the area vector $\vec{A}=A\hat{n}$. Its magnitude is simply the area A and the direction is along the unit vector $\hat{n}$ perpendicular to the area

$$\boxed{\phi_E = \vec{E}.\vec{A}}$$

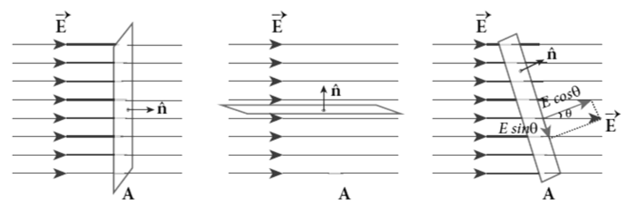
$\theta = 0°$; $\phi_E = E\,A\cos 0° = EA$

$$\boxed{\phi_E = EA}$$

$\theta = 90°$; $\phi_E = E\,A\cos 90°$

$$\boxed{\phi_E = 0}$$

Here, $\vec{A}=A\hat{n}$



(a)Electric flux = *EA* (b)Electric flux = 0 (c)Electric flux = (*E cos*θ)A

70. ELECTRIC FLUX IN A NON – UNIFORM ELECTRIC FIELD AND AN ARBITRARILY SHAPED AREA:

Suppose the electric field is not uniform and the area A is not flat then the entire area is divided into 'n' small area segments $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, \ldots \Delta\vec{A}_n$ *such that each area element is almost flat and the electric field over each area element is considered to be uniform. The electric flux for the entire area A is approximately written as*

$$\phi_E = \vec{E}_1.\Delta\vec{A}_1 + \vec{E}_2.\Delta\vec{A}_2 + \vec{E}_3.\Delta\vec{A}_3 + \ldots \vec{E}_n.\Delta\vec{A}_n$$

$$\phi_E = \sum_{i=1}^{n} \vec{E}_i.\Delta\vec{A}_i \longrightarrow (a)$$

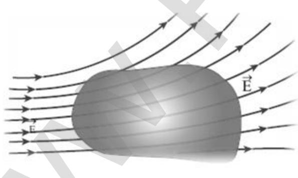
By taking $\Delta\vec{A}_i \to 0$ *, the summation in equation* ***(a)*** *becomes an integral,*

$$\phi_E = \int \vec{E}.d\vec{A}$$

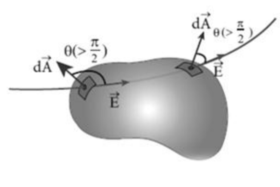
Thus electric flux depends on the electric field on the surface and the orientation of the surface with the electric field .

71. ELECTRIC FLUX FOR CLOSED SURFACE

Consider a closed surface in a region of non – uniform electric field as shown in figure.



(a)



(b)

The total electric flux over the closed surface is

$$\phi_E = \oint \vec{E}.d\vec{A}$$

Let the flux lines enter and leave the surface through an elementary area $d\vec{A}$ *, as shown in figure* ***(b)*** *. For each elemental area, the outward normal is the direction of* $d\vec{A}$*. In the figure* ***(b)*** *the angle between* $d\vec{A}$ *and* $\vec{E}$ *is greater than 90° for* *one area element less than 90° for the other elementary area. For $\theta < 90°$, the electric flux is positive and for $\theta > 90°$, the electric field is negative.*

[$\emptyset = EACOS\theta, COS\theta$ has negative values if $\theta > 90°$, and the value is positive for $\theta > 90°$]

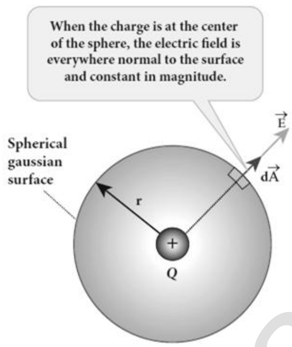
Thus , the electric flux $\emptyset_E$ is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

$$\emptyset_E = \oint \vec{E}.d\vec{A}$$

$$\boxed{\emptyset_E = \oint E.\mathrm{dA}\ \cos\theta}$$

θ is the angle between $\vec{E}$ and $d\vec{A}$.

72. GAUSS LAW PROOF



Consider a point +q placed at the centre of a sphere , as shown in figure . The electric field $\vec{E}$ is directed radially outward at all points on the surface of the sphere. Let $d\vec{A}$ be the elementary area on the surface of the sphere . The direction of $d\vec{A}$ is along the same direction as the electric field $\vec{E}$. Hence the angle between $\vec{E}$ and $d\vec{A}$ is zero. (i.e) $\theta=0°$.

$$\emptyset_E = \oint \vec{E}.d\vec{A} = \oint E.dA\cos\theta$$

$$\theta=0°.$$

$$\emptyset_E = \oint E.dA\cos\theta$$

$$\emptyset_E = \oint E.dA$$

As the electric field has the same magnitude along the surface of the sphere .

$$\emptyset_E = E\oint .dA = EA$$

A → Area of the sphere ,

r → radius of the sphere , then $A = 4\pi r^2$

$$\emptyset_E = EA = E.\,4\pi r^2$$

E is the electric field due to point charge Q

$$\emptyset_E = \frac{1}{4\pi\varepsilon o}\frac{Q}{r^2}4\pi r^2$$

$$\boxed{\emptyset_E = \frac{Q}{\varepsilon o}}$$

This law is applicable for any arbitrary shaped closed surfaces which encloses a charge Q.

73. IMPORTANT ASPECTS OF GAUSS LAW :

1. *From the equation* $\emptyset_E = \frac{Q}{\varepsilon o}$ *, it is observed that the total electric flux depends only on the charges enclosed by the closed surface and independent of the charges present outside the surface , and the surface of the closed surface .*
2. *The total electric flux is independent of the location of the charges inside the closed surface*
3. ***Gaussian Surface*** *:*

The imaginary surfaces chosen to find the electric field using gauss law is called a ***Gaussian surface*** *depends on the type of charge configuration and the type of symmetry existing in that charge distribution .*

a) *For a point charge and spherical charge distribution the Gaussian surfaces is in the form of a sphere concentric to the charge distribution*

b) *For a cylindrical charge distribution the Gaussian surface is taken in the form of a cylinder*

c) *For a plane charge distribution , the Gaussian surface is taken in the form of a Pill box.*

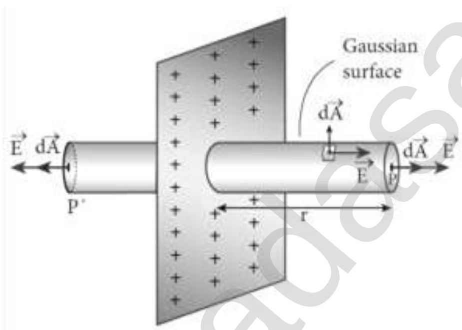
4. *The electric field $\vec{E}$ depends only on the charges enclosed by the closed surfaces*

5. *Gaussian surface is not applicable for discrete charges ,because the electric field is not well defined . Gaussian surface is applicable for continuous charge distribution .*

6. *Gauss law is another form of Coulomb's law and it is also applicable to the charges in motion. Thus, Gauss law is treated as a more general law than Coulomb's law.*

74. ELECTRIC FIELD DUE TO A CHARGED INFINITE SHEET

Consider an infinite plane sheet of charge as shown in the fig . Let σ be the surface charge density.



Consider a point P at a distance r from the sheet . The electric field points radially outward on both side of the sheet . should be same at equal distance on both sides. The Gaussian surface is taken in the form of cylinder of length 2r extending at equal distance of r on both sides of the sheet .Let A be the area of the flat surface of the cylinder . The plane sheet passes perpendicularly through the middle of Gaussian surface . Use Gauss law for the cylindrical surface.

$$\emptyset_E = \oint \vec{E}.d\vec{A}$$

$$\emptyset_E = \underbrace{\int \vec{E}.d\vec{A}}_{\text{Curved surface}} + \int_{P} \vec{E}.d\vec{A} + \int_{P^I} \vec{E}.d\vec{A}$$

$$\emptyset_E = \frac{Q_{enc}}{\varepsilon o}$$

$\vec{E}$ is perpendicular to the element area at all points on the curved surface.

$$\vec{E}.d\vec{A}=0$$

$$\emptyset_E = \int_P \vec{E}.d\vec{A} + \int_{P^l} \vec{E}.d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

P and P^l → $\vec{E}$ and $d\vec{A}$ are parallel.

$$\vec{E}\,.\,d\vec{A} = |\vec{E}|\,|d\vec{A}|\cos 0^\circ \qquad \cos 0^\circ=1$$

$$\vec{E}\,.\,d\vec{A} = E.dA$$

$$\emptyset_E = \int_P E.dA + \int_{P^l} E.dA = \frac{Q_{enc}}{\varepsilon_0}$$

Since the magnitude of the electric field at these two equal surfaces [P and P^l] is uniform E can be taken out of the intergration

$$\emptyset_E = E\int_P dA + E\int_{P^l} dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$\emptyset_E = 2E\int_P dA = \frac{Q_{enc}}{\varepsilon_0}$$

Charge density , $\sigma = \frac{dQ}{dA}$

$$dQ = \sigma.dA$$

$$\int dQ = \int \sigma.dA$$

$$Q = \int \sigma.dA$$

$$\emptyset_E = 2E\int_P dA = \frac{\int \sigma.dA}{\varepsilon_0}$$

$$2EA = \frac{\sigma}{\varepsilon_0}\int dA$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$\hat{n}$ → unit vector normal to the plane

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}} \longrightarrow \text{(a)}$$

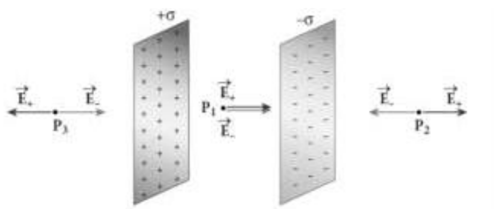
$\vec{E}$ due to an infinite plane sheet of charge depends on the surface charge density ‘ ’

and independent of 'r' .

1. *If σ > 0, $\vec{E}$ at any point P points perpendicularly outwards to the point* *($\hat{n}$) and if σ > 0, $\vec{E}$at any point P points perpendicularly inwards to the point (-$\hat{n}$)*
2. *Eq (a) is true only in the middle region of the plane and at points far away* *from both ends.*

75. ELECTRIC FILED DUE TO TEO PARALLEL CHARGED INFINITE SHEETS

Consider two infinitely charged plane sheets with equal and opposite charge density + and – placed parallel to each other as shown in figure .



The electric field between the plates and outside the plates is found using Gauss law

The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_o}$ and it points perpendicular outward is σ > 0 and points inwards if σ < 0

Consider a point P_1 in between the plates . The electric field $\overrightarrow{E_+}$due to +σ point outwards and the electric field $\overrightarrow{E_-}$ due to - σ points radially inwardsThe electric field at the point P_1 due to + σ and – σ are in the same direction .That towards the right .

The electric field $\overrightarrow{E_+}$ at P_1 due to + σ is

$$\overrightarrow{E_+} = \frac{\sigma}{2\varepsilon_o}$$

The electric field − at P_1 due to - σ is

$$\overrightarrow{E_-} = \frac{\sigma}{2\varepsilon_o}$$

Total electric field at the point P1 is,

$$E_{\text{inside}} = \frac{\sigma}{2\varepsilon o} + \frac{\sigma}{2\varepsilon o}$$

$$E_{\text{inside}} = \frac{\sigma + \sigma}{2\varepsilon o}$$

$$E_{\text{inside}} = \frac{2\sigma}{2\varepsilon o}$$

$$E_{\text{inside}} = \frac{\sigma}{\varepsilon o}$$

The direction of the electric field the plates is directed from positively charged plate is the plates is directed from positively charged to negative charged plate and is uniform everywhere inside the plate.

Consider the two points P_2 and P_3 on the right and left sides of the plates .

The electric field at P_2 due to + σ and – σ are $\overrightarrow{E_+}$ and $\overrightarrow{E_-}$

$\overrightarrow{E_+}$ and $\overrightarrow{E_-}$ are equal in magnitude and opposite in direction . Hence the net electric field at P_2 is zero .

Similarly, the electric field at 'P_3'

$\overrightarrow{E_+}$ and $\overrightarrow{E_-}$ are equal in magnitude and opposite in direction . Hence the net electric field at P_3 is zero .

Hence, the net field $\vec{E}$ is zero at the points P_2 and P_3 outside the plates .

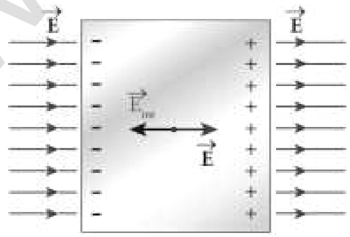
Hence , the electric field due to two parallel charged infinite sheets is,

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{n}}$$

76.PROPERTIES OF CONDUCTOR AT ELECTROSTATIC EQUILIBRIUM:

i. The electric field $\vec{E}$ is zero everywhere inside a conductor .This is true for all conductors , whether it is solid or hollow.

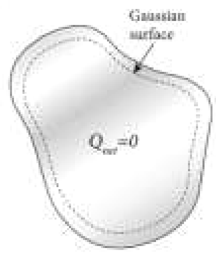
EXPERIMENTAL PROOF:



Consider a conductor placed in an external electric field as shown in the figure . Before the application of electric field $\vec{E}$ the free electrons are uniformly distributed in all the possible directions in the conductor .After the application of electric field $\vec{E}$, the negative electrons are accelerated to the left side of the plate and the positive charges are accelerated *to the right side of the plate .[This is because the external field has its positive potential in the left and negative potential in the rights side].*

Due to this alignment of free electrons an internal field $\vec{E}_{int}$ is created inside the conductor in a direction opposite to the applied field .This internal field $\vec{E}_{int}$ increases, until it nullifies the external field. Hence, the net field inside the conductor is zero At this stage the conductor is said to be in electrostatic equilibrium . The time taken by the conductor to reach electrostatic equilibrium state is instantaneous of the order of 10^{-16} sec.

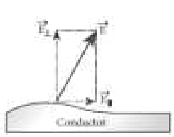
ii. *These is no net charge inside the conductors . The charges resides only on the surface of the conductors .*

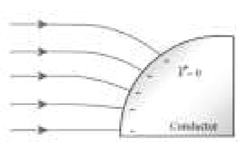


PROOF :

Consider an arbitrarily shaped conductor as shown in figure . To find the effect of charges inside the conductor .Imagine , a Gaussian surface inside the conductor such that it is very close to the surface of the conductor . The electric field $\vec{E}$ is zero everywhere inside a conductor , which shows that there is no net charge inside the conductor .No charge is enclosed by the Gaussian surface .Even if any charge is introduced inside the conductor , it immediately searches the surface of the conductor .

iii. *The electric field outside the conductor is perpendicular to the surface of the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_o}$,where σ is the surface charge density at that point.*

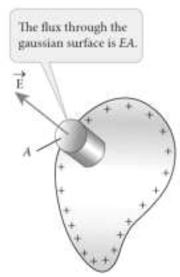




PROOF :

To show that the electric filed is perpendicular to the surface of the conductor at electrostatic equilibrium . The electric field on the surface of the conductor may have parallel and perpendicular components as shown in figure .If the electric field has components parallel to the surface this filed would exert a force on the neighbouring charges. This creates an acceleration of the free electrons on the surface. This means that the conductor is not in electrostatic equilibrium .Hence the electric field must be perpendicular to the surface as shown in figure.When the conductor is at electrostatic equilibrium.

77. TO SHOW THAT E= $\frac{\sigma}{\varepsilon_o}$ AT ANY POINT OUTSIDE THE SURFACE OF THE CONDUCTOR:



Consider a conductor as shown in figure. To find the electric field at a point outside the conductor imagine a Gaussian surface in the form of a cylinder , such that one half of the cylinder is embedded inside the conductor since the electric field is normal to the surface of the conductor ,the curved part of the cylindrical surface has zero electric flux .The bottom part of the Gaussian surface is inside the conductor where the electric field is zero. The top flat surface of the cylinder alone contributes to the total electric flux. let A be the area of cross section of the top surface . Applying Gauss law ,

$$\emptyset_E = \oint E.dA = \frac{Q}{\varepsilon_o}$$

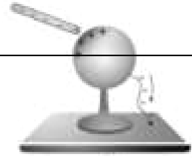
Q is the charge over the area A. If is the surface density of charges .

$$\sigma = \frac{Q}{A}(or)\ Q = \sigma A.$$

$$E \oint dA = \frac{Q}{\varepsilon_o}$$

$$E.A = \frac{Q}{\varepsilon_o}$$

$$E.A = \frac{\sigma A}{\varepsilon_o}$$



$$E = \frac{\sigma}{\varepsilon_o}$$

$$\boxed{\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{n}}$$

$\hat{n}$ is the unit vector pointing outwards normal to the surface of the conductor .

If $\sigma > 0$, then the electric field points inwards and $\hat{n}$ = - $\hat{n}$

iv. The electrostatic potential has the same value on the surface and inside of the conductor

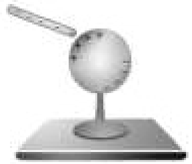
EXPLANATION

The electric field outside the conductor is perpendicular to the surface of the conductor. Therefore, electric field has no parallel component on the surface . The charges on the surface does not experience any field in the parallel direction. Since there is no force due to the field in the parallel direction .The work done along the direction parallel to the surface of the conductor will be zero . Hence charges can be moved on the surface without doing any work. As w=qv ,this is possible only if the Potential V is constant at all points on the surface, and there is no potential difference between any two points on the surface.

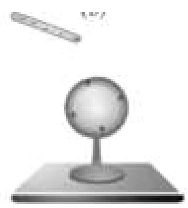
The electric field inside the conductor is zero. The potential is the same on the surface of the conductor .Thus at electrostatic equilibrium, the conductor is always at equipotential.

78. EXPERIMENT TO DEMONSTRATE ELECTROSTATIC INDUCTION

Consider an uncharged equal distribution of charges cancel each other conducting sphere at rest on an insulating stand as shown in figure. Let a negatively charged conducting rod is brought near the sphere without touching it . The negative charges in the rod repels the negative charges of the sphere. As a result, the negative charges are moved further side, But the total charge is zero.



Now connect the conducting sphere to the ground, through a wire. This is called grounding. [Ground can receive any amount of electrons] The electrons in the sphere are removed from the sphere as shown in the figure .The positive charges remain near the region of the rod , as they are attracted by the negative charges of the rod



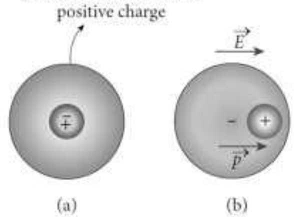
The grounding wire is now removed as shown in the figure .The sphere has positive charges alone near the region of the rod.

Now that charged rod is taken away from the conductor. The positive charge are now uniformly distributed on the surface of the conductor as shown in figure.

The neutral conducting sphere is now positively charged without direct contact between the rod and the sphere for an arbitrary shaped sphere conductor distribution of positive charges is not be uniform.

79. EFFECT OF ELECTRIC FIELD ON NON- POLAR MOLECULES

The atoms constituting the molecule has positively charged nucleus and orbiting electrons . Each atom is considered as a dipole. The nucleus is at the centre and the moving electrons create an electron cloud. The centre of the positive nucleus and the centre of the negatively electron clouds coincides in a non-polar molecule without any external electric field as shown in the figure.



When an external electric field is applied the positive charge moves a bit to the right and the negative charge moves a bit to the left .The positive and negative charges are separated by a small distance . This indicates a dipole moment in the direction of the external field as shown in the figure . The dielectric is now said to be the polarized. Thus ,a non-polar molecule is polarized by an external field

80. INDUCED ELECTRIC FIELD INSIDE A DIELECTRIC :

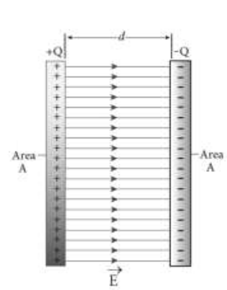
Consider a rectangular dielectric slab placed between two oppositely charged capacitor plates as shown in figure . The opposite charges on the plates provide external electric field for the dielectric. The dipole try to align in the direction of the external electric field $\vec{E}_{ext}$ *as shown in figure. Consider a linear string of dipoles in the field as shown in the figure .Each positive charge canels with neighbour negative charge at the centre .But at the ends there are no neighbour charges to get cancelled .Hence a negative charge and positive charge are left at both the ends. These charges cannot be removed and they are cancelled bound charges. So, the dielectric in the external field is equivalent to two oppositely charged sheets with surface charge densities* $+\sigma_b$ *and* $-\sigma_a$ *. These bound charges are not free to move like free electrons in conductors. These Bound charges also produce electric field within the dielectric.*

EXAMPLE

A charged balloon after rubbing sticks on a wall ,because the negatively charged balloon when brought near the wall indicates opposite charges on the surface of the wall, which attracts the balloon.

81. CAPACITANCE OF A PARALLEL PLATE CAPACITOR

Consider a capacitor with two parallel plates as shown in the figure.



Let 'A' be the cross sectional area of the plates and 'd' is the distance between them . The electric field between these two infinite parallel plates which is uniform and is given by $E = \frac{\sigma}{\varepsilon_0}$

$\sigma = \frac{Q}{A}$; If d is much smaller than A then the above result is used even for finite sized parallel plate capacitor .Using Gauss law ,the electric field E between the plates is,

$$\int \vec{E}.d\vec{A} = \frac{Q}{\varepsilon o}$$

$$E.A = \frac{Q}{\varepsilon o}$$

$$\boxed{E = \frac{Q}{\varepsilon o A}}$$

In the above equation the R.H.S is a constant .

$\vec{E}$ is thus uniform ,

$$V = E.d$$

$$V = \frac{Q.d}{\varepsilon o A}$$

Capacitance C of a capacitor is $C = \frac{Q}{V}$

$$C = \frac{Q}{\left[\frac{Q.d}{\varepsilon o A}\right]} = \frac{\varepsilon o A}{d}$$

$$\boxed{C = \frac{\varepsilon o A}{d}}$$

From above equation ,the capacitance C is directly proportional to area of cross section and inversely proportional to the distance between the plates.

From above equation, as the area of cross section of the capacitor plates is increased ,more charges are distributed for same potential difference V. As a result, the capacitance is increased.

From above equation , V=E.d , if the distance between the plates 'd' is reduced, the potential difference decreases with 'E' constant voltage difference V increases to keep E constant. This leads to an additional form of charges to the battery, till the voltage on the capacitor is equal to the battery voltage with the distance 'd' is increased [V $\propto$ d],the capacitor voltage increases and when it is greater than the battery voltage, then the charges flow from capacitor plates to battery till both the voltages are equal.

82. ENERGY STORED IN A CAPACITOR :

Capacitor not only store charges, but it also stores energy. Consider a capacitor connected to a battery. The electrons from the plate connected to the negative terminal of the battery will be transferred to the plate connected to the positive terminal. Let the total charge transferred is -Q. To transfer the charge work is done by the battery and this work done is stored as potential energy in the capacitor.

Let the potential difference V transfers a small infinitesimal charge dQ and let dw the work done to transfer this charge.

$$W=VQ$$

$$dw = V.dQ$$

$$V=\frac{Q}{C}.$$

$$dw=\frac{Q}{C}.\ dQ$$

$$\int dw = \int \frac{Q}{C}.dQ$$

$$W = \int \frac{Q}{C}.dQ = \frac{1}{C}\frac{Q^2}{2}.$$

This workdone is stored as electrostatic potential energy (U_E) in the capacitor.

$$U_E = \frac{Q^2}{2C}$$

$$Q = VC$$

$$U_E = \frac{V^2C^2}{2C}$$

$$\boxed{U_E = \frac{1}{2}CV^2}$$

The energy stored (i.e) potential energy is proportional to the capacitance of the capacitor and square of the voltage between the plates.

83. EXPRESSION FOR ENERGY DENSITY (U_E)

For a parallel plate capacitor of capacitance is , $C = \frac{\varepsilon_0 A}{d}$

A → Area of the plates

d → separation between the plates The potential V in between the plates is

$$V=E.d$$

The electrostatic potential energy U_E of the capacitor is

$$U_E= ½\ CV^2$$

$$U_E = ½\ [\frac{\varepsilon_0 A}{d}]\ [E.d]^2$$

Where A_d is the volume of the space between the capacitor plates. From the definition of energy density.

$$U_E = \frac{U}{Volume}$$

Volume = Ad ; $$U_E = \frac{\frac{1}{2}\left[\frac{\varepsilon_o A}{d}\right][E.d]^2}{Ad}$$

$$\boxed{U_E = \frac{1}{2}\varepsilon_o E^2}$$

Above equation shows that the energy is stored in the electric field existing in between the plates of the capacitor. Thus, the energy density depends only on the electric field and not on the size of the capacitor plates .Equation holds for any types of charge configurations.

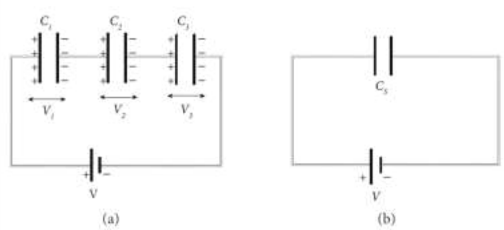
84. CAPACITOR IN SERIES :

Consider three capacitors C_1, C_2 and C_3 are connected in series with a battery of voltage V as shown in the figure. Let electrons of charge - Q are transferred from the negative terminal of the battery to the right plate of C_3. [This is because the negative potential repels the negative electrons away from the battery] . This induces an equal amount of positive charge on the left plate of C_3. The transferred – Q electrons from the right place of the C_3 pushes the same amount of - Q to the right plate of C_2 due to electrostatic induction. Similarly, the positive charges induced on the left to plate of C_2

pushes the equal amount of charge - Q on the right plate of C_1 .At the same time the electron of charge – Q are transferred from left plate of C_1 to the positive terminal of battery. Hence the same amount of charge Q is store in all the capacitors C_1, C_2 and C_3

But the voltage across the capacitor varies .Let V_1, V_2 and V_3 be the voltage across C_1, C_2 and C_3 .

The battery voltage across each capacitor is equal to the battery voltage v



(a) Capacitors connected in series (b) Equivalence capacitors C_s

$$V = V_1 + V_2 + V_3$$

As Q= cV $\longrightarrow$ $V = \frac{Q}{C}$

$V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$ as the charge Q remains the same in C_1, C_2 and C_3 .

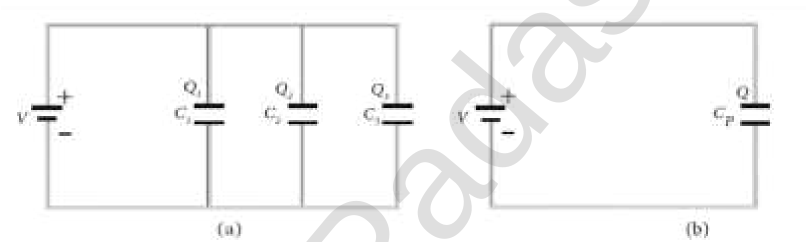
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$V = \frac{Q}{C_S}$, $C_S \longrightarrow$ Equivalent capacitor when the capacitors are connected in series.

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad \boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

The inverse of equivalent capacitance 'C_3' of the three capacitors connected in series is equal to the sum of the inverse of each capacitance. The equivalent capacitance C_3 is always less than the smallest individual capacitance in series.

85. CAPACITORS IN PARALLEL



(a) capacitors in parallel (b) equivalent capacitance with the same total charge

Consider three capacitors C_1, C_2 and C_3 connected in parallel with the battery of voltage V are shown in the figure .Since C_1, C_2 and C_3 connected across the battery, the potential across C_1, C_2 and C_3 will be same equal to the battery voltage V.Since the capacitance of the capacitor are different , the charges flowing through the capacitors varies. According to the conservation of charge, the sum of charges through C_1, C_2 and C_3 is equal to the total charge transferred by the battery.

i.e) $\qquad Q = Q_1 + Q_2 + Q_3$

Since , Q= CV $\longrightarrow Q_1 = C_1V$, $Q_2 = C_2V$ *and* $Q_3 = C_3V$

Let the combined capacitance of the parallel combination be C_P. Q=

$C_P V$

$C_P V = C_1V + C_2V + C_3V$ C_P

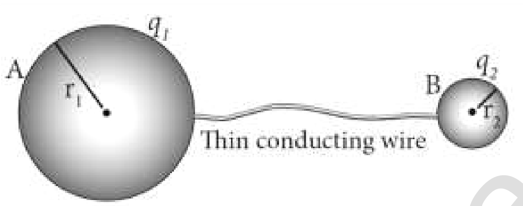
$V = V [C_1 + C_2 + C_3]$

$$\boxed{C_P = C_1 + C_2 + C_3}$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of individual capacitors C_P is always larger than the largest individual capacitance.

86. DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

Considering two conducting spheres A and B of radii and r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in figure.



The distance between the sphere is much greater than the radii of the spheres.

If a charge Q is introduced into any one of the spheres, this charge Q is redistribute into both the sphere such that the electrostatic potential is same in both the space .They are now uniformly charged and attain electrostatic equilibrium .Let be there charge residing on the surface of sphere A and is the charge residing on the surface of sphere B such that the $Q = q_1 + q_2$ The charges are distributed only on the surface and there is no net charge inside the conductor The Electrostatic potential at the surface of the sphere A is given by,

$$\boxed{V_A = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1}}$$

The Electrostatic potential at the surface of the sphere B is given by,

$$\boxed{V_B = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_2}}$$

The surface of the conductor is an equipotential . Since the spheres are connected by the conducting wire , the surfaces of both the spheres together from an equipotential surface. This implies that,

$$V_A = V_B$$

$$\frac{1}{4\pi\varepsilon o}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon o}\frac{q_2}{r_2}$$

$$\boxed{\frac{q_1}{r_1} = \frac{q_2}{r_2}}$$

Let us take the charge density on the surface of sphere A and $\sigma_1$charge density on the surface of sphere B and σ_2 implies that $q_1 = 4\pi r_1^2\sigma_1$ and $q_2 = 4\pi r_2^2\sigma_2$.

Substituting these values,

$$\frac{4\pi r_1^2\sigma_1}{r_1} = \frac{4\pi r_2^2\sigma_2}{r_2}$$

$$\boxed{\sigma_1 r_1 = \sigma_2 r_2}$$

From which we concluded that,

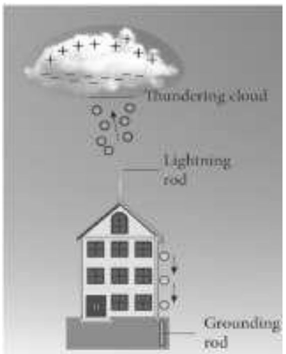
$$\boxed{\sigma r = constant}$$

Thus, the surface charge density 'σ ' is inversely proportional to the radius of the sphere.

For a smaller radius , the charge density will be larger and vice versa.

87. LIGHTNING ARRESTED [OR]LIGHTNING CONDUCTOR

This is a device used to protect tall building from lightning strikes .It works on the principle of action at points on Corona discharge This device consists of a long thick copper rod passing from top of the building to the ground .The upper end of the rod has a sharp spike or a sharp needle as shown in the figure.



The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building ,it induces a positive charge on the spike. Since the induced charge density on the sharp spike is large , it results in corona discharge .This positively charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud .The negative charge pushes to the spikes passes through the copper rod and is safety diverted to the earth. The Lightning arrester does not stop lightning ;rather it diverts the lightning to the ground safely.

PART –D (5 MARKS)

1. COULOMB'S LAW

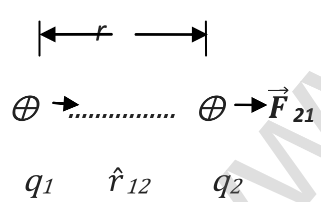
STATEMENT

The force of attraction or repulsion between two point charges [q_1 and q_2] separated by a distance 'r' is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charges.

PROOF

Consider two charges → q_1 and q_2

distance → r



The force on the point charge 'q_1' exerted by charging 'q_1' is

$$\vec{F}_{21} \propto \frac{q1q2}{r^2} \; \hat{r}_{12}$$

$$\vec{F}_{21} = K\frac{q1q2}{r^2} \; \hat{r}_{12}$$

In the similar manner the force on the charge 'q_1' exerted by 'q_2' is,

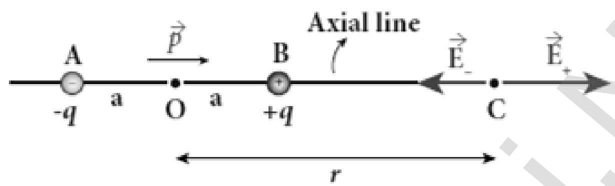
$$\vec{F}_{12} = K\frac{q1q2}{r^2}\ \hat{r}_{21}$$

$$K = \frac{1}{4\pi\varepsilon o} = 9 \times 10^{9}\ Nm^2C^{-2}$$

ε_o → *Permittivity of free space* → $8.85 \times 10^{-12}\ C^2N^{-1}m^2$

2. *ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE AT POINTS ON THE LINE*

Consider an electric dipole separated by a distance 2a placed on x-axis .



*A point **C** is located at a distance **r** from the midpoint **O** along the axial line.*

*The electric field at the point **C** is obtained by summing the field due to **+q** and **–q** at **C**.*

*The charge **+q** is at a distance **r** a from C and **–q** is at a distance **r + a** from **C** .*

$\vec{E}_+$ *due to **+q** at the point **C***

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon o}\ \frac{q}{(r-a)2}\ \text{along BC}$$

$\vec{P}$ *is -q to +q and is along BC.*

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon o}\ \frac{q}{(r-a)2}\ \hat{P}$$

$\vec{E}_-$ *due to **-q** at the point **C***

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon o}\ \frac{q}{(r+a)2}\ \hat{P}\ \text{along BC}$$

Since +q is closes to the point C than –q , $\vec{E}_+$ is stronger than $\vec{E}_-$. The length of the vector is a measure of its magnitude. Therefore, the length of $\vec{E}_+$ vector is drawn larger than that of the point C is calculated from the superposition principle of the *electric field as ,*

$$\vec{E}_{tot} = \vec{E}_{+} + \vec{E}_{-}$$

$$= \frac{1}{4\pi\varepsilon o}\frac{q}{(r-a)^2}\hat{P} - \frac{1}{4\pi\varepsilon o}\frac{q}{(r+a)^2}\hat{P}$$

$$= \frac{q}{4\pi\varepsilon o}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\varepsilon o}\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\varepsilon o}\left[\frac{r^2+2ra+a^2-r^2+2ra+r^2}{(r-a)^2(r+a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\varepsilon o}\left[\frac{4ra}{(r-a)^2(r+a)^2}\right]\hat{P}$$

$$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon o}\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{P}$$

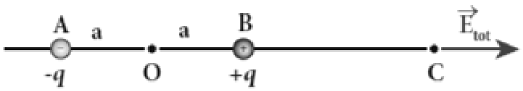
i. ***'C' is very far away from dipole , r >> a***

$$(r^2 - a^2)^2 \approx r^4$$

$$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon o}\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{P}$$

$$\vec{P} = 2aq.\,\hat{P}$$

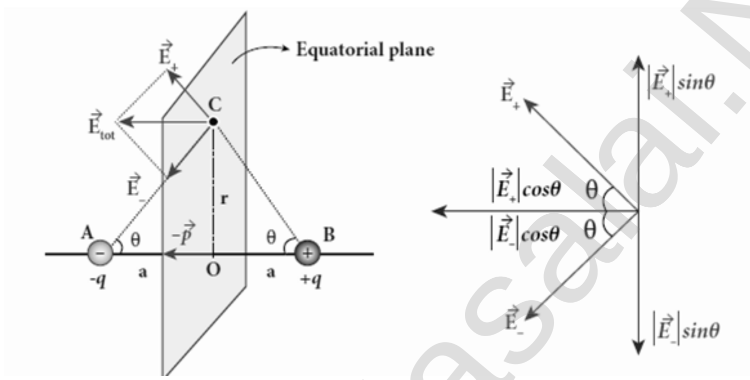
$$\boxed{\vec{E}_{TOT} = \frac{1}{4\pi\varepsilon o}\cdot\frac{2\vec{P}}{r^3}} \longrightarrow (a)$$



ii. ***Points 'C' is left side of the dipole . $\vec{E}_{TOT}$ still in the direction of $\vec{P}$***

3. ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE AT A POINT ON THE EQUATORIAL PLANE

Consider a dipole separated by a distance '2a'. A point 'C' at a distance 'r' from the m midpoint 'O' of the dipole on the equatorial plane. Since C is at equal distance from the charges +q and –q, the magnitude of the electric field $\vec{E}_+$ and $\vec{E}_-$ will be the same. $\vec{E}_+$ is along BC, $\vec{E}_-$ is along CA.



$\vec{E}_+$ and $\vec{E}_-$ are resolved into two components, one component parallel to the dipole axis and the other perpendicular to it. The perpendicular components are oppositively dissected and cancel each other. Hence the magnitude of the total electric field at C is the sum of parallel components of $\vec{E}_+$ and $\vec{E}_-$. Total electric field $\vec{E}_{tot}$ are along $-\vec{P}$

$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\hat{P} - |\vec{E}_-|\cos\theta\hat{P}$$

*Electric field $\vec{E}_+$ at **C** due to **+q** is*

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_+^2} \quad \text{along BC}$$

r_+ is the distance between +q and C.

From the triangle OCB $r_+^2 = r^2 + a^2$

$$\left|\vec{E}+\right| = \frac{1}{4\pi\varepsilon o}\ \frac{q}{r^2+a^2}$$

Electric field $\vec{E}_-$ *at* **C** *due to* **-q** *is*

$$\left|\vec{E}-\right| \ = \frac{1}{4\pi\varepsilon o}\ \frac{q}{r_-^2} \quad \text{along } CA$$

From the triangle OCB $r_-^2 = r^2 + a^2$

$$\left|\vec{E}-\right| = \frac{1}{4\pi\varepsilon o}\ \frac{q}{r^2+a^2}$$

$$\left|\vec{E}+\right| = \left|\vec{E}-\right| = \frac{1}{4\pi\varepsilon o}\ \frac{q}{r^2+a^2}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon o}\ \frac{q}{(r^2+a^2)} \cos\theta\,\hat{P} - \frac{1}{4\pi\varepsilon o}\ \frac{q}{(r^2+a^2)} \cos\theta\,\hat{P}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon o}\ \frac{2q}{(r^2+a^2)} \cos\theta\,\hat{P}$$

In ΔOCB, OB=a ; BC² = r² + a²

$$\cos\Theta = \frac{OB}{BC} = \frac{q}{\sqrt{r^2+a^2}}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon o}\ \frac{2qa}{\sqrt{r^2+a^2}(r^2+a^2)}\hat{P}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon o}\ \frac{2qa}{(r^2+a^2)^{\frac{3}{2}}}\hat{P}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon o}\ \frac{\vec{P}}{(r^2+a^2)^{\frac{3}{2}}} \qquad \vec{P} = 2qa\hat{P}$$

Point 'C' is very far away from the dipole

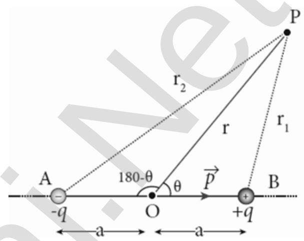
$$r >> a \ \rightarrow (r^2+a^2)^{\frac{3}{2}} \approx (r^2+a^2)^{\frac{3}{2}} \approx r^3$$

$$\boxed{\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon o}\frac{\vec{P}}{r3}} \longrightarrow \text{(b)}$$

4. ELECTROSTATIC POTENTAIL AT THE POINT DUE TO AN ELECTRIC DIPOLE

Consider two equal and opposite charges separated by a small distance 2a.

The point P is located at a distance r from the midpoint of the dipole . Let ϑ be the angle between the line OP and dipole axis AB.



$BP = r_1$; $AP = r_2$

Potential at P due to +q = $\frac{1}{4\pi\varepsilon o}\frac{q}{r_1}$

Potential at P due to - q = $-\frac{1}{4\pi\varepsilon o}\frac{q}{r_2}$

Total potential at the point P,

$$V = \frac{1}{4\pi\varepsilon o}\frac{q}{r_1} - \frac{1}{4\pi\varepsilon o}\frac{q}{r_2}$$

$$\boxed{V = \frac{q}{4\pi\varepsilon o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

$a << r$

By the cosine law for triangle BOP,

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$r_1^2 = r^2\left[1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right]$$

$a << r$, the term $\frac{a^2}{r^2}$ is very small and can be neglected

$$r_1^2 = r^2\left[1 - \frac{2a}{r}\cos\theta\right]$$

$r_1 = r\left[1 - \frac{2a}{r}\cos\theta\right]^{\frac{1}{2}}$

$\frac{1}{r_1} = \frac{1}{r\left[1 - \frac{2a}{r}\cos\theta\right]^{\frac{1}{2}}}$

$\frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2a}{r}\cos\theta\right]^{\frac{-1}{2}}$

$\frac{a}{r} << 1$, we can use binomial theorem .

$$\boxed{\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{a}{r}\cos\theta\right]}$$

Similarly applying the cosine law for triangle AOP,

$r_2^2 = r^2 + a^2 - 2ra\cos(180 - \theta)$

$r_2^2 = r^2 + a^2 + 2ra\cos\theta$ $\left[\cos(180 - \theta) = -\cos\theta\right]$

$r_2^2 = r^2\left[1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right]$

$r_2^2 = r^2\left[1 + \frac{2a}{r}\cos\theta\right]$ Neglecting $\frac{a^2}{r^2}$ [a<<r]

$r_2 = r\left[1 + \frac{2a}{r}\cos\theta\right]^{\frac{1}{2}}$

$\frac{1}{r_2} = \frac{1}{r}\left[1 + \frac{2a}{r}\cos\theta\right]^{\frac{-1}{2}}$

Use Binomial theorem,

$$\boxed{\frac{1}{r_2} = \frac{1}{r}\left[1 - \frac{a}{r}\cos\theta\right]}$$

$V = \frac{q}{4\pi\varepsilon o}\Big[\frac{1}{r}(1 + \frac{a}{r}\cos\theta) - \frac{1}{r}(1 - \frac{a}{r}\cos\theta)\Big]$

$V = \frac{q}{4\pi\varepsilon o}\Big[\frac{1}{r}(1 + \frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta)\Big]$

$V = \frac{1}{4\pi\varepsilon o}\frac{2aq}{r_2}\cos\theta$ $P = 2aq$

$\boxed{V = \frac{1}{4\pi\varepsilon o}\frac{P}{r_2}\cos\theta}$ $P\cos\theta = \overrightarrow{P}.\vec{r}$

$\boxed{V = \frac{1}{4\pi\varepsilon o}\frac{\vec{P}.\vec{r}}{r_2}}$

Special cases :

Case (1)

If the point P lies on the axial line of the dipole on the side of +q ,then ϑ=0.

$\boxed{V = \frac{1}{4\pi\varepsilon o}\frac{P}{r_2}}$ *COS 0°=1*

Case (2)

The point P lies on the axial line of the dipole on the side of –q, then ϑ=180°.

$\boxed{V = -\frac{1}{4\pi\varepsilon o}\frac{P}{r_2}}$ *COS 180°=-1*

Case (3)

The point P lies on the axial line of the equatorial line the dipole , then ϑ=90°.

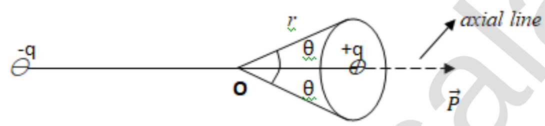
$\boxed{V = 0}$ *COS 90°=0*

IMPORTANT POINTS

1. *The potential due to an electric dipole falls as* $\frac{1}{r^2}$ *and the potential due to*

single point charge falls as $\frac{1}{r}$ .Thus, the potential point charge falls faster than that due to a monopole . As the distance increase from electric dipole , the effects of positive and negative charges nullify each other .

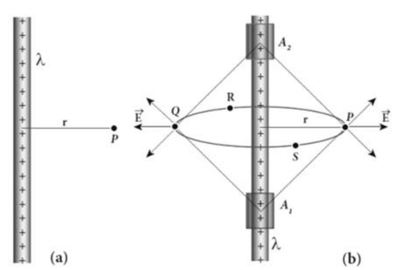
2. The potential due to a point charge is spherically symmetric since it depends only on the distance r. But the potential due to a dipole is not spherically symmetric because the potential depends on the angle between $\vec{P}$ and position vector $\vec{r}$ of the point .



However, the dipole potential is axially symmetric , If the position vector $\vec{r}$ is rotated about $\vec{P}$ by keeping θ fixed, then all points, on the cone potential as shown in figure.

5. ELECTRIC FIELD DUE TO AN INFINITY LONG WIRE (or) CYLINDRICAL CHARGE DISTRIBUTION

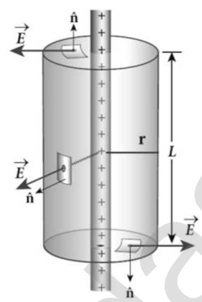
Consider an infinity long straight wire with linear charge density λ



The electric field due to the charged wire at a point P which at a *distance r from the wire is determined using Gauss law. Consider two charge elements A_1 and A_2 on the wire which are at equal distance from P. The resultant field due to these charge elements points radially outwards from the wire. Moreover, the magnitude of the electric field at a distance r is the same around the wire .Hence , the wire possess cylindrical symmetry .*

To find the electric field $\vec{E}$

Consider a cylindrical Gaussian surface of radius r and length L.



Because for cylindrical symmetry , the Gaussian surface should be taken in the form of a cylinder.

From Gauss law,

$$\phi_E = \oint \vec{E}.d\vec{A}$$

The cylindrical charge distribution contains three surfaces , top, bottom and curved surface .All these surfaces contribute to the total electric flux

$$\phi_E = \underbrace{\int \vec{E}.d\vec{A}}_{\text{Curved surface}} + \underbrace{\int \vec{E}.d\vec{A}}_{\text{top surface}} + \underbrace{\int \vec{E}.d\vec{A}}_{\text{Bottom surface}}$$

For the top and bottom surface $\vec{E}$ and $\hat{n}$ vectors are perpendicular .

Therefore, $\vec{E}$ *and* $d\vec{A}$ *are perpendicular .*

$$\vec{E} \,.\, d\vec{A} = |\vec{E}|\,|d\vec{A}| \cos 90$$

$$\vec{E} \,.\, d\vec{A} = E \times dA \times 0$$

$$\vec{E} \,.\, d\vec{A} = 0$$

For the top and bottom surfaces ,

$$\int_{\text{top surface}} \vec{E}.d\vec{A} = 0 \quad \text{and} \quad \int_{\text{Bottom surface}} \vec{E}.d\vec{A}$$

$$\emptyset_E = \int_{\text{Curved surface}} \vec{E}.d\vec{A}$$

For the curved surface , the parallel to .Hence, is parallel to

$$\vec{E} \,.\, d\vec{A} = |\vec{E}|\,|d\vec{A}|. = \cos 0°$$

$$\vec{E} \,.\, d\vec{A} = E \,.dA \times 1$$

$$\vec{E} \,.\, d\vec{A} = E \,.dA$$

$$\emptyset_E = \int E.dA$$

$\vec{E}$ *has constant magnitude around the closed surface.*

$$\emptyset_E = E \int dA$$

$$\emptyset_E = E\,A$$

Charge density $\lambda = \frac{dQ}{dl}$

; dQ= λ *dLUsing Gauss law,*

$$\emptyset_E = \int \vec{E}.d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Curved surface

$$\emptyset_E = EA = \frac{\int \lambda.dL}{\varepsilon_0}$$

$$EA = \frac{1}{\varepsilon_0}\lambda \int dL$$

$$EA = \frac{1}{\varepsilon_0}\lambda L.$$

A → Area of the curved surface.

A=2πrL [circumference X Lenght]

$$E.\ 2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

$$E. = \frac{\lambda}{2\pi r\varepsilon_0}$$

Vector form,

$$\boxed{\vec{E} = \frac{1}{2\pi r\varepsilon_0}\frac{\lambda}{r}\hat{r}} \longrightarrow \text{(a)}$$

The electric filed due to an infinitely long charged wire is $\vec{E} = \frac{1}{2\pi r\varepsilon_0}\frac{\lambda}{r}\hat{r}$ *,* $\vec{E}$ *is depends on* $\frac{1}{r}$

This equation shows that ,

1 $\vec{E}$ *is always perpendicular to the length of the wire .*

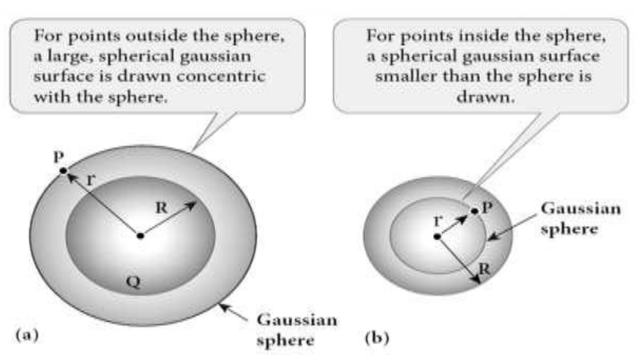
2 *If* $\lambda > 0$ *,* $\vec{E}$ *points perpendicular in outward direction* $(\hat{r})$

3 *If* $\lambda < 0$ *,* $\vec{E}$ *points perpendicular in inward direction* $(-\hat{r})$

4 *Eq (a) is only for an infinitely long charged wire. If the length of the wire is true around the midpoint of the wire and far away from the both ends of the wire.*

6. ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

Consider a uniformly charged spherical shell as shown in figure. The electric field points radically outwards in all directions around the shell . For the spherical charge *distribution , the Gaussian surface is taken in the form of a sphere concentric to the charge distribution .*



e) ELECTRIC FIELD AT A POINT OUTSIDE THE SHELL : (r > R)

Consider a point P at a distance r from the centre of the shell , asshown in figure (a) .The charges are uniformly distributed on the spherical surface .

If Q is positively (Q > 0) the electric field points radially outwards and if Q is negative (Q <0) the electric field points radially inwards .Applying Gauss law to the Gaussian surface of radius r.

$$\oint \vec{E}.d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$\vec{E}$ and $d\vec{A}$ points in the same direction radially outwards at all points on to Gaussian surface.

Angle between $\vec{E}$ and $d\vec{A}$ is θ=0°

Hence , $\vec{E}\,.\,d\vec{A} = |\vec{E}|\,|d\vec{A}|\cos 0°$ cos0°=1

$$\vec{E}\,.\,d\vec{A} = E.dA$$

$$\oint E.dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E.A = \frac{Q_{enc}}{\varepsilon_0}$$

$A = \pi r^2$ *A → Surface area of the sphere.*

$$E.4\,\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$$

The electric field at a point outside the shell is ,

$$E = \frac{1}{4\pi\varepsilon o}\frac{Q}{r^2}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon o}\frac{Q}{r^2}\hat{r}}$$

If Q>0, $\vec{E}$ points outwards and if Q<0, then $\vec{E}$ points inwards.

The electric field $\vec{E}$ at a point outside a charged shell looks as through the entire charges are concentrated at the centre of the spherical sphere.

b) ELECTRIC FIELD AT A POINT ON THE SURFACE OF THE SPHERICAL SHELL (r =R)

On the surface of the shell r = R.

The electric field at points on the spherical shell is,

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon o}\frac{Q}{R^2}\hat{r}}$$

c) ELECTRIC FIELD AT A POINT ON THE SURFACE OF THE SPHERICAL SHELL (r < R)

Consider a point P inside the shell at a distance r from the centre.

Imagine a Gaussian surface of radius 'r' passing through the point P, Applying Gauss law to the Gaussian surface .

$$\emptyset_E = \oint \vec{E}.d\vec{A} = \frac{Q}{\varepsilon o}$$

$\vec{E}$ and $d\vec{A}$ points in the same direction radially outwards at all points on to Gaussian surface.

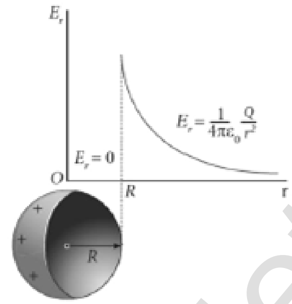
θ°=0.

$\vec{E}.\,d\vec{A}= |\vec{E}|\,|d\vec{A}|$ cos0°= E.dA

$$\emptyset_E = \oint \vec{E}.d\vec{A} = \frac{Q}{\varepsilon o}$$

$$E\oint dA = \frac{Q}{\varepsilon o}$$

$$EA == \frac{Q}{\varepsilon o}$$



$A = 4\pi r^2$

$$E.\,4\pi r^2 = \frac{Q}{\varepsilon o}$$

$$E = \frac{1}{4\pi\varepsilon o}\frac{Q}{r^2}$$

Since Gaussian surface encloses no charges ,

$$Q = 0$$

$$\boxed{E = 0}$$

The electric field due to the charged spherical sphere is zero at all the points inside the shell.

As $\vec{E} \propto \frac{1}{r^2}$ the electric field decreases as r increases and vice versa. The plot of 'E' versus 'r' of a spherical shell. From the graph, $\vec{E}$ inside the sphere is zero, and it maximum at the surface (r = R) and decreases exponentially with distance r.

7. *EFFECT OF DIELECTRIC IN CAPACITORS*

The capacitance of a capacitor is altered by the insertion of dielectric materials like mica ,glass or paper in between the plates. The dielectric can be inserted in two ways,

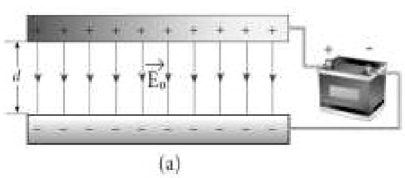
i. *When the capacitor is disconnected from the battery.*

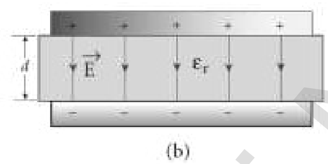
ii. *When the capacitor is connected to the battery.*

(i) WHEN THE CAPACITOR IS DISCONNECTED FROM THE BATTERY

Consider a capacitor of cross- sectional area A and separated by a distance d. Let the capacitor is charged by a voltage V_O. Q_O is that charge stored in the capacitor and. E_O is the electric field between the plates. The space between the plate is empty .The capacitance of a capacitor is taken as C_O .

The battery is now disconnected from the capacitor and let a dielectric is inserted





between the plates are shown in the figure .The dielectric constant of the dielectric is $\boldsymbol{\varepsilon_r}$. The introduction of dielectric modifies the electric field . Let the new field be E.

$$E = \frac{\varepsilon_o}{\varepsilon_r}$$

E_O is the electric field inside the capacitor plates without the dielectric. For any dielectric material , $\boldsymbol{\varepsilon_r} > 1$. Hence, $E < E_O$. The new electric field E decreases when the dielectric is inserted. The new potential difference V is,

$$V=Ed.$$

$$V=\frac{\varepsilon_o}{\varepsilon_r} d$$

$$V=\frac{V_o}{\varepsilon_r}$$

$V < V_O$, The potential decreases by the introduction of dielectric.

Capacitance increases when the dielectric is introduced between the plates of the capacitor

PROFF :

The new capacitance in the presence of dielectric is

$$C = \frac{Q_O}{V} = \frac{Q_O}{\frac{V_O}{\varepsilon_r}} = \frac{\varepsilon_r Q_O}{V}$$

$$\boxed{C= \varepsilon_r C_O}$$

Since $\varepsilon_r > 1$, $C > C_O$. Thus , the capacitance of the capacitor is increased in the presence of dielectric.

$$C_o = \frac{\varepsilon_o A}{d}$$

$$C = \frac{\varepsilon_r \varepsilon_o A}{d}$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o} \; ; \qquad \varepsilon = \varepsilon_o \varepsilon_r$$

$$C = \frac{\varepsilon A}{d}$$

$\varepsilon \rightarrow$ Permittivity of the dielectric medium.

ENERGY STORED IN THE CAPACITOR BEFORE AND AFTER THE INSERTION

Before the insertion of dielectric ,the energy stored in the capacitor U_o is,

$$U_o = \frac{1}{2}\frac{Q_o^2}{Co}$$

After the insertion of dielectric the charge Q_o remains constant and the capacitance is increased by an amount C= $\varepsilon_r \varepsilon_o$. The energy stored after the insertion of dielectric is

$$U = \frac{1}{2}\frac{Q_o^2}{C}$$

$$U = \frac{1}{2}\frac{Q_o^2}{\varepsilon_r Co} = \frac{Uo}{\varepsilon_r}$$

$$\boxed{U = \frac{Uo}{\varepsilon_r}}$$

As $\varepsilon_r > 1$, $U < U_o$. Thus, the energy stored in the capacitor decreases as the dielectric introduced. This decrease is due to the fact that the capacitor spends some energy in pulling the dielectric inside.

(ii) WHEN THE BATTERY REMAINS CONNECTED TO THE CAPACITOR

Consider a dielectric inserted between the plates of a capacitor, when a battery of voltage V_o is connected across it as shown in the figure. The charge stored in a capacitor increases by a factor when a dielectric is introduced between the plates.

(a) (b)

$Q = \boldsymbol{\varepsilon_r} Q_o$

As Q is increased ,the capacitance C is also increases . The new capacitance C is,

$$C = \frac{Q}{V_0} = \frac{\varepsilon_r Qo}{V_0}$$

$$C_o = \frac{Qo}{V_0}$$

$$\boxed{C = \boldsymbol{\varepsilon_r}.C_o}$$

As the battery is connected, the potential remains the same .The capacitance is increased by the introduction of dielectric.

ENERGY STORED IN A CAPACITOR BEFORE AND AFTER THE INSERTION OF DIELECTRIC

Before inserting the dielectric ,the energy stored in the capacitor is,

$$U_o = \frac{1}{2} Co \boldsymbol{V_0^2}$$

After the insertion of dielectric, the capacitance increases as C= C$_o$. Hence, the energy stored is

$$U = \frac{1}{2} C V_o^2 = \frac{1}{2} \boldsymbol{\varepsilon_r} C_o V_o^2$$

$$\boxed{U = \varepsilon_r.U_o}$$

The energy density U is given by

$$\boxed{U = \frac{1}{2} \varepsilon E_o^2}$$

Where, ε is permittivity of the dielectric material.

Since $\varepsilon_r > 1$. $U > U_O$. The energy (U) of the capacitor increases after the introduction of dielectric . Since the voltage between the capacitor plates V_o is constant, the electric field (Eo) also remains constant.

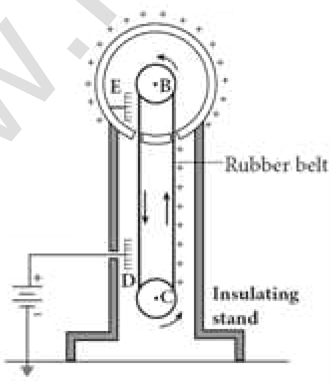
S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

7. ***VAN DE GRAFF GRNERATOR***

In the year 1929, Robert van de Graaff designed a machine which produce large amount of electrostatic potential difference, up to several million volts [10^7]. This van de graaff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in the figure .A pulley B is mounted at the centre of a hollow sphere and another pulley C is fixed at the bottom.A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor .Two comb shaped metallic conductors E and D are fixed near the pulley

The comb D is maintained at a positive potential of 10^4 V by a power supply .The upper



comb E is connected to the inner side of the hollow metallic sphere.

Because of the highest field near comb D air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and Negative charges are attracted towards the comb D. The positive charges stick to the belt and move up .When the positive charges reach the *comb E, a large amount of negative and positive charges are induced on either side of comb E,due to electrostatic induction .As a result ,the positive charges are pushed away from the comb E and they reach the outer surface of the sphere .Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere .At the same time, the negative charges nullify the positive charge in the belt due to Corona discharge before it passes over the pulley.*

When the belt descends, it has almost no net charge. At the bottom, it again gains a Large positive charge. The belt goes up and delivers the positive charge to the outer surface of the sphere .This process continues until the outer surface produces the potential difference of the order of 10^{7} *V. Which is the limiting value we can't store charges beyond this limit. Since the extra charges starts leaking to the surroundings due to ionization of air .The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.*

The high voltage produced in this Van De graaff generator is used to accelerator positive ions [protons and deutrons] for nuclear disintegration and other applications

UNSOLVED PROBLEMS

1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

Given data: q = 50 nc (or) $q = 50 \times 10^{-9} C$, $e = 1.6 \times 10^{-9} C$, n = ?

Solution :

$$q = ne \Rightarrow n = \frac{q}{e}$$

$$n = \frac{50 \times 10^{-9}}{1.6 \times 10^{-19}} = 31.25 \times 10^{10} \text{electrons} = 3.125 \times 10^{11} \text{electrons}$$

2. The total number of electrons in the human body is typically in the order of 10^{28}. Suppose, due to some reason, you and your friend lost 1% of this number of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

Given

$n = 10^{28}$, r= 1m ; m =60 kg

$n^{l} = 1\% + 10^{28} = \frac{1}{100} \times 10^{28} = 10^{26}$ [n also loss 1% of e^-'s → $n = 10^{26}$]

Solution :

i) $\boxed{F_e = \frac{1}{4\pi\varepsilon_o} \frac{qq^l}{r^2}}$

$q = ne = 10^{26} \times 1.6 \times 10^{-19} = 1.6 \times 10^{7} C$

$q^l = ne = 10^{26} \times 1.6 \times 10^{-19} = 1.6 \times 10^{7} C$

$\therefore F_e = \frac{9 X 10^9 X 1.6 X 10^7 X 1.6 X 10^7}{1^2} = 9 X 2.56 X 10^{23} = 23.04 X 10^{23}$

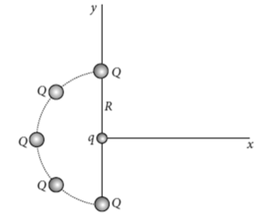
$\boxed{F_e = 23 X 10^{23} N}$

ii)Weight , $\boxed{W = mg}$

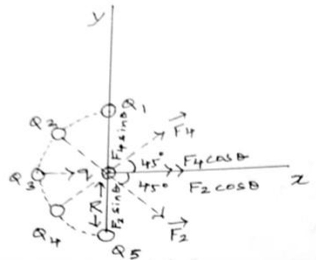
$W = 60 \times 9.8 = 588 N$

$\therefore \frac{Fe}{W} = \frac{23 X 1023}{588} = 0.039 X 10^{23} = \underline{3.9 X 10^{21}}$

3. Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the centre of the circle of radius R. Calculate the electrostatic force experienced by the charge q.



Solution :



$Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = Q$

1. Force acting on q due to Q_1 and Q_5 are equal & opposite .Hence $\vec{F_1}$ & $\vec{F_5}$ get cancelled.

   Net force is zero.

2. Force acting on q due to Q_2 [$\vec{F_2}$] and Q_4 [$\vec{F_4}$] is resolved into two components.

   $F_2 \sin\theta$ and $F_4 \sin\theta$ are equal in magnitude but opposite in direction ,gets cancelled.

   $F_2 cos$ and $F_4 \cos\theta$ are horizontal components acts in same direction , gets added.

3. Force acting on q due to Q_3 is $\vec{F_3}$

   ∴Total force acting on 'q' is $\vec{F} = \vec{F_3} + F_2 cos\theta\ \hat{\imath} + F_4 \cos\theta\ \hat{\imath}$

$$\vec{F} = k\frac{qQ}{R^2}\hat{\imath} + k\frac{qQ}{R^2}cos45°\ \hat{\imath} + k\frac{qQ}{R^2}\cos 45°\ \hat{\imath} \qquad [\theta = 45°]$$

$$= k\frac{qQ}{R^2}\hat{\imath}\left[1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] = = k\frac{qQ}{R^2}\hat{\imath}\left[1 + \frac{2}{\sqrt{2}}\right] == k\frac{qQ}{R^2}\hat{\imath}\left[1 + \sqrt{2}\right]$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^2}\left[1 + \sqrt{2}\right]\hat{\imath}\ N$$

4. Suppose a charge +q on Earth's surface and another +q charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change?

   (Take $m_E = 5.9 \times 10^{24}$ kg, $m_M = 7.9 \times 10^{22}$ kg)

Given Data:

Gravitational Constant $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Mass of Earth $m_E = 5.9 \times 10^{24} kg$

Mass of moon $m_M = 7.9 \times 10^{22}$ kg

a) $F_e = F_G$

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = \frac{Gm_E m_M}{r^2}$$

$$\frac{1}{4\pi\varepsilon_0}q^2 = Gm_E m_M$$

$$9\times10^9 . q^2 = 6.67 \times 10^{-11} \times 5.9 \times 10^{24} \times 7.9 \times 10^{22}$$

$$q^2 = \frac{6.67 \times 10^{-11} \times 5.9 \times 10^{24} \times 7.9 \times 10^{22}}{9 \times 10^9}$$

$$q = \sqrt{\frac{6.67\ X\ 5.9\ X\ 7.9\ X\ 10^{26}}{9}} = 10^{13}\sqrt{\frac{6.67\ X\ 5.9\ X\ 7.9}{9}}$$

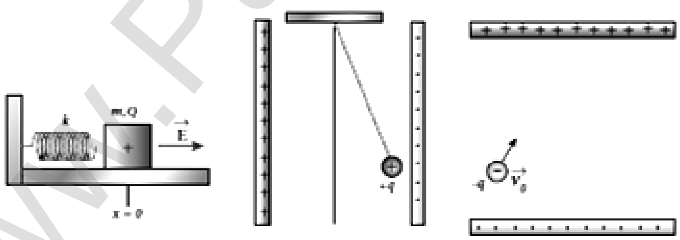
$$\boxed{q = 5.87\ X\ 10^{13}\,C}$$

b) $kq^2 = GM_EM_m$

If $r = \frac{r}{2}$

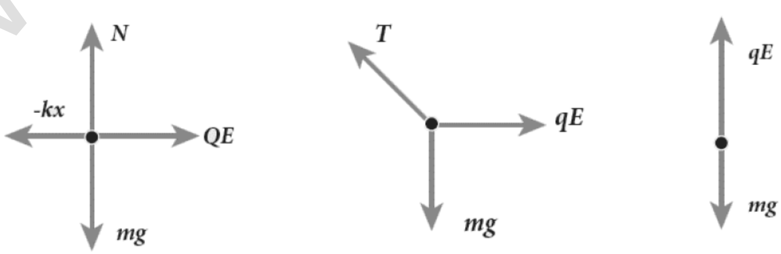
∴The charge 'q' will not change.

5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).



(a) (b) (c)

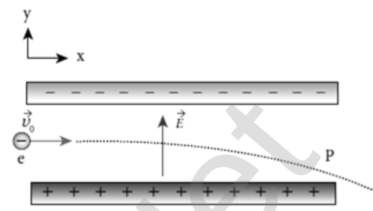
Free body Diagram



(a) (b) (c)

6. Consider an electron travelling with a speed v_0 and entering into a uniform electric field $\vec{E}$ which is perpendicular to $\vec{v}_0$ as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.

Solution :



a) Acceleration ($\vec{a}$)

F =eE

$$\vec{a} = \frac{eE}{m}(-\hat{j}) = -\frac{eE}{m}\hat{j}$$

b) Velocity ($\vec{V}$)

$$\vec{v} = \vec{u} + \vec{a}t = v_o\hat{\imath} + \frac{eE}{m}t(-\hat{j})$$

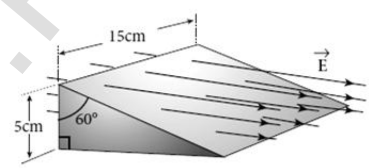
$$= v_o\hat{\imath} - \frac{eE}{m}t$$

c) Position Vector ($\vec{r}$)

$$s = ut + \frac{1}{2}at^2$$

$$\vec{r} = v_o t\hat{\imath} + \frac{1}{2}\frac{eE}{m}t^2(-\hat{j}) = v_o t\hat{\imath} - \frac{1}{2}\frac{eE}{m}t^2(\hat{j})$$

7. A closed triangular box is kept in an electric field of magnitude E = 2 × 10^3 N C^{-1} as shown in the figure.



Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface.

Solution :

a) Vertical rectangular surface

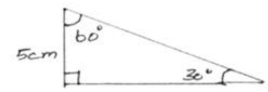
$\phi_E = EA\cos\theta$; A = 15 cm X 5 cm

$$\phi_E = 2 \times 10^3 \times 15 \times 10^{-2} \times 5 \times 10^{-2} \times \cos 0^0$$

$$\phi_E = 150 \times 10^{-1} \quad \text{(or) } 15\ Nm^2C^{-1}$$

b) Slanted surface

$$\phi_E = EA\cos\theta \quad ; \; A = 15 \times 10^{-2} \times 10 \times 10^{-2} = 150 \times 10^{-4}\ m^2$$
$$\phi_E = 2 \times 10^3 \times 15 \times 10^{-2} \times 10 \times 10^{-2} \times \cos 60^0$$
$$\phi_E = 300 \times 10^{-1} \times \frac{1}{2}$$
$$= 150 \times 10^{-1} \text{ (or) } 15\ Nm^2C^{-}$$



$\sin\theta = \frac{opp}{hyp}$

$\sin 30^\circ = \frac{5 \times 10^{-2}}{hyp}$

$hyp = \frac{5 \times 10^{-2}}{1/2}$

$hyp = 10 \times 10^{-2}\ m$

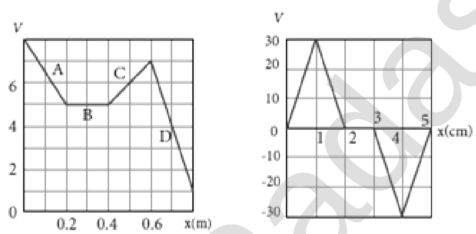
c) Entier surface

(inward flux = outward flux)

$\phi_E = 0$

Net flux is zero.

8. **The electrostatic potential is given as a function of *x* in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of *x* for the figure (b).**



(a) (b)

Solution :

For fig (a)

$$E = -\frac{dV}{dx}$$

I. (a) Region A: **From 0 to 0.2 m ; Slope is negative**

$$E = \frac{dV}{dx} = \frac{3}{0.2} = 15$$

$$E_x = +15\ vm^{-1}$$

(b) Region B: **From 0.2 to 0.4 m ; Potential is Constant (V =5)**

$$E = \frac{dV}{dx} = \frac{0}{0.2} = 0$$

$$E_x = 0$$

(c) Region C: **From 0.4 to 0.6 m ; Slope is Positive**

$$E = \frac{dV}{dx} = \frac{2}{0.2} = 10$$

$$E_x = -10\ vm^{-1}$$

(d) Region D: **From 0.6 to 0.8 m ; Slope is Negative**

$$E = \frac{dV}{dx} = \frac{6}{0.2} = 30$$

$$E_x = +30\ vm^{-1}$$

For fig (b)

(a) From 0 to 1cm ; Slope is Positive

$$E_x = -\,{}^{dv}/_{dx} = -\,{}^{30}/_{1}$$

$$E_x = -30\ Vm^{-1}$$

(b) From 1 to 2cm ; Slope is Negative

$$E_x = -\left[-{}^{dv}/_{dx}\right] = {}^{dv}/_{dx} = {}^{30}/_{1}$$

$$E_x = 30\ Vm^{-1}$$

(c) From 2 to 3 cm ; V =0

$$E_x = -\,{}^{dv}/_{dx} = 0$$

(d) From 3 to 4cm ; Slope is Negative

$$E_x = -\left[-{}^{dv}/_{dx}\right] = {}^{dv}/_{dx} = {}^{30}/_{1}$$

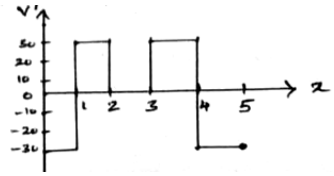
$$E_x = 30\ Vm^{-1}$$

(e) From 4 to 5 cm ; Slope is Positive

$$E_x = -\,{}^{dv}/_{dx} = -\,{}^{30}/_{1}$$

$$E_x = -30\ Vm^{-1}$$

Electric field as a function of 'x' diagram for 'b' is



9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure To create the spark, an electric field of magnitude 3 × 10^6 Vm^{-1} is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same?

(c) find the potential difference if the gap is 1 mm.

Solution :

d = $0.6\text{ mm (or) } 6 \times 10^{-4} m\,,\ E = 3 \times 10^{6} V m^{-1}$

a) Potential Difference , V=E x d

$$V = 3 \times 10^{6} \times 6 \times 10^{-4}\text{ m} = 1800V$$

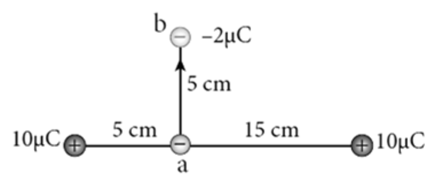
b) $V \propto dx$

so, if gap increased , V also increased.

c) If d = 1mm, V=E x d

$$V = 3 \times 10^{6} \times 10^{-3}$$
$$V = 3000V$$

10. A point charge of +10 μC is placed at a distance of 20 cm from another identical point charge of +10 μC. A point charge of -2 μC is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.



Given $q_1 = q_2 = 10\ \mu c$, $q = -2\ \mu c$

r_1 =5 cm ; $r_1^l = \sqrt{5^2+5^2} = \sqrt{50} = 5\sqrt{2}\ cm$

r_2 =15cm ; $r_2^l = \sqrt{15^2+15^2} = \sqrt{250} = 5\sqrt{10}\ cm$

Solution :

(i) Intial Potential energy (U_i) when -2 μc charge at 'a'

$$U_i = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q}{r_1} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q}{r_2}$$

$$= \frac{q_1 q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} + \frac{1}{r_2}\right] \qquad [q_1 = q_2]$$

$$U_i = 9\,X\,10^9\,X\,(-2\,X\,10^{-6})X\,10\,X10^{-6}\,X\left[\frac{1}{5}+\frac{1}{15}\right] X \frac{1}{10^{-2}}$$

$$= -18 \,X\, 10^{-2} X \left[\frac{20}{75}\right] X \frac{1}{10^{-2}} = -\frac{24}{5}$$

$\boxed{U_i = -4.85}$

(ii) Final potential energy (Uf) when a point charge (-2 μC) moved at 'b'

$$U_f = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q}{r_1^l} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q}{r_2^l}$$

$$= \frac{q_1 q}{4\pi\varepsilon_o}\left[\frac{1}{r_1^l} + \frac{1}{r_2^l}\right] \qquad [q_1 = q_1]$$

$$= 9\,X\,10^9\,X(-2\,X\;10^{-6})X\,10\,X\,10^{-6}\,X\left[\frac{1}{5\sqrt{2}} + \frac{1}{5\sqrt{10}}\right]$$

$$= \frac{-18\,X\,10^{-2}}{5\,X10^{-2}}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{10}}\right]$$

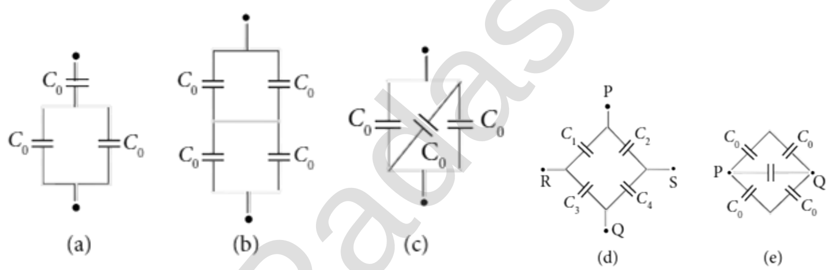
$$= -3.6\,X\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{10}}\right]$$

$$= -3.6\,X\left[\frac{1.414}{2} + \frac{3.16}{10}\right]$$

$$= -3.6\,X\,[0.707 + 0.316]$$

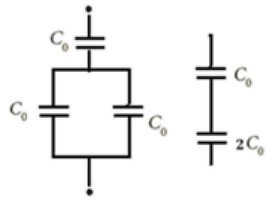
$$\boxed{U_f = -3.683\text{ J}}$$

11. Calculate the resultant capacitances for each of the following combinations of capacitors.



Solution :

(a)

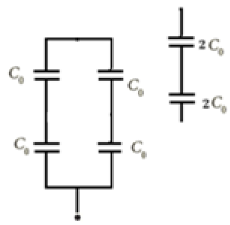


$C_p = C_0 + C_0 = 2C_0$

$\frac{1}{C_S} = \frac{1}{C_0} + \frac{1}{2C_0} = \frac{2+1}{2C_0} = \frac{3}{2C_0}$

$\boxed{C_S = \frac{2C_0}{3}}$

(b)

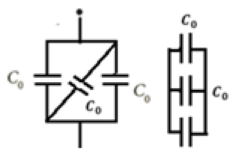


$C_p = C_0 + C_0 = 2C_0$

$\frac{1}{C_S} = \frac{1}{2C_0} + \frac{1}{2C_0} = \frac{2}{2C_0} = \frac{1}{C_0}$

$\boxed{C_S = C_0}$

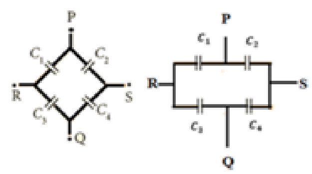
(c)



$C_p = C_0 + C_0 + C_0$

$\boxed{C_p = 3C_0}$

(d)



C_1 and C_3 are in series: $\frac{1}{C_{S_1}} = \frac{1}{C_1} + \frac{1}{C_3} = \frac{C_1 + C_3}{C_1 C_3}$

$\boxed{C_{S_1} = \frac{C_1 C_3}{C_1 + C_3}}$

C_2 and C_4 are in series: $\frac{1}{C_{S_2}} = \frac{1}{C_2} + \frac{1}{C_4} = \frac{C_2 + C_4}{C_2 C_4}$

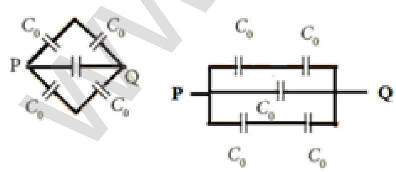
$\boxed{C_{S_2} = \frac{C_2 C_4}{C_2 + C_4}}$

C_{S_1} and C_{S_2} are in parallel ,

$C_{PQ} = C_{S_1} + C_{S_2} = \frac{C_1 C_3}{C_1 + C_3} + \frac{C_2 C_4}{C_2 + C_4} = \frac{C_1 C_3 (C_2 + C_4) + C_2 C_4 (C_1 + C_3)}{(C_1 + C_3)(C_2 + C_4)}$

$\boxed{C_{PQ} = \frac{C_1 C_2 C_3 + C_1 C_3 C_4 + C_1 C_2 C_4 + C_2 C_3 C_4}{(C_1 + C_3)(C_2 + C_4)}}$

(e)



$\frac{1}{C_{S_1}} = \frac{1}{C_0} + \frac{1}{C_0} = \frac{2}{C_0}$

$\boxed{C_{S_1} = \frac{C_0}{2}}$

$\frac{1}{C_{S_2}} = \frac{1}{C_0} + \frac{1}{C_0} = \frac{2}{C_0}$

$\boxed{C_{S_2} = \frac{C_0}{2}}$

$C_P = \frac{C_0}{2} + C_0 + \frac{C_0}{2}$

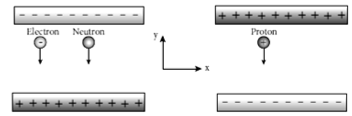
$C_P = 2C_0$

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance h = 1 mm as shown in the figure.

(a) Calculate the time of flight for both electron and proton

(b) Suppose if a neutron is allowed to fall, what is the time of flight

(c) Among the three, which one will reach the bottom first?



(Take m_p = 1.6 × 10^{-27} kg, m_e = 9.1 × 10^{-31} kg and g = 10 ms^{-2})

Given

$$m_e = Mass\ of\ electron = 9.1 \times 10^{-31} kg$$
$$m_p = Mass\ of\ proton = 1.6 \times 10^{-27} kg$$

Solution :

a) h = 1mm = 1×10^{-3} m ; V=5v

$$E = \frac{V}{x} = \frac{5}{10^{-3}} = 5 \times 10^3 Vm^{-1}$$

i) Time of flight for electron (t_e)

$$s = ut + \frac{1}{2}at^2$$

$$S = h ;\ u = 0 ;\ a = \frac{F}{m} = \frac{eE}{m_e} ;\ t = t_e$$

$$h = 0 + \frac{1}{2}\left[\frac{eE}{m_e}\right]t_e^2 = \frac{1}{2}\left[\frac{eE}{m_e}\right]t_e^2$$

$$t = \sqrt{\frac{2hm}{eE}}$$

$$t_e = \sqrt{\frac{2 \times 10^{-3} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 5 \times 10^3}}$$

$$t_e = \sqrt{\frac{18.2}{8} \times 10^{-18}} = \sqrt{2.275 \times 10^{-18}} = 1.5 \times 10^{-9} s$$

$$\boxed{t_e = 1.5\ ns}$$

ii) Time of flight for proton (t_P)

$$s = ut + {}^1/_2 at^2$$

$$S = h ; u = 0 ; a = \frac{F}{m} = \frac{eE}{mp} ; t = t_p$$

$$h = 0 + {}^1/_2 \left[\frac{eE}{m_p}\right] t_e^2 = {}^1/_2 \left[\frac{eE}{m_p}\right] t_p^2$$

$$t = \sqrt{\frac{2hm}{eE}}$$

$$t_p = \sqrt{\frac{2 \times 10^{-3} \times 1.6 \times 10^{-27}}{1.6 \times 10^{-19} \times 5 \times 10^3}}$$

$$t_p = \sqrt{\frac{3.2}{8} \times 10^{-14}} = \sqrt{0.4 \times 10^{-14}} = 0.63 \times 10^{-7} s$$

$$\boxed{t_p = 63 \text{ ns}}$$

b) Time of neutron for proton (t_n)

$$s = ut + {}^1/_2 at^2$$

$$u = 0 ; a = g ; s = h ; t = t_n$$

$$t_n = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 10^{-3}}{10}} = \sqrt{2 \times 10^{-4}}$$

$$= 1.414 \times 10^{-2} s \quad \text{(or)}\ 14.14 \times 10^{-3} s$$

$$\boxed{t_n = 14.14 \text{ms}}$$

c)

$t_n > t_p > t_e$

So, electron will reach first.

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air (3×10^6 Vm^{-1}), lightning will occur.

(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25 C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Solution ;

Given $E = 3\times 10^{6} Vm^{-1}, d = 1000m, q = 25C$

i) Electric potential difference

$$V = E \times d$$
$$V = 3 \times 10^{6} \times 10^{3}$$
$$\boxed{V = 3 \times 10^{9} V}$$

ii) Electro static potential energy (U)

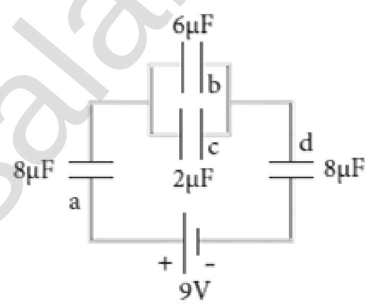
$$u = Vq$$
$$u = 3 \times 10^{9} \times 25$$
$$\boxed{V = 75 \times 10^{9} J}$$

14. For the given capacitor configuration

(a) Find the charges on each capacitor

(b) potential difference across them

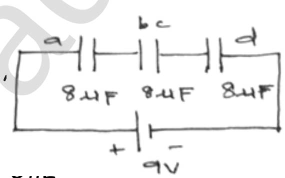
(c) energy stored in each capacitor



Solution :

V =9V

C_a =8μF , C_b =6μF, C_c =2μF ,



C_b and C_c are in parallel

$C_{bc} = C_b + C_c = 6 + 2 = 8\mu F$

Potential difference in each capacitor :

$V_a = V/3 = 9/3 = 3V$

$V_b = V_c = V_d = V/3 = 9/3 = 3V$

$\boxed{V_a = V_b = V_c = V_d = 3Volt}$

Charge on each capacitor :

$q_a = C_a V_a = 8 \times 10^{-6} \times 3 = 24 \times 10^{-6}$ C

$q_b = C_b V_b = 6 \times 10^{-6} \times 3 = 18 \times 10^{-6}$ C

$q_c = C_c V_c = 2 \times 10^{-6} \times 3 = 6 \times 10^{-6}$ C

$q_d = C_d V_d = 8 \times 10^{-6} \times 3 = 24 \times 10^{-6}$ C

$\boxed{q_a = 24\mu F}$ $\boxed{q_b = 18\mu F}$ $\boxed{q_c = 6\mu F}$ $\boxed{q_d = 24\mu F}$

Energy stored in each capacitor :

$U_a = {}^1/_2 C_a V_a^2 = {}^1/_2 \times 8 \times 10^{-6} \times [3]^2 = 36 \times 10^{-6}$ J

$U_b = {}^1/_2 C_b V_b^2 = {}^1/_2 \times 6 \times 10^{-6} \times [3]^2 = 27 \times 10^{-6}$ J

$U_c = {}^1/_2 C_c V_c^2 = {}^1/_2 \times 2 \times 10^{-6} \times [3]^2 = 9 \times 10^{-6}$ J

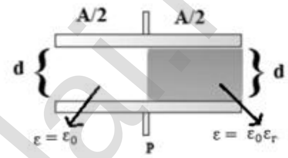
$U_d = {}^1/_2 C_d V_d^2 = {}^1/_2 \times 8 \times 10^{-6} \times [3]^2 = 36 \times 10^{-6}$ J

$\boxed{U_a = 36\mu J}$ $\boxed{U_b = 27\ \mu J}$ $\boxed{U_c = 9\ \mu J}$ $\boxed{U_d = 36\mu J}$

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ϵ_r as shown in the figure. Calculate the capacitance of capacitors P and Q.

Solution :

Capacitor 'P' :



Capacitor, $C_1 = \frac{\varepsilon_0 A}{d}$,

With dielectric $C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d}$

In capacitor 'P' : C_1 and C_2 are in parallel

$$C_1 = \frac{\varepsilon_0 A}{d},\ C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\boxed{C_P = C_1 + C_2}$$

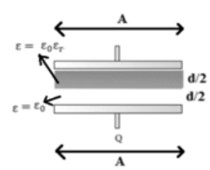
Here A = A/2

$$C_1 = \frac{\varepsilon_0 A}{2d},\ C_2 = \frac{\varepsilon_0 \varepsilon_r A}{2d}$$

$$C_P = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 \varepsilon_r A}{2d}$$

$$\boxed{C_P = \frac{\varepsilon_0 A}{2d}(1+\varepsilon_r)}$$

Capacitor 'Q' :



$$C_1 = \frac{2\varepsilon_0 \varepsilon_r A}{d},\ C_2 = \frac{2\varepsilon_0 A}{d}$$

In capacitor 'Q' : C_1 and C_2 are in series

$$\boxed{C_Q = \frac{C_1 C_2}{C_1 + C_2}}$$

Here A = A ; d = d/2

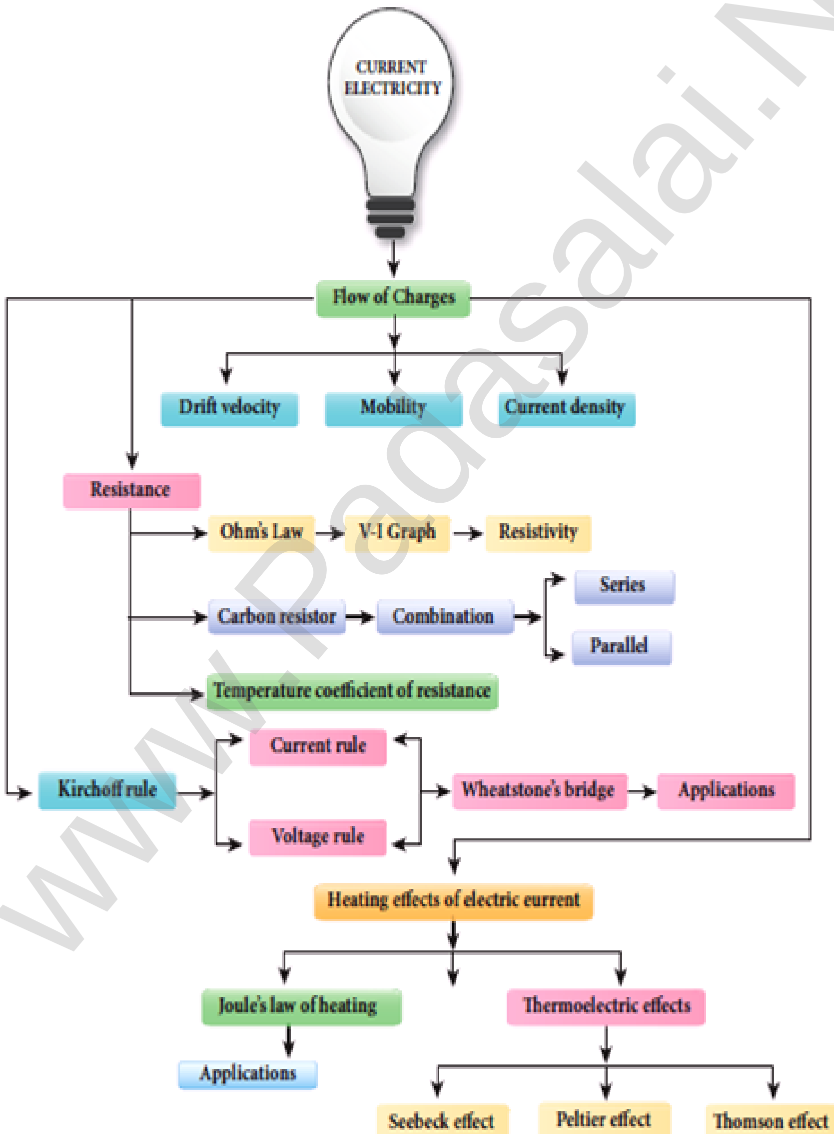
$$C_1 = \frac{2\varepsilon_0 \varepsilon_r A}{d},\ C_2 = \frac{2\varepsilon_0 A}{d}$$

$$\frac{1}{C_Q} = \frac{d}{2\varepsilon_r \varepsilon_0 A} + \frac{d}{2\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left[\frac{1+\varepsilon_r}{\varepsilon_r}\right]$$

$$\boxed{C_Q = \frac{2\varepsilon_0 A}{d}\left(\frac{\varepsilon_r}{1+\varepsilon_r}\right)}$$

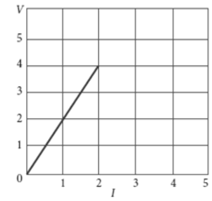
UNIT 2

CURRENT ELECTRICITY



PART –A (1 MARK)

1. The following graph shows current versus voltage values of some unknown Conductor. What is the resistance of this conductor?



(a)2 ohm (b) 4 ohm (c) 8 ohm (d)1 ohm

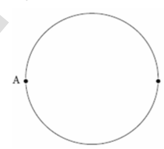
Solution:

$R=\frac{dV}{dI}$

$=\frac{4-0}{2-0}=2$

ANSWER: (a)2 ohm

2. A wire of resistance 2 ohms per meter is bent to form a circle of radius 1m. The equivalent resistance between its two diametrically opposite points, A and B as shown in the figure is



a)$\pi\ \Omega$ b) $\frac{\pi}{2}\Omega$ c) $2\pi\ \Omega$ d)$\frac{\pi}{4}\ \Omega$

Solution:

$L=2\pi R=2\pi x1=2\pi$

Total resistance $R=2\pi\ x2=4\pi\ \Omega$

Resistance in each part$=\frac{4\pi}{2}=2\pi\ \Omega$

Equivalent resistance are connected

in parallel $R_P=\frac{R}{n}=\frac{2\pi}{2}$

$R_P=\pi\ \Omega$ **ANSWER: a)π Ω**

3. A toaster operating at 240 V has a resistance of 120 Ω. The power is

a) 400 W b) 2 W c) 480 W d) 240 W

Solution:

$P=\frac{V^2}{R}$

$=\frac{240\ x240}{120}$

$P=480W$

ANSWER : c) 480 W

4. A carbon resistor of (47 ± 4.7) k Ω to be marked with rings of different colours for its identification. The colour code sequence will be

a) Yellow – Green – Violet – Gold   b) Yellow – Violet – Orange – Silver

c) Violet – Yellow – Orange – Silver   d) Green – Orange – Violet – Gold

Solution:

Yellow – Violet – Orange – Silver

$(47\text{x}10^3) \pm 10\%$=

Tolerance in % =47000x10%=

4700x0.1=470 (47000 ± 470) ohm

Or (47 ± 4.7)1000 ohm

ANSWER:

b) Yellow – Violet – Orange – Silver

5. What is the value of resistance of the following resistor?
   (a)100 k Ω (b)10 k Ω (c) 1k Ω (d)1000 k Ω

Solution:

Brown: 1

Black:0

Yellow:10^4 or 10000

Value of resistance : $10\text{x}10^4$

ANSWER : (a)100 k Ω

6. Two wires of A and B with circular cross section made up of the same material with equal lengths. Suppose $R_A = 3\ R_B$,then what is the ratio of radius of wire A to that of B?
   (a)3   (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

Solution:

$R_A = 3\ R_B$

$R \propto \frac{l}{A}$; length are common

$\frac{R_A}{R_B} = \frac{\pi r_2^2}{\pi r_1^2}$

$\frac{r_1}{r_2} = \sqrt{\frac{R_B}{R_A}}$

$= \sqrt{\frac{R_B}{3R_B}} = \frac{1}{\sqrt{3}}$

ANSWER: c)$\frac{1}{\sqrt{3}}$

7. A wire connected to a power supply of 230 V has power dissipation P_1. Suppose the wire is cut into two equal pieces and connected parallel to the same power supply. In this case power dissipation is P_2. The ratio ${P_2}/{P_1}$ is
   (a)1 (b) 2 (c) 3 (d) 4

Solution:

Before cutting the wire, resistance- R

After cutting the wire, resistance of each pieces- $\frac{R}{2}$

Two equal pieces are connected parallel is

$$R_P=\frac{R_1R_2}{R_1+R_2}$$

$$R_P=\frac{\frac{R}{2}\,X\,\frac{R}{2}}{\frac{R}{2}+\frac{R}{2}} = \frac{\frac{R^2}{4}}{R} = \frac{R^2}{4R} = \frac{R}{4}$$

Power $P=\frac{V^2}{R}$

potential is common

ratio $\frac{P_2}{P_1} = \frac{R}{R_P} = \frac{4R}{R}=4$

ANSWER : (d) 4

8. In India electricity is supplied for domestic use at 220 V. It is supplied at 110 V in USA. If the resistance of a 60W bulb for use in India is R, the resistance of a 60W bulb for use in USA will be
   (a) R (b) 2R (c)R/4 (d) R/2

Solution:

Power $P=\frac{V^2}{R}$

For india $P_1=\frac{V_1^2}{R_1} = \frac{V_1^2}{R}$

For USA $P_2=\frac{V_2^2}{R_2}$

$P_1= P_2$

$$\frac{V_1^2}{R}=\frac{V_2^2}{R_2}$$

$$R_2 = \frac{V_2^2}{V_1^2} x R$$

$$=\frac{110x110x\,R}{220x220} = \frac{R}{4}$$

ANSWER: (c) $\frac{R}{4}$

9. In a large building, there are 15 bulbs of 40W, 5 bulbs of 100W, 5 fans of 80W and 1 heater of 1kW are connected. The voltage of electric mains is 220V. The minimum capacity of the main fuse of the building will be

a) 14 A (b) 8 A (c) 10 A (d) 12 A

Solution:

Total power consumed P

=(15x40)+(5x100)+(5x80)+(1x100)

P=2500W

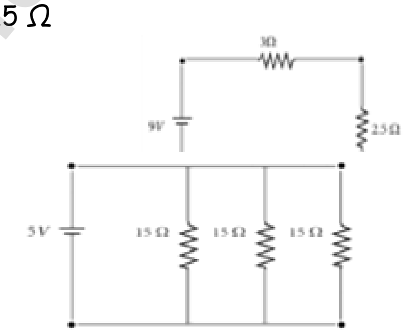
Current $I=\frac{P}{V}=\frac{2500}{220}$

I=11.36 A **ANSWER: (d) 12 A**

10. There is a current of 1.0 A in the circuit shown below. What is the resistance of P ?

a) 1.5 Ω b) 2.5 Ω c) 3.5 Ω d) 4.5 Ω

Solution:



Resistance $R=\frac{V}{I}=\frac{9}{1}=9\ \Omega$

Three resistance are connected in series $R= R_1+R_2+P$

9=3+2.5+P

P=9-5.5=3.5 Ω **ANSWER: (c) 3.5**

11. What is the current out of the battery?

a)1A b) 2A c) 3A d) 4A

Solution:

The equivalent resistance are connected in parallel $R_P=\frac{R}{n}$

$R_P=\frac{15}{3}=5\ \Omega$

Current $I=\frac{V}{R_P}=\frac{5}{5}=1A$ **ANSWER: (a)** 1A

12. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K, its resistance is 1 Ω. The Resistance of the wire will be 2 Ω at
    a) 1154 K b) 1100 K c) 1400 K d) 1127 K

Solution:

Temperature coefficient

$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$

$(T_2 - T_1) = \frac{R_2 - R_1}{R_1 \alpha}$

300 K=27^0C

$(T_2 - 27^0) = \frac{2-1}{1X0.00125}$

$(T_2 - 27^0 = \frac{1}{0.00125} = 800^0$C

$T_2 = 800^0C + 27^0C = 827^0C$

In kelvin scale

T_2=827+273=1100 K

ANSWER : b) 1100 K

13. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is
    a) 0.2 Ω b) 0.5 Ω c) 0.8 Ω d) 1.0 Ω

Solution:

$r = \left(\frac{\xi - V}{V}\right)R$

V=IR=0.2X10=2V

$r = \left(\frac{2.1-2}{2}\right)10 = \left(\frac{1}{2}\right) = 0.5\ \Omega$

ANSWER : b) 0.5 Ω

14. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
    a) each of them increases b) each of them decreases
    c)copper increases and germanium Decreases d) copper decreases and germanium increases

Solution:

For conductors, resistivity is directly proportional to temperature, $\rho \propto T$

For copper, Temperature cooled (decrease)-Resistance decrease

For semiconductor, resistivity is inversely proportional to temperature, $\rho \propto \frac{1}{T}$

For **germanium ,Temperature cooled (decrease)-Resistance increase**

ANSWER : d) copper decreases and germanium increases

15. In Joule's heating law, when R and t are constant, if the H is taken along the y axis and I2 along the x axis, the graph is
    a) straight line b) parabola c) circle d) ellipse

Solution:

when R and t are constant,

$H \propto I^2$ Graph : straight line

ANSWER : a) straight line

PART – B (2 MARKS)

1. CURRENT

If a net charge Q passes through any cross section of a conductor in time t.

$$\boxed{I = \frac{Q}{t}}$$

It is a scalar quantity.

S.I unit ampere (A)

2. ONE AMPERE

1 coulomb of charge passing through a perpendicular cross section in 1 second.

$$\boxed{1A = \frac{1c}{1s}}$$

3. DRIFT VELOCITY

The average velocity acquired by the electrons inside the conductor when it is subjected to an electric field.

$$\boxed{\vec{V}_d = \vec{a}\tau} \quad \boxed{\vec{V}_d = -\mu\vec{E}}$$

(or)

Unit → $\frac{m}{s}$ *(or) ms*$^{-1}$

4. MOBILITY [μ]

The magnitude of the drift velocity per unit electric field.

$$\boxed{\mu = \frac{|\vec{V}_d|}{|\vec{E}|}}$$

Unit → $\frac{m^2}{Vs}$ *(or) ms*$^{-1}$

5. CURRENT DENSITY

The current per unit area of cross section of the conductor.

$$\boxed{\vec{J} = ne\vec{V}_d}$$

$$J = \frac{I}{A}$$

It is the vector quantity.

SI unit → $\frac{A}{m^2}$ *(or) Am^{-2}*

Substituting $\vec{V}_d$

$$\vec{J} = -\frac{n.e^2\tau}{m}\vec{E}$$

$$\vec{J} = -\sigma\vec{E}$$

$\sigma = \frac{n.e^2\tau}{m}$ *is called conductivity .*

Resistivity (ρ)

The inverse of conductivity

$$\rho = \frac{1}{\sigma} = \frac{m}{n.e^2\tau}$$

6. RESISTIVITY (ρ)

The resistance offered to current flow by a conductor of unit length having unit area of cross section .

$$\boldsymbol{\rho} = \frac{RA}{l}$$

SI Unit → *ohm meter (Ωm)*

7. TEMPERATURE COEFFICIENT OF RESISTIVITY [∝]

The ratio of increase in resistivity per degree rise in temperature to its resistivity at T_o.

$$\boxed{\propto = \frac{\rho_T - \rho_o}{\rho_o[T - T_o]} = \frac{\Delta\rho}{\rho_{o\Delta T}}}$$

Unit → */°c*

∝ of conductors

∝ is positive, if the temperature of a conductor increases, the resistivity increases.

∝ of semiconductors

∝ is negative, the resistivity decreases with increase in temperature.

8. *DISTINGUISH BETWEEN ENERGY AND POWER IN ELECTRICAL CIRCUTIS:*

S.N0	ELECTRIC POWER	ELECTRIC ENERGY
1	*The rate at which the electrical potential energy is delivered.*	*Multiplying the power and duration of the time.*
2	$P = V I$	*Energy = VIT*
3	*Watt (or)* $\frac{J}{s}$	*Joules (or) Watt hour* *In practice ,* $1\ Kwh = 1000\ wh = 3.6 \times 10^6\ J$

9. *ELETROMOTIVE FORCE [ε]*

The amount of work a battery or cell does to move a certain amount of charge around the circuit.

UNIT → Volt (V)

10. *INTERNAL RESISTANCE [r]*

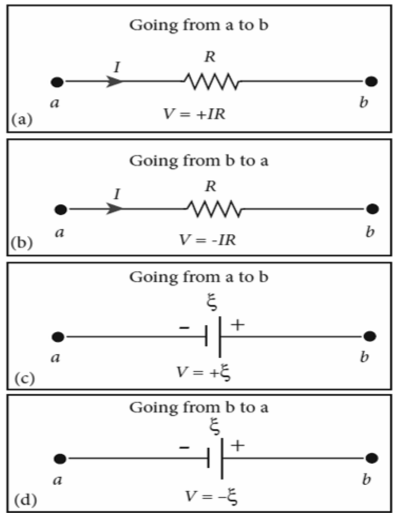
There is resistance to the flow of charges within the battery.

UNIT → Ohm (Ω)

11. *VOLTAGE RULE OR LOOP RULE :*

In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system.

The product of current and resistance is taken as positive when the direction of current followed. Suppose, if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistance is negative.



The emf is considered positive when proceeding from negative to the positive terminal of the cell .This rule applied only when all currents in the circuit reach a steady state condition.

12. WHY IS COPPER WIRE NOT SUITABLE FOR A POTENTIOMETER?

1. *The value of temperature coefficient of resistance for copper is **high**.*
2. *Further its resistivity is **low**.*

13. HEATING EFFECT OF ELECTRIC CURRENT

When current flows through a resistor, some of the electrical energy delivered on to the system is converted into heat energy and is dissipated .This heating effect of current is known as Joule's heating effect.

14. JOULE'S LAW

The heat developed in an electrical circuit due to the flow of current varies directly as,

1. *The square of the current.*
2. *The resistance of the circuit and*
3. *The time of flow.*

$$\boxed{H = I^2RT}$$

This relation was experimentally verified by Joule and is known as Joule's law of heating.

15. WHY NICHROME IS USED AS A HEATING ELEMENT ?

1. *High specific resistance.*
2. *very high temperature. (high melting point)*
3. *Without oxidation*

16. THERMOELECTRIC EFFECT

Conversion of temperature difference into electrical voltage and vice versa is known as ***thermoelectric effect***

17. SEEBECK EFFECT

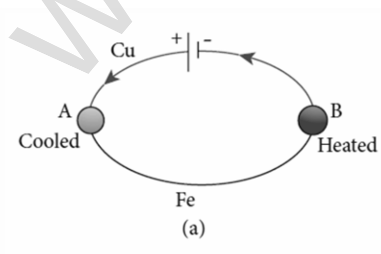
A closed circuit consisting of two dissimilar metals ,when the junctions are maintained at different temperatures an emf is developed .That current is called thermoelectric current .The two dissimilar metals connected to form two junctions is known as Thermocouple.It is reversible.

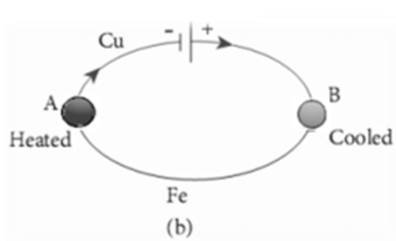
18. APPLICATION OF SEEBECK EFFECT

1. *Seebeck effect is used in electric generators.*
2. *This effect is utilised in automobiles as automotive thermoelectric generators for increasing fuel efficiency.*
3. *Seebeck effect is used in thermocouples and thermophiles to measure the temperature difference between two objects.*

19. PELTIER EFFECT

An electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and observed at the other Junction.





In the cu-Fe Thermocouple junctions A and B are maintained at the same *temperature. Let a current from your battery flow through the thermocouple . At the junction A, where the current flows from cu to Fe, heat is observed and the junction A becomes cold At the junction B, where the current flows form Fe to cu heat is liberated and it becomes hot. When the direction of current is reversed , junction A gets heated junction B gets cooled. Hence peltier effect is reversible.*

20. THOMSON EFFECT

If two points in a conductor are at different temperatures ,the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is reversible.

21. WHAT ARE THE FACTORS DEPENDS ON THE MAGNITUDE OF THE THERMOELECTRIC EMF?

i. *The nature of the metal forming the couple.*

ii. *The temperature difference between the junctions.*

PART – C (3 MARKS)

1. COLOUR CODE FOR THE CARBON RESISITORS

Carbon resistor consists of a ceramic core ,on which a thin layer of Crystalline carbon is deposited

USES

1. *Inexpensive.*
2. *Stable.*
3. *Compact in size.*

VALUE OF THE RESISTANCE :

Using colour rings.

Three coloured rings are used.

First two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth colour, silver or Gold shows the tolerance of resistor at 10 % or 5% . If there is no forth ring, the tolerance is 20 %.

EXAMPLE

First ring → green

Second ring → blue

Third ring → orange

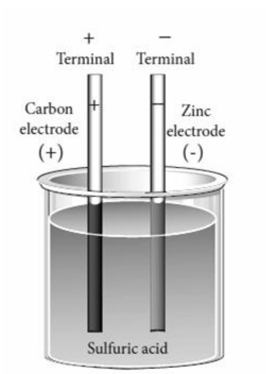
Tolerance → gold

The value of resistance

$5 \quad 6 \quad 10^3 \quad \pm 5\,\% \rightarrow 56 \times 10^3\ \Omega$ (or) $56\ k\Omega \pm 5\%$

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Sliver		10^{-2}	10%
Colorless			20%

2. ELECTRIC CELLS AND BATTERIES:

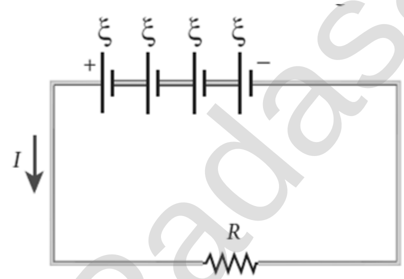


An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrode immersed in an electrolyte. Several electric cells connected together form a *battery. When it's connected to a circuit, electrons flow from the positive to negative terminal through the circuit. By using chemical reactions, a battery produces potential difference across its terminals . It's provides the energy to move the electrons through the circuit.*

3. CELL IN SERIES

In series connection ,the negative terminal of one cell is connected to the positive terminal of second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free net use terminal of the last cell become the terminals of the battery.

Suppose, n cells, each of EMF ξ volts and internal resistance 'r' ohms are connected in series with an external resistance 'R'.



Cells in series (circuit diagram)

The total emf of the battery = ξn

The total resistance in the circuit = nr + R

By ohm 's law,

$$I = \frac{total\ \ emf}{total\ \ resistance} = \frac{n\xi}{nr+R}$$

Case (a) : [r << R]

$$\boxed{I = \frac{n\xi}{R} \approx nI_1}$$

I_1 is the current due to a single cell = $\frac{\xi}{R}$

The current supplied by the battery is 'n' times that supplied by a single cell.

Case (b) : [r >>R]

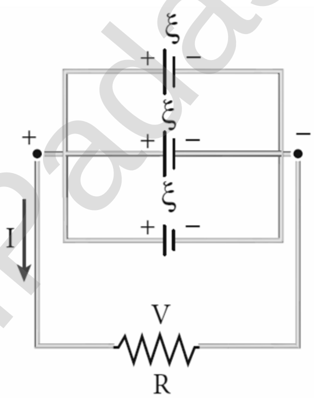
$$\boxed{I = \frac{n\xi}{nr} \approx \frac{\xi}{r}}$$

It is the current due to single cell.

There is no advantage in connecting several cells.

Thus series connection of cells is advantageous only when the effective internal resistance of the cell is negligible small compared with 'R' .

4. CELLS IN PARALLEL :



cells in parallel (Circuit diagram)

In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and the negative terminal of the battery.

Let 'n' cells can be connected in parallel between the points A and B and a resistance R is connected between that points A and B.

$\xi \rightarrow$ *EMF of the cell*

$r \rightarrow$ *Internal resistance .*

The equivalent resistance,

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \ldots\ldots\ldots \frac{1}{r} = [n\ terms\ ] = \frac{n}{r}$$

$$r_{eq} = \frac{r}{n}$$

The total resistance $= R + \frac{r}{n}$

The total emf is the potential difference between the points A and B which is equal to ξ

$$\boxed{I = \frac{\xi}{R + \frac{r}{n}} = \frac{n\,\xi}{r + nR}}$$

Case (a) : [r >> R]

$$\boxed{I = \frac{n\,\xi}{r} = nI_1}$$

The current through the external resistance due to the whole battery is n times the current due to a single cell.

Case (a) : [r >> R]

$$\boxed{I = \frac{\xi}{R}}$$

The current due to the whole battery is the same as that due to a single cell.

Hence it is advantages to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

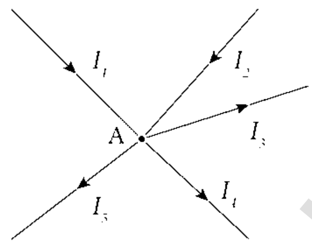
5. KIRCHHOFF'S RULES

Ohm's law is useful only for simple circuits .For more Complex circuits, kirchoff's law can be used to find current and voltage.

1. *Current rule*
2. *Voltage rule*

6. CURRENT RULE OR JUCTION RULE :

The algebraic sum of the currents at any junction of a circuit is zero. current entering the junction is taken as positive and current leaving the junction is taken as negative.



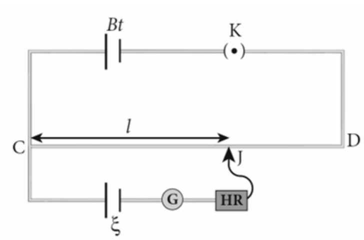
$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

$$\boxed{I_1 + I_2 = I_3 + I_4 + I_5}$$

The sum of the entering current in a Junction is equal to the sum of the leaving current from the junction. This is follow from the law of conservation of charges.

7. POTENTIOMETER

Potential is used for the accurate measurement of potential difference, current and resistance. It consists of ten metre long uniform wire of Maganin or constantan stretched in parallel rows each of one metre length, on a wooden board. The two free ends A and B are brought to the same side and fixed to Copper stripes with binding screws. A metre scale is fixed parallel to the wire. A Jockey is provided for making contact.



PRINCIPLE OF THE POTENTIOMETER.

A steady current is maintained across the wire CD by a battery Bt. The battery , key and potentiometer wire are connected in series forms the primary circuit. The positive terminal of the cell of emf ξ *is connected to the point C and negative terminal is connected to jockey through a galvanometer G and a High resistance HR. This forms the secondary circuit.*

Let Contact be make at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell ξ *then no current will flow through the Galvanometer and it will show zero deflection. CJ is the balancing length l. The potential difference across CJ is equal to Irl. Where 'I' is the current flowing through the wire and 'r' is the resistance per unit length of the wire.*

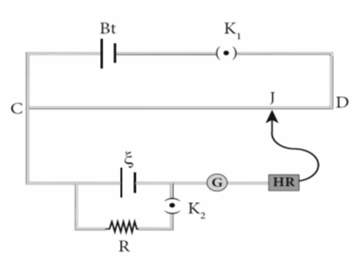
$$\boxed{\xi = Irl}$$

Since I and r constant, $\boxed{\xi \propto l}$

The emf of the cell is directly proportional to the balancing length.

8. MEASUREMENT OF INTERNAL RESISTANCE OF A CELL BY POTENTIOMETER

The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and negative terminal of the battery is connected to the end D through a key K_1*. This forms the primary circuit .*



The positive terminal of the cell ε whose internal resistance is to be determined is also contain to the end C of the wire. The negative terminal of the cell ε is connected to a jockey through a galvanometre and a high resistance. A resistance box R and key K_2 are connected across the cell . With key K_2 open, the balancing point J is obtained and the balancing length CJ =l_1 is measured. Since the cell is in open circuit ,its emf is

$$\boxed{\varepsilon \propto l_1}$$

A suitable resistance [10 Ω] is included in the resistance box and K_2 is closed.

The current is,

$$I = \frac{\xi}{R+r}$$

The potential difference across R is,

$$V = \frac{\xi}{R+r}R$$

This potential difference is balanced on the Potentiometer wire[l_2]

Then

$$\boxed{\frac{\xi}{R+r}R \propto l_2}$$

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2}$$

$$\boxed{r = R\left[\frac{l_1}{l_2} - 1\right] = R\left[\frac{l_1-l_2}{l_2}\right]}$$

Substituting the values of R, L_1 and L_2 the value r determined .The experiment can be repeated for different values of R. The external resistance increases and also increases the internal resistance.

9. APPLICATION OF JOULE'S HEATING EFFECT

1. ELECTRIC HEATERS:

Electric iron ,electric heater , electric toaster arr some of the home appliances that utilise the heating effect of current. The heating elements are made of nichrome and alloy of Nickel and chromium.

2. ELECTRICAL FUSES

Fuses are connected in series in a circuit to protect the electrical devices from the heat development by the passage of excessive current.It is short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain Value lead and copper wire melts And burns out when the current increases above 5A and 35 A respectively.

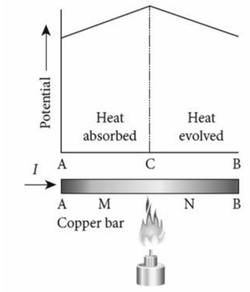
3. ELECTRICAL FURNACE

Furnace are used to manufacture such as Steel ,silicon carbide, quarts , gallium arsenide etc... To produce temperature of 1500℃ molybdenum - nichrome Wire wound on a silica tube is used carbon are furnaces produce temperature upto 3000 ℃.

4. ELECTRICAL LAMP

It consists of tungsten filament [melting point 3380℃] kept inside a glass bulb and heated to incandescent by current . Incandescent electric lamps only about 5 % of electrical energy is converted into light and the rest is wasted as heat. And also electric discharge lamps, electric welding and electric arc .

10. POSITIVE THOMSON EFFECT

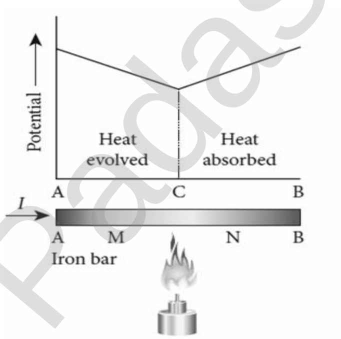


(a) Positive thomson effect

If current is passed through a copper bar AB which is heated at the middle point C , the point C will be at higher potential This indicates that the heat is observed along AC and evolved along CB of the conductor .Thus heat is transferred due to the current flow in the direction of the current. It is called positive effect.

EX : *Silver, zinc and cadmium.*

11. NEGATIVE THOMSON EFFECT



(b) Negative thomson effect

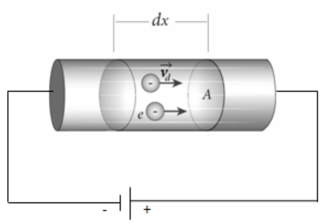
When the copper bar is replaced by an iron bar , heat is evolved along CA and absorbed along BC.Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect.

EX: *Platinum ,Nickel , Cobalt and mercury .*

PART – D (5 MARKS)

1. MICROSCOPIC MODEL OF CURRENT

Consider a conductor with area of cross section ' A ' and an electric field $\vec{E}$ applied from right to left.



V_d ➔ *drift velocity of the electron*

n ➔ *Electrons per unit volume in the conductor*

Electrons move a distance 'dx' with in a small interval of 'dt'.

$$V_d = \frac{dx}{dt} \;, \; dx = V_d\, dt.$$

The electrons available in the volume of length dx is,

= volume x number of a chance per unit volume.

$= Adx \;\; X \;\; n$

$= [\, A\, V_d\, dt \,]\, n$

Total charge in volume element [dQ]

= (charge) x (number of electrons in the volume element)

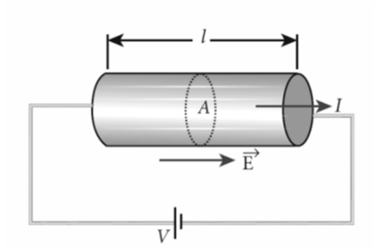
$dQ = e \;\; X\, [\, A\, V_d\, dt \,]\, n$

$$I = \frac{dQ}{dt} = \frac{neAV_d dt}{dt}$$

$$\boxed{I = neAV_d}$$

2. *OHM 'S LAW*

Consider a segment of wire of length ' r ' and cross section area ' A '.



A potential difference 'V' is applied across the wire , a net electric field is created in the wire which constitutes the current.

$$V = E.l$$

$$J = \sigma E = \sigma \frac{V}{l} \qquad J = \frac{I}{A}$$

$$\frac{I}{A} = \sigma \frac{V}{l}$$

$$V = I\left[\frac{l}{\sigma A}\right]$$

The quantity $\frac{l}{\sigma A}$ *is called resistance of the conductor. [R]*

$$R = \frac{l}{\sigma A} \qquad R \propto \frac{l}{A}$$

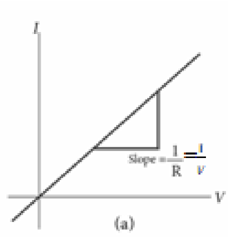
Macroscopic form of ohm's law.

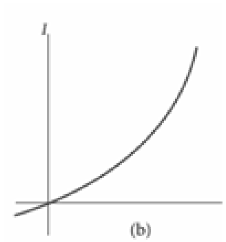
$$\boxed{V = IR}$$

The resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.

$$\boxed{R = \frac{V}{I}}$$

S.I Unit $\rightarrow$ *ohm* (Ω)





The graph between current verses voltage is straight line with a slope equal to the inverse of resistance R of the conductor. It is show in the graph (a) . Material for which the current against voltage graph is a straight line through the origin are said to obey Ohm's law and their behaviour is said to be ohmic materials or device that do not follow Ohm's law are said to be non-ohmic .They do not have a constant resistance.

3. RESISITORS IN SERIES AND PARALLEL

RESISTORS IN SERIES :

Consider three resistors R_1, R_2 and R_3 connected in series. The amount of charge passing through resistors R_1 must be pass through resistors R_2 and R_3.

Since ,the charges cannot accumulate anywhere in the circuit . Due to this reason,this current I passing through all the three resistors is the same. According to Ohm's law ,if same current pass through different resistors of different values ,then the potential difference across each resistor must be different. Let V_1 ,V_2 and V_3 be the potential difference across each of resistor R_1, R_2 and R_3 respectively. $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$ .But the total voltage V is equal to the sum of voltage across each resistor .

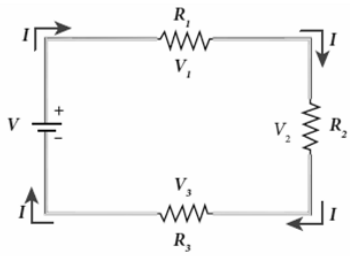
$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$V = I (R_1 + R_2 + R_3)$$

$$V = IR_s$$

$$\boxed{R_s = R_1 + R_2 + R_3}$$

Where R_s is Equivalent resistance.



(a) Three resistors in series

(b) Equivalent resistance (R_S) has the same current

The total or equivalent resistance is the sum of the individual resistance.

The value of equivalent resistance in series connection will be greater than each individual resistance.

RESISTORS IN PARALLEL :

Consider three resistors R_1, R_2 and R_3 are in parallel when they are connected across the same potential difference.

The total current I that leave the battery is split into three separate paths. Let I_1, I_2 and I_3 be the current through the resistors R_1, R_2 and R_3 respectively. Due to the conservation of charges , total current in the I is equal to sum of the current through each of the three resistors.

$$I = I_1 + I_2 + I_3$$

Since, the voltage across each resistor is the same, applying ohm 's law to each resistors.

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3}$$

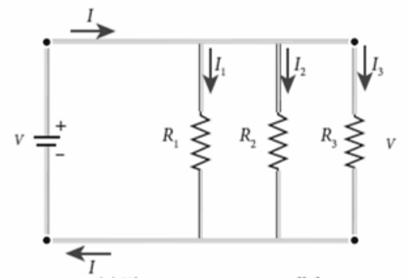
$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

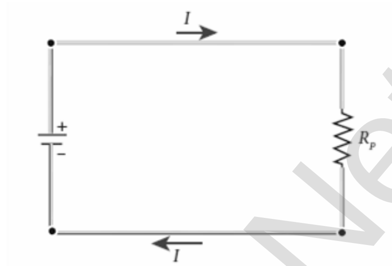
$$I = \frac{V}{R_P}$$

$$\boxed{\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

R_p → Equivalent resistance of the parallel combination of resistors.



(a) Three resistors in parallel

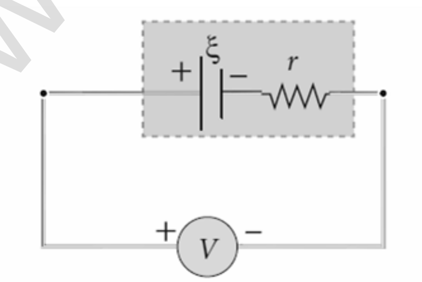


(b) Equivalent resistance (R_p) has the same current

The reciprocal of the effective resistance is equal to the sum of the reciprocal of the values of resistance of the individual resistor. The value of equivalent resistance will be lesser then each individual resistance

4. DETERMINATION OF INTERNAL RESISTANCE

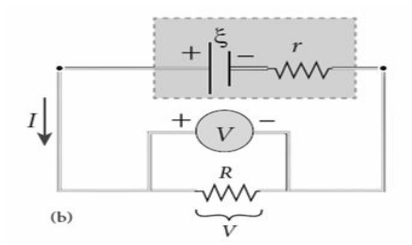
The EMF of cell ξ is measured by connecting a high resistance voltmeter across it without connecting the external resistance R.[Fig (A)]



(a) Voltmeter

Hence, the voltmeter reading gives the EMF of the cell .Then,external resistance R is included in the circuit and current I is established in the circuit.

The potential difference across R is equal to the potential difference across the cell (V) . [Fig (b)]



V=IR

Due to internal resistance 'r' of the cell, the voltmeter reads a value V, which is less than the EMF of cells certain amount of voltage(Ir) has dropped.

$$V = \xi - Ir$$

$$Ir = \xi - V$$

$$\frac{Ir}{IR} = \frac{\xi - V}{V}$$

$$\boxed{r = \left[\frac{\xi - V}{V}\right] R}$$

Since ξ , V and R an internal resistance r can be determined.

Due to this internal resistance ,the power delivered to the circuit is not equal to power rating mentioned in the battery.

The power,

$$P = I\xi = I\,[V + Ir]$$

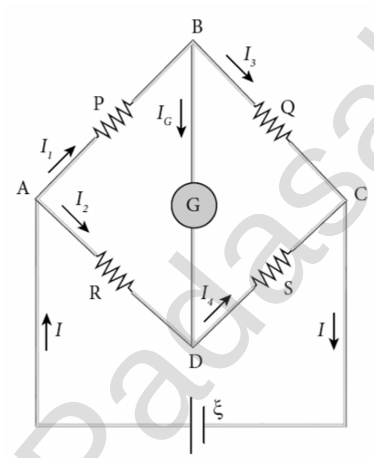
$$P = I\,[IR + Ir]$$

$$\boxed{P = [I^2R + I^2r]}$$

Hence , I^2r is the power delivered to the internal resistance. I^2R is the power delivered to the electrical circuit. For a good battery, the internal resistance 'r' is very small, then $I^2r << I^2R$ and almost entire power is delivered to the resistance.

5. WHEATSTONE'S BRIDGE

An important application of kirchhoff 's rules is the wheatstone's Bridge. The Bridge consists of four resistance P , Q ,R and S connected.A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current through the Galvanometer is 'I_G' and its resistance is 'G' .



Applying Kirchhoff 's current rule to junction B,

$$I_1 - I_G - I_3 = 0 \longrightarrow (1)$$

Applying Kirchhoff's current rule to junction D,

$$I_2 + I_G - I_4 = 0 \longrightarrow (2)$$

Applying voltage rule to loop ABDA,

$$I_1P + I_GG - I_3R = 0 \longrightarrow (3)$$

Applying voltage rule to loop ABCDA,

$$I_1P + I_3Q - I_4 S - I_2R = 0 \longrightarrow (4)$$

The points B and D are at same potential the bridge is said to be balanced. No current flows through galvanometer (I_G =0). Substituting I_G=0 in above equation (1) , (2) and (3)

$$I_1 = I_3$$

$$I_2 = I_4$$

$$I_1 P = I_2 R \longrightarrow (5)$$

Above equation substituting eq (4)

$$I_1 P + I_3 Q - I_4 S - I_2 R = 0$$

$$I_1 (P + Q) = I_2 (S + R)$$

Dividing equation (6) by eq (5)

$$\frac{P+Q}{P} = \frac{R+S}{R}$$

$$1 + \frac{Q}{P} = 1 + \frac{S}{R}$$

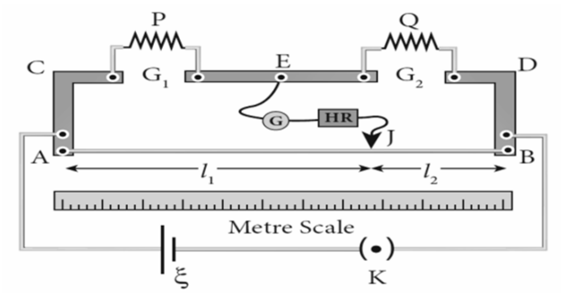
$$\frac{Q}{P} = \frac{S}{R}$$

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

This is the bridge balance condition .Only under this condition, Galvanometer shows a deflection. If three of the resistance are known ,the value of unknown resistance can be determined.

6. METER BRIDGE

The metre bridge is another form of wheatstone's bridge. It consists of a uniform maganin wire AB ofone meter length. This wire is stretched along a metre scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps G_1 and G_2. An unknown resistance 'P' is connected in G_1 and a standard resistance Q is connected in G_2. A Jockey is connected to the terminal E on the central copper strip through a galvonometer (G) and high resistance(HR) . The exact *position of Jockey on the wire can be read on the scale . A lechlanche cell and a key [K] are connected across the ends of bridge wire.*



A position of the jockey on the wire is adjusted so that the Galvanometer shows zero deflection. Let the point be J. The lengths AJ and JB of the bridge wire now replaced a resistance R and S of the wheatstone's bridge.

$$\frac{P}{Q} = \frac{R}{S} = \frac{R^l.AJ}{R^l.JB}$$

R^l ⟶ *Resistance per unit length of wire.*

$$\frac{P}{Q} = \frac{AJ}{JB} = \frac{l_1}{l}$$

$$\boxed{P = Q\frac{l_1}{l_2}}$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistance. This error can be eliminated, if another set of readings are taken with P and Q interchange and the average value of Pis found.

To find the specific resistance of the material of the wire in the coil P.

r ⟶ *Radius of the wire.*

l ⟶ *Length of the wire*

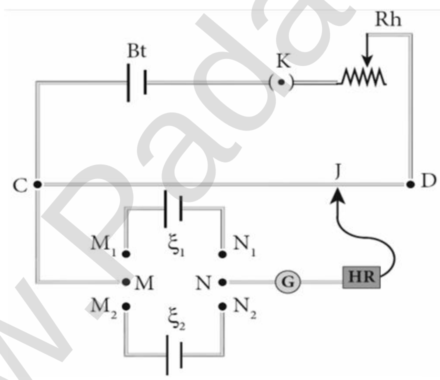
$$Resistance = \rho \frac{l}{A}$$

$$\rho = P\frac{A}{l}$$

$$\boxed{\rho = P\frac{\pi r^2}{l}}$$

7. COMPARISON OF EMF OF TWO CELLS WITH A POTENTIOMETER

To compare the EMF of two cells the circuit connections are made as shown in figure. Potentiometer wire CD is connected to a battery Bt and and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of DPDT [double pole double throw] switch and the other terminal N is connected to a jockey through a galvanometer G and high resistance HR. The cells whose emf ξ_1 and ξ_2 *to be compared are connected to the terminals* $M_1 N_1$ *and* $M_2 N_2$ *of the DPDT switch .*



The DPDT switch is pressed towards M_1 , N_1 *so that cell* ξ_1 *is included in the secondary circuit and balancing length* l_1 *is found by adjusting then jockey for zero deflection Then the second cell* ξ_2 *is included in the circuit and the balancing length* l_2 *is determined. Let r be the resistance per unit length of the Potentiometer wire and I be the current flowing through the wire.*

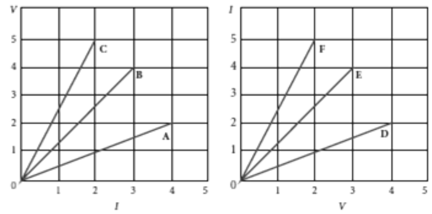
$\xi_1 = Irl_1$ $\xi_2 = Irl_2$

$$\boxed{\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}}$$

By including a Rheostat [Rh] in the primary circuit, the experiment can be repeated several times by charging the current flowing through it

UNSOLVED PROBLEM

1. ***The following graphs represent the current versus voltage and voltage versus current for the six conductors A, B, C, D, E and F. Which conductor has least resistance and which has maximum resistance?***



Solution

According to ohm's law,

$V = IR; \quad R = \frac{V}{I}$

Graph-I:

Conductor A, I = 4 A and V = 2 V

$R = \frac{V}{I} = \frac{2}{4} = 0.5\ \Omega$

Conductor B, I = 3 A and V = 4 V

$R = \frac{V}{I} = \frac{4}{3} = 1.33\ \Omega$

Conductor C, I = 2 A and V = 5 V

$R = \frac{V}{I} = \frac{5}{2} = 2.5\ \Omega\ \Omega.$

Graph-II:

Conductor D, I = 2 A and V = 4 V

$R = \frac{V}{I} = \frac{4}{2} = 2\ \Omega$ d

Conductor E, I = 4A and V = 3 V

$R = \frac{V}{I} = \frac{3}{4} = 1.75\ \Omega$

Conductor F, I = 5A and V = 2 V

$R = \frac{V}{I} = \frac{2}{5} = 0.4\ \Omega$

Conductor F has least resistance, $R_F = 0.4\ \Omega$,

Conductor C has maximum resistance, $R_C = 2.5$ G.

2. ***Lightning is very good example of natural current. In typical lightning, there is 10^9 J energy transfer across the potential difference of 5×10^7 V during a time interval of 0.2 s. Using this information, estimate the following quantities (a) total amount of charge transferred between cloud and ground (b) the current in the lightning bolt (c) the power delivered in 0.2 s.***

Solution

$N = 10^9\ J\ ;\ V = 5 \times 10^7\ V\ ;\ t = 0.2\ s = 2 \times 10^{-1}\ s.$

During the lightning energy, $E = 10^9$ J

Potential energy, $V = 5 \times 10^7$ V

Time interval, t = 0.2 s

(a) Amount of charge transferred between cloud and ground,

q = It

(b) Current in the lighting bolt, E = VIt

I = $\frac{E}{Vt}$ = $\frac{10^9}{5\times10^7\times0.2}$ = 1 × 10^9 × 1^{-7}

I = 1 × 10^2

I = 100 A

∴ q = It = 100 × 0.2

q = 20 C.

(c) Power delivered, E = VIt

P = VI = 5 × 10^7 × 100 = 500 × 10^7

I = 5 × 10^9 W

P = 5 GW.

3. ***A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of free electrons per cubic meter in the wire is 8 × 10^{28}, calculate the current density and average drift velocity of electrons.***

Solution

Cross-sections area of copper wire, A = 10^6 m^2

I = 2 A

Number of electron, n = 8 x 10^{28}

Current density, J = $\frac{1}{A}$ = $\frac{2}{10^{-6}}$

J = 2 × 10^6 Am^{-2}

Average drift velocity, V_d = $\frac{1}{neA}$

e is the charge of electron = 1.6 × 10^{-9} C

V_d = $\frac{2}{8\times10^{28}\times1.6\times10^{-19}\times10^{-6}}$ = $\frac{1}{64.\times103}$

V_d = 0.15625 × 10^{-3}

V_d = 15.6 × 10^{-5} ms^{-1}

4. ***The resistance of a nichrome wire at 20^0C is 10 Ω. If its temperature coefficient of resistivity of nichrome is $0.004/^0C$, find the resistance of the wire at boiling point of water. Comment on the result.***

Solution

Resistance of a nichrome wire at 0°C, R_0 = 10 Ω

Temperature co-efficient of resistance, α = 0.004/°C

Resistance at boiling point of water, RT = ?

Temperature of boiling point of water, T = 100 °C

$R_T = R_0 (1 + \alpha T) = 10[1 + (0.004 \times 100)]$

$R_T = 10(1 + 0.4) = 10 \times 1.4$

$R_T = 14\ \Omega$

As the temperature increases the resistance of the wire also increases.

5. The rod given in the figure is made up of two different materials.

Both have square cross sections of 3 mm side. The resistivity of the first material is 4 × 10^-3 Ωm and that of second material has resistivity of 5 × 10^-3 Ωm. What is the resistance of rod between its ends?

<u>Solution</u>

Square cross section of side, a = 3 mm = 3×10^{-3} m

Cross section of side, A = $a^2 = 9 \times 10^{6}$ m

First material:

Resistivity of the material, $\rho_1 = 4 \times 10^{-3}\ \Omega$m

length, l_1 = 25 cm = 25×10^{-2} m

Resistance of the lord, $R_1 = \frac{\rho_1 l_1}{A} = \frac{4\times10^{-3}\times25\times10^{-2}}{9\times10^{-6}} = \frac{100\times10^{-5}\times10^{6}}{9}$

$R_1 = 11.11 \times 10^1\ \Omega$

Second material:

Resistivity of the material, $\rho_2 = 5 \times 10^{-3}\ \Omega$m

length, l_2 = 70 cm = 70×10^{-2} m

Resistance of the rod, $R_2 = \frac{\rho_2 l_2}{A} = \frac{5\times10^{-3}\times70\times10^{-2}}{9\times10^{-6}} = \frac{350\times10^{-5}\times10^{6}}{9}$

$R_2 = 38.88 \times 10^1$

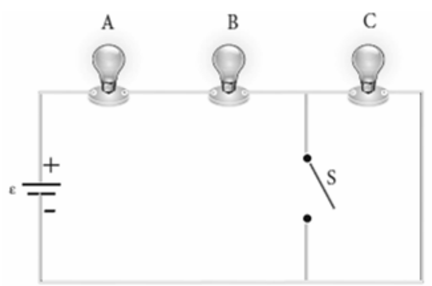
$R_2 = 389\ \Omega$

Total resistance between the ends f the rods

$R = R_1 + R_2 = 111 + 389$

$= 500\ \Omega$

6. *Three identical lamps each having a resistance R are connected to the battery of emf ε as shown in the figure.Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A,B and C. (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decrease, increase or remain same?*



<u>Solution</u>

Resistance of the identical lamp = R

Emf of the battery = ξ

According to Ohm's Law, ξ = IR

(a) Current:

When Switch is open— The current in the circuit. Total resistance of the bulb,

$R_s = R_1 + R_2 + R_3$

$R_1 = R_2 = R_3 = R$

$R_s = R + R + R = 3R$

∴ Current, $I = \frac{\xi}{R_s}$

$\Rightarrow I_0 = \frac{\xi}{3R}$

Switch is closed— The current in the circuit. Total resistance of the bulb,

$R_s = R + R = 2R$

Current $I = \frac{\xi}{R_s}$

$I_c = \frac{\xi}{2R}$.

(b) Intensity:

When switch is open — All the bulbs glow with equal intensity.

When switch is closed — The intensities of the bulbs A and B equally increase. Bulb C will not glow since no current pass through it.

(c) Voltage across three bulbs:

When switch is open — Voltage across bulb A, $V_A = I_0 R = \frac{\xi}{3R} \times R = \frac{\xi}{3}$

similarly:

Voltage across bulb B, $V_B = \frac{\xi}{3}$

Voltage across bulb C, $V_C = \frac{\xi}{3}$

When switch is closed— Voltage across bulb A, $V_A = I_c R = \frac{\xi}{2R} \times \frac{\xi}{2}$

similarly:

Voltage across bulb B, $V_B = I_c R \; \frac{\xi}{2}$

Voltage across bulb C, $V_C = 0$

(d) Power delivered to the circuit,

When switch is opened — Power P, = VI

$$P_A = V_A I_0 = \frac{\xi}{3} \times \frac{\xi}{3R} = \frac{\xi^2}{9R}$$

Similarly: $P_B = \frac{\xi^2}{9R}$ and $P_c = \frac{\xi^2}{9R}$

When switch is closed — Power P, = VI

$$P_A = V_A I_c = \frac{\xi}{2} \times \frac{\xi}{2R} = \frac{\xi^2}{4R}$$

Similarly: $P_B = \frac{\xi^2}{4R}$ and $P_c = 0$

(e) Total power delivered to circuit increases.

7. ***An electronics hobbyist is building a radio which requires 150 Ω in her circuit. But she has only 220 Ω, 79 Ω and 92 Ω resistors available. How can she connect the available resistors to get the desired value of resistance?***

Solution

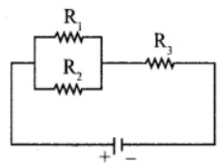
Required effective resistance = 150 Ω

Given resistors of resistance, R_1 = 220 Ω, R_2 = 79 Ω, R_3 = 92 Ω

Parallel combination R_1 and R_2

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{220} + \frac{1}{79} = \frac{79+220}{220\times 79}$$

R_p = 58 Ω



Parallel combination R_p and R_3

$R_s = R_p + R_3 = 58 + 92$

$R_s = 150\ \Omega$

Therefore , parallel combination of 230 Ω and 79 Ω in series with 92 Ω

8. A cell supplies a current of 0.9 A through a 2 Ω resistor and a current of 0.3 A through a 7 Ω resistor. Calculate the internal resistance of the cell.

Solution

Current from the cell, $I_1 = 0.9$ A

Resistor, $R_1 = 2\ \Omega$

Current from the cell, $I_2 = 0.3$ A

Resistor, R_2 $7\ \Omega$

Internal resistance of the cell, r = ?

Current in the circuit $I_1 = \frac{\xi}{r+R_1}$

$\xi = I_1 (r + R_1)$ ...... (1)

Current in the circuit, $I_2 = \frac{\xi}{r+R_2}$ ...... (2)

Equating equation (1) and (2),

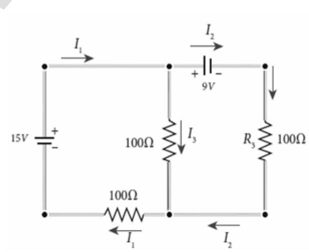
$I_1 r + I_1 R_1 = I_2 R_2 + I_2 r$

$(I_1 - I_2)r = I_2 R_2 - I_1 R_1$

$$r = \frac{I_2R_2 - I_1R_1}{I_1 - I_2} = \frac{(0.3\times7)-(0.9\times2)}{0.9-0.3} = \frac{2.1-1.8}{0.6} = \frac{0.3}{0.6}$$

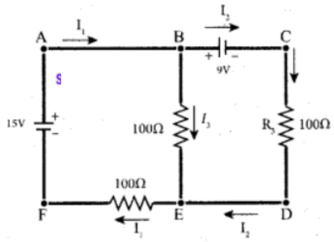
$r = 0.5\ \Omega$.

9. *Calculate the currents in the following circuit.*



Solution

Applying Kirchoff's 1st Law at junction B



$I_1 - I_1 - I_3 = 0$

$I_3 = I_1 - I_2$ ...... (1)

Applying Kirchoff's IInd Law at junction in ABEFA

$100\ I_3 + 100\ I_1 = 15$

$100\ (I_3 + I_1) = 15$

$100\ I_1 - 100\ I_2 + 100\ I_1 = 15$

$200\ I_1 - 100\ I_2 = 15$ ...... (2)

Applying Kirchoff's IInd Law at junction in BCDED

$-100 I_2 + 100\ I_3 = 9$

$-100\ I_2 + 100(I_1 - I_2) = 9$

$100 I_1 - 200\ I_2 = 9$

Solving equating (2) and (3)

$$\begin{array}{lrcl} \text{equ. (2)} \times 2 & 400\ I_1 - \cancel{200\ I_2} & = & 30 \\ & 100\ I_1 - \cancel{200}\ I_2 & = & 9 \\ & (-) \quad (+) & = & (-) \\ \hline & 300\ I_1 & = & 21 \\ & I_1 & = & 0.07\ \text{A} \end{array}$$

Substitute I_1 values in equ (2)

$200(0.07) - 100\ I_2 = 15$

$14 - 100\ I_2 = 15$

$-\ 100\ I_2 = 15 - 14$

$I_2 = \frac{-1}{100}$

$I_2 = -0.01\text{A}$

Substitute I_1 and I_2 value in equ (1), we get

$I_3 = I_1 - I_2 = 0.07 - (-0.01)$

$I_3 = 0.08$ A.

10. A wire has a length of 4 m and resistance of 20 Ω. It is connected in potentiometer series with resistance of 2980 Ω and a cell of emf 4 V. Calculate the potential gradient along the wire

<u>Solution</u>

Length of the potential wire, l = 4 m

Resistance of the wire, r = 20 Ω

Resistance connected series with potentiometer wire, R = 2980 Ω

Emf of the cell, ξ = 4 V

Effective resistance, R_s = r + R = 20 + 2980 = 3000 Ω

Current flowing through the wire, $I = \frac{\xi}{r+R_s} = \frac{4}{3000}$

$I = 1.33 \times 10^{-3}$ A

Potential drop across the wire, V = Ir

$= 1.33 \times 10^{-3}3 \times 20$

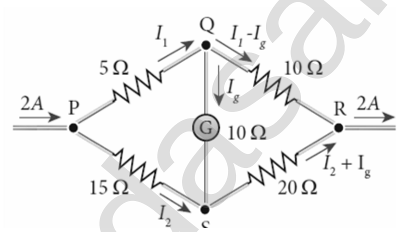
$V = 26.6 \times 10^{-3}$ V

$$\text{Potential gradient} = \frac{\text{Potential drop across the wire}}{\text{length of the wire}} = \frac{V}{l} = \frac{26.6\times10^{-3}}{4}$$

$= 6.65 \times 10^{-3}$

Potential garadient = 0.66×10^{-2} Vm^{-1}

11. Determine the current flowing through the galvanometer (G) as shown in the figure.



Solution

Current flowing through the circuit, I = 2A

Applying Kirchoff's I[st] law at junction P, $I = I_1 + I_2$

$I_2 = I - I_1$ ...(1)

Applying Kirchoff's I[nd] law at junction PQSP

$5 I_1 + 10 I_g - 15 I_2 = 0$

$5 I_1 + 10 I_g - 15(I - I_1) = 0$

$20 I_1 + 10 I_g = 15 I$

$20 I_1 + 10 I_g = 15 \times 2$

÷ by 10 $21 I_1 + I_g = 3$ ... (2)

Applying Kirchoff's II[nd] law at junction QRSQ

$10(I_1 - I_g) - 20(I - I_1 - I_g) - 10 I_g = 0$

$10 I_1 - 10_g - 20(I - I_1 - I_g) - 10 I_g = 0$

$10 I_1 - 10_g - 20 I + 20 I_1 - 20I_g - 10 I_g = 0$

$30 I_1 - 40 I_g = 20 I$

÷ by 10 ⇒ $3 I_1 - 4 I_g = 20 I$

$3 I_1 - 4 I_g = 2 I$

$3 I_1 - 4 I_g = 2 \times 2 = 4$

$$\begin{array}{ll} \text{equ (2)} \times 3 & 6 I_1 + 3 I_g = 9 \\ \text{equ (3)} \times 2 & 6 I_1 - 8 I_g = 8 \\ & (-) \quad (+) = (-) \\ \hline & 11 I_g = 1 \end{array}$$

$$I_g = \frac{1}{11} \text{ A}$$

12. ***Two cells each of 5V are connected in series with a 8 Ω resistor and three parallel resistors of 4 Ω, 6 Ω and 12 Ω. Draw a circuit diagram for the above arrangement. Calculate i) the current drawn from the cells (ii) current through each resistor***

<u>Solution</u>

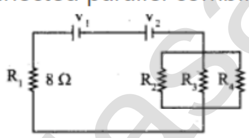
$V_1 = 5$ V; $V_2 = 5$ V

$R_1 = 8\ \Omega$; $R_2 = 4\ \Omega$; $R_3 = 6\ \Omega$; $R_4 = 12\ \Omega$

Three resistors R_2, R_3 and R_4 are connected parallel combination

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{3}{12} + \frac{2}{12} + \frac{1}{12} = \frac{6}{12}$$



Resistors R_1 and R_p are connected in series combination

$R_s = R_1 + R_p = 8 + 2 = 10$

$R_s = 10\Omega$

Total voltage connected series to the circuit

$V = V_1 + V_2$

$= 5 + 5 = 10$

$V = 10$ V.

(i) Current through the circuit, $I = \frac{V}{R_s} = \frac{10}{10}$

$I = 1$ A

Potential drop across the parallel combination,

$V' = IR_p = 1 \times 2$

$V'\ 2$ V

(ii) Current in 4 Ω resistor, $I = \frac{V'}{R_2} = \frac{2}{4} = 0.5$ A

Current in 6 Ω resistor, $I = \frac{V'}{R_3} = \frac{2}{6} = 0.33$ A

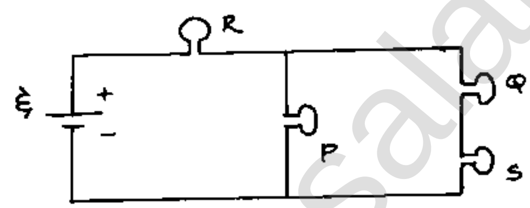
Current in 12 Ω resistor, $I = \frac{V'}{R_4} = \frac{2}{12} = 0.17$ A

13. ***Four bulbs P, Q, R, S are connected in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behaviour is observed.***

	P	Q	R	S
P removed	*	on	on	on
Q removed	on	*	on	off
R removed	off	off	*	off
S removed	on	off	on	*

Draw the circuit diagram for these bulbs.

Solution



14. ***In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?***

Solution

Emf of the cell$_1$, ξ_1 = 1.25 V

Balancing length of the cell, l_1 = 35 cm = 35 x 10^{-2} m

Balancing length after interchanged, l_2 = 63 cm = 63 x 10^{-2} m

Emf of the cell$_1$, ξ_2 = ?

The ration of emf's, $\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}$

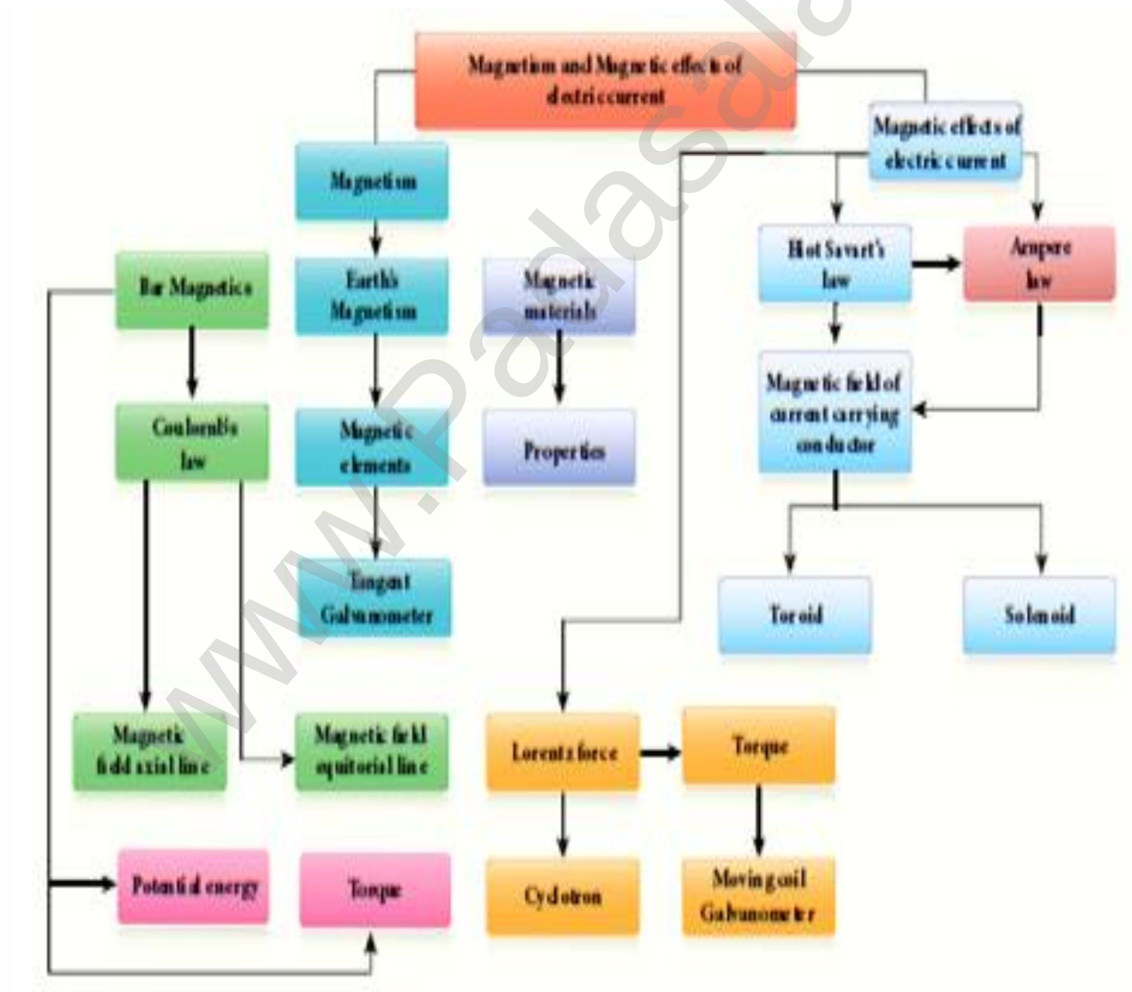
The ration of emf's, $\xi_2 = \xi_1 = \left(\frac{l_2}{l_1}\right)$

$= 1.25 \times \left(\frac{63\times10^{-2}}{35\times10^{-2}}\right) = 12.5 \times 1.8$

ξ_2 = 2.25 V.

UNIT 3

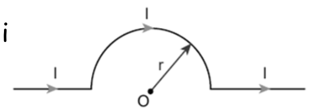
MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT



Mr.Thiviyaraj V.M.Sc.,M.Phill,B.Ed

PART –A (1 MARK)

1. The magnetic field at the center O of the following current loop i



(a) $\frac{\mu_0 I}{4r}\otimes$ (b) $\frac{\mu_0 I}{4r}\odot$ (c) $\frac{\mu_0 I}{2r}\otimes$ (d) $\frac{\mu_0 I}{2r}\odot$

Solution :

(According to right hand thumb rule B acts inwards)

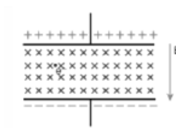
$$\mathbf{B} = \oint dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl \quad (\theta = 90^\circ)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r^2}\int \pi r \quad = \frac{\mu_0 I}{4r}$$

ANSWER :a) $\frac{\mu_0 I}{4r}\otimes$

2. An electron moves straight inside a charged parallel plate capacitor of uniform charge density σ. The time taken by the electron to cross the parallel plate capacitor when the plates of the capacitor are kept under constant magnetic field of induction $\vec{B}$ is



a) $\varepsilon_0\frac{elB}{\sigma}$ b) $\varepsilon_0\frac{lB}{\sigma l}$ c) $\varepsilon_0\frac{lB}{e\sigma}$ d) $\varepsilon_0\frac{lB}{\sigma}$

Solution :

Velocity = $\frac{\textbf{displacement}}{time} = \frac{l}{t}$

$\frac{E}{B} = \frac{l}{t}$ $\quad [E = \frac{\sigma}{\varepsilon_0}]$

t=$\frac{B}{E}$xl = $\frac{B\varepsilon_0}{\sigma}$xl

t= $\varepsilon_0\frac{lB}{\sigma}$

ANSWER: d) $\varepsilon_0\frac{lB}{\sigma}$

3. The force experienced by a particle having mass m and charge q accelerated through a potential difference V when it is kept under perpendicular magnetic field is $\vec{B}$

a) $\sqrt{\frac{2q^3BV}{m}}$ b) $\sqrt{\frac{q^3B^2V}{2m}}$ c) $\sqrt{\frac{2q^3B^2V}{m}}$ d) $\sqrt{\frac{2q^3BV}{m^3}}$

Solution :

$\frac{1}{2}mv^2$=qV

$v^2=\frac{2qV}{m}$ => $v = \sqrt{\frac{2qV}{m}}$

$$F = Bqv = Bq\sqrt{\frac{2qV}{m}}$$

$$F = \sqrt{\frac{2q^3B^2V}{m}}$$

ANSWER: *c*) $\sqrt{\frac{2q^3B^2V}{m}}$

4. A circular coil of radius 5 cm and 50 turns carries a current of 3 ampere. The magnetic dipole moment of the coil is
   (a) 1.0 amp – m^2 (b) 1.2 amp – m^2 (c) 0.5 amp – m^2 (d) 0.8 amp – m^2

Solution :

$P_m = NIA = NI\pi r^2$

$= 50 x 3 x 3.14 (5 x 10^{-2})^2$

$11775 x 10^{-4} = 1.2\ am^2$

ANSWER: (b) 1.2 amp – m^2

5. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 mA (milli ampere). The radii of inside and outside turns are a = 50 mm and b = 100 mm respectively. The magnetic induction at the center of the spiral is
   (a) 5 μT (b) 7 μT (c) 8 μT (d) 10 μT

Solution :

Magnetic induction

$B = \oint dB = \frac{\mu_0 NI}{2x} dx$

$dB = \int_a^b \frac{\mu_0 NI}{2x} dx = \frac{\mu_0 NI}{2x} \int_a^b dx$

$= \frac{\mu_0 NI}{2x} \ln \frac{b}{a}$

$n = \frac{N}{b-a}$

$$\frac{4\pi x 10^{-7} x 100 x 8 x 10^{-3} x 2.303 x log(2)}{2 x 10^{-7} x 10^{-2}}$$

$= 6.96 x 10^{-6} T = 7\ \mu T$

ANSWER: (b) 7 μT

6. Three wires of equal lengths are bent in the form of loops. One of the loops is circle, another is a semi-circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque?
   (a) circle (b) semi-circle (c) square (d) all of them

Solution :

Torque α Area ; τ α A

Area of circle > Area of squre>

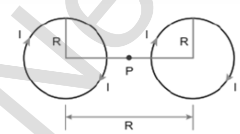
Area of semi-circle

$\pi r^2 > r^2 > \frac{1}{2}\pi r^2$

$\tau_{circle} > \tau_{squre} > \tau_{semi\ circle}$

ANSWER: a) circle

7. Two identical coils, each with N turns and radius R are placed coaxially at a distance R as shown in the figure. If I is the current passing through the loops in the same direction, then the magnetic field at a point P which is at exactly at R/2 distance between two coils is



a) $\frac{8\mu_0 NI}{\sqrt{5}R}$ b) $\frac{8N\mu_0 I}{5^{3/2}R}$ c) $\frac{8N\mu_0 I}{5R}$ d) $\frac{4N\mu_0 I}{\sqrt{5}R}$

Solution :

$B_1 = \frac{\mu_0 NI}{2}\frac{R^2}{(R^2+Z^2)^{3/2}}$ $[Z=\frac{R}{2}]$

$= \frac{\mu_0 NI}{2}\frac{R^2}{[(R^2+(\frac{R}{2})^2]^{3/2}}$

$= \frac{\mu_0 NI}{2}\frac{R^2}{[(R^2+\frac{R^2}{4}]^{3/2}}$

$= \frac{\mu_0 NI}{2}\frac{R^2}{[\frac{5R^2}{4}]^{3/2}}$

$= \frac{\mu_0 NI}{2}\frac{R^2 x 8}{R^3 5^{3/2}}$

$B_1 = \frac{4N\mu_0 I}{5^{3/2}R}$; $B_1 = B_2$

Total magnetic field at p B= $B_1 + B_2$

B=$2B_1$=$2x\frac{4\mu_0 NI}{R5^{3/2}} = \frac{8N\mu_0 I}{5^{3/2}R}$

ANSWER : b) $\frac{8N\mu_0 I}{5^{3/2}R}$

8. A wire of length l carries a current I along the Y direction and magnetic field is given by $\vec{B} = \frac{B}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}+\hat{k})$ The magnitude of Lorentz force acting on the wire is

a) $\sqrt{\frac{2}{3}}$ BIl   b) $\sqrt{\frac{1}{3}}$ BIl   c) $\sqrt{2}$ BIl   d) $\sqrt{\frac{1}{3}}$ BIl

Solution :

Magnetic field $\vec{B}=\frac{B}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}+\hat{k})$;

F=$(I\vec{l}x\vec{B})$= $Il\hat{\jmath}$ x $\frac{B}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}+\hat{k})$

=$\frac{BIl}{\sqrt{3}}(\hat{\jmath}x\hat{\imath}+\hat{\jmath}x\hat{\jmath}+\hat{\jmath}\,x\hat{k})$;

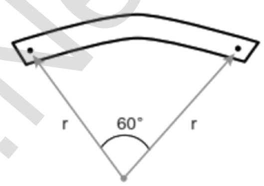
=$\frac{BIl}{\sqrt{3}}$(-$\hat{k}$+0+$\hat{\imath}$)

=$\frac{BIl}{\sqrt{3}}\sqrt{-1^2+1^2}$=

F =$\frac{BIl}{\sqrt{3}}\sqrt{2}$=$\sqrt{\frac{2}{3}}$ BIl

ANSWER : (a)$\sqrt{\frac{2}{3}}$ BIl

9. A bar magnet of length l and magnetic moment M is bent in the form of an arc shown in figure. The new magnetic dipole moment will be



(a) M (b) $\frac{3}{\pi}$ M (c) $\frac{2}{\pi}$ M (c) $\frac{1}{2}$ M

Solution :

For straight magnet,

magnetic moment M=q_ml

For new magnet,

magnetic moment M'=q_ml'

l'=2r sin30⁰=r

arc length =radius x $\boldsymbol{\theta}$

l=r $\boldsymbol{\theta}$; l= r $\frac{\pi}{3}$

r=$\frac{3l}{\pi}$=l'

For new magnet, magnetic moment

M'=q_ml'=$\frac{3q_ml}{\pi}$ =$\frac{3M}{\pi}$

ANSWER : (b) $\frac{3}{\pi}$ M

10. A non-conducting charged ring of charge q, mass m and radius r is rotated with constant angular speed ω. Find the ratio of its magnetic moment with angular momentum is

a) $\frac{q}{m}$ b) $\frac{2q}{m}$ c) $\frac{q}{2m}$ d) $\frac{q}{4m}$

Solution :

Magnetic moment μ_L=I.A μ_L=$\frac{q}{T}$ πr^2

$\left(T=\frac{2\pi}{\omega}\right)$

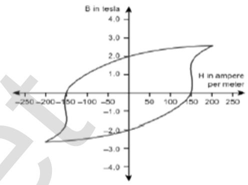
μ_L=$\frac{q\omega\,\pi r^2}{2\pi}$

Angular momentum L= $mr^2\omega$

$\frac{\mu_L}{L} = \frac{q\omega\ \pi r^2}{2\pi\ mr^2\omega} = \frac{q}{2m}$ **ANSWER : c)** $\frac{q}{2m}$

11. The BH curve for a ferromagnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns/cm. The current that should be passed in the solenoid to demagnetize the ferromagnetic completely is

    (a)1.00mA (b)1.25mA (c)1.50 mA (d) 1.75 mA



Solution :

n=1000 turns/cm, H=150(graph)

H =nI

$I = \frac{H}{n} = \frac{150}{10^5} = 150 x\ 10^{-5}$

$I = 1.50 x\ 10^{-3}$ A

ANSWER : (c)1.50 mA

12. Two short bar magnets have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centers is (Horizontal components of Earth's magnetic induction is 3.6×10^{-5} Wb m^{-2})

    (a) 3.60×10^{-5} Wb m^{-2} (b) 3.5×10^{-5} Wb m^{-2}(c) 2.56×10^{-4} Wb m^{-2} (d) 2.2×10^{-4} Wb m^{-2}

Solution :

P_{m1}=1.20 Am^2 ; P_{m2}=1.00 Am^2

Distance from common magnetic

equator =20cm/2=10cm=10^{-1}m

$B_H = 3.6X10^{-5}$ Wb m^{-2}

Resultant magnetic field

$B=B_1+B_2+B_H$

$=\frac{\mu_0 P_{m1}}{4\pi r^3} + \frac{\mu_0 P_{m2}}{4\pi r^3} + B_H$

$B = \frac{\mu_0}{4\pi r^3}(P_{m1} + P_{m2}) + B_H$

$=\frac{4\pi X10^{-7}}{4\pi X10^{-3}}(1.2 + 1) + 3.6X10^{-5}$

$2.2x10^{-4} + 0.36X10^{-4}$

$B = 2.56 \times 10^{-4}$ Wb m^{-2}

ANSWER : (c) 2.56×10^{-4} Wb m^{-2}

13. The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?
a) 30° (b) 45° (c) 60° (d) 90°

Solution :

$B_V = B_H$

$\text{Tan}\,\theta = \frac{B_V}{B_H} = 1$

$\theta = \text{Tan}^{-1}(1) = 45^\circ$ **ANSWER :** (b) 45°

14. A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an axis perpendicular to its plane passing through the center with angular velocity ω. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic field whose strength is B which is directed perpendicular to the axis of rotation
(a) $\frac{1}{4}\sigma\omega\pi BR$ (b) $\frac{1}{4}\sigma\omega\pi BR^2$ (c) $\frac{1}{4}\sigma\omega\pi BR^3$ (d) $\frac{1}{4}\sigma\omega\pi BR^4$

Solution :

Total charge on the disc

$Q = \sigma A = \sigma 2\pi r dr$

Time period $T = \frac{2\pi}{\omega}$

Current in the ring $dI = \frac{dQ}{T}$

$dI = \frac{\sigma 2\pi r dr\ \omega}{2\pi} = \sigma\omega r dr$

magnetic moment of the ring dM

$= dI\ \pi r^2$

$M = \int_0^R \pi\sigma\omega r^3 dr = \frac{\pi}{4}\sigma R^4\omega$

$M = \frac{1}{4}QR^2\omega \quad (Q = \sigma\pi R^2)$

$\tau = P_m B\ \text{Sin}\,\theta\ ; \quad \theta = 90^\circ$

$\tau = P_m B = \frac{1}{4}\sigma A R^2 \omega B$

$\tau = \frac{1}{4}\sigma\pi R^2 R^2 \omega B$

ANSWER : (d) $\frac{1}{4}\sigma\omega\pi BR^4$

15. A simple pendulum with charged bob is oscillating with time period T and let θ be the angular displacement. If the uniform magnetic field is switched ON in a direction perpendicular to the plane of oscillation then
(a) time period will decrease but θ will remain constant
(b) time period remain constant but θ will decrease
(c) both T and θ will remain the same
(d) both T and θ will decrease

Solution :

Magnetic field is perpendicular to the plane of oscillation, there is no work. so both T and θ will remain the same

ANSWER: (C) both T and θ will remain the same

PART-B (2 MARKS)

1. GEOMAGNETISM OR TERRESTRIAL MAGNETISM

The branch of Physics which deals with the Earth's magnetic field is called ***Geomagnetism or Terrestrial magnetism****.*

2. DECLINATION OR MAGNETIC DECLINATION [D]

The angle between magnetic meridian at a point and geographical Meridian is called the ***declination****.*

3. DIP OR MADNETIC INCLINATION [I]

The angle subtended by the Earth's total magnetic field with the horizontal direction in the magnetic meridian is called ***dip or magnetic inclination****.*

4. HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD [B_H]

The component of Earth's magnetic field along the horizontal direction in the magnetic meridian.

$$\boxed{B_H = B_E \cos I}$$

5. MAGNETIC DIPOLE MOMENT [$\vec{P_m}$]

The product of its pole strength and magnetic length.

$$\vec{P_m} = q_m \vec{d}$$

$$\boxed{P_m = 2q_m l}$$

SI unit → Am^2

6. MAGNETIC FIELD [$\vec{B}$]

A force experienced by the bar magnet of unit pole strength.

$$\vec{B} = \frac{1}{q_m}\vec{F}$$

Unit → $NA^{-1}m$

7. MAGNETIC FLUX [$\emptyset_B$]

The number of magnetic field lines crossing per unit area is called magnetic flux $\emptyset_B$.

$$\boxed{\emptyset_B = \vec{B} \cdot \vec{A} = BA \cos\theta}$$

θ *is angle between* $\vec{B}$ *and* $\vec{A}$

a) $\vec{B}$ *is normal to the surface*

$\theta = 0°$ $\boxed{\emptyset_B = BA}$

b) $\vec{B}$ *is parallel to the surface*

$\theta = 90°$ $\boxed{\emptyset_B = 0}$

Suppose the magnetic field is not uniform over the surface,

$$\boxed{\emptyset_B = \int \vec{B} \cdot d\vec{A}}$$

SI unit → *Weber [wb]*

It is Scalar quantity.

1 wb = 10^8 *Maxwell [CGS unit]*

8. MAGNETIC FLUX DENSITY

The number of magnetic field lines crossing unit area kept normal to the direction of line of force.

unit → *wb* m^{-2} *[or] tesla*

9. COULOMB'S INVERSE SQUARE LAW OF MAGNETISM

The force of attraction or repulsion between two magnetic force is directly proportional to the product of the of their pole strengths and inversely proportional to the square of the distance between them.

$$\vec{F} \propto \frac{q_{mA} q_{mB}}{r^2} \hat{r}$$

$$\boxed{\vec{F} = k \frac{q_{mA} q_{mB}}{r^2} \hat{r}}$$

$$k \approx \frac{\mu_o}{4\pi} \approx 10^{-7}\ Hm^{-1}$$

[μ_o → absolute permeability of free space]

10. TANGENT LAW

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields , it will come to rest in the direction of the resultant of the two fields .

$$B = B_H \tan\theta$$

11. RIGHT HAND RULE

If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flows ,then the fingers encircling the wire points in the direction of magnetic field lines produced.

12. .MAXWELL'S RIGHT HAND CORK SCREW RULE

If we rotate a right-handed screw then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field.

13. SIMILARITIES BETWEEN COULOMB'S LAW AND BIOT SAVORT LAW

Electric and magnetic field

⇨ *Obey inverse Square Law, so they are long-range fields.*

⇨ *Obey the principle of superposition and are linear with respect to source.*

$E \propto q$

$B \propto Idl$

14. RIGHT HAND THUMB RULE

If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.

15. END RULE

A current in circular loop in anticlockwise direction ,the polarity is North Pole.

A current in circular loop in clockwise direction, the polarity is South Pole .

16. AMPERE'S CIRCUITAL LAW

The line integral of magnetic field over a closed loop is μ_o times net current enclosed by the loop.

$$\oint_c \vec{B}\,\overrightarrow{dl} = \mu_o I \text{ enclosed}$$

17. SOLENOID

A solenoid is a long coil of wire closely wound in the form of helix .

The magnetic field of the solenoid is due to the superposition of magnetic fields of each turn of the solenoid .

Direction *: By right hand palm - rule .*

18. FLEMING'S LEFT HAND RULE

Stretch out forefinger , the middle finger and that thumb of the left-hand such that they are in three mutually perpendicular directions. If the fore finger points in the *direction of magnetic field the middle finger ,in the direction of electric current. Then thumb will point in the direction of the force experienced by the conductor.*

19. LORENTZ FORCE

If the charges moves into the magnetic field, It experiences a force. This force is known as Magnetic Lorentz Force

$$\vec{F} = q\,[\,\vec{V} \times \vec{B}\,]$$

If the charge is moving in both the magnetic and electric fields , the total force experienced by the charge is,

$$\vec{F} = q\,[\vec{E} + \vec{V} \times \vec{B}\,]$$

20. TESLA

If a unit charge moving in it with unit velocity experiences unit force then the magnetic field strength is 1 tesla

$$1\,T = \frac{1\,NS}{Cm} = 1\,\frac{N}{Am} = 1\,N\,A^{-1}m^{-1}$$

21. LIMITATIONS OF CYCLOTRON

a) *The speed of the ion is limited.*
b) *Electron cannot be accelerated.*
c) *Uncharged particles cannot accelerated.*

22. AMPERE

A constant current which when passed through each of the two infinitely long parallel straight conductors kept side-by-side parallelly at a distance of one metre apart in air or vacuum causes each conductor to experience a force of 2×10^{-7} Newton per metre length of conductor .

23. FIGURE OF MERIT OF A GALVANOMETER

The current required to produce a deflection of one scale division in the galvanometer

24. CURRENT SENSITIVITY

The deflection produced per unit current flowing through the galvanometer.

$$I_s = \frac{\theta}{I} = \frac{NAB}{k} => I_s = \frac{1}{G}$$

25. HOW CAN BE INCREASE THE CURRENT SENSITIVITY OF A GALVANOMETER?

CURRENT SENSITIVITY $I_s = \frac{\theta}{I} = \frac{NAB}{K}$

The current sensitivity of the Galvanometer can be increased by

(i) *Increasing the number of turns ,N*
(ii) *increasing their magnetic induction ,B*
(iii) *increasing the area of the coil ,A*
(iv) *decreasing the couple per unit Twist of the suspension wire ,k*

26. WHY IS PHOSPHOR - BRONZE WIRE USED AS THE SUSPENSION WIRE ?

Because the couple per unit test is very small, so we use the moving coil galvanometer.

VOLTAGE SENSITIVITY

The deflection produced per unit voltage applied across the galvanometer.

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR_g} = \frac{NAB}{KR_g}$$

$$V_s = \frac{1}{GR_g} = \frac{I_S}{R_g}$$

27. CURIE'S LAW

When temperature is increased, thermal vibration will upset the alignment of magnetic dipole moments. Therefore, the magnetic susceptibility decreases with increase in temperature. In many cases, the susceptibility of the materials is

$$\chi_m \propto \frac{1}{T} (or) \chi_m = \frac{c}{T}$$

This relation is called Curie's law

28. CURIE-WEISS LAW

As temperature increases, the ferromagnetism decreases due to the increased thermal agitation of the atomic dipoles. At a particular temperature, ferromagnetic material becomes paramagnetic. This temperature is known as Curie temperature TC. The susceptibility of the material above the Curie temperature is given by

$$\chi_m = \frac{c}{T - T_c}$$

This relation is called Curie-Weiss law

29. MAGNETISING FIELD

The magnetic field which is used to magnetize a sample or specimen is called the magnetising field.

Unit: *A m^{-1}.*

30. MAGNETIC PERMEABILITY

The magnetic permeability is the measure of ability of the material to allow the passage of magnetic field lines through it or measure of the capacity of the substance to take magnetisation or the degree of penetration of magnetic field through the substance.

31. RELATIVE PERMEABILITY [μ_r]

The ratio between absolute permeability of the medium to the permeability of free space.

$\mu_r = \frac{\mu}{\mu_o}$ $\quad\quad$ μ_r = [AIR]

32. INTENSITY OF MAGNETISATION

The net magnetic moment per unit volume of the material .

$\vec{M} = \frac{\vec{F_M}}{V}$ $\quad\quad$ *Unit: A m^{-1}.*

The pole strength per unit area

$$|\vec{M}| = M = \frac{q_m}{A}$$

33. MAGNETIC INDUCTION [or] TOTAL MAGNETIC FIELD

The sum of the magnetic field ρ Bo produced in vacuum due to the magnetising field and the magnetic field $\overrightarrow{B_m}$ due to the induced magnetism of the substance.

$$\vec{B} = \overrightarrow{B_o} + \overrightarrow{B_m} = \mu_o [\vec{H} + \vec{M}]$$

MAGNETIC SUSCEPTIBILITY

The ratio of the intensity of magnetisation ($\vec{M}$) induced in the material to the magnetising field ($\vec{H}$)

$$\chi_m = \frac{|\vec{M}|}{|\vec{H}|}$$

PART-C (3 MARKS)

1. PROPERTIES OF MAGNET

1. *A freely suspended bar magnet will always point along the north-south direction*
2. *A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet.*
3. *When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.*
4. *Two poles of a magnet have pole strength equal to one another.*
5. *The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called a magnetic length.*

Magnetic length < geometrical length.

The ratio of magnetic length and geometrical length is = 0.833.

2. MAGNETIC FIELD LINES

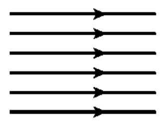
1. *Magnetic field lines are continuous closed curves.The direction of magnetic field lines is from north pole to south pole of the magnet and South Pole to north pole inside the magnet.*
2. *The direction of magnetic field at any point on the curve is known by drawing*

tangent to the magnetic line of force at that point.

3. *Magnetic field lines never intersect each other.*

4. *The degree of closeness of the field line determines the relative strength of the magnetic field .The magnetic field is strong where magnetic field lines crowd and weak magnetic field lines thin out.*

3. UNIFORM MAGNETIC FIELD



Magnetic field is said to be uniform if it has same magnitude and direction at all the points in a given region.

Ex: *local Earth's magnetic field is uniform.*

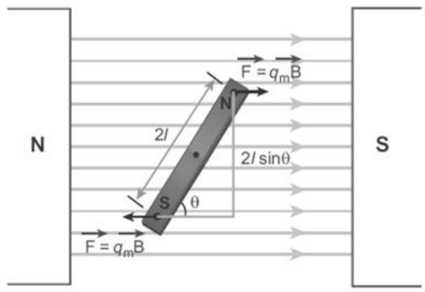
NON-UNIFORM MAGNETIC FIELD

Magnetic field is said to be non-uniform if the magnitude and direction or both where is at all its points.

Ex: *magnetic field of a bar magnet.*

4. TORQUE ACTING ON A BAR MAGNET IN UNIFORM MAGNETIC FIELD

Consider a magnet of length 2ℓ of pole strength q_m kept in a uniform magnetic field $\vec{B}$. Each pole experiences a force of magnitude $q_m B$ but acts in opposite direction. Therefore , the net force exerted on the magnet is zero, so that there is no translatory motion. These two forces constitutes a couple which will rotate and try to align in the direction of the *magnetic field* $\vec{B}$.



The force experienced by the North and South Pole,

$$\vec{F}_N = q_m \vec{B} \qquad ; \qquad \vec{F}_S = - q_m \vec{B}$$

The net force acting on a dipole,

$$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}$$

The moment of force or torque experienced by North and South Pole about O point is,

$$\vec{\tau} = \overrightarrow{ON} \; X \; \vec{F}_N + \overrightarrow{OS} + \vec{F}_S$$

$$= \overrightarrow{ON} \; X \; q_m \vec{B} \; + \; \overrightarrow{OS} + [-q_m \vec{B}]$$

By using right hand corkscrew rule ,

$$|\overrightarrow{ON}| = |\overrightarrow{OS}| = l \quad and \quad |q_m \vec{B}| = |-q_m \vec{B}|$$

The magnitude of total point about O is,

$$\tau = \ell \; x \; q_m \, B \sin\theta + \; \ell \; x \; q_m \, B \sin\theta \qquad [q_m \, X \, 2\ell = P_m]$$

$$= 2\,\ell \; x \; q_m \, B \sin\theta$$

$$\boxed{\tau = P_m \, B \sin\theta}$$

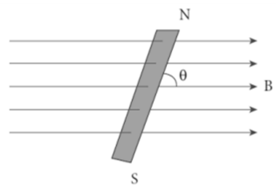
In vector notation,

$$\boxed{\vec{\tau} = \overrightarrow{P_m} \; x \; \vec{B}}$$

5. POTENTIAL ENERGY OF A MAGNET IN A UNIFORM MAGNETIC FIELD

When a bar magnet of dipole moment $\overrightarrow{P_m}$ *is held at an angle* θ *with the direction of a uniform magnetic field* $\vec{B}$. *The magnitude of the torque acting on the dipole is,*

$$|\vec{\tau}_B| = |\overrightarrow{P_m}|\,|\vec{B}|\ \sin\theta$$



If the dipole is rotated through a very small angular displacement ' dθ ' against the torque τ_B *at constant angular velocity , then the work done by external torque,*

$$dw = |\vec{\tau}_{ext}|\, d\theta$$

$$|\vec{\tau}_B| = |\vec{\tau}_{ext}|$$

$$dw = P_m\, B\, \sin\theta\, d\theta$$

Total work done in rotating the dipole from θ^l *to* θ *is,*

$$W = \int_{\theta^l}^{\theta} \tau\, d\theta = \int_{\theta^l}^{\theta} P_m\ B\ \sin\theta\, d\theta$$

$$= P_m\, B\ \left[-\cos\theta \right]_{\theta^l}^{\theta}$$

$$W = -\, P_m\, B\ [\ \cos\theta - \cos\theta^l\ ]$$

This work done is stored as potential energy in bar magnet at an angle θ.

$$U = -\ P_m\, B\ [\ \cos\theta - \cos\theta^l\ ]$$

The reference point $\theta^l = 90°$

$$\boxed{U = -\, P_m\, B\cos\theta}$$

The potential energy stored in a bar magnet in a uniform magnetic field is given by,

$$\boxed{U = -\,\vec{P}_m\, \vec{B}}$$

Case 1:

(i) $\theta = 0°$

$U = -\ P_m B\ [\ \cos 0°\ ]$

$U = -\ P_m B$

The bar magnet is aligned along the $\vec{B}$

The potential energy is maximum.

Case 2:

(ii) $\theta = 180°$

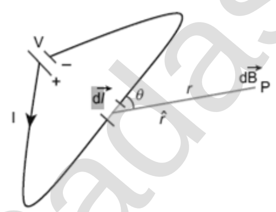
$U = -\ P_m B\ [\ \cos 180°\ ]$

$U = \ P_m B$

The bar magnet is aligned anti- parallel to external magnetic field.

The potential energy is maximum

6. ***BIOT-SAVART LAW***



The magnitude of magnetic field $\overrightarrow{dB}$ *at a point P at a distance r from the small elemental length taken on a conductor carrying current varies ,*

(i) *Directly as the strength of the current I*

(ii) *Directly as the magnitude of the length element* $\overrightarrow{dl}$

(iii) *Directly as the sine of the angle between* $\overrightarrow{dl}$ *and* $\vec{r}$

(iv) *Inversely as the square of the distance between the point P and length element* $\overrightarrow{dl}$

$$dB \propto \frac{Idl}{r^2} \sin\theta$$

$$dB = K\frac{Idl}{r^2} \sin\theta$$

$K = \frac{\mu_0}{4\pi}$ *in SI units & K = 1 in CGS units.*

$\overrightarrow{dB}$ *is perpendicular to both I* $\overrightarrow{dl}$ *and the unit vector* $\hat{r}$ *director from* $\overrightarrow{dl}$

towards a point P.

The net magnetic field at P due to the conduct is obtained from principle of superposition by considering the contribution from all current element I $\vec{dl}$

$$\vec{B} = \int \vec{dB} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

Cases :

1. *If the point P lies on the conductor*

$$\theta = 0°$$

$$\vec{dB} = 0$$

2. *If the point P lies in perpendicular conductor*

$$\theta = 90°$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \hat{n}$$

$\hat{n} \rightarrow$ *Unit vector perpendicular to both $\vec{dl}$ and $\hat{r}$*

7. DIFFERENCE BETWEEN COULOMB'S LAW AND BIOT SAVORT LAW

S.NO	ELECTRIC FIELD	MAGNETIC FIELD
1	*Produced by a scalar source i.e., an electric charge q*	*Produced by a vector source i.e., an current element I $\vec{dl}$*
2	*It is directed along the position vector joining the source and the point at which the field is calculated*	*It is directed perpendicular to the position vector $\vec{r}$ and the current element I $\vec{dl}$*
3	*Does not depend on angle*	*Depends on the angle between the position vector $\vec{r}$ and the current element I $\vec{dl}$*

8. CURRENT LOOP AS A MAGNETIC DIPOLE

The magnetic field at a point on the axis of the current-carrying circular loop,

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{[R^2+Z^2]^{3/2}} \quad \hat{k}$$

At larger distance z >> R, $R^2 + Z^2 \approx Z^2$

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{Z^3} \ \hat{k} \ \text{[or]} \ \vec{B} = \frac{\mu_0 I}{2\pi} \frac{\pi R^2}{Z^3} \ \hat{k}$$

Let A be the area of the circular loop , $A = \pi R^2$

$$\vec{B} = \frac{\mu_0 I}{2\pi} \frac{A}{Z^3} \ \hat{k} \ \text{[or]} \ \vec{B} = \frac{\mu_0}{4\pi} \frac{2IA}{Z^3} \ \hat{k}$$

$$P_m = IA$$

$P_m \Longrightarrow$ *magnetic dipole moment*

Vector notation,

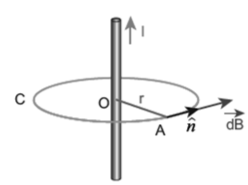
$$\boxed{\vec{P}_m = \mathrm{I}\vec{A}}$$

A current carrying circular loop behaves as a magnetic dipole of magnetic moment $\vec{P}_m$

The magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.

DIRECTION : Using right hand thumb rule

9. ***MAGNETIC FIELD DUE TO THE CURENET CARRYING WIRE OF INFINITE LENGHT USING AMPERE'S LAW***



Consider a straight conduct of infinite length carrying current I and the direction of magnetic field lines is shown in figure. Since the wire is geometrically cylindrical in the shape and symmetrical about its axis, we construct an amperian loop in the *form of a circular shape of at a distance r from the centre of the conductor .*

From the Ampere 's law,

$$\oint_C \vec{B}\,\vec{dl} = \mu_o I$$

$\vec{dl}$ *is the line element along Amperian loop. Hence the angle between magnetic field vector and the line element is zero*

$$\oint_C \vec{B}\,dl = \mu_o I$$

$$B \oint_C dl = \mu_o I$$

B ➝ *Uniform over the Amperian loop*

For a circular loop,

$$B \int_0^{2\pi R} dl = \mu_o I$$

$$B\,[\,2\pi R\,] = \mu_o I$$

$$B = \frac{\mu_o I}{2\pi R}$$

In vector form,

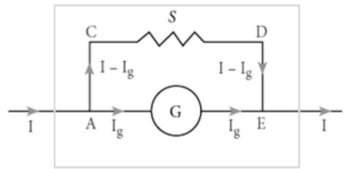
$$\vec{B} = \frac{\mu_o I}{2\pi R}\,\hat{n}$$

$\hat{n}$ ➝ *Unit vector along that tangent on the Amperian loop*

10. CONVERSION OF GALVANOMETER TO AN AMMETER

Ammeter is a instrument used to measure current flowing in the electrical circuit .

A galvanometer is converted into an ammeter by connecting yellow resistance in parallel with the galvanometer. This low resistance is called shunt resistance 'S'. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.



Ammeter

When current I reaches the junction A it divides into two components. Let I_g be the current passing through the Galvanometer of resistance Rg through a path AGE and the remaining current [I -I_g] passes along the path ACDE through shunt resistance S. The value of shunt resistance is so adjusted that current Ig produces full scale deflection in the galvanometer.

$$V_{galvanometer} = V_{shunt}$$

$$I_g R_g = [I - I_g]\, S$$

$$\boxed{S = \frac{I_g}{[I - I_g]} R_g}$$

$$I_g = \frac{S}{[S + R_g]} I$$

Since,

$$\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I$$

The deflection produced in the galvanometer is a measure of the current I passing through the circuit.

Shunt resistance is connected in parallel to Galvanometer, the resistance of ammeter [R_a] is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{s} \Rightarrow R_{eff} = \frac{R_g s}{R_g + s}$$

$$\boxed{R_a = \frac{R_g s}{R_g + s}}$$

R_a is small,the resistance offered by the ammeter is small. So when we connect

ammeter in series with ammeter will not changed appreciably the current in the circuit.

For an ideal ammeter ,the resistance must be equal to zero . But in reality ,the reading in ammeter is always less than the actual current in the circuit.

I_{ideal} *=> Current in ideal ammeter*

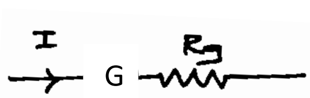
I_{actual} *=>Actual current in the circuit*

The percentage error,

$$\frac{\Delta I}{I} X\ 100\ \% = \frac{I_{ideal} - I_{actual}}{I_{ideal}}\ x\ 100\ \%$$

In order to increase the range of an ammeter n turns ,the value of shunt resistance to be connected in parallel is , $S = \frac{R_g}{n-1}$

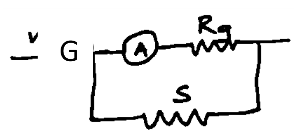
NOT FOR EXAM



V= I R_g ; I = $\frac{V}{R_g}$

$$\frac{V}{\frac{R_g S}{R_g + S}} = nI$$

$$\frac{V}{\frac{R_g S}{R_g + S}} = n\frac{V}{R_g}$$



$$\frac{R_g + S}{R_g\ S} = \frac{n}{R_g}$$

$$R_g + S = nS = nS - S$$

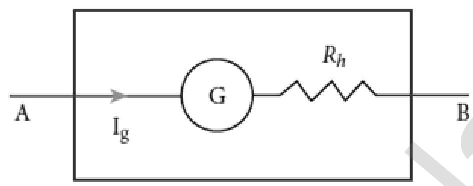
$$R_g = S\ [\ n - 1\ ]$$

11. CONVERSION OF GALVANOMETER TO AN VOLTMETER

A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits.

A Galvanometer is converted into a voltmeter by connecting high resistance R_h in series with galvanometer.

The value of resistance is so adjusted so that current `$I_g$` produces full scale deflection in the galvanometer .



Voltmeter

R_g => *Resistance of galvanometer.*

$$I = I_g = \frac{potential\ differnce}{total\ resistance}$$

Effective resistance in the circuit gives the resistance of voltmeter.

$$R_v = R_g + R_h$$

$$I_g = \frac{V}{R_g + R_h}$$

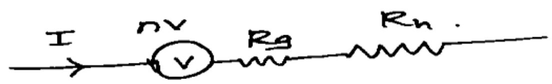
$$R_g = \frac{V}{I_g} - R_g \qquad\qquad I_g \propto V$$

The resistance of voltmeter is very large, a voltmeter connected in parallel in an electrical circuit means least current in the circuit. An <u>ideal voltmeter</u> is one which has <u>infinite resistance.</u>

In order to increase the range of voltmeter 'n' times the value of resistance to be connected in series with the Galvanometer is

$$R_h = [\, n - 1]\ R_g$$

NOT FOR EXAM



$V = IR_g$; $I = V/R_g$.

$$nV = I\,[R_g + R_h]$$

$$\frac{nV}{R_g+R_h} = \frac{V}{R_g}$$

$$nR_g = R_g + R_h$$

$$R_h = nR_g - R_g$$

$$R_h = [\,n - 1]\,R_g$$

12. PROPERTIES OF DIAMAGNETIC MATERIALS

- *i) Magnetic susceptibility is negative.*
- *ii) Relative permeability is slightly less than unity.*
- *iii) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.*
- *iv) Susceptibility is nearly temperature independent.*

13. PROPERTIES OF PARAMAGNETIC MATERIALS

- *i) Magnetic susceptibility is positive and small.*
- *ii) Relative permeability is greater than unity.*
- *iii) The magnetic field lines are attracted into the paramagnetic materials when placed in a magnetic field*
- *iv) Susceptibility is inversely proportional to temperature.*
- *v)*

14. PROPERTIES OF FERROMAGNETIC MATERIALS

- *i) Magnetic susceptibility is positive and large.*
- *ii) Relative permeability is large.*
- *iii) The magnetic field lines are strongly attracted into the ferromagnetic materials when placed in a magnetic field.*
- *iv) Susceptibility is inversely proportional to temperature.*

15. APPLICATIONS OF HYSTERESIS LOOP

The significance of hysteresis loop is that it provides information such as retentivity, coercivity, permeability, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material. Therefore, the study of hysteresis loop will help us in selecting proper and suitable material for a given purpose. Some examples:

i) ***PERMANENT MAGNETS:***

The materials with high retentivity, high coercivity and low permeability are suitable for making permanent magnets.

Examples: *Carbon steel and Alnico*

ii) ***ELECTROMAGNETS:***

The materials with high initial permeability, low retentivity, low coercivity and thin hysteresis loop with smaller area are preferred to make electromagnets.

Examples*: Soft iron and Mumetal (Nickel Iron alloy).*

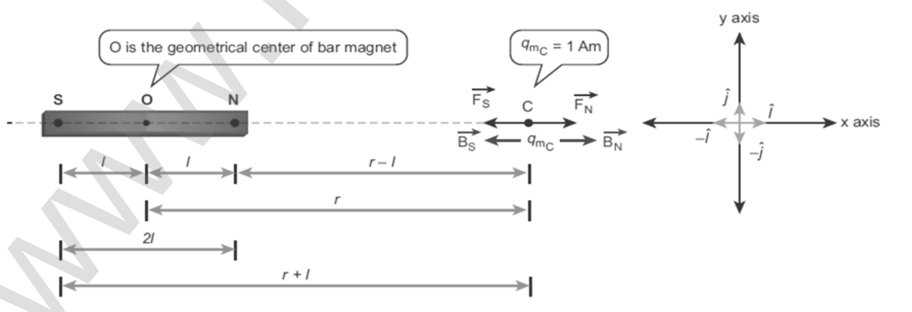
iii) ***CORE OF THE TRANSFORMER****:*

The materials with high initial permeability, large magnetic induction and thin hysteresis loop with smaller area are needed to design transformer cores.

Examples*: Soft iron*

PART-D (5 MARKS)

1. MAGNETIC FIELD AT A POINT ALONG THE AXIAL LINE OF THE MAGNETIC DIPOLE



Consider a bar magnet NS shown in the figure. let N be the north pole and S be the South Pole of the bar magnet, which of pole strength q_m and separated by a distance of 2 ℓ. The magnetic field at a point C at a distance from the geometrical centre O of the bar magnet can be computed by keeping unit north pole [q_{mc} = 1 Am] at C.

The force of repulsion between north pole of the bar magnet and unit north pole at point .[Due to coulomb's law]

$$\overrightarrow{F}_N = \frac{\mu_o}{4\pi}\frac{q_m}{[r-l]^2}\ \hat{\imath}$$

The force of attraction between south pole of the bar magnet and unit north pole at point C.

$$\overrightarrow{F}_S = -\frac{\mu_o}{4\pi}\frac{q_m}{[r+l]^2}\ \hat{\imath}$$

Net force at point C ,

$$\overrightarrow{F} = \overrightarrow{F}_N + \overrightarrow{F}_S$$

$$\overrightarrow{F} = \overrightarrow{B}$$

$$\overrightarrow{B} = \frac{\mu_o}{4\pi}\frac{q_m}{[r-l]^2}\ \hat{\imath} + \left[-\frac{\mu_o}{4\pi}\frac{q_m}{[r+l]^2}\ \hat{\imath}\ \right]$$

$$\overrightarrow{B} = \frac{\mu_o q_m}{4\pi}\left[\frac{1}{[r-l]^2} - \frac{1}{[r+l]^2}\right]\hat{\imath}$$

$$\overrightarrow{B} = \frac{\mu_o 2r}{4\pi}\left[\frac{q_m.[2l\,]}{[r^2-l^2]^2}\right]\hat{\imath}$$

$$\overrightarrow{B} = \frac{\mu_o}{4\pi}\left[\frac{2rP_m}{[r^2-l^2]^2}\right]\hat{\imath} \qquad |\vec{P}_m| = P_m = q_m.2\ell$$

$$[r^2 - l^2]^2 \approx r^4 \qquad r >> \ell$$

$$\vec{B}_{axial} = \frac{\mu_o}{4\pi}\left[\frac{2P_m}{r^3}\right]\hat{\imath} \qquad \hat{P}_m = P_m\,\hat{\imath}$$

$$\boxed{\vec{B}_{axial} = \frac{\mu_o}{4\pi}\frac{2}{r^3}\overrightarrow{P_m}}$$

2. MAGNETIC FIELD AT A POINT ALONG THE EQUATORIAL LINE DUE TO A MAGNETIC DIPOLE

Consider a bar magnet NS. Let N be the north pole and S be South Pole *of the bar magnet , each with pole strength q_m and separated by a distance 2ℓ. The magnetic field at a point c at distance r from the geometrical centre O of the bar magnet can be computed by keeping unit north pole [q_{mc} = 1 Am] at c . The force of repulsion between North pole of the bar magnet and unit north pole at a point c.*

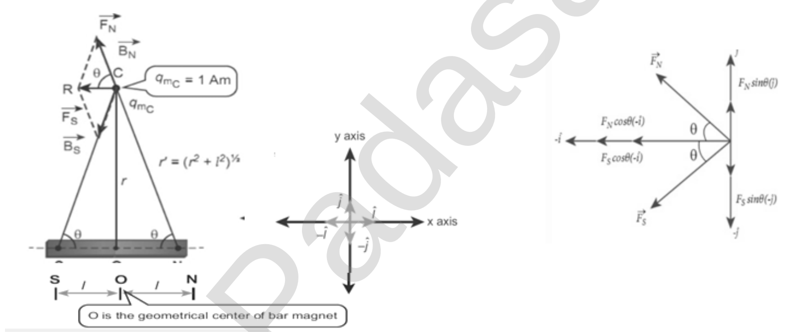
$$\vec{F}_N = - F_N \cos\theta\,\hat{\imath} + F_N \sin\theta\,\hat{\jmath}$$

where $F_N = \frac{\mu_o}{4\pi}\frac{q_m}{r^{l\,2}}$

The force of attraction between South Pole of the bar magnet and unit north pole at a point c.

$$\vec{F}_S = - F_S \cos\theta\,\hat{\imath} - F_S \sin\theta\,\hat{\jmath}$$

where $F_S = \frac{\mu_o}{4\pi}\frac{q_m}{r^{l\,2}}$



The net force at a point c is,

$$\vec{F} = \vec{F}_N + \vec{F}_S$$

This net force is equal to the magnetic field at the point c

$$\vec{B} = -[F_N + F_S]\cos\theta\,\hat{\imath}$$

$$F_N = F_S$$

$$\vec{B} = -\frac{2\mu_o}{4\pi}\frac{q_m}{r^{l\,2}}\cos\theta\,\hat{\imath}$$

$$= -\frac{2\mu_o}{4\pi}\frac{q_m}{[r+l\,]^2}\cos\theta\,\hat{\imath}$$

In a right angle triangle NOC,

$$\cos\theta\,\hat{\imath} = \frac{l}{r^l} = \frac{l}{[r^2 + l^2]^{1/2}}$$

$$\vec{B} = -\frac{\mu_o}{4\pi}\frac{q_m \ x\,[\,2l\,]}{[r^2 + l^2]^{3/2}}\,\hat{\imath}$$

$$|\vec{P}_m| = P_m = q_m 2l$$

$$\vec{B}_{equa} = -\frac{\mu_o}{4\pi}\frac{P_m}{[r^2 + l^2]^{3/2}}\,\hat{\imath}$$

$$[r^2 + l^2]^{3/2} \approx r^3 \qquad r >> \ell$$

$$\vec{B}_{equa} = -\frac{\mu_o}{4\pi}\frac{P_m}{r^3}\,\hat{\imath}$$

$$P_m\hat{\imath} = \vec{P}_m$$

$$\boxed{\vec{B}_{equa} = -\frac{\mu_o}{4\pi}\frac{\vec{P}_m}{r^3}}$$

Magnitude of the B $_{axial}$ is twice that of magnitude of B $_{equa}$ and the direction of B $_{axial}$ and B $_{equa}$ are opposite.

3. TANGENT GALVANOMETER [TG]

Tangent Galvanometer is a device used to measure very small current . It is a moving type galvanometer. Its working is based on tangent law.

CONSTRUTION

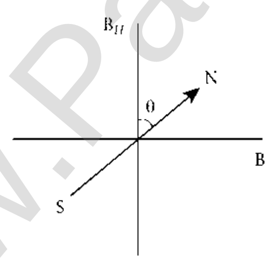
It consists of copper coil wounded on non-magnetic circular frame. The same is made up of brass or wood which is mounted vertically on a horizontal base table with three leveling screws. The tangent galvanometer is provided with two or more coils of different number of turns consists of 2 turns ,5 turns and 50 turns which are of different thickness and or used for measuring currents for different strengths.

At the centre of turntable, small upright projection is seen on which a compass box is placed. Compass box consists of a small magnetic needle which is pivoted at the centre, such that arrangement shows the centre of both magnetic needle and circular coil *exactly coincide . A thin aluminium pointer is attached to the magnetic needle normally and moves over circular scale. The surface scale is divided into four quadrant and graduated in degrees which are used to measure the deflection of aluminium pointer on a circular degree scale. In order to avoid parallel error, in measure mirror is placed below the aluminium pointer.*

THEORY

When no current is passed through the coil , the small magnetic needle life along horizontal component of Earth's magnetic field. When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field. Now there are two field which are active mutually perpendicular to each other.

1. *The magnetic field (**B**) due to the electric current in the coil acting# normal to the plane of the coil.*
2. *The component of Earth's magnetic field ($\boldsymbol{B_H}$)*



Because of these crossed fields, the pivoted magnetic needle deflects and angle θ ,

From tangent law,

$$B = B_H \tan\theta$$

When an electric current is passed through a circular coil of radius R having N turns , the magnitude of magnetic field at the centre is,

$$B = \frac{\mu_o NI}{2R}$$

$$\frac{\mu_o NI}{2R} = B_H \tan\theta$$

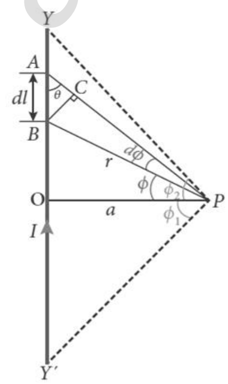
The horizontal component of Earth's magnetic field

$$\boxed{B_H = \frac{\mu_o NI}{2R} \frac{1}{\tan\theta}}$$ *in tesla*

PRECAUTIONS

1. *All the nearby magnets and magnetic materials are kept away from the instrument.*
2. *Using spirit level, the levelling screws at the base are adjusted so that the small magnetic needle is exactly horizontal and also coil is exactly vertical .*
3. *The coil remains in magnetic meridian.*
4. *The compass box is rotated such that the point reads 0 ° -0 °*

4. MAGNETIC FIELD DUE TO LONG STRAIGHT CONDUCTOR CARRYING CURRENT



Let YY^1 be an infinitely long straight conductor and I be the steady current through the conductor. A point P which is at a distance a from the wire, let us consider a small line element dl.

According to Biot- savart law, The magnetic field at point 'P' due to the element is ,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\overrightarrow{dl}\sin\theta}{r^2}\,\hat{n}$$

$\hat{n} \rightarrow$ unit vector , θ is the angle between Idl and line joining dl and the point P.

Let 'r' be the distance between line element at A to the point P.

To apply trigonometry, draw a perpendicular AC to the line BP

$$\Delta ABC, \sin\theta = \frac{AC}{AB}$$

$$AC = AB\sin\theta$$

$$AB = dl \Rightarrow AC = dl\sin\theta$$

Let $d\emptyset$ be the angle subtended between AP and BP

$$< APB = < BPC = d\emptyset$$

$$\Delta APC;\ \sin[d\emptyset] = \frac{AC}{AP}$$

$d\emptyset$ is very small, $\sin[d\emptyset] \approx d\emptyset$

$$AP = r \Rightarrow AC = rd\emptyset$$

$$AC = dl\sin\theta = rd\emptyset$$

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi r^2}\ [rd\emptyset]\,\hat{n}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{Id\emptyset}{r}\,\hat{n}$$

Let $\emptyset$ be the angle between BP and OP

In a ΔOPA, $\cos\emptyset = \frac{OP}{BP} = \frac{a}{r}$

$$r = \frac{a}{\cos\emptyset}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I}{\frac{a}{\cos\emptyset}}\,d\emptyset\,\hat{n}$$

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi a}\cos\theta\ d\emptyset\,\hat{n}$$

The total magnetic field at P due to the conductor YY^l

$$\vec{B} = \int_{\emptyset_1}^{\emptyset_2} \overrightarrow{dB} = \int_{\emptyset_1}^{\emptyset_2} \frac{\mu_0 I}{4\pi a} \cos\emptyset . d\emptyset \, \hat{n}$$

$$= \frac{\mu_0 I}{4\pi a} [sin\emptyset]_{-\emptyset_1}^{\emptyset_2} \, \hat{n}$$

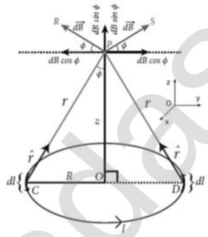
$$\vec{B} = \frac{\mu_0 I}{4\pi a} [\, sin\emptyset_1 + sin\emptyset_2] \, \hat{n}$$

For an Infinitely long conductor $\emptyset_1 = \emptyset_2 = 90°$

$$\vec{B} = \frac{\mu_0 I}{4\pi a} \, x \, 2\hat{n}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi a} \hat{n}}$$

5. *MAGNETIC FIELD PRODUCED ALONG THE AXIS OF THE CURRENT CARRYING CIRCULAR COIL*



Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction .

The magnetic field at a point 'P' on the axis of the circular coil at a distance z from its centre of the coil O is computed by taking two diametrically opposite line element of the coil each of length $\overrightarrow{dl}$ *at c and D . let* $\vec{r}$ *be the vector joining the current element*[$I\overrightarrow{dl}$] *at C to the point P.*

According to Biot- Savart's law,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \frac{I\overrightarrow{dl} \, x \, \hat{r}}{r^2}$$

The magnetic field of $\overrightarrow{dB}$ *is,*

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

θ is angle $I\vec{dl}$ and $\vec{r}$; $\theta = 90°$

The direction of $\overrightarrow{dB}$ is perpendicular to the current element $I\vec{dl}$ and CP. It is therefore along PR perpendicular to CP.

The magnitude of magnetic field at P due to current element at D is same as that for the element at C because of equal distances from the coil. But its direction is along PS.

$\overrightarrow{dB}$ resolved into two components ;

$dB\cos\emptyset$ along Y – direction

$dB\sin\emptyset$ along Z – direction

The Horizontal components cancels out.

The Vertical components along contribute to net magnetic field $\vec{B}$ at the point P.

$$\vec{B} = \int \overrightarrow{dB} = \int dB\sin\emptyset\,\hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}\sin\emptyset\,\hat{k}$$

From ΔOCP,

$$\sin\emptyset = \frac{R}{[R^2+Z^2]^{1/2}} \text{ . and } r^2 = R^2 + Z^2$$

Substituting above equation,

$$\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R}{[R^2+Z^2]^{3/2}}\int_0^{2\pi R} dl\ \hat{k}$$

We integrate the line element from O to $2\pi R$

$$\vec{B} = \frac{\mu_0 NI}{2}\frac{R^2}{[R^2+Z^2]^{3/2}}\ \hat{k}$$

N → Turns in coil

The magnetic field points at the centre of the coil is ,

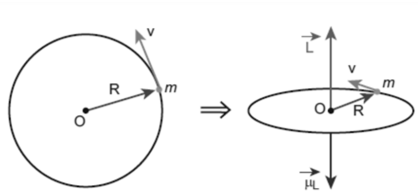
$$\boxed{\vec{B} = \frac{\mu_0 NI}{2R}\ \hat{k}}$$

[Z = 0]

6. MAGNETIC DIPOLE MOMENT OF REVOLVING ELECTRON

An electron undergoes circular motion around the nucleus . The circulating electron in a loop is like current in a circular loop . The magnetic dipole moment due to current carrying circular loop is ,

$$\vec{\mu}_L = I\vec{A}$$



In Magnitude, $\mu_L = IA$

T→ *time period of revolution of an electron,*

The current, $I = -\frac{e}{T}$

e → *Charge of an electron*

R → *radius of the circular Orbit*

V → *Velocity of the electron in the circular Orbit*

$$T = \frac{2\pi R}{V}$$

$$\mu_L = -\frac{2\pi R}{V}\pi R^2 = -\frac{eVR}{2}$$

$A = \pi R^2$ → *Area of the circular loop*

Angular momentum of electron about O.

$$\vec{L} = \vec{R} \times \vec{P}$$

In magnitude , L = RP =mVR

$$\frac{\mu_L}{L} = -\frac{eVR/2}{mVR} = -\frac{e}{2m}$$

$$\vec{\mu}_L = -\frac{e}{2m}\vec{L}$$

The negative sign indicates that the magnetic moment and angular momentum are in *opposite direction.*

In magnitude , $\frac{\mu_L}{L} = \frac{e}{2m} = \frac{1.6\ X\ 10^{-19}}{2\ X\ 9.11\ X\ 10^{-31}}$

$$\frac{\mu_L}{L} = 0.0878\ X\ 10^{12}\ ckg^{-1} = 8.78 \times 10^{10}\ ckg^{-1} \text{ constant.}$$

The ratio $\frac{\mu_L}{L}$ *is a constant known as* *gyro -magnetic ratio*

According to Neil 's Bohr quantisation rule,

$$L = n\hbar = \frac{nh}{2\pi}$$

$h \rightarrow$ *Planck's Constant* $\rightarrow$ 6.63×10^{-34} *Js*

$n \rightarrow$ *orbit number* $[\ n = 1, 2, 3,..\ ]$

$$\mu_L = \frac{e}{2m} L = n\frac{eh}{4\pi m}$$

$$\mu_L = n \times \frac{[\ 1.6\ X\ 10^{-19}]h}{4\pi m}\ Am^2$$

$$\mu_L = n \times \frac{[\ 1.6\ X\ 10^{-19}][6.63\ x\ 10^{-34}]}{4\ X\ 3.14\ X\ [\ 9.11\ x\ 10^{-31}\ ]}$$

$$\boxed{\mu_L = n \times 9.27\ X\ 10^{-24}\ Am^2}$$

The minimum value of magnetic moment, n = 1

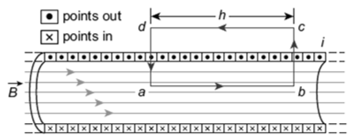
$$\mu_L = 9.27\ X\ 10^{-24}\ Am^2 = 9.27\ X\ 10^{-24}\ JT^{-1}$$

$$[\mu_L]_{min} = \mu_B$$

$\mu_B = \frac{eh}{4\pi m} = 9.27\ X\ 10^{-24}\ Am^2$ *is called* *Bohr magnetron* *which is used to measure atomic magnetic moments.*

7. MAGNETIC FIELD DUE TO A LONG CURRENT CARRYING SOLENOID

Consider a solenoid of length L having N turns. The diameter of the solenoid is assumed to be much smaller when compared to its length and that is wound very closely.



Magnetic field of a solenoid

To calculate the magnetic field at any point inside the solenoid ,using ampere's circuital law , consider a rectangle look abcd

From Ampere's circuital law ,

$$\oint_C \vec{B}\, dl = \mu_o I_{enclosed}$$

LHS

$$\oint_C \vec{B}\, dl = \int_a^b \vec{B}\,\vec{dl} + \int_b^c \vec{B}\,\vec{dl} + \int_c^d \vec{B}\,\vec{dl} + \int_d^a \vec{B}\,\vec{dl}$$

Since the elemental length along bc and da are perpendicular to the magnetic field which is along the axis of the solenoid.

$$\int_b^c \vec{B}\,\vec{dl} = \int_b^c B.dl\cos 90° = 0;$$

$$lll^{ly} \quad \int_d^a \vec{B}\,\vec{dl} = 0;$$

The magnetic field outside of the solenoid is zero,

$$\int_c^d \vec{B}\,\vec{dl} = 0$$

For the path along ab ,

$$\int_a^b \vec{B}\,\vec{dl} = B\int_a^b dl\cos 0° = B\int_a^b dl$$

$$B\int_a^b dl = BL$$

Let I be the current passing through the solenoid of N turns ,

$$\int_a^b \vec{B}\,\vec{dl} = BL = \mu_0 NI$$

$$B = \mu_0 \frac{NI}{L}$$

The number of turns per unit length is,

$$\frac{N}{L} = n$$

$$B = \mu_0 \frac{nLI}{L} = \mu_0 nI$$

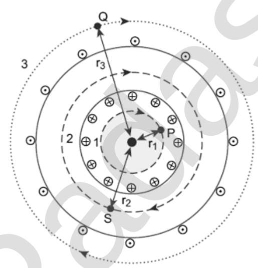
n & $\mu_0 \longrightarrow$ *constant*

For a fixed current I , the magnetic field inside the solenoid is also a constant .

8. TOROID

A Solenoid is bent in such a way its ends are joined together to form a closed ring shape , is called a toroid.

The magnetic field has constant magnitude inside the toroid whereas in the interior region [At Point P]and exterior region [At Point Q] the magnetic field is zero .



a) OPEN SPACE INTERIOR TO THE TOROID

Let us calculate the magnetic field B_P at point P . We construct an amperian loop1 of radius r_1 around the point P we take a circular loop so that the length of the look is its circumference .

$$L_1 = 2\pi r_1$$

Ampere's circuital law for the loop 1 is ,

$$\oint_{\text{Loop 1}} \vec{B}_P . \vec{dl} = \mu_o I_{enclosed}$$

Loop 1 encloses no current, $I_{enclosed} = 0$

$$\oint_{\text{Loop 1}} \vec{B}_P . \overrightarrow{dl} = 0$$

This is possible only if the magnetic field at a point P vanishes ,

$$\vec{B}_P = 0$$

b) OPEN SPACE EXTERIOR TO THE TOROID

Let us calculate the magnetic field B_Q at a point Q. We construct an amperian loop 3 of radius of r_3 around the point Q the length of the loop is

$$L_3 = 2\pi r_3$$

Ampere's circuital law for the loop 3 is ,

$$\oint_{\text{Loop 3}} \vec{B}_Q . \overrightarrow{dl} = \mu_o I_{enclosed}$$

Each turn of the loop , current coming out of the plane of paper is cancelled by the current going into the plane of the paper

$$I_{enclosed} = 0$$

$$\oint_{\text{Loop 3}} \vec{B}_Q . \overrightarrow{dl} = 0$$

The magnetic field at a point Q vanishes ,

$$\vec{B}_Q = 0$$

c) INSIDE THE TOROID

Let us calculate the magnetic field B_S at a point s by constructing an amperian loop 2 of radius r_2 around the point S. The length of the loop is

$$L_2 = 2\pi r_2$$

Ampere's circuital law for the loop 2 is ,

$$\oint_{\text{Loop 2}} \vec{B}_S . \overrightarrow{dl} = \mu_o I_{enclosed}$$

Let I be the current passing through the toroid and N is no of turns

$$I_{enclosed} = NI$$

$$\underset{\text{Loop 2}}{\oint} \vec{B}_s . \vec{dl} = \underset{\text{Loop 2}}{\oint} B_s . dl \cos\theta = B_s 2\pi r_2$$

$$B_s 2\pi r_2 = \mu_o \, NI$$

$$B_s = \mu_o \frac{NI}{2\pi r_2}$$

No of turns per unit length $n = \frac{N}{2\pi r_2}$

$$\boxed{B_s = \mu_o \, n\mathrm{I}}$$

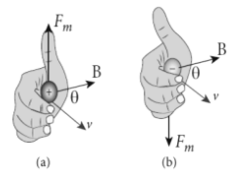
9. FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

When an electric charge q is moving with velocity $\vec{V}$ *in the magnetic field* $\vec{B}$ *it experiences a force called magnetic Lorentz Force* $\vec{F}_m$

$$\vec{F}_m = q\,[\,\vec{V} \times \vec{B}\,]$$

In magnetic,

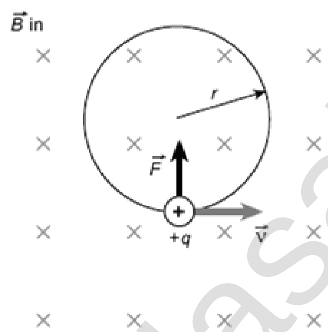
- ii. $\vec{F}_m$ *is directly proportional to the magnetic field* $\vec{B}$
- iii. $\vec{F}_m$ *is directly proportional to the velocity* $\vec{V}$ *of the moving charge*
- iv. $\vec{F}_m$ *is directly proportional to the sine of the angle between the velocity and magnetic field*
- v. $\vec{F}_m$ *is directly proportional to the magnitude of the charge q*
- vi. *The direction of* $\vec{F}_m$ *is always perpendicular to* $\vec{V}$ *and* $\vec{B}$ *as* $\vec{F}_m$ *is a cross product of* $\vec{V}$ *and* $\vec{B}$



vii. *The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge*

viii. *If the velocity $\vec{V}$ of charge q is along magnetic field $\vec{B}$ then $\vec{F}_m$ is zero*

10. MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD



Consider a charged particles of charge q having mass m entering into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{V}$ such that velocity is perpendicular to the magnetic field. As soon as the particle enters into the field. Lorentz Force acts on it in a direction perpendicular to both magnetic field and velocity $\vec{V}$

As a result, the charged particle moves into a circular orbit. The Lorentz Force on the charged particle is given by ,

$$\vec{F} = q\,[\,\vec{V} \times \vec{B}\,]$$

Since , Lorentz Force alone acts on the particle, the magnitude of the net force on the particle is

$$\sum_i F_i = F_m = qvB$$

This Lorentz Force acts as centripetal force for the particle causing it to execute circular motion. Therefore ,

$$qvB = \frac{mv^2}{r}$$

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{P}{qB}$$

Linear momentum of a particle , P= mV

Let T be the time taken by the particle to finish one complete circular motion, then

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{qB}$$

T is called the cyclotron period. The reciprocal of time period is the frequency f,

$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi r}$$

In terms of angular frequency,

$$w = 2\pi f = \frac{q}{m}B$$

These are called as cyclotron frequency or Gyro frequency.

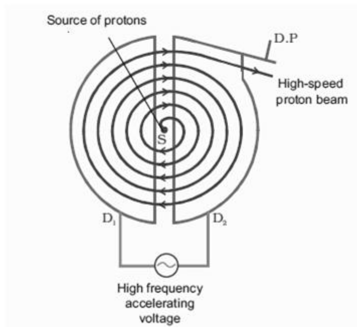
The component of velocity of the particle perpendicular to the field keeps charging due to Lorentz Force, it is a helical around the field lines.

11. CYCLOTRON

To accelerate the charged particles to gain large kinetic energy. It is called as high energy accelerator .

PRINCIPLE

When a charged particle moves perpendicular to the magnetic field, it experiences magnetic Lorentz Force .



CONSTRUCTION

The particles are allowed to move in between two semicircular metal containers called Dees .Dees are interest in an evacuated chamber and it is kept in your region with uniform magnetic field controlled by an Electromagnet. The direction of magnetic field is normal to the plane of the dees. The two dees are kept separated with a gap and the source s is placed at the centre in the gap between the dees. Dees are connected to high frequency alternating potential difference.

WORKING

The ion ejected from source S is positively charged. It is as directed towards a Dee -I which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees ,the ion moves in a circular path. After one semicircular path inside Dee –I, the ion searches that gap between Dees . At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee -2 with greater velocity. For this circular motion, the centripetal force on charged particle q is provided by Lorentz Force.

$$\frac{mv^2}{r} = Bqv$$

$$r = \frac{m}{qB} v$$

$$r \propto v$$

The increases in velocity increases the radius of the circular path. This process continues and hence the particle moves in spiral path of increasing radius . Increase it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T.

The important condition the frequency f at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator. This is called resonance condition.

$$f_{osc} = \frac{qB}{2\pi m}$$

The time period of oscillation is,

$$T = \frac{2\pi m}{qB}$$

The kinetic energy of a charged particle is,

$$KE = \frac{1}{2} mv^2$$

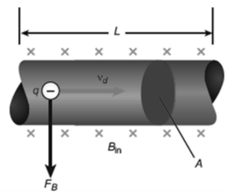
$$= \frac{q^2 m^2 r^2}{2m}$$

LIMITATIONS OF CYCLOTRON

a) *The speed of the ion is limited.*
b) *Electron cannot be accelerated.*
c) *Uncharged particles cannot accelerated.*

12. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN THE MAGNETIC FIELD

When a current carrying conductor is placed in a magnetic field ,the force experienced by the conductor is equal to the sum of Lorentz forces on the individual charge carriers in the conductor. Consider a small segment of conductor of length dl with cross sectional area A and current I.



The free electrons drift opposite to the direction of current. So the relation between current and drift velocity.

$$I = nAeVd$$

If the conductor is kept in a magnetic field $\vec{B}$, then average force experienced by the electron in the conductor is

$$\vec{f} = - e[\vec{V}_d \, X \, \vec{B}]$$

n ➔ Number of free electrons present in unit volume

$$n = \frac{N}{V}$$

N ➔ Number of free electrons in the small element of volume

$$V = A.dl$$

Hence Lorentz Force on the elementary section of length dl is the product of the number of the electrons and the force acting on each electron .

$$\vec{F} = - enAdl[\vec{V}_d \, X \, \vec{B}]$$

The current element in the conductor is $I\overrightarrow{dl} = -enA\vec{V}_d dl$ *.Therefore, the force on the smaller elemental section of the current carrying conductor is,*

$$\overrightarrow{dF} = [\, I\overrightarrow{dl} \times \vec{B}\,]$$

The force on a conductor carrying current of length 'ℓ' placed in a uniform magnetic field is

$$\vec{F}_{total} = [\, I\vec{l} \times \vec{B}\,]$$

In magnitude, $F_{total} = BI\ell \sin\theta$

a) *The conductor is placed along the direction of the magnetic field,* $\theta=0°$

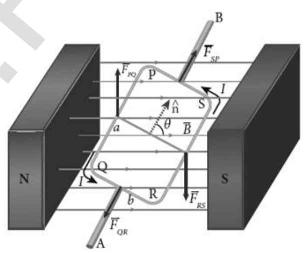
$$F = 0$$

b) *the conductor is placed perpendicular to the magnetic field,* $\theta = 90°$

$$F_{total} = BI\ell \quad [\text{ Maximum }]$$

13. TORQUE ON A CURRENT LOOP PLACED IN A MAGNETIC FIELD

Consider a rectangular loop PQRS carrying current I is placed in a uniform magnetic field $\vec{B}$ *. Let a and b be the length and breadth of a rectangular loop respectively. The unit vector normal* $\hat{n}$ *to the plane of the loop makes an angle* θ *with the magnetic field.*



The magnitude of the magnetic force acting on the arm PQ and RS is

$$F_{PQ} = I_a B \sin[\tfrac{\pi}{2}] = IaB$$

The direction of the force is upwards in arm PQ and downwards in [using right hand corkscrew rule] in the arm RS.

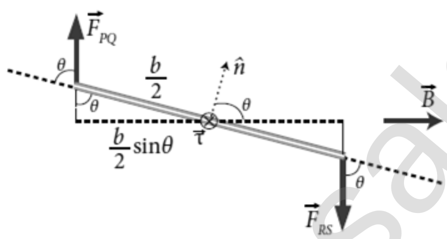
The magnitude of the magnetic force acting on the arm QR is,

$$F_{QR} = I_b B \sin\left[\frac{\pi}{2} - \theta\right] = I\,bB\cos\theta$$

The force acting on the arm SP is,

$$F_{SP} = I_b B \sin\left[\frac{\pi}{2} + \theta\right] = I\,bB\cos\theta$$

Since the forces F_{QR} and F_{SP} are equal ,opposite and collinear, the cancer each other. But the forces F_{PQ} and F_{RS} which are equal in magnitude and opposite in direction of not acting along same straight line. The four constituted a couple which exerts a torque on the loop .



The magnitude of torque acting on the PQ about AB is $\tau_{PQ} = \left[\frac{b}{2}\sin\theta\right] I_{aB}$ *and it points in the direction of AB.*

The magnitude of torque acting on the arm RS about AB is $\tau_{RS} = \left[\frac{b}{2}\sin\theta\right] I_{aB}$ *and it points also in the same direction AB.*

The total torque acting on the entire loop about an axis AB is

$$\tau = \left[\frac{b}{2}\sin\theta\right] F_{PQ} + \left[\frac{b}{2}\sin\theta\right] F_{RS}$$

$$= I_a\,[b\sin\theta\,]\,B$$

$= I\,AB\sin\theta$ *along the direction AB.*

$$\vec{\tau} = [\,I\,\vec{A}\,]\; X\;\vec{B}$$

$$\vec{\tau} = [\vec{P}_m]\; X\;\vec{B} \qquad [\vec{P}_m = I\vec{A}]$$

The torque is to rotate the loop so as to align its normal vector with direction *of the magnetic field.*

$\tau = NIAB \sin\theta$ *[N –turns is the rectangular loop]*

<u>Special cases</u>

a) $\theta = 90°$ *,The plane of the loop is parallel to the* $\vec{B}$*, torque is maximum*

$$\tau_{max} = IAB$$

b) $\theta = 90° / 180°$ *,The plane of the loop is perpendicular to the* $\vec{B}$*, torque is zero*

$$\tau = 0$$

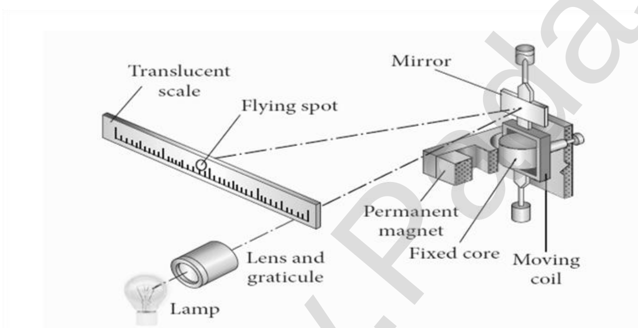
14. MOVING COIL GALVANOMETER

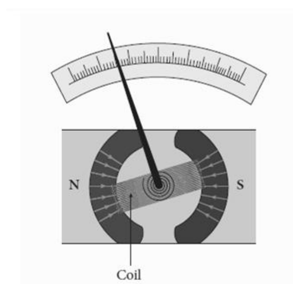
Used to detect the flow of current in an electric circuit .

PRINCIPLE

When a current carrying loop is placed in a uniform magnetic field, it experiences a torque.

CONSTRUCTION





A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns would over a light metallic frame. The cylindrical soft- iron core placed is symmetrical inside the coil. The rectangular coil is suspended freely between two pole pieces of a horseshoe magnet.

The upper end of the rectangular coil is attracted to one end of the points of phosphor bronze and the lower end of the coil is connected to hair spring which also made up of phosphor bronze. In a fine suspension strip, a small plane mirror is attached in order to *measure the deflection of the coil with the help of the lamp and scale arrangement. The other end of the mirror is connected to the Torsion head. In order to pass electric current through the Galvanometer the suspension strip and the spring ' s ' are connected to terminals.*

WORKING

Consider a single turn of the rectangular coil PQRS when length is ℓ and breadth b.

PQ=RS = l and QR =SP= b.

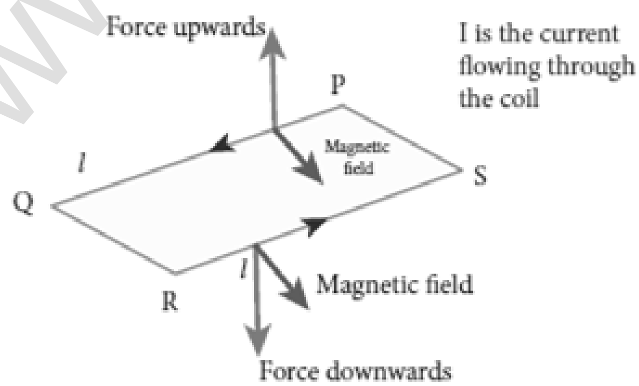
Let I be the electric current flowing through the rectangular coil PQRS. The horse shoe magnet have a hemispherical magnetic poles which produces a radial magnetic field ,the sides QR and SP are parallel to the magnetic field B and experiences noforce. The sides PQ and Rs are always perpendicular to the magnetic field and experience equal forces in opposite direction .Due to this ,torque is produced.

For single turn, the deflecting torque is

$$\tau = bF = bBI\ell = [\ell b] BI = ABI \qquad [A = \ell b]$$

For coil with N turns ,

$$\tau = NABI$$



Due to this deflecting torque, the coil gets twisted under restoring torque is *developed.Hence the moment of the restoring torque is proportional to the amount of Twist θ,*

$$\tau = k\theta$$

$k \rightarrow$ Restoring couple per unit twist

At equilibrium ,

The deflection couple= the storing couple.

$$NABI = k\theta$$

$$I = \frac{k}{NAB}\theta$$

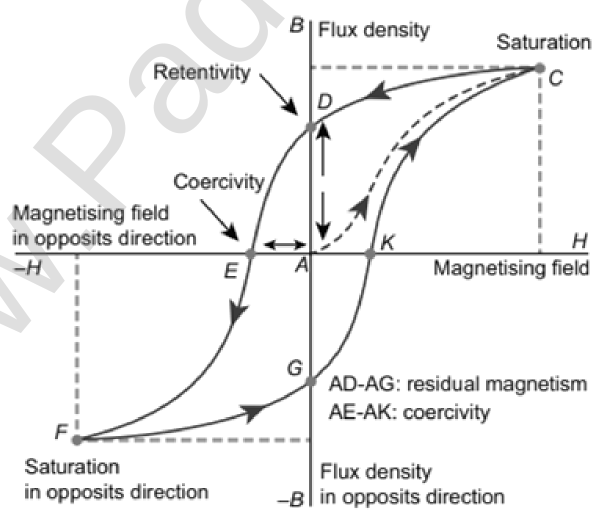
$$I = G\,\theta$$

$G = \frac{k}{NAB} \rightarrow$ *Galvanometer constant [or] current reducing factor of the galvanometer.*

15. HYSTERESIS OF A FERROMAGNETIC MATERIAL

The phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.

Explanation:



- ✓ *A ferromagnetic material (Iron) is magnetised slowly by a magnetising field $\vec{H}$.*
- ✓ *The magnetic induction $\vec{B}$ of the material increases from point A with the magnitude of the magnetising field and then attains a saturation level.*

SATURATION MAGNETIZATION

Saturation magnetization is defined as the maximum point up to which the material can be magnetised by applying the magnetising field.

- ✓ *If the magnetising field is now reduced, the magnetic induction also decreases but does not retrace the original path CA. It takes different path CD. When the magnetising field is zero, the magnetic induction is not zero and it has positive value.*

RETENTIVITY [AD]

It is defined as the ability of the materials to retain the magnetism in them even after the magnetising field disappears.

- ✓ *The magnetising field is gradually increased in the reverse direction. Now the magnetic induction decreases along DE and becomes zero at E.*

COERCITIVITY[AE]

The magnitude of the reverse magnetising field for which the residual magnetism of the material vanishes is called its coercivity.

- ✓ *Further increase of ρ H in the reverse direction causes the magnetic induction to increase along EF until it reaches saturation at F in the reverse direction. If magnetising field is decreased and then increased with direction reversed, the magnetic induction traces the path FGKC.*
- ✓ *This closed curve ACDEFGKC is called hysteresis loop and it corresponds to one cycle of magnetization*

HYSTERESIS LOSS

It is found that the energy lost (or dissipated) per unit volume of the material when it is carried through one cycle of magnetisation is equal to the area of the hysteresis loop.

16. DIFFERENCES BETWEEN SOFT AND HARD FERROMAGNETIC MATERIALS

S.No.	Properties	Soft ferromagnetic materials	Hard ferromagnetic materials
1	When external field is removed	Magnetisation disappears	Magnetisation persists
2	Area of the loop	Small	Large
3	Retentivity	Low	High
4	Coercivity	Low	High
5	Susceptibility and magnetic permeability	High	Low
6	Hysteresis loss	Less	More
7	Uses	Solenoid core, transformer core and electromagnets	Permanent magnets
8	Examples	Soft iron, Mumetal, Stalloy etc.	Carbon steel, Alnico, Lodestone etc.

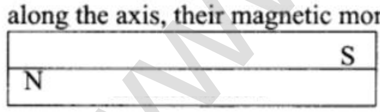
UNSOLVED PROBLEMS

1. ***A bar magnet having a magnetic moment $\vec{M}$ is cut into four pieces i.e., first cut into two pieces along the axis of the magnet and each piece is further cut along the axis into two pieces. Compute the magnetic moment of each piece.***

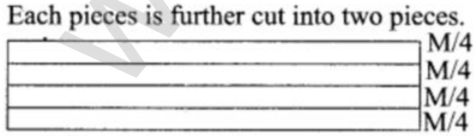
Solution

Consider a bar magnet of magnetic moment $\vec{M}$. When a bar magnet first cut in two pieces

along the axis, their magnetic moment is $\frac{\overline{M}}{2}$



Each pieces is further cut into two pieces.

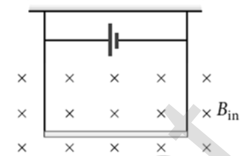


Their magnetic moment of each pieces $\frac{\vec{M}}{4}$

Their magnetic moment of each pieces $\vec{M}_{new}$ $\frac{1}{4}$ $\vec{M}$

2. *A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume g = 10 m s^{-2}*

Solution



Downward force, F = mg

Linear mass density, $\frac{m}{l}$ = 0.2 gm^{-1}

$\frac{m}{l}$ = 0.2 x 10^{-3} kg m^{-1}

m = (0.2 x 10^{-3} x l) kg m^{-1}

F = (0.2 x 10^{-3} x l x 10) N

Upward magnetic force acting on the wire

F = BIl

0.2 x 10^{-3} x l x 10 = 1 x I x l

I = 2 x 10^{-3}

I = 2 m A

3. *A circular coil with cross-sectional area 0.1 cm2 is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate (a) total torque on the coil (b) total force on the coil (c) average force on each electron in the coil due to the magnetic field. (The free electron density for the material of the wire is 10^{28} m^{-3}).*

Solution

Cross sectional area of coil, A = 0.1 cm^2

A = 0.1 x 10^{-4}m^2

Uniform magnetic field of strength, B = 0.2T

Current passing in the coil, I = 3A

Angle between the magnetic field and normal to the coil, θ = 0°

(a) Total torque on the coil,

τ = ABI sin θ = 0.1 x 10^{-4} x 0.2 x 3 sin0° sin0° = 0

τ = 0

(b) Total force on the coil

F = BIl sin θ = 0.2 x 3 x l x sin 0°

F = 0

(c) Average force:

F = qV_dB

drift velocity, $V_d = \frac{1}{neA}$

[∵ q = e]

F = $e\left(\frac{1}{neA}\right)B$

[∵n = 10^{28} m^{-3}

$\frac{IB}{nA} = \frac{3\times0.2}{10^{28}\times0.1\times10^{-4}}$ = 6 x 10^{-24}

F_{av} = 0.6 x 10^{-23} N

4. ***A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T.If the bar magnet is oriented at an angle 30° with the external field experiences a torque of 0.2 Nm. Calculate: (i) the magnetic moment of the magnet (ii) the work done by the magnetic field in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.***

Solution

Uniform magnetic field strength B = 0.8T

Bar magnet orient an angle with magnetic field θ = 30°

Torque τ = 0.2 Nm

(i) Magnetic moment of the magnet,

Torque τ = P_m B Sin θ

∴ Magnetic moment, $P_m = \frac{\tau}{B\sin\theta} = \frac{0.2}{0.8\times Sin30^{\circ}} = \frac{0.2}{0.4}$

P_m = 0.5 Am^2

(ii) Work done by external torque is stored in the magnet as potential energy.

W = U = – P_m B Sin θ

Here, applied force acting on magnet its moving from most stable θ′ to most unstable θ.

θ′= 0° and θ = 180°

So, workdone W = U = – P_m B (Cosθ – Cosθ′)

= – P_m B (Cos 180° – Cos 0°) = – 0.5 x 0.8 ((-1) – 1) = – 0.4 (-2)

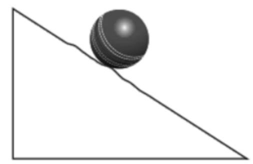
W = U = 0.8

W= 0.8 J

5. ***A non - conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform***

magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current I required to rest the sphere in equilibrium.

<u>Solution</u>



At equilibrium

f_s R-p_m B sin θ = 0

mgR = NBAI

$I = \frac{mgR}{NBA} = \frac{mgR}{NB\pi R^2}$

$I = \frac{mg}{\pi RNB}$

Mass of the sphere, m= 100 g = 100 x 10^{-3} kg

Radius of the sphere R = 20 cm = 20 x 10^{-2} m

Number of turns n = 5

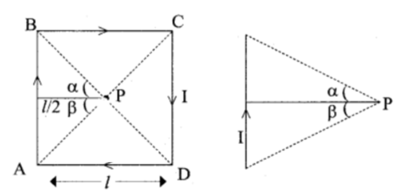
Uniform magnetic field B = 0.5 T

$I = \frac{100\times10^{-3}\times10}{\pi\times20\times10^{-2}\times5\times0.5} = \frac{1000\times10^{-3}}{\pi\times50\times10^{-2}} = \frac{20\times10^{-1}}{\pi}$

$I = \frac{2}{\pi}$ A.

6. ***Calculate the magnetic field at the centre of a square loop which carries a current of 1.5 A, length of each side being 50 cm.***

<u>Solution</u>



Current through the square loop, I = 1.5A

Length of each loop, l = 50cm 50 x 10^{-2} m

According to Biot-Savart Law.

Magnetic field due to a current carrying straight wire

$$B = \frac{\mu_0 I}{4\pi a}(\sin\alpha + \sin\beta) = \frac{4\pi\times10^{-7}\times1.5}{4\pi\times\left(\frac{l}{2}\right)}(\sin 45^o + \sin 45^o)$$

$$= \frac{2\times1.5\times10^{-7}}{l}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \frac{2\times1.5\times10^{-7}}{50\times10^{-2}}\left(\frac{2}{\sqrt{2}}\right)$$

B =0.084866 x 10^{-5} T

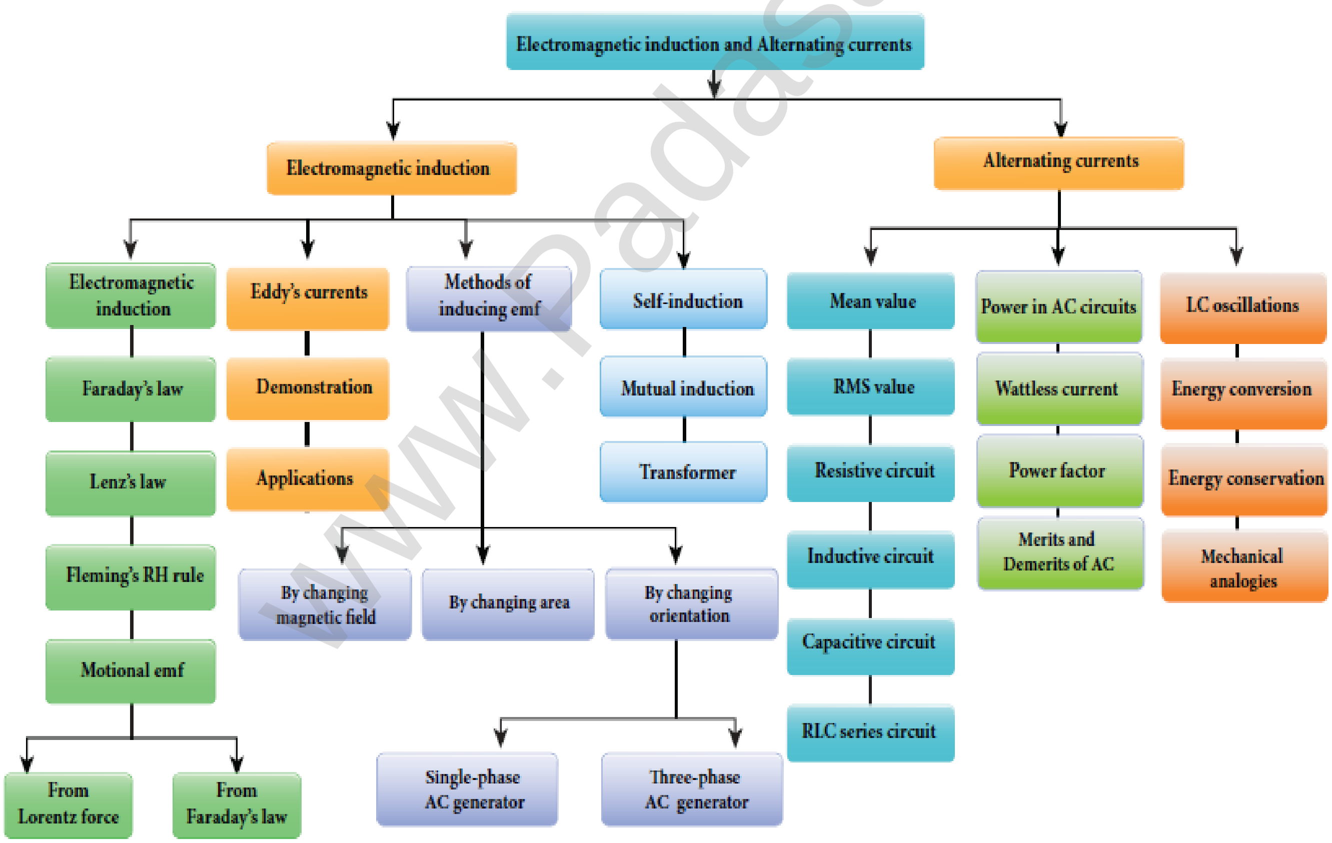
Magnetic field at a point p? of centre of current carrying square ioop

B' =4 sides x B

= 4 x 0.08487 x 10^{-5} = 0.33948 x 10^{-5}

UNIT 4

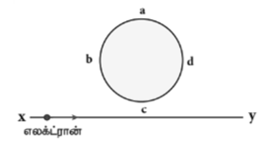
ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT



PART – A (1 MARKS)

1. An electron moves on a straight line path XY as shown in the figure. The coil abcd is adjacent to the path of the electron. What will be the direction of current, if any, induced in the coil?

   a)The current will reverse its direction as the electron goes past the coil
   b) No current will be induced c) abcd d) adcb



Solution :-

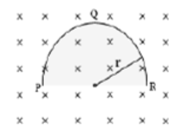
1. When electron moves towards the loop flux increases , induced current flows in anticlockwise direction(abcd)
2. When electron moves away flux decrease, induced current flows clock wise direction.(dcba).

ANSWER :

a)The current will reverse its direction as the electron goes past the coil

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.The potential difference developed across the ring when its speed *v*, is

   a) Zero b) $\frac{BV\pi r^2}{2}$ and P is at higher potential
   c) $\pi r B v$ and R is at higher potential d) $2rBv$ and R is at higher potential



Solution :-

ε =B l and R are higher potential

ANSWER :

d)$2rBv$andRisathigherpotential

3. The flux linked with a coil at any instant t is given by $\Phi_B = 10t^2 - 50t + 250t$. The induced emf at t = 3s is

   (a) -190 *V* (b) -10 *V* (c) 10 *V* (d) 190 *V*

Solution :-

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\varepsilon = -\frac{d}{dt}\,(10t^2-50t+250) = -[20t-50]$$

$t = 3s$; ε =-[(20x3)-50]=

=-60+50 = -10V

ANSWER : (b) -10 *V*

4. When the current changes from +2A to -2A in 0.05 s, an emf of 8 V is induced in a coil. The co-efficient of selfinductionof the coil is

   (a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H

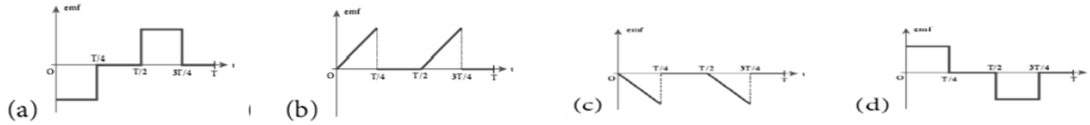
Solution :-

$$\varepsilon = -L\frac{dI}{dt}$$

$$L = -\frac{\varepsilon}{\frac{dI}{dt}} = \frac{-8}{\frac{-4}{0.05}} = \frac{0.4}{4} = 0.1\text{H}$$

ANSWER : (d) 0.1 H

5. The current i flowing in a coil varies with time as shown in the figure. The variation of induced emf with time would be

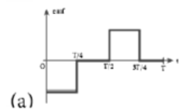


Solution :-

$\varepsilon \propto -\frac{dI}{dt}$

- 0 to T/4 –flux changes current induces opposite side
- T/4 to T/2 –No change in flux,so no current induced
- T/2 to 3T/4 – Flux change current induces positive region

ANSWER :



6. A circular coil with a cross-sectional area of 4 cm2 has 10 turns. It is placed at the centre of a long solenoid that has 15 turns/cm and a cross-sectional area of 10 cm^2. The axis of the coil coincides with the axis of the solenoid. What is their mutual inductance?

   (a) 7.54 μH (b) 8.54 μH (c) 9.54 μH (d) 10.54 μH

Solution :-

$M = \frac{\mu_0 n_1 n_2 A_2 l}{l} = \mu_0 N_1 N_2 A_2$

$= 4\pi \times 10^{-7} \times 15 \times 10^2 \times 10 \times 4 \times 10^{-4}$

$= 7.54 \times 10^{-6} H = 7.54 \mu H$

ANSWER : (a) 7.54 μH

7. In a transformer, the number of turns in the primary and the secondary are 410 and 1230 respectively. If the current in primary is 6A, then that in the secondary coil is

   (a) 2 A (b) 18 A (c) 12 A (d) 1 A

Solution :-

$\frac{i_p}{i_s} = \frac{N_s}{N_p} => i_s = \frac{N_p}{N_s} i_p = \frac{410}{1230} x6 = 2A$

ANSWER : (a) 2 A

8. A step-down transformer reduces the supply voltage from 220 V to 11 V and increase the current from 6 A to 100 A. Then its efficiency is

   (a) 1.2 (b) 0.83 (c) 0.12 (d) 0.9

Solution :-

$\eta = \frac{E_S I_S}{E_P I_P} = \frac{11X100}{220X6} = 0.83$

ANSWER : (b) 0.83

9. In an electrical circuit, R, L, C and AC voltage source are all connected in series. When L is removed from the circuit, the phase difference between the voltage and current in the circuit is $\pi/3$. Instead, if C is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is

(a) $1/2$ (b) $1/\sqrt{2}$ c)1 (d) $\sqrt{3}/2$

Solution :-

Phase by removing the inductor = Phase by removing the capacitor

$X_L = X_C$;

power factor $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

$Z = R$

ANSWER : c)1

10. In a series RL circuit, the resistance and inductive reactance are the same. Then the phase difference between the voltage and current in the circuit is

(a) $\pi/4$ (b) $\pi/2$ c) $\pi/6$ (d) 0

Solution :-

$R = X_L \Rightarrow \tan\phi = \frac{X_L}{R} = 1$

$\Phi = \tan^{-1}(1) = 45^o = \frac{\pi}{4}$

ANSWER : (a)$\frac{\pi}{4}$

11. In a series resonant RLC circuit, the voltage across 100 Ω resistor is 40 V. The resonant frequency ω is 250 rad/s. If the value of C is 4 μF, then the voltage across L is

(a) 600 V (b) 4000 V (c) 400V (d) 1 V

Solution :-

At resonance $X_L = X_C$

$X_L = L\omega_r = \frac{1}{C\omega_r} = \frac{1}{4X10^{-6}X250}$

$X_L = 10^3\Omega$

$I = \frac{V}{R} = \frac{40}{100} = 0.4A$

Voltage across $L = IX_L$

$= 0.4X10^3 = 400V$

ANSWER : (c) 400V

12. An inductor 20 mH, a capacitor 50 μF and a resistor 40Ω are connected in series across a source of emf v = 10 sin 340 t. The power loss in AC circuit is

(a) 0.76 W (b) 0.89 W (c) 0.46 W (d) 0.67 W

Solution :-

$X_L = L\omega = 20X10^{-3}X340 = 6.8\Omega$

$V_{rms} = \frac{V_0}{\sqrt{2}} = 10x0.707 = 7.07$

$X_C = \frac{1}{C\omega} = \frac{1}{50X10^{-6}X340} = 58.8\Omega$

$X_L - X_C = 58.8 - 6.8 = 52\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} =$

$\sqrt{40^2 + 52^2} = \sqrt{1600 + 2704} = 65.6\Omega$

$P_{AV} = I_{Rms}^2 .R = \left[\frac{E_{Rms}}{Z}\right]^2 x\ R = \left[\frac{7.07}{65.6}\right]^2 x\ 40$

$= (0.107)^2 x\ 40 = 0.46W$

ANSWER : (c) 0.46 W

13. The instantaneous values of alternating current and voltage in a circuit are $i = \frac{1}{\sqrt{2}} \sin(100\pi t)$A and $V = \frac{1}{\sqrt{2}} \sin(100\pi t + \pi/3)$V. The average power in watts consumed in the circuit is

(a) $1/4$ (b) $\sqrt{3}/4$ c) $1/2$ (d) $1/8$

Solution :-

$P_{AV} = \frac{I_0 E_0}{2} \cos\phi$

$= \frac{1}{2} X \frac{1}{\sqrt{2}} X \frac{1}{\sqrt{2}} \cos\frac{\pi}{3}$

$= \frac{1}{2} X \frac{1}{2} X \frac{1}{2} = \frac{1}{8}$

ANSWER : (d)$\frac{1}{8}$

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

(a) $Q/2$ (b) $Q/\sqrt{3}$ (c) $Q/\sqrt{2}$ (d)Q

Solution :-

$U_E = \frac{Q^2}{2C}$; $U'_C = \frac{Q^{2\prime}}{2C}$

Energy stored equally

$U'_C = \frac{1}{2} U_C = \frac{1}{2} X \frac{Q^2}{2C}$

$\frac{Q^{2\prime}}{2C} = \frac{1}{2} X \frac{Q^2}{2C}$ $\Rightarrow Q^{2\prime} = \frac{Q^2}{2}$

$Q' = \frac{Q}{\sqrt{2}}$

ANSWER : c) $\frac{Q}{\sqrt{2}}$

15. $\frac{20}{\pi^2}$H inductor is connected to a capacitor of capacitance C. The value of C in order to impart maximum power at 50 Hz is

(a) 50 μF (b) 0.5 μF (c) 500 μF (d) 5 μF

Solution :-

Maximum power,

At resonance $X_L = X_C$

$L\omega_r = \frac{1}{C\omega_r}$

$C = \frac{1}{L\omega_r^2} = \frac{\pi^2}{20X4\pi^2X50^2} = \frac{1}{20X4X2500} = \frac{1}{200000}$

$= 0.5 \times 10^{-5}$F $= 5\mu$F

ANSWER : (d) 5 μF

PART – B (2 MARKS)

1. ***MAGNETIC FLUX [$\emptyset_B$]***

The number of magnetic field lines passing through that area normally.

$$\emptyset_B = \int_A \vec{B}.\vec{dA} = BA\cos\theta$$

SI unit => Tm^2 [or] weber

2. ***ELECTROMAGNETIC INDUCTION***

Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This current is called an induced current and the emf giving rise to such current is called an induced emf. This phenomenon is known as electromagnetic induction.

3. ***FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION***

FIRST LAW

Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit which lasts in the circuit as long as the magnetic flux is changing.

SECOND LAW

The magnetic of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit

$$\xi = \frac{d\phi_B}{dt}$$

$$\xi = N\frac{d\phi_B}{dt} \qquad N => \text{No of turns in a coil}$$

4. FLUX LINKAGE [$N\phi_B$]

The product of number of turns 'N' of the coil and the magnetic flux linking each turn of the coil ϕ_B

5. LENZ'S LAW

The direction of the induced current is such that it always opposes the cause responsible for its production.

$$\xi = -\frac{d[N\phi_B]}{dt}$$

6. FLEMING'S RIGHT HAND RULE

The thumb, index finger and the middle finger of right hand are stretched out in mutually perpendicular direction. If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of their conductors, then the middle finger will indicate the direction of the induced current.

7. UNIT OF INDUCTANCE (or) ONE HENRY OF SELF-INDUCTANCE

One henry if a current changing at the rate of 1 As^{-1} induces an opposing emf of 1 V in it.

$$1\ H = 1wb\ A^{-1} = 1\ VsA^{-1}$$

8. MUTUAL INDUCTION

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as ***mutual induction*** *and the emf induced is called mutually induced emf.*

9. CO-EEFICIENT OF MUTUAL INDUCTION OF THE COIL (or) MUTUAL INDUCTANCE

The opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is $1As^{-1}$.

10. UNIT OF MUTUAL - INDUCTANCE (or) ONE HENRY OF MUTUAL INDUCTANCE

The mutual inductance between two neighbouring coils is one henry if a current changing at the rate of $1A\ s^{-1}$ *in one coil induces an opposing emf of 1V in neighbouring coil.*

11. METHODS OF PRODUCING INDUCED EMF

$$\xi = \frac{d}{dt}[\,BA\ COS\theta\,]$$

1. *By changing the magnetic field B*
2. *By changing the area A of the coil and*
3. *By changing the relative orientation θ of the coil with magnetic field*

12. PRODUCTION OF INDUCED EMF BY CHANGING THE MAGNETIC FIELD

The change in flux is brought about by

(i) relative motion between the circuit and the magnet

(ii) variation in current flowing through the nearby coil

13. POLY-PHASE AC GENERATOTOR

Some AC generators may have more than one coil in the armature core and each coil produces an alternating emf. In these generators, more than one emf is produced. Thus they are called ***poly-phase generators****.*

14. ADVANTAGES OF THREE PHASE GENERATOR

1) *For a given dimension of the generator, it produces higher power output .*
2) *For the same capacity, three-phase alternator is smaller in size when compared to single-phase alternator.*
3) *Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.*

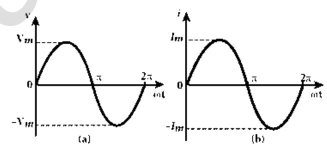
15. ALTERNATING CURRENT

An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.

16. SINUSOIDAL ALTERNATING VOLTAGE

If the wave form of alternating voltage is a sine wave , then it is known as in sinusoidal alternating voltage

$$V = V_m \sin\omega t$$



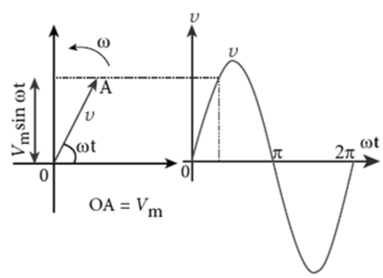
Alternating current $\quad i = I_m \sin\omega t$

17. PHASOR

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a phasor.

18. PHASOR DIAGRAM

The diagram which shows various phasors and their phase relations is called phasor diagram.



19. APPLICATION OF SERIES RLC RESONANT CIRCUIT

1. RLC circuits have many applications like filter circuits, oscillators, voltage multipliers etc.
2. Tuning circuits of radio and TV systems

   The tuning is commonly achieved by varying capacitance of a parallel plate variable capacitor.

20. QUALITY FACTOR (or) Q-FACTOR

The ratio of voltage across L or C at resonance to the applied voltage.

$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ or } C \text{ at resonance}}{\text{Applied voltage}}$$

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

21. POWER FACTOR

i. Power factor = $\cos\phi$ = cosine of the angle of lead or lag

ii. Power factor = $R/2$ = $Resistance/impedance$

iii. Power factor = $P_{av}/V_{RMS}I_{RMS}$ = $\frac{True\ power}{apparent\ power}$

EXAMPLES:

i. Power factor = $\cos 0° = 1$

   pure resistive circuit ϕ between V and i is zero.

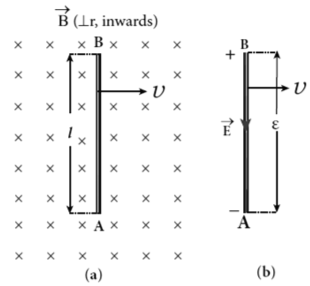
ii. Power factor = $\cos\left[\pm\pi/2\right] = 0$

   purely inductive or capacitive circuit ϕ between V and i is $\pm\pi/2$

iii. Power factor lies between 0 and 1 for a circuit having R, L and C in varying proportions

PART – C (3 MARKS)

1. MOTIONAL EMF FROM LORENTZ FORCE



Consider a straight conductor rod AB of length l in a uniform magnetic field $\vec{B}$ which is directed perpendicular into the plane of the paper .The length of the rod is normal to the magnetic field .Let the rod move with a constant velocity $\vec{V}$ towards rights side .The Lorentz force acts on free electrons in the direction from B to A and is,

$$\vec{F}_B = -e\,[\vec{V} X \vec{B}\,]$$

The action of this Lorentz force is to accumulate the free electrons at the end A and produces a potential difference across the rod which in turn establishes an electric field $\vec{E}$ directed along BA the coulomb force starts along AB.

$$\vec{F}_E = -e\,\vec{E}$$

At equilibrium, the magnetic Lorentz Force $\vec{F}_B$ and the coulomb force $\vec{F}_E$ balance each other and no further accumulation of free electrons at the end A takes place,

$$|\vec{F}_B| = |\vec{F}_E|$$

$$|-e\,[\vec{V} X \vec{B}\,]| = |-e\,\vec{E}|$$

$$VB\sin\theta = E$$

$$VB = E \qquad [\,\theta = 90°]$$

The potential difference between two ends of the rod is ,

$$V = El$$

$$V = VBl$$

The Lorentz force on the free electron is responsible to maintain this potential difference and hence produces an emf.

$$\xi = BlV$$

As this emf is produced due to the movement of the rod, it is often called as motional emf .If the ends A and B are connected by an external circuit of total resistance R,

then current $i = \frac{\xi}{R}$

$i = \frac{Blv}{R}$ *flows in it.*

The direction of the current is found from right -hand thumb rule.

2. EDDY CURRENT

An emf induced in a conductor when the magnetic flux passing through it changes

.

3. SELF – INDUCTION

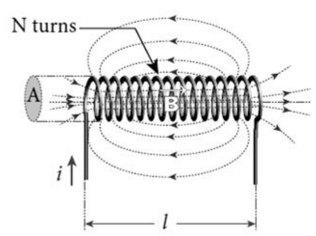
An electric current flowing through a coil will set up a magnetic field around it. Therefore, the magnetic flux of the magnetic field is linked with that coil itself. If this flux is changed by changing the current, an emf is induced in that same coil . This phenomenon is known as ***self-induction.***

4. CO-EEFICIENT OF SELF-INDUCTION OF THE COIL (or) SELF INDUCTANCE

The opposing emf induced in the coil when the rate of change of current through the coil is 1 A s⁻¹.

$$L = -\xi$$

5. SELF INDUCTANCE OF A LONG SOLENOID



Consider a long solenoid of length l and cross-sectional area A. Let n be the number of turns per unit length (or turn density) of the solenoid. When an electric current i is passed through the solenoid, a magnetic field produced inside is almost uniform and is directed along the axis of the solenoid

$$B = \mu_o n i$$

The magnetic flux passing through each turn is

$$\emptyset_B = \int_A \vec{B}\,\vec{dA} = BA\,COS\theta$$

$$= BA \qquad [\theta = 0°]$$

$$\emptyset_B = [\mu_o n i]A$$

The total magnetic flux linked or flux linkage of the solenoid with N turns

$$N\emptyset_B = [nl][\mu_o n i]A$$

$$N\emptyset_B = [\mu_o n^2 A l]\, i$$

$$N\emptyset_B = L\, i \qquad \boxed{L = \mu_o n^2 A \ell}$$

If the solenoid is filled with a dielectric medium of relative permeability μ_r,

$$L = \mu n^2 A l \quad [\text{ or }]$$

$$\boxed{L = \mu_o \mu_r n^2 A \ell}$$

6. ENERGY STORED IN AN INDUCTOR

Whenever a current is established in the circuit, the inductance opposes the growth of the current. In order to establish a current in the circuit, work is done against this opposition by some external agency. This work done is stored as magnetic potential energy.

Let us assume that electrical resistance of the inductor is negligible and inductor effect alone is considered. The induced emf ξ at any instant 't' is

$$\xi = -L\frac{di}{dt}$$

Let dW be work done in moving a charge dq in a time dt against the opposition, then

$$dw = -\xi\, dq$$

$= - \xi\ i\ dt$

$dw = - [- L \frac{di}{dt}]\ i\ dt$

$dw = L\ i\ dt$

The Total work done ,

$$W = \int dw = \int_0^i L\ i\ dt = L\ [\frac{i^2}{2}]_0^i$$

$$W = \frac{1}{2} L i^2$$

This work done is stored as magnetic potential energy

$$\therefore \quad U_B = \frac{1}{2} L i^2$$

The energy density is the energy stored per unit volume of the space,

$u_B = \frac{U_B}{Al}$ *[volume of the solenoid = Al]*

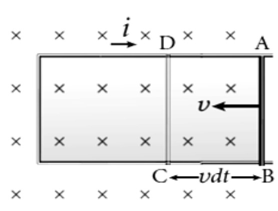
$u_B = \frac{Li^2}{2Al}$

$= \frac{[\mu_o n^2 Al] i^2}{2Al}$ $L = \mu_o n^2 Al$

$= \frac{\mu_o n^2 i^2}{2}$ $B = \mu_o ni$

$$\boxed{u_B = \frac{B^2}{2\mu_o}}$$

7. PRODUCTION OF INDUCED EMF BY CHANGING THE AREA OF THE COIL



Consider a conducting rod of length 'l' moving with a velocity 'V' towards left on a rectangular fixed metallic framework. The whole arrangement is placed in a uniform magnetic field $\vec{B}$ whose magnetic lines are perpendicularly directed into the plane of the paper.

As the rod moves from AB to DC in a time dt, the area enclosed by the loop and hence the magnetic flux through the loop decreases.

The change in magnetic flux in time dt is

$$d\emptyset_B = B \, X \, [dA] \text{ change in area}$$

$$= B \text{ x Area ABCD} \qquad [\text{Area ABCD} = l\,(Vdt\,)\,]$$

$$d\emptyset_B = B\, l\, V\, dt$$

The induced emf is ,

$$\xi = \frac{d\emptyset_B}{dt}$$

$$\xi = B\, l\, V$$

This emf is known as Motional emf.

The direction of induced current is found to be clockwise from Fleming's right hand rule.

Induced current in the loop is,

$$i = \frac{\xi}{R}$$

$$i = \frac{B\, l\, V}{R}$$

ENERGY CONSERVATION

The current-carrying movable rod AB kept in the perpendicular $\vec{B}$ experiences a force $\vec{F}_B$ in the outward direction,

$$\vec{F}_B = i\, \ell\, B$$

A constant force that is equal and opposite to the magnetic force, must be applied.

$$|\vec{F}_{app}| = |\vec{F}_B| = i\, \ell\, B$$

∴ Mechanical work is done by the applied force to move the rod. The rate of doing work or power is

$$P = \vec{F}_{app} \,.\, \vec{V} = F_{app} \,.\, V \cos\theta$$

$$= i\, \ell\, BV \qquad [\,\theta = 0°\,]$$

$$= [\,\frac{BlV}{R}\,]\, \ell\, BV$$

$$P = \frac{B^2 l^2 V^2}{R}$$

Joule heating takes place in the loop ,

$$P = I^2 R$$

$$= \left[\frac{BlV}{R}\right]^2 R$$

$$P = \frac{B^2 l^2 V^2}{R}$$

The mechanical energy needed to move the rod is converted into electrical energy which then appears as thermal energy in the loop.

8. AC GENERATOR

1. *It is an energy conversion device.*
2. *It converts mechanical energy used to rotate the coil or field magnet into electrical energy.*
3. *Alternator produces a large scale electrical power for use in homes and industries.*

PRINCIPLE

Electromagnetic induction.

CONSTRUCTION

Two major parts ,

1. *Stator*
2. *Rotor*

- ***STATOR***

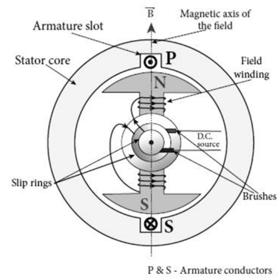
The stationary part which has armature windings mounted in it is called stator.

(i) Stator core

It is made up of iron or steel alloy. It is a hollow cylinder and is laminated to minimize eddy current loss. The slots are cut on inner surface of the core to accommodate armature windings.

(ii) Armature winding

It is the coil, wound on slots provided in the armature core



- ***ROTOR***

It contains magnetic field windings. The magnetic poles are magnetized by DC source. The ends of field windings are connected to a pair of slip rings, attached to a common shaft about which rotor rotates. Slip rings rotate along with rotor. To maintain connection between the DC source and field windings, two brushes are used which continuously slide over the slip rings. This is 2-pole rotor

9. *ENERGY LOSSES IN THE TRANSFORMER*

i) *Core loss or Iron loss*

This loss takes place in transformer core. Hysteresis loss and eddy current loss are known as core loss or Iron loss. When transformer core is magnetized and demagnetized repeatedly by the alternating voltage applied across primary coil, hysteresis takes place due to which some energy is lost in the form of heat. Hysteresis loss is minimized by using steel of high silicon content in making transformer core.

Alternating magnetic flux in the core induces eddy currents in it. Therefore there is energy loss due to the flow of eddy current, called eddy current loss which is minimized by using very thin laminations of transformer core.

ii) *Copper loss*

Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to Joule heating. This energy loss is called copper loss which is minimized by using wires of larger diameter.

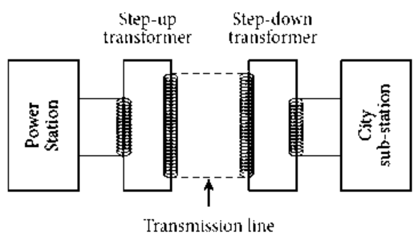
iii) *Flux leakage*

Flux leakage happens when the magnetic lines of primary coil are not completely linked with secondary coil. Energy loss due to this flux leakage is minimized by winding coils one over the other.

10. ADVANTAGES OF AC IN LONG DISTANCE POWER TRANSMISSION

The electric power generated is transmitted over long distances through transmission lines to reach towns or cities where it is actually consumed. This process is called power transmission.

At the transmitting point, the voltage is increased and the corresponding current is decreased by using step-up transformer



Then it is transmitted through transmission lines. This reduced current at high voltage reaches the destination without any appreciable loss. At the receiving point, the voltage is decreased and the current is increased to appropriate values by using step-down transformer and then it is given to consumers. Thus power transmission is done efficiently and economically.

ILLUSTRATION

An electric power of 2 MW is transmitted to a place through transmission lines of total resistance, say R = 40 Ω, at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.

Case (i):

$P = 2\ MW;\ R = 40\ \Omega;\ V = 10\ Kv$

Power , $P=VI$

$$\text{Current } I = \frac{P}{V} = \frac{2 \times 10^{6}}{10 \times 10^{3}} = 200A$$

Power loss = Heat produced = I^2R

$$=[200]^2 \times 40 = 1.6 \times 10^{6}\ W$$

$$\%\text{ of power loss} = \frac{1.6 \times 10^{6}}{2 \times 10^{6}} \times 100\%$$

$$= 0.8 \times 100\% = 80\%$$

Case (ii):

$P = 2\ MW;\ R = 40\ \Omega;\ V = 100\ kV$

$$I = \frac{P}{V} = \frac{2 \, X \, 10^{6}}{100 \, X \, 10^{3}} = 20A$$

Power loss = I^2R

$$= [20]^2 \, X \, 40 = 0.016 \, X \, 10^{6} \, W$$

$$\% \text{ of power loss} = \frac{0.016 \, X \, 10^{6}}{2 \, X \, 10^{6}} \, x \, 100\%$$

$$= 0.008 \, x \, 100\% = 0.8\%$$

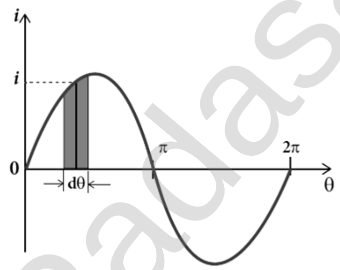
Thus it is clear that when an electric power is transmitted at higher voltage, the power loss is reduced to a large extent.

11. MEAN OR AVERAGE OF AC

The average value of alternating current is defined as he average of all values of current over a postive half -cycle or negative half -cycle.

The insantaneous value of sinusoidal alternating current is

$$i = I_m \sin\omega t \text{ or } i = I_m \sin\theta$$



The sum of all current over a half -cycle is given by area of postive half -cycle

$$I_{av} = \frac{Area \ of \ positive \ half-cycle \ [or \ negative \ half-cycle]}{base \ length \ of \ half-cycle}$$

Consider an elementary strip of thickness $d\theta$ in the positive half-cycle of the current wave . Let i be the mid-ordinate of that strip.

Area of the elementary strip = $i \, d\theta$

Area of positive half-cycle = $\int_0^{\pi} i \, d\theta = \int_0^{\pi} I_m \sin\theta \, d\theta$

$$= I_m \, [-\cos\theta]_0^{\pi}$$

$$= I_m \, [\cos\pi - \cos 0]$$

$$= 2 \, I_m$$

The base length of half-cycle is π

Average value of AC, $I_{av} = \frac{2 \, I_m}{\pi}$

$$I_{av} = 0.637 I_m$$

Hence the average value of AC is 0.637 times the maximum value I_m of the alternating current.

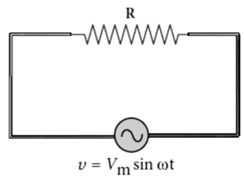
For negative half-cycle, $I_{av} = -0.637 I_m$

'n' current in a half –cycle of AC,

$$I_{av} = \frac{\text{sum of all current over half cycle}}{\text{number of current}}$$

$$I_{av} = \frac{i_1+i_2+\cdots i_n}{n}$$

12. *AC CIRCUIT CONTAINING PURE RESISTOR*



Consider a circuit containing a pure resistor of resistance R connected across an alternating voltage source . The instantaneous value of the alternating voltage

$$V = V_m \sin\omega t$$

An alternating current i flowing in the circuit due to this voltage, develops a potential drop across R and is,

$$V_R = iR$$

Kirchoff 's loop rule , For this resistive circuit,

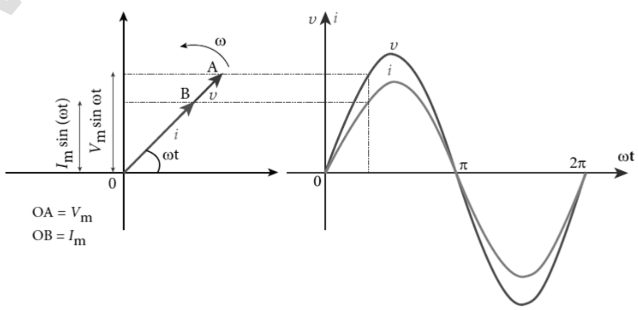
$$V - V_R = 0$$

$$V_m \sin\omega t = iR$$

$$i = \frac{V_m}{R}\sin\omega t$$

$$i = I_m \sin\omega t$$

$I_m = \frac{V_m}{R}$ => *the peak value of alternating current in the circuit.*



The applied voltage and the current are in phase with each other in a resistive circuit. It means that they reach their maxima and minima simultaneously. This is indicated in the phasor diagram . The wave diagram also depicts that current is in phase with the applied voltage.

13. RESONANCE IN SERIES RLC CIRCUIT :

RESONANCE FREQUENCY

*The frequency of the applied alternating source ω_r is equal to the natural frequency $1/\sqrt{LC}$ of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in **electrical resonance**. The frequency at which resonance takes place is called **resonant frequency.***

Resonant angular frequency, $\omega_r = 1/\sqrt{LC}$

Or $f_r = \frac{1}{2\pi\sqrt{LC}}$

At series resonance,

$$\omega_r = 1/\sqrt{LC} \quad (or)\ \omega_r^2 = 1/LC$$

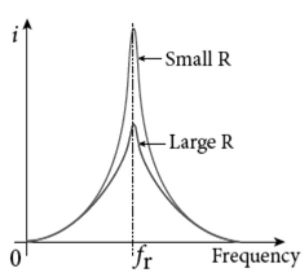
$$\omega_r L = 1/\omega_r C \qquad (or)\ X_L = X_C$$

This is the condition for resonance in RLC circuit.

Since X_L and X_C are frequency dependent, the resonance condition ($X_L = X_C$)can be achieved by varying the frequency of the applied voltage.

14. EFFECTS OF SERIES RESONANCE [OR] RESONANCE CURVE

When series resonance occurs, the impedance of the circuit is minimum and is equal to the resistance of the circuit. As a result of this, the current in the circuit becomes maximum. This is shown in the resonance curve drawn between current and frequency



At resonance, the impedance is

$$Z = \sqrt{R^2[X_L - X_C]^2} = R \text{ since } X_L = X_C$$

The current in the circuit is,

$$I_m = \frac{V_m}{\sqrt{R^2[X_L - X_C]^2}}$$

$$I_m = \frac{V_m}{R}$$

The maximum current at series resonance is limited by the resistance of the circuit. For smaller resistance, larger current with sharper curve is obtained and vice versa.

15. POWER IN AC CIRCUITS

The rate of consumption of electric energy in that circuit.

The alternating voltage and alternating current in the series inductive RLC circuit at an instant

$V = V_m \sin\omega t$ and $i = I_m \sin[\omega t + \phi]$

ϕ = *Phase angle between V and i*

The instantaneous power is,

$$P = Vi$$

$$= V_m \sin\omega t. \; I_m \sin[\omega t + \phi]$$

$$= V_m I_m \sin\omega t\, [\sin\omega t.\cos\phi + \cos\omega t \sin\phi\,]$$

$$P = V_m I_m\ [\cos\phi \sin^2\omega t + \sin\omega t \cos\omega t \sin\phi]$$

Here the average of $\sin^2\omega$ *over a cycle is ½ and that of* $\sin\omega t \cos\omega t \sin\phi$ *is zero.*

$$P_{av} = V_m I_m \cos\phi \times ½$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P_{av} = V_{RMS} I_{RMS} \cos\phi$$

$V_{RMS} I_{RMS}$ => *Apparent power*

$\cos\phi$ => *Power factor*

The average power of an AC circuit is also known as the true power of the circuit.

SPECIAL CASES

1. *For a purely resistive circuit, the phase angle between voltage and current is zero and cos ϕ = 1.*

$$\therefore P_{av} = V_{RMS}\, I_{RMS}$$

2. *For a purely inductive or capacitive circuit, ϕ is $\pm\,{}^{\pi}/_{2}$ and cos $\left[\pm\,{}^{\pi}/_{2}\right]$= 0*

$$\therefore P_{av} = 0$$

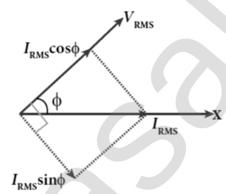
3. *For series RLC circuit,*

$$\phi = tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

4. *For series RLC circuit at resonance, ϕ = 0 ; cos ϕ = 1.*

$$\therefore P_{av} = V_{RMS}\, I_{RMS}$$

16. WATTFUL CURRENT AND WATTLESS CURRENT



*The component of current [I_{RMS} cosϕ] which is in phase with the voltage is called active component. The power consumed by this current= V_{RMS} I_{RMS} cosϕ. So that it is also known as **'Wattful' current.***

*The other component [I_{RMS} sinϕ] which has a phase angle of ${}^{\pi}/_{2}$ withthe voltage is called reactive component. The power consumed is zero. Hence it is also known as **'Wattless' current**.*

The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This wattless current occurs in a purely inductive or capacitive circuit

17. ADVANTAGES AND DISADVANTAGES OF AC OVER DC

ADVANTAGES

(i) *The generation of AC is cheaper than that of DC.*

(ii) *When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.*

(iii) AC can easily be converted into DC with the help of rectifiers.

DISADVANTAGES

(i) Alternating voltages cannot be used for certain applications such as charging of batteries, electroplating, electric traction etc.

(ii) At high voltages, it is more dangerous to work with AC than DC.

18. CONSERVATION OF ENERGY IN LC OSCILLATIONS

During LC oscillations in LC circuits, the energy of the system oscillates between the electric field of the capacitor and the magnetic field of the inductor. Although, these two forms of energy vary with time, the total energy remains constant.

$$\text{Total energy}, U = U_E + U_B = {q^2}/{2c} + {1}/{2}\, LI^2$$

Case (i)

When the charge in the capacitor, $q = Q_m$ and the current through the inductor, $i = 0$, the total energy is

$$U = {Q_m^2}/{2c} + 0 = {Q_m^2}/{2c}$$

The total energy is wholly electrical.

Case (ii)

When $q = 0$; $i = I_m$,

The total energy is , $U = 0 + {1}/{2}\, LI_m^2 = {1}/{2}\, LI_m^2$

$$= {L}/{2} \times \left[{Q_m^2}/{LC}\right] \qquad \left\{I_m = Q_m w = \frac{Q_m}{\sqrt{LC}}\right\}$$

$$= {Q_m^2}/{2c}$$

The total energy is wholly electrical.

Case (iii)

When charge = q; current = i,

The total energy is

$$U = {q^2}/{2c} + {1}/{2}\, LI^2$$

Since $q = Q_m \cos\omega t$

$i = -\,{}^{dq}/_{dt} = Q_m\omega \sin \omega t$

[$-ve$ sign in current indicates that the charge in the capacitor decreases with time]

$$U = \frac{Q_m^2 \cos^2 \omega t}{2c} + \frac{L\omega^2 Q_m^2 \sin^2 \omega t}{2} \qquad \left[\omega^2 = \frac{1}{LC}\right]$$

$$= \frac{Q_m^2 \cos^2 \omega t}{2c} + \frac{LQ_m^2 \sin^2 \omega t}{2LC}$$

$$= {}^{Q_m^2}/_{2c}\,(\cos^2 \omega t + \sin^2 \omega t)$$

$$U = {}^{Q_m^2}/_{2c}$$

From above these cases, the total energy of the system remains constant.

19. ANALOGIES BETEEN LC OSCILLATIONS AND SIMPLE HARMONIC OSCILLATIONS

QUALITATIVE TREATMENT

The electromagnetic oscillations of LC system can be compared with the mechanical oscillations of a spring-mass system.

There are two forms of energy involved in LC oscillations. One is electrical energy of the charged capacitor; the other magnetic energy of the inductor carrying current.

Likewise, the mechanical energy of the spring-mass system exists in two forms; the potential energy of the compressed or extended spring and the kinetic energy of the mass.

Energy in two oscillatory systems

	LC oscillator		Spring-mass system
Element	**Energy**	**Element**	**Energy**
Capacitor	Electrical Energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Spring	Potential energy $= \frac{1}{2}k\,x^2$
Inductor	Magnetic energy $= \frac{1}{2}Li^2 \quad i = \frac{dq}{dt}$	Mass	Kinetic energy $= \frac{1}{2}mv^2 \quad v = \frac{dx}{dt}$

Analogies between electrical and mechanical quantities

Electrical system	Mechanical system
Charge q	Displacement x
Current $i = \frac{dq}{dt}$	Velocity $v = \frac{dx}{dt}$
Inductance L	Mass m
Reciprocal of capacitance $\frac{1}{C}$	Force constant k
Electrical energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Potential energy $= \frac{1}{2}kx^2$
Magnetic energy $= \frac{1}{2}Li^2$	Kinetic energy $= \frac{1}{2}mv^2$
Electromagnetic energy $U = \frac{1}{2}\left(\frac{1}{C}\right)q^2 + \frac{1}{2}Li^2$	Mechanical energy $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

The angular frequency of oscillations of a spring-mass is

$$w = \sqrt{\frac{k}{m}}$$

$k \Rightarrow \frac{1}{c}$ *and* $m \Rightarrow L$

The angular frequency of LC oscillations is,

$$w = \frac{1}{\sqrt{LC}}$$

PART – D (5 MARKS)

1. APPLICATION OF EDDY CURRENT

Though the production of eddy current is undesirable in some cases, it is useful in some other cases. A few of them are

i. *Induction stove*
ii. *Eddy current brake*
iii. *Eddy current testing*
iv. *Electromagnetic damping*

i. Induction stove

Induction stove is used to cook the food quickly and safely with less energy consumption. Below the cooking zone, there is a tightly wound coil of insulated wire. The *cooking pan made of suitable material, is placed over the cooking zone. When the stove is switched on, an alternating current flowing in the coil produces high frequency alternating magnetic field which induces very strong eddy currents in the cooking pan. The eddy currents in the pan produce so much of heat due to Joule heating which is used to cook the food*

ii. Eddy current brake

This eddy current braking system is generally used in high speed trains and roller coasters. Strong electromagnets are fixed just above the rails. To stop the train, electromagnets are switched on. The magnetic field of these magnets induces eddy currents in the rails which oppose or resist the movement of the train. This is Eddy current linear brake .In some cases, the circular disc, connected to the wheel of the train through a common shaft, is made to rotate in between the poles of an electromagnet. When there is a relative motion between the disc and the magnet, eddy currents are induced in the disc which stop the train. This is Eddy current circular brake

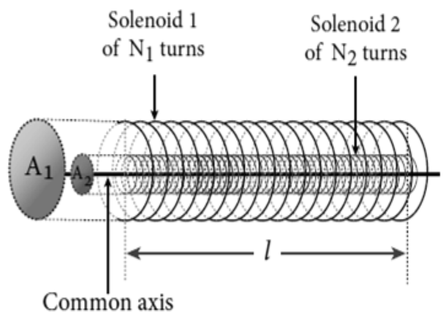
iii. Eddy current testing

It is one of the simple non–destructive testing methods to find defects like surface cracks, air bubbles present in a specimen. A coil of insulated wire is given an alternating electric current so that it produces an alternating magnetic field. When this coil is brought near the test surface, eddy current is induced in the test surface. The presence of defects causes the change in phase and amplitude of the eddy current that can be detected by some other means.In this way, the defects present in the specimen are identified

iv. Electro magnetic damping

The armature of the galvanometer coil is wound on a soft iron cylinder. Once the armature is deflected, the relative motion between the soft iron cylinder and the radial magnetic field induces eddy current in the cylinder The damping force due to the flow of eddy current brings the armature to rest immediately and then galvanometer shows a steady deflection. This is called electromagnetic damping.

2. ***MUTUAL INDUCTANCE BETWEEN THE LONG CO-AXIAL SOLENOIDS.***



Consider two long co-axial solenoids of same length l. The length of these solenoids is large when compared to their radii so that the magnetic field produced inside the solenoids is uniform and the fringing effect at the ends may be ignored. Let A_1 and A_2 be the area of cross section of the solenoids with A_1 being greater than A_2. The turn density of these solenoids are n_1 and n_2 respectively.

Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_0 n_1 i_1$$

$\vec{B}_1$ are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to current i_1,

$$\emptyset_{21} = \int_{A_2} \vec{B}_1 \overrightarrow{dA} = B_1 A_1 \qquad [\theta = 0°]$$

$$= [\mu_0 n_1 i_1]\, A_2$$

The flux linkage with solenoid 2 ,

$$N_2\,\emptyset_{21} = [n_2\ell][\mu_0 n_1 i_1]\, A_2 \qquad [\; N_2 = n_2\ell\;]$$

$$N_2\,\emptyset_{21} = [\mu_0 n_1 n_2 A_2 \ell]\; i_1$$

$$N_2\,\emptyset_{21} = M_{21}\, i_1$$

$$M_{21} = \mu_0 n_1 n_2 A_2 \ell$$

Mutual inductance M_{21} of the solenoid 2 with respect to solenoid 1

The magnetic field produced by the solenoid 2 when carrying a current i_2

$$B_2 = \mu_0 n_2 i_2$$

This magnetic field B_2 is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area A_2 is the effective area over which the magnetic field B_2 is present, not area A_1. Then the magnetic flux $\emptyset_{12}$

$$\emptyset_{12} = \int_{A_2} \vec{B}_1 \overrightarrow{dA} = B_2 A_2 \qquad [\ \theta = 0°\ ]$$

$$= [\mu_O n_1 i_1]\, A_2$$

The flux linkage of solenoid 1 with total turns N_1 is

$$N_1 \emptyset_{12} = [n_1\ell][\mu_O n_2 i_Q]\, A_2 \qquad [N_2 = n_2\ell\ ]$$

$$N_1 \emptyset_{12} = [\mu_O n_1 n_2 A_2 \ell]\ i_1$$

$$N_1 \emptyset_{12} = M_{12}\, i_1$$

$$M_{12} = \mu_O n_1 n_2 A_2 \ell$$

$$M_{12} = M_{21} = M$$

The mutual inductance between two long co-axial solenoids

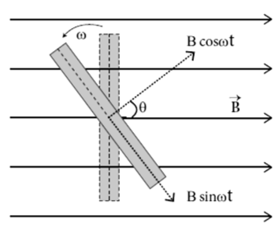
$$M = \mu_O n_1 n_2 A_2 \ell$$

If a dielectric medium of relative permeability μ_r is present inside the solenoids,

$$M = \mu n_1 n_2 A_2 \ell$$

$$M = \mu_O \mu_r n_1 n_2 A_2 \ell$$

3. PRODUCTION OF INDUCED EMF BY CHANGING RELATIVE ORIENTATION OF THE COIL WITH THE MAGNETIC FIELD



Consider a rectangular coil of N turns kept in a uniform magnetic field $\vec{\square}$ The coil rotates in anti-clockwise direction with an angular velocity ω about an axis, perpendicular to the field and to the plane of the paper.

At time t = 0, the plane of the coil is perpendicular to the field and the flux linked with the coil has its maximum value

The change in magnetic flux in time dt is

$d\phi_B$ = B X [dA] change in area

= B x Area ABCD [Area ABCD = ℓ (Vdt)]

$$d\phi_B = B\,\ell\,V\,dt$$

The induced emf is ,

$$\xi = \frac{d\phi_B}{dt}$$

$$\xi = B\,\ell\,V$$

This emf is known as Motional emf.

The direction of induced current is found to be clockwise from Fleming's right hand rule.

Induced current in the loop is,

$$i = \frac{\xi}{R}$$

$$i = \frac{B\,l\,V}{R}$$

ENERGY CONSERVATION

The current-carrying movable rod AB kept in the perpendicular $\vec{B}$ experiences a force $\vec{F}_B$ in the outward direction,

$$\vec{F}_B = i\,\ell\,B$$

A constant force that is equal and opposite to the magnetic force, must be applied.

$$\left|\vec{F}_{app}\right| = \left|\vec{F}_B\right| = i\,\ell\,B$$

∴ Mechanical work is done by the applied force to move the rod. The rate of doing work or power is

$$P = \vec{F}_{app} \cdot \vec{V} = F_{app} \cdot V\cos\theta$$

$$= i\,\ell\,BV \qquad [\,\theta = 0°\,]$$

$$= [\frac{BlV}{R}]\ \ell BV$$

$$P = \frac{B^2 l^2 V^2}{R}$$

Joule heating takes place in the loop ,

$$P = l^2\ R$$

$$= [\frac{BlV}{R}]^2\ R$$

$$P = \frac{B^2 l^2 V^2}{R}$$

The mechanical energy needed to move the rod is converted into electrical energy which then appears as thermal energy in the loop.

4. ADVANTAGES OF STATIONARY ARMATURE – ROTATING FIELD ALTERNATING

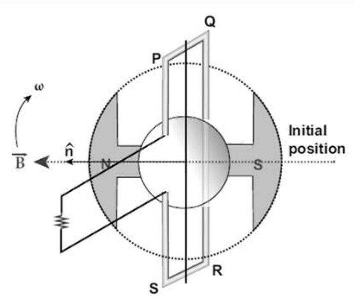
1) *The current is drawn directly from fixed terminals on the stator without the use of brush contacts.*
2) *The insulation of stationary armature winding is easier.*
3) *The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.*
4) *Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.*

5. SINGLE PHASE AC GENERATOR

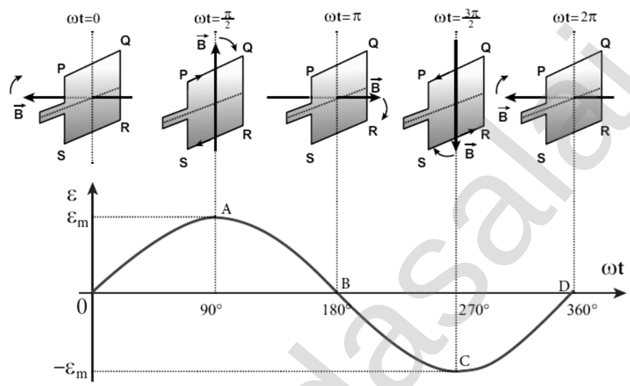
The armature conductors are connected in series so as to form a single circuit which generates a single-phase alternating emf and hence it is called single-phase alternator.

A single-turn rectangular loop PQRS is mounted on the stator. The field winding is fixed inside the stator and it can be rotated about an axis, perpendicular to the plane of the paper.

The loop PQRS is stationary and is also perpendicular to the plane of the paper. When field windings are excited, magnetic field is produced around it. Let the field magnet be rotated in clockwise direction by some external means



Assume that initial position of the field magnet is horizontal. At that instant, the direction of magnetic field is perpendicular to the plane of the loop PQRS. The induced emf is zero. This is represented by origin O in the graph drawn between induced emf and time angle.



When field magnet rotates through 90°, magnetic field becomes parallel to PQRS. The induced emfs across PQ and RS would become maximum. Since they are connected in series, emfs are added up and the direction of total induced emf is given by Fleming's right hand rule. The direction of the induced emf is at right angles to the plane of the paper. For PQ, it is inwards and for RS it is outwards. Therefore, the current flows along PQRS. The point A in the graph represents this maximum emf.

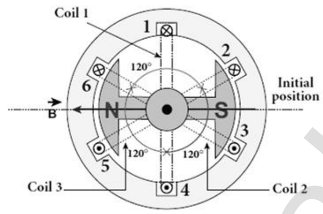
For the rotation of 180° from the initial position, the field is again perpendicular to PQRS and the induced emf becomes zero. This is represented by point B.

The field magnet becomes again parallel to PQRS for 270° rotation of field magnet. The induced emf is maximum but the direction is reversed. Thus the current flows along SRQP. This is represented by point C.

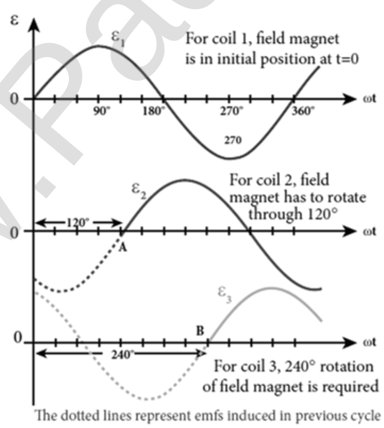
On completion of 360°, the induced emf becomes zero and is represented by the point D. From the graph, it is clear that emf induced in PQRS is alternating in nature. Therefore, when field magnet completes one rotation, induced emf in PQRS finishes one cycle.

6. THREE PHASE AC GENERATOR

The armature core has 6 slots, cut on its inner rim. Each slot is 60° away from one another. Six armature conductors are mounted in these slots. The conductors 1 and 4 are joined in series to form coil 1. The conductors 3 and 6 form coil 2 while the conductors 5 and 2 form coil 3. So, these coils are retangular in shape and are 120° apart from one another



The initial position of the field magnet is horizontal and field direction is perpendicular to the plane of the coil 1. As it is seen in single phase AC generator, when field magnet is rotated from that position in clockwise direction, alternating emf ξ_1 in coil 1 begins a cycle from origin O.



An alternating emf ξ_2 in coil 2 starts at point A after field magnet has rotated through 120°. Therefore, the phase difference between ξ_1 and ξ_2 is 120°. Similarly, emf _3 in coil 3 would begin its cycle at point B after 240° rotation of field magnet from initial position. Thus these emfs produced in the three phase AC generator have 120° phase difference between one another.

7. TRANSFORMER

Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency.

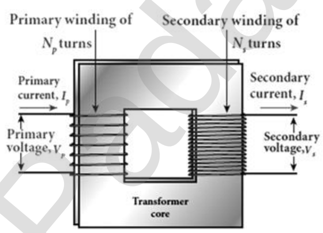
If the transformer converts an alternating current with low voltage into an alternating current with high voltage, it is called ***step-up transformer****. On the contrary, if the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called* ***step-down transformer****.*

PRINICIPLE

The principle of transformer is the mutual induction between two coils.

CONSTRUCTION

There are two coils of high mutual inductance wound over the same transformer core. The core is generally laminated and is made up of a good magnetic material like silicon steel. Coils are electrically insulated but magnetically linked via transformer core



The assembled core and coils are kept in a container which is filled with suitable medium for better insulation and cooling purpose.

WORKING

The coil across which alternating voltage is applied is called primary coil P and the coil from which output power is drawn out is called secondary coil S.

an alternating magnetic flux is set up in the laminated core. If there is no magnetic flux leakage, then whole of magnetic flux linked with primary coil is also linked with secondary coil. This means that rate at which magnetic flux changes through each turn is same for both primary and secondary coils. The emf induced in the primary coil or back emf ξ_P is

$$\xi_P = - N_P \frac{d\phi_B}{dt}$$

But the voltage applied V_p across the primary is equal to the back emf .

$$Vp = - N_p \frac{d\phi_B}{dt}$$

The frequency of alternating magnetic flux in the core is same as the frequency of the applied voltage. Therefore, induced emf in secondary will also have same frequency as that of applied voltage. The emf induced in the secondary coil ξ_S is given by

$$\xi_S = - N_S \frac{d\phi_B}{dt}$$

The voltage across secondary coil,

$$V_s = - N_S \frac{d\phi_B}{dt}$$

[N_S and N_S are the numbers of turns in the primary and secondary coil]

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = K$$

$K \rightarrow$ voltage transformer ratio

For an ideal transformer ,

Input power $V_P i_P$ = output power $V_s i_s$

Where i_P and i_s are the currents in the primary and secondary coil respectively

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S} = K$$

i. *$K > 1$ [$N_S > N_P$, $V_S > V_p$, and $I_S < I_p$]*

   This is the case of step-up transformer in which voltage is increased and the corresponding current is decreased

ii. *$K < 1$ [$N_S < N_P$, $V_S < V_p$, and $I_S > I_p$]*

   This is step-down transformer where voltage is decreased and the current is increased.

EFFICIENCY OF THE TRANSFORMER

The ratio of the useful output power to the input power

$$\eta = \frac{Output\ power}{input\ power}$$

Transformers are highly efficient devices having their efficiency in the range of 96 – 99%.

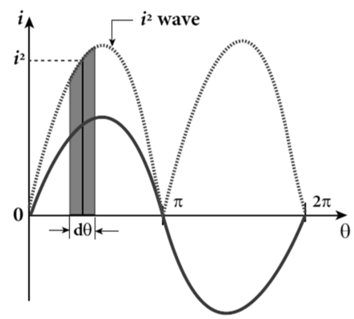
8. ***RMS VALUE OF AC [I_{RMS}]***

The square root of the mean of the squares of all currents over one cycle

For alternating voltages ,V_{RMS}

Alternating current $\quad i = I_m \sin\omega t$

$$i = I_m \sin\theta$$



The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave

$$I_{RMS} = \sqrt{\frac{\text{area of one cycle of squared wave}}{\text{Base length of one cycle}}}$$

An elementary area of thickness 'dθ'. Let i^2 be the mid orginate of the element .

Area of the element $= i^2 d\theta$

Area of one cycle of squared wave

$$= \int_0^{2\pi} i^2 \, d\theta$$

$$= \int_0^{2\pi} I_m^2 \sin^2\theta \, . d\theta$$

$$= I_m^2 \int_0^{2\pi} \sin^2\theta \, . d\theta$$

$$= I_m^2 [\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta \,] \qquad [\sin^2\theta = \frac{1-\cos\theta}{2}\,]$$

$$= I_m^2 [0 - \frac{\sin 2\theta}{2}]_0^{2\pi} \,]$$

$$= I_m^2 \left\{ [2\pi - \frac{\sin 2 \, x \, 2\pi}{2}] - [\,0 - \frac{\sin\theta}{2}] \right\}$$

$$= \frac{I_m^2}{2} \, x \, 2\,\pi = I_m^2 \, \pi \qquad [\sin 0 = \sin 4\pi = 0]$$

The base length of one cycle is 2π

$$I_{RMS} = \sqrt{\frac{I_m^2 \, \pi}{2\,\pi}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$I_{RMS} = 0.707\, I_m$$

A symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

For alternating voltage,

$$I_{RMS} = 0.707\, I_m$$

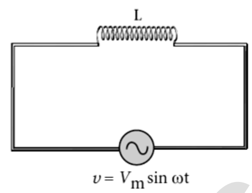
For ex ;

If we consider 'n' current ,in one cycle of AC,

$$I_{RMS} = \sqrt{\frac{sum\ of\ squares\ of\ all\ \ current\ \ over\ \ one\ \ cycle}{number\ of\ current}}$$

$$I_{RMS} = \sqrt{\frac{i_1+i_2+\cdots i_n}{n}}$$

9. AC CIRCUIT CONTAINING ONLY AN INDUCTOR



Consider a circuit containing a pure inductor of inductance L connected across an alternating voltage source . The instantaneous value of the alternating voltage is

$$V = V_m \sin\omega t$$

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit.

$$\xi = -L\frac{di}{dt}$$

By applying Kirchoff 's loop rule,

$$V + \xi = 0$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t.dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L}\int \sin\omega t.dt$$

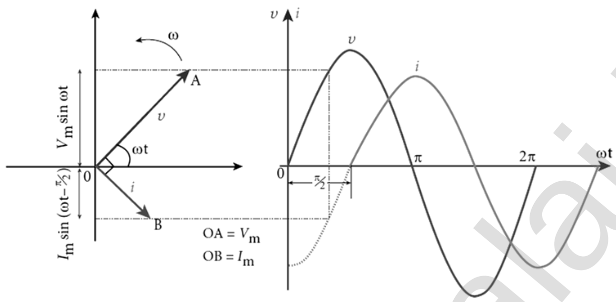
$$i = \frac{V_m}{L\omega}[\,-\cos\omega t\,] + constant$$

The integration constant in the above equation is independent of time. So constant as zero.

$i = \frac{V_m}{L\omega}\ sin[\omega t\ -\frac{\pi}{2}\ ]$ $\qquad \left(-\ cos\omega t\ = -\ sin\left[\frac{\pi}{2} - \omega t\right] = \ sin[\omega t\ -\frac{\pi}{2}\ ]\right)$

(or) $i = I_m\ sin[\omega t\ -\frac{\pi}{2}\ ]$

The peak value of the alternating current is $I_m = \frac{V_m}{L\omega}$*. From the above equation that current lags behind the applied voltage by* $\frac{\pi}{2}$ *in an inductive circuit.*



INDUCTIVE REACTANCE [X_L]

$I_m = \frac{V_m}{\omega L}$ *, ωL plays the same role as the resistance in resistive circuit. This is the resistance offered by the inductor, called inductive reactance **[X_L]***

***UNIT:** ohm.*

$X_L = \omega L$

The inductive reactance (X_L) varies directly as the frequency.

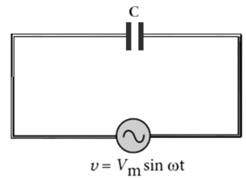
$X_L = 2\pi fL$

f => frequency of the alternating current. For a steady current, f = 0; $X_L = 0$*. Thus an ideal inductor offers no resistance to steady DC current.*

10. AC CIRCUIT CONTAINING ONLY A CAPACITOR

Consider a circuit containing a capacitor of capacitance C connected across an alternating voltage source .The instantaneous value of the alternating voltage is given by

$V = V_m\ sin\omega t$



Let q be the instantaneous charge on the capacitor. The emf across the capacitor at that instant is $^q/_C$.

According to Kirchoff 's loop rule,

$$V - {}^q/_C = 0$$

$$q = CV$$

$$q = CV_m \sin\omega t$$

$$i = \frac{dq}{dt} = \frac{d}{dt}\,[CV_m \sin\omega t]$$

$$= CV_m \frac{d}{dt}\sin\omega t$$

$$= CV_m \cos\omega t$$

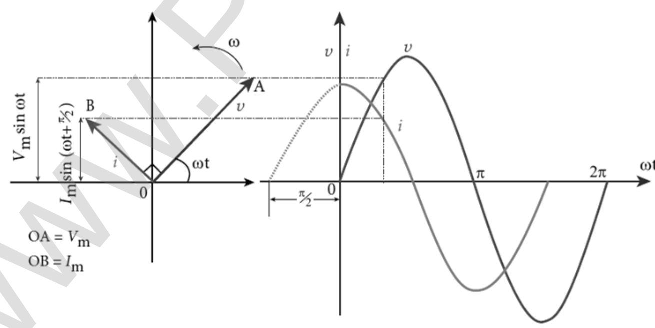
$$i = \frac{V_m}{{}^1/_{C\omega}}\ \sin[\omega t + \frac{\pi}{2}]$$

Instantaneous value of current,

$$i = I_m\ \sin[\omega t + \frac{\pi}{2}]$$

Peak value of the current $I_m = \frac{V_m}{{}^1/_{\omega C}}$

Here, current leads the applied voltage by $\frac{\pi}{2}$ *in a capacitive circuit.*



CAPACITIVE REACTANCE

The peak value of current Im is given by $I_m = \frac{V_m}{{}^1/_{\omega C}}$. *Let us compare this equation with* $I_m = \frac{V_m}{R}$ *for a resistive circuit. The quantity* ${}^1/_{\omega C}$ *plays the same role as the resistance R in resistive circuit. This is the resistance offered by the capacitor, called capacitive reactance* (X_C).

UNIT *ohm.*

$$X_C = \frac{1}{\omega C}$$

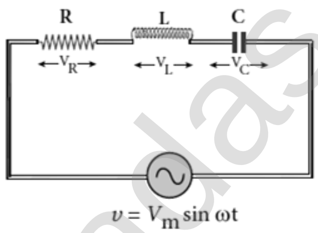
The capacitive reactance (X_C) varies inversely as the frequency. For a steady current, f = 0.

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{0} = \infty$$

Thus a capacitive circuit offers infinite resistance to the steady current. So that steady current cannot flow through the capacitor.

11. AC CIRCUIT CONTAINING A RESISTOR ,AN INDUCTOR AND A CAPACITOR IN SERIES –SERIES RLC CIRCUIT

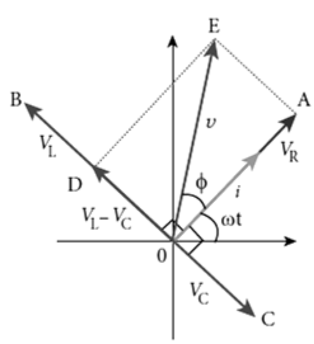
Consider a circuit containing a resistor of resistance R, an inductor of inductance L and a capacitor of capacitance C connected across an alternating voltage source



The instantaneous value of the alternating voltage is given by

$$V = V_m \sin\omega t$$

Let i be the resulting current in the circuit at that instant. As a result, the voltage is developed across R, L and C.



From phasor diagram,

The voltage across R [V_R] = OA = I_mR

[V_R is in phase with i]

The voltage across L [V_L] = OB = I_mL

[V_L leads i by $\frac{\pi}{2}$]

The voltage across C [V_c] = OC = I_mC

[V_c lags behind i by $\frac{\pi}{2}$]

OI = I_m

The circuit is either effectively inductive or capacitive or resistive depending on the value of V_L or V_C.

Let us assume that $V_L > V_C$ Therefore, net voltage drop across L-C combination is $V_L - V_C$ which is represented by a phasor $\overrightarrow{OD}$

By parallelogram law, the diagonal $\overrightarrow{OE}$ gives the resultant voltage v of V_R and (V_L - V_C) and its length OE is equal to V_m.

$$V_m^2 = V_R^2 + (V_L - V_C)^2$$

$$V_m = \sqrt{(I_mR)^2 + (I_mX_L - I_mX_C)^2}$$

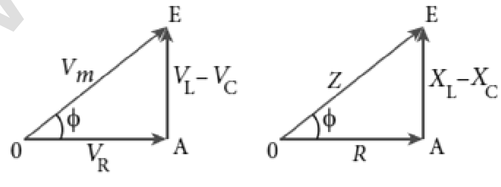
$$= I_m\sqrt{R^2 + (X_L - X_C)^2} \text{ or}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \text{ or}$$

$$I_m = \frac{V_m}{Z}$$

$$\text{where } \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Z => Impedance of the circuit which refers to the effective opposition to the current by the series RLC circuit.



From phasor diagram, the phase angle between v and i is

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

SPECIAL CASES

(i) $X_L > X_C$

$(X_L - X_C)$ is positive and phase angle ϕ is also positive.The applied voltage leads the current by ϕ and The circuit is inductive.

$$\therefore V = V_m \sin\omega t \; ; \quad i = I_m \sin[\omega t - \phi]$$

(ii) $X_L < X_C$

$(X_L - X_C)$ is negative and ϕ is also negative. The current leads voltage by ϕ and the circuit is capacitive.

$$\therefore V = V_m \sin\omega t \; ; \quad i = I_m \sin[\omega t + \phi]$$

(iii) $X_L = X_C$

ϕ is zero. Therefore current and voltage are in the same phase and the circuit is resistive.

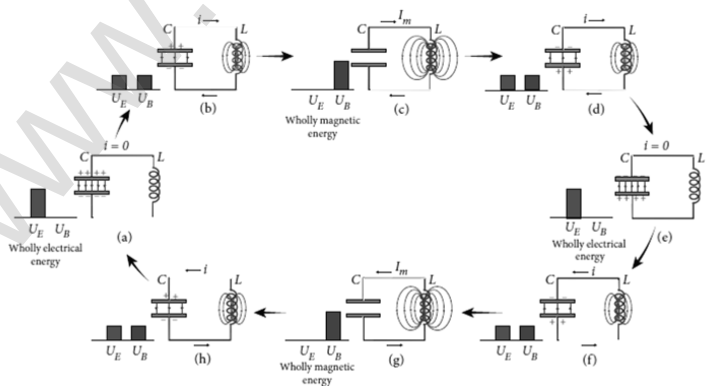
$$\therefore V = V_m \sin\omega t \; ; \quad i = I_m \sin\omega t$$

12. ENERGY CONVERSION DURING LC OSCILLATION

LC OSCILLATION

*The energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called **LC oscillations**.*

GENERATION OF LC OSCILLATION



Let us assume that the capacitor is fully charged with maximum charge Q_m at the initial stage. So that the energy stored in the capacitor is maximum and is given by $U_E = \frac{Q_m^2}{2C}$.

As there is no current in the inductor, the energy stored in it is zero i.e., $U_B = 0$. Therefore, the total energy is wholly electrical.

The capacitor now begins to discharge through the inductor that establishes current i in clockwise direction. This current produces a magnetic field around the inductor and the energy stored in the inductor is given by $U_B = {Li^2}/{2}$. As the charge in the capacitor decreases, the energy stored in it also decreases and is given by $U_E = {q^2}/{2C}$. Thus there is a transfer of some part of energy from the capacitor to the inductor. At that instant, the total energy is the sum of electrical and magnetic energies.

When the charges in the capacitor are exhausted, its energy becomes zero i.e., $U_E = 0$. The energy is fully transferred to the magnetic field of the inductor and its energy is maximum. This maximum energy is given by $U_E = {LI_m^2}/{2}$ where I_m is the maximum current flowing in the circuit. The total energy is wholly magnetic Even though the charge in the capacitor is zero, the current will continue to flow in the same direction because the inductor will not allow it to stop immediately .The current is made to flow with decreasing magnitude by the collapsing magnetic field of the inductor. As a result of this, the capacitor begins to charge in the opposite direction. A part of the energy is transferred from the inductor back to the capacitor. The total energy is the sum of the electrical and magnetic energies

When the current in the circuit reduces to zero, the capacitor becomes fully charged in the opposite direction. The energy stored in the capacitor becomes maximum. Since the current is zero, the energy stored in the inductor is zero. The total energy is wholly electrical .

The state of the circuit is similar to the initial state but the difference is that the capacitor is charged in opposite direction. The capacitor then starts to discharge through the inductor with anti-clockwise current. The total energy is the sum of the electrical and magnetic energies

As already explained, the processes are repeated in opposite direction. Finally, the circuit returns to the initial state . Thus, when the circuit goes through these stages, an alternating current flows in the circuit. As this process is repeated again and again, the electrical oscillations of definite frequency are generated. These are known as LC oscillations.

In the ideal LC circuit, there is no loss of energy. Therefore, the oscillations will continue indefinitely. Such oscillations are called undamped oscillation

UNSOLVED PROBLEMS

1. *A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30o to the field. Calculate the magnetic flux through the coil.*

Solution

Square coil of side (a) = 30 cm = 30×10^{-2}m

Area of square coil (A) = a^2 = $(30 \times 10^{-2})2 = 9 \times 10^{-2}$ m^2

Number of turns (N) = 500

Magnetic field (B) = 0.4 T

Angular between the field and coil (θ) = 90 – 30 = 60°

Magnetic flux (Φ) = NBA cos 0 = $500 \times 0.4 \times 9 \times 10^{-2} \times \cos 60° = 18 \times \frac{1}{2}$

Φ = 9 W b

2. *A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire.*

Solution

Magnetic flux (Φ) = 4 m Wb = 4×10^{-3} Wb

time (t) = 0.4 s

The magnitude of induced emf (e) = $\frac{d\Phi}{dt} = \frac{4\times10^{-3}}{0.4}$ 10^{-2}

e = 10 mV

3. *The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\phi_B = (2t^3 + 4t^2 + 8t + 8)$Wb. If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time t = 3 second.*

Solution

Magnetic flux (Φ_B) = $(2t^3 + 8t^2 + 8t + 8)$Wb

Resistance of the coil (R) = 5 Ω

time (t) = 3 second

Induced current through the coil, I = $\frac{e}{R}$

Induced emf, e = $\frac{d\Phi_B}{dt} = \frac{d}{dt}((2t^3 + 4t^2 + 8t + 8) = 6t^2 + 8t + 8$

Here time (t) = 3 second

e = $6(3)^2 + 8 \times 3 + 8 = 54 + 24 + 8 = 86$ V

∴ Induced current through the coil, I = $\frac{e}{R} = \frac{86}{5} = 17.2$ A

4. *A closely wound circular coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced in it. Calculate the number of turns in the coil.*

Solution

Radius of the coil (r) = 0.02 m

Area of the coil (A) = $\pi r^2 = 3.14 \times (0.02)^2 = 1.256 \times 10^{-3}$ m^2

Change in magnetic field, dB = 8000 – 2000 = 6000 T

Time, dt = 6 second

Induced emf, e = 44 V

$\theta = 0°$

Induced emf in the coil, $e = NA \frac{d}{dt} \cos\theta \cdot dt$

$44 = N \times 1.256 \times 10^{-3} \times \frac{600}{6} \times \cos 0°$

$$N = \frac{44}{1.256\times10^{-3}\times1000\times1} = \frac{44}{1.256\times10^{-3}\times10^{3}}$$

Number of turns N = 35 turns

5. ***A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180^o. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.***

Solution

Rectangular coil of their area, $A = 6\ cm^2 = 6 \times 10^{-4}\ m^2$

Number of turns N = 3500 turns

Magnetic field, B = 0.4 T

Resistance of the coil, R= 35 Ω

Induced emf (e) = change in flux per second = $\Phi_2 - \Phi_1$

e = NAB cos 180° – NBA cos 0° = -NBA – NBA = – 2 NBA

$= -2 \times 3500 \times 0.4 \times 6 \times 10^{-4} - 16800 \times 10^{-4} = -1.68$ V

Current flowing the coil, $I = \frac{e}{R} = \frac{-1.68}{35} = 0.048$

Magnitude of the current, $I = 48 \times 10^{-3}$ A

Amount of charge flowing through the coil, $q = It = 48 \times 10^{-3} \times 1 = 48 \times 10^{-3}$ C

6. ***An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.***

Solution

Induced current, I = 2.5 mA

Resistance of conductor, R = 100 Ω

∴ The rate of change of flux, $\frac{d\Phi_B}{dt} = e$

$\frac{d\Phi_B}{dt} = e = IR = 2.5 \times 10\text{-}3 \times 100 = 250 \times 10^{-3}$ dt

$\frac{d\Phi_B}{dt} = 250\ mWb\ s^{-1}$

7. ***A fan of metal blades of length 0.4 m rotates normal to a magnetic field of 4×10^{-3}T. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.***

Solution

Length of the metal blade, l = 0.4 m

Magnetic field, B = 4 x 10^{-3} T

Induced emf, e = 0.02 V

Rotational area of the blade, A = πr^2 = 3.14 x $(0.4)^2$ = 0.5024 m^2

Induced emf in rotational of the coil, e = NBA ω sin θ

$$\omega = \frac{e}{NBA\sin\theta} \qquad [N = 1, \theta = 90°, \sin 90° = 1]$$

$$\omega = \frac{0.02}{1\times4\times10^{-3}\times0.5024\times\sin 90°} = \frac{0.02}{2.0096\times10^{-3}}$$

= 9.95222 x 10^{-3} x 10^3

= 9.95 revolutions/second

Rate of rotational of the blade, ω = 9.95 revolutions/second

8. ***A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of 4 x 10^{-5} T. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.***

Solution

Length of the metal spokes, l = 1 m

Rotational area of the spokes, A = π^2 = 3.14 x $(1)^2$ = 3.14 m^2

Horizontal component of Earth's field, B = 4 x 10^{-5} T

Induced emf, e = 31.4 mV

The rate of revolution of wheel,

$$1, \omega = \frac{e}{NBA\sin\theta} \qquad [N = 1, \theta = 90°, \sin 90° = 1]$$

$$= \frac{31.4\times10^{-3}}{1\times4\times10^{-5}\times3.14\times\sin 90°} = \frac{31.4\times10^{-3}}{12.56\times10^{-5}} = 2.5\times10^2$$

ω = 250 revolutions / second

9. ***Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m.***

Solution

Length of the air core solenoid, l = 2 m

Diameter, d = 0.04 m

Radius, r = $\frac{d}{2}$ = 0.02 m

Area of the air core solenoid, A = π^2 = 3.14 x $(0.02)^2$ = 1.256 x 10^{-3} m^2

Number of Turns, N = 4000 turns

Self inductance, L = $\mu_0 n^2$ Al

$$= \mu_0 \frac{N^2}{l^2} \times Al \qquad [n = \frac{N}{l}, \mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}]$$

$$= \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (4000)^2 \times 1.256 \times 10^{-3}}{2}$$

$$= \frac{252405760 \times 10^{-10}}{2}$$

$$= 126202880 \times 10^{-10} = 12.62 \times 10^{-3} = 12.62\ mH$$

10. ***A coil of 200 turns carries a current of 4 A. If the magnetic flux through the coil is 6×10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil.***

Solution

Number of turns of the coil, N = 200

Current, I = 4 A

Magnetic flux through the coil, $\Phi = 6 \times 10^{-5}$ Wb

Energy stored in the coil, $U = \frac{1}{2} LI^2 = \frac{1}{I2}$

Self inductance of the coil, $L = \frac{N\Phi}{I}$

$U = \frac{1}{2} \frac{N\Phi}{I} \times I^2 = \frac{1}{2} N\Phi I = \frac{1}{2} \times 200 \times 6 \times 10^{-5} \times 4$

$U = 2400 \times 10^{-5} = 0.024$ J (or) joules.

11. ***A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux linked with each turn when it carries a current of 1 A.***

Solution

Length of the solenoid, $l = 50$ cm $= 50 \times 10^{-2}$ m

Number of turns per cm, N = 400

Number of turns in 50 cm, N = 400 x 50 = 20000

Diameter of the solenoid, d = 0.04 m

Radius of the solenoid, $r = \frac{d}{2} = 0.02$ m

Area of the solenoid, $A = \pi^2 = 3.14 \times (0.02)^2 = 1.256 \times 10^{-3}\ m^2$

Current passing through the solenoid, I = 1 A

Magnetic fluex,

$$\phi = \frac{\mu_0 N^2 AI}{l} = \frac{4\pi \times 10^{-7} \times (20000)^2 \times 1.256 \times 10^{-3} \times 1}{50 \times 10^{-2}}$$

$$= \frac{4 \times 3.14 \times 4 \times 10^{8} \times 10^{-10} \times 1.256}{50 \times 10^{-2}} = \frac{63.10144 \times 10^{-2}}{50 \times 10^{-2}}$$

$$\phi = 1.262\ Wb$$

12. ***A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with each turn of the coil, find the inductance of the coil.***

Solution

Number of turns, N = 200; Current, I = 0.4 A

Magnetic flux linked with coil, Φ = 4 mWb = 4 x 10^{-3} Wb

Induction of the coil, $L = \frac{N\Phi}{I} = \frac{200\times4\times10^{-3}}{0.4} = \frac{800\times10^{-3}}{0.4}$ 2 H

13. ***Two air core solenoids have the same length of 80 cm and same cross-sectional area 5 cm^2. Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns.***

Solution

Length of the solenoids, l = 80 cm = 8 x 10^{-2} m

Cross sectional area of the solenoid, A = 5 cm^2 = 5 x 10^{-4} m^2

Number of turns in the Ist coil, N_1 = 1200

Number of turns in the IInd coil, N_2 = 400

Mutual inductance between the two coils,

oils, $M = \frac{\mu_0 N_1 A l N_2}{l^2} = \frac{\mu_0 N_1 A N_2}{l}$

$= \frac{4\pi\times10^{-7}\times1200\times400\times5\times10^{-4}}{80\times10^{-2}} = \frac{30144000\times10^{-11}}{80\times10^{-2}}$

$= 376800 \times 10^{-9} = 0.38 \times 10^{-3} = 0.38$ mH

14. ***A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm2 is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.***

Solution

Number of turns of long solenoid per cm = $\frac{400}{10^{-2}}$; N_2 = 400 x 10^2

Number of turns inside the solenoid, N_2 = 100

Cross-sectional area of the coil, A = 4 cm^2 = 4 x 10^{-4} m^2

Current through the solenoid, I = 2A; time, t = 0.04 s

Induced emf of the coil, e = -M $\frac{dI}{dt}$

Mutual inductance of the coil,

$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi\times10^{-7}\times400\times10^2\times100\times4\times10^{-4}}{1}$

$= 2009600 \times 10^{-9} = 200.96 \times 10^{-5} = 2$ mH

Induced emf of the coil,

$e = -M\frac{dI}{dt} = -2\times10^{-3}\times\frac{2}{0.04} = \frac{-4\times10^{-3}}{0.04}$

$= -0.10 = -0.1$ V

The current through the solenoid reverse its direction if the induced emf, e = -0.2 V

15. ***A 200 turn circular coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil and the solenoid.***

Solution

Number of turns of the solenoid, N_2 = 200

Radius of the solenoid, r = 2cm = 2 x 10^2 m

Area of the solenoid, A = πr^2= 3.14 x (2 x $10^{-2})^2$ = 1.256 x 10^{-3} m^2

Turn density of long solenoid per cm, N_1 = 90 x 10^2

Mutual inductance of the coil,

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi\times10^{-7}\times90\times10^2\times200\times1.256\times10^{-3}}{1}$$

= 283956.48 x 10^{-8} ⇒ M = 2.84 mH

16. ***The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. Their areas of their cross-section and their lengths are the same and are 4 cm^2 and 0.04 m respectively. If the number of turns in S_1 is 200 and that in S_2 is 800, calculate the mutual inductance between the solenoids. If the current in solenoid 1 is increased form 2A to 8A in 0.04 second, calculate the induced emf in solenoid 2.***

Solution

Relative permeability of iron core, μ_r = 900

Number of turns of solenoid S_1, N_1 = 200

Number of turns of solenoid S_2, N_2 = '800

Area of cross section, A = 4 cm^2 = 4 x 10^{-4} m^2

Length of the solenoid S_1, l_1 = 0.04 m

current, I = $I_2 - I_1$ = 8 – 2 = 6A

time taken, t = 0.04 second

emf induced in solenoid S_2 e = -M $\frac{dI}{dt}$

Mutual inductance between the two coils, M = $\frac{\mu_0 \mu_r N_1 N_2 A}{l}$

$$= \frac{4\pi\times10^{-7}\times900\times200\times800\times4\times10^{-4}}{0.04}$$

$$= \frac{4\times3.14\times10^{-7}\times576\times10^{6}\times10^{-4}}{0.04} = \frac{7234.56\times10^{-5}}{0.04}$$

M = 180864 x 10^{-5} = 1.81 H

Emf induced in solenoid S_2, e = -M$\frac{dI}{dt}$ = -1.81 x $\frac{6}{0.04}$

Magnitude of emf, e = 271.5 V

17. ***A step-down transformer connected to main supply of 220 V is used to operate 11V,88W lamp. Calculate (i) Voltage transformation ratio and (ii) Current in the primary.***

Solution

Voltage in primary coil, V_p = 220 V

Voltage in secondary coil, V_s = 11 V

Output power = 88 W

(i) To find transformation ratio, $k = \frac{V_s}{V_p} = \frac{11}{220} = \frac{1}{20}$

(ii) Current in primary, $I_p = \frac{V_s}{V_p} \times I_s$

So, I_s = ?

Outputpower = $V_s I_s$

$\Rightarrow 88 = 11 \times I_s$

$I_s = \frac{88}{11} = 8A$

Therefore, $I_p = \frac{V_s}{V_p} \times I_s = \frac{11}{220} \times 8 = 0.4$ A

18. ***A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.***

Solution

Primary voltage, V_p = 200 V

Secondary voltage, V_s = 120 V

Efficiency, η = 90%

Secondary resistance, R_s = 40 Ω

Current drawn by the primary of the transformc, $I_p = \frac{V_s}{R_s} \times I_s$

Output power = $V_s I_s$

$$\text{From Ohm's law, } I_s = \frac{V_s}{R_s} = \frac{V_s^2}{R_s} = \frac{(120)^2}{40} = 360 \text{ W}$$

$$\therefore I_s = \frac{\text{Output power}}{V_s} = \frac{360}{120} = 3 \text{ A}$$

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_s I_s}{V_p I_p}$$

$$I_p = \frac{V_s I_s}{V_p \eta} = \frac{120 \times 3 \times 100}{200 \times 90} = 2\text{A}$$

19. ***The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.***

Solution

Efficiency, η = 80% = $\frac{80}{100}$

Number of turns in primary, N_p = 300

Number of turns in secondary, N_s = 1200

Resistance in primary, R_p = 0.82 Ω

Resistance in secondary, $R_s = 6.2\ \Omega$

Secondary voltage, $V_s = 1600$ V

Output power = 32 kW

Output power = $V_s I_s$

$$I_s = \frac{\text{Output power}}{V_s} = \frac{32\times10^3}{1600} = 20\text{ A}$$

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\frac{80}{100} = \frac{32\times10^3}{\text{Input power}}$$

Input power = 40 kW

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} \qquad \Rightarrow V_p = \frac{V_s N_p}{N_s} = \frac{1600\times300}{1200}$$

$$V_p = 400\text{ V}$$

Input power = $V_p I_p$

$$I_p = \frac{\text{Input power}}{V_p} = \frac{40000}{400} = 100\text{ A}, \qquad [I_p = 100\text{ A}]$$

Power loss in primary = $I_p^2 R_p$ = $(100)^2$ x 0.82 = 8200 = 8.2 kW

Power loss in secondary = $I_s^2 R_s$ = $(20)^2$ x 6.2 = 2480 = 2.48 kW

20. *Calculate the instantaneous value at 60°, average value and RMS value of an alternating current whose peak value is 20 A.*

Solution

Peak value of current, $I_m = 20$ A

Angle, $\theta = 60°$ [$\theta = \omega t$]

(i) Instantaneous value of current,

$i = I_m \sin \omega t = I_m \sin\theta$

$= 20 \sin 60° = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} = 10 \times 1.732$

$i = 17.32$ A

(ii) Average value of current,

$I_{av} = \frac{2I_m}{\pi} = \frac{2\times20}{3.14}$

$I_{av} = 12.74$ A

(iii) RMS value of current,

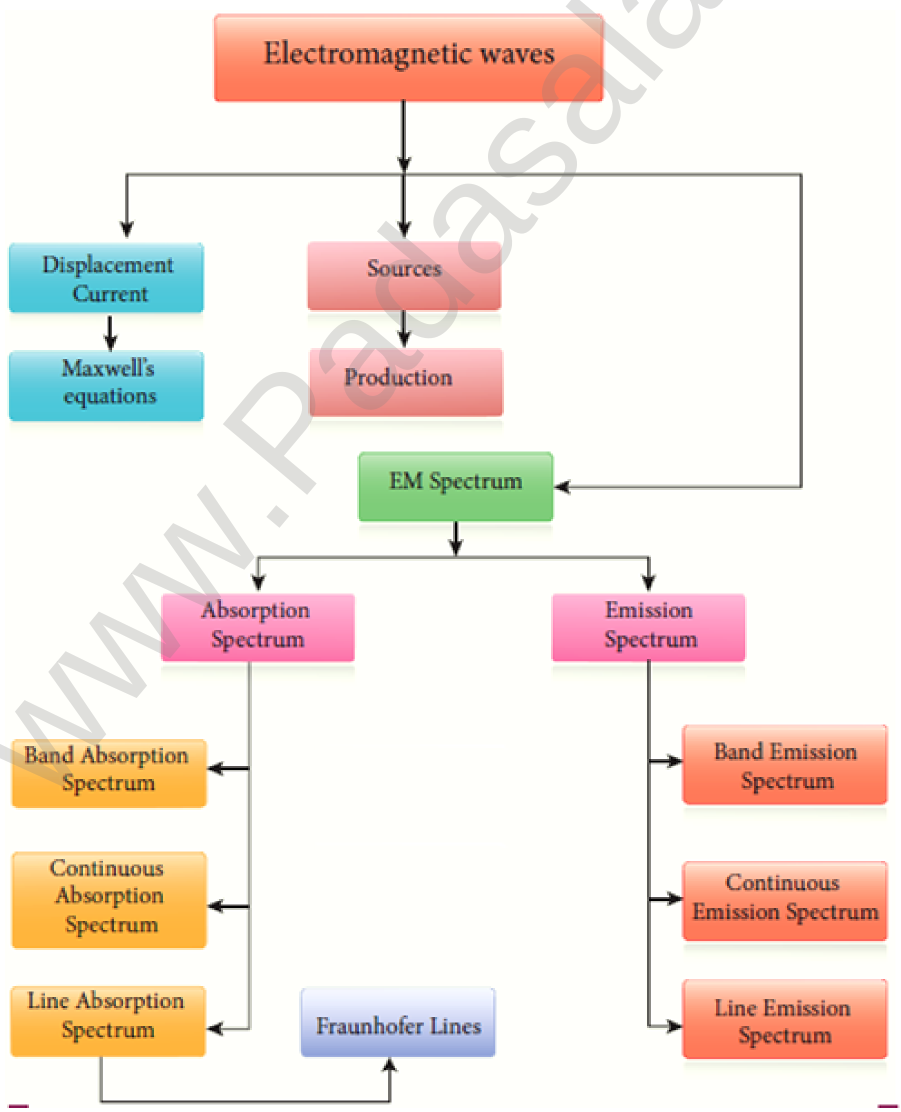
$I_{RMS} = 0.707\ I_m$

or $\frac{I_m}{\sqrt{2}} = 0.707 \times 20$

$I_{RMS} = 14.14$ A

UNIT 5

ELECTROMAGNETIC WAVES



PART – A (1 MARKS)

1. The dimension of is____________
   a)$[LT^{-1}]$ b) $[L^2T^{-2}]$ c) $[L^{-1}T]$ d) $[L^{-2}T^{-2}]$

Solution :

$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ => $c^2 = \frac{1}{\mu_0\varepsilon_0}$

$c^2 = [LT^{-1}]^2 = [L^2T^{-2}]$

ANSWER : b) $[L^2T^{-2}]$

2. If the amplitude of the magnetic field is 3×10^{-6}T then amplitude of the electric field for a electromagnetic waves is
   (a) 100 V m^{-1} (b) 300 V m^{-1} (c) 600 V m^{-1} (d) 900 V m^{-1}

Solution :

$c = \frac{E}{B}$ => $E = B x c = 3 \times 10^{-6} x 3 \times 10^{8}$

$= 9 \times 10^2 = 900$ V m^{-1}

ANSWER : (d) 900 V m^{-1}

3. Which of the following electromagnetic radiation is used for viewing objects through fog
   (a)microwave (b)gammarays (c)X-rays (d)infrared

   ANSWER : (d) infrared

4. Whichofthefollowingarefalseforelectromagneticwaves
   (a)transverse (b) non mechanical wave (c)longitudinal(d)producedbyacceleratingcharges
   ANSWER : (c)longitudinal

5. Consideranoscillatorwhichhasachargedparticle and oscillates about its mean position with a frequency of 300 MHz.The wavelength of electromagnetic waves produced by this oscillator is
   (a)1m (b)10m (c)100m (d)1000 m

Solution :

$\upsilon = c\lambda$ => $\lambda = \frac{\upsilon}{c} = \frac{3x10^8}{3x10^8} = 1$m

ANSWER : (a) 1 m

6. The electric and the magnetic field, associated with an electromagnetic wave, propagating along X axis can be represented by

   (a) $\vec{E}$=E$_0\hat{j}$ and $\vec{B}$=B$_0\,\hat{k}$ (b) $\vec{E}$=E$_0\hat{k}$ and $\vec{B}$=B$_0\,\hat{j}$

   (c) $\vec{E}$=E$_0\hat{i}$ and $\vec{B}$=B$_0\,\hat{j}$ (d) $\vec{E}$=E$_0\hat{j}$ and $\vec{B}$=B$_0\,\hat{i}$

Solution :

$\vec{E}$X$\vec{B}$ = E$_0\hat{j}$ X B$_0\,\hat{k}$ = E$_0$ X B$_0(\hat{j}\,\hat{k})$ =

E$_0$ B$_0(\hat{j}x\hat{k})$ = E$_0$ B$_0(\hat{I})$

i – x direction

ANSWER : (a) $\vec{E}$=E$_0\hat{j}$ and $\vec{B}$=B$_0\,\hat{k}$

7. In an electromagnetic wave in free space the rms value of the electric field is 3 V m^{-1} . The peak value of the magnetic field is

   (a) 1.414×10^{-8} T (b) 1.0×10^{-8} T (c) 2.828×10^{-8} T (d) 2.0×10^{-8} T

Solution :

$B_{Rms} = \frac{B_0}{\sqrt{2}}$ => $B_0 = B_{Rms}\sqrt{2}$

$c = \frac{E_{Rms}}{B_{Rms}}$

$B_{Rms} = \frac{E_{Rms}}{c}$

$B_0 = \frac{E_{Rms}}{c}\sqrt{2}$

$\frac{3}{3x10^8}$ x 1.414

$=1.414 \times 10^{-8}$T

ANSWER : (a) 1.414×10^{-8} T

8. During the propagation of electromagnetic waves in a medium:

   (a) electric energy density is double of the magnetic energy density

   (b) electric energy density is half of the magnetic energy density

   (c) electric energy density is equal to the magnetic energy density

   (d) both electric and magnetic energy densities are zero

Solution :

Magnetic energy $u_B = \frac{B^2}{2\mu_0}$

Electric energy $u_E = \frac{1}{2}\varepsilon_0 E^2$

$u_E = \frac{1}{2}\varepsilon_0 (Bc)^2$

$u_E = \frac{1}{2}\varepsilon_0 B^2c^2 = \frac{1}{2}\varepsilon_0 B^2 x\, c^2$

$c^2 = \frac{1}{\mu_0\varepsilon_0}$

$=\frac{1}{2}\,\varepsilon_0 B^2 x \frac{1}{\mu_0\varepsilon_0}$

$u_E = \frac{B^2}{2\mu_0}$

$u_E = u_B$

ANSWER : (c) electric energy density is equal to the magnetic energy density

9. If the magnetic monopole exists, then which of the Maxwell's equation to be modified?

(a) $\oint \vec{E}\, d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$ (b) $\oint \vec{E}\, d\vec{A} = 0$ (c) $\oint \vec{E}\, d\vec{A} = \mu_0 I_{enclosed} + \mu_0\, \varepsilon_0 \frac{d}{dt}\int \vec{E}\, d\vec{A}$ (d) $\oint \vec{E}\, d\vec{l} = \frac{d}{dt}\Phi_B$

Solution :

The magnetic lines of force form a continuous closed path. No isolated magnetic monopole exists. Total magnetic flux=0

$\oint \vec{E}\, d\vec{A} = 0$

ANSWER : (b) $\oint \vec{E}\, d\vec{A} = 0$

10. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is

(a) $\frac{E}{c}$ (b) $2\frac{E}{c}$ (c) Ec (d) $\frac{E}{c^2}$

Solution :

Change in momentum $p = p_f - p_i$

$= \frac{E}{c} - (-\frac{E}{c}) = \frac{E}{c} + \frac{E}{c} = 2\,\frac{E}{c}$

ANSWER : (b) $2\frac{E}{c}$

11. Which of the following is an electromagnetic wave?

(a) α – rays (b) β- rays (c) γ – rays (d) all of them

Solution :

α – rays : helium nucleus,
β – rays –electron,
γ – rays- electromagnetic wave

ANSWER : (c) γ – rays

12. Which one of them is used to produce a propagating electromagnetic wave?.

(a) an accelerating charge (b) a charge moving at constant velocity
(c) a stationary charge (d) an uncharged particle

ANSWER : (a) an accelerating charge

13. Let E =Eo sin[10^6x-ωt] be the electric field of plane electromagnetic wave, the value of ω is
(a) 0.3 × 10^{-14} rad s^{-1} (b) 3 × 10^{-14} rad s^{-1}(c) 0.3 × 10^{14} rad s^{-1} (d) 3 × 10^{14} rad s^{-1}

Solution :

$E = E_o\sin[kx-\omega t]$ ---→ (1)

$E = E_o\sin[10^6x-\omega t]$ → (2)

From eqn (1) and (2)

$k = 10^6$

$k = \frac{\omega}{c}$

$\omega = kxc = 10^6 x 3x10^8$

$\omega = 3x10^{14}$ rad s^{-1}

ANSWER : (d) 3 × 10^{14} rad s^{-1}

14. Which of the following is NOT true for electromagnetic waves?.
(a) it transport energy (b) it transport momentum
(c) it transport angular momentum
(d) in vacuum, it travels with different speeds which depend on their frequency

Solution :

1)In vacuum, it travels with different speeds which depend on their frequency

2)Velocity of electromagnetic wave(light) is constant in vacuum.

ANSWER : (d) in vacuum, it travels with different speeds which depend on their frequency

15. The electric and magnetic fields of an electromagnetic wave are
(a) in phase and perpendicular to each other
(b) out of phase and not perpendicular to each other
(c) in phase and not perpendicular to each other
(d) out of phase and perpendicular to each other.

ANSWER : (a) in phase and perpendicular to each other

PART – B (2S)

1. ELECTROMAGNETIC WAVES

Electromagnetic waves are non-mechanical waves which move with speed equals to the speed of light. It is a transverse wave.

2. SPECTRUM

The definite pattern of colours obtained on the screen after dispersion.

3. FRAUNHOFER LINES

The spectrum obtained from the Sun is examined, it consists of large number of dark lines. These dark lines in the solar spectrum are known as Fraunhofer lines.

***USES** : Identifying elements present in the Sun's atmosphere.*

PART - C (3 MARKS)

1. MAXWELL'S EQUATION IN INTEGRAL FORM

Maxwell's equations completely explain the behaviour of charges, currents and properties of electric and magnetic fields. These equations can be written in integral form or derivative form.

FIRST EQUATION

The Gauss's law of electricity ,

$$\oint_s \vec{E}.\vec{dA} = \frac{Q_{enclosed}}{\varepsilon_0 t}$$

$\vec{E} \rightarrow Electric\ field$

$Q \rightarrow Net\ charge\ enclosed\ by\ the\ surface$

This equation is true for both discrete and continuous distribution of charges.

It also indicates that the electric field lines start from positive charge and terminate at negative charge.

SECOND EQUATION

This law is similar to Gauss's law for electricity. So this law can also be called as Gauss's law for magnetism. The surface integral of magnetic field over a closed surface is zero.

$$\oint_s \vec{B}.\vec{dA} = 0$$

$\vec{B} \rightarrow Magnetic\ field$

This equation implies that the magnetic lines of force form a continuous closed path.

THIRD EQUATION

It is Faraday's law of electromagnetic induction. This law relates electric field with the changing magnetic flux,

$$\oint_l \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$$

$\vec{E} \rightarrow Electric\ field$

This equation implies that the line integral of the electric field around any closed path is equal to the rate of change of magnetic flux through the closed path bounded by the surface.

FOURTH EQUATION

It is modified Ampere's circuital law. This is also known as Ampere – Maxwell law. This law relates the magnetic field around any closed path to the conduction current and displacement current through that path.

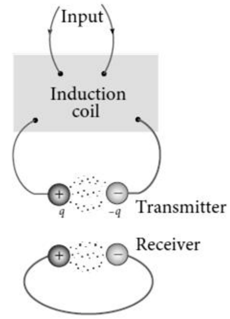
$$\oint_l \vec{B}.\vec{dl} = \mu_0\, i_c + \mu_0\, \varepsilon_0 \frac{d}{dt} \oint_s \vec{E}.\vec{dA}$$

This equation shows that both conduction current and displacement current produce magnetic field.

These four equations are known as Maxwell's equations in electrodynamics. This equation ensures the existence of electromagnetic waves. The entire communication system in the world depends on electromagnetic waves.

2. *PRODUCTION OF ELECTROMAGNETIC WAVES*

Maxwell's prediction was experimentally confirmed by Heinrich Rudolf Hertz in 1888.



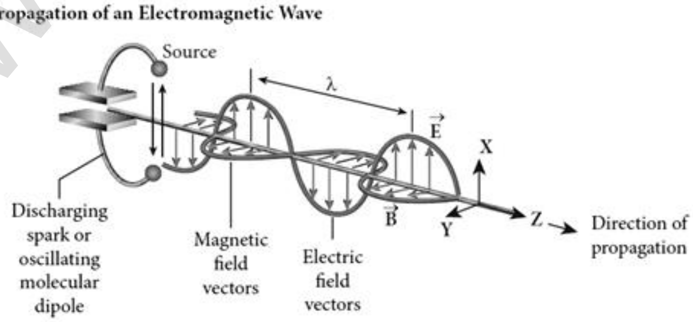
It consists of two metal electrodes which are made of small spherical metals. These are connected to larger spheres and the ends of them are connected to induction coil with very large number of turns. This is to produce very high electromotive force (emf).

Since the coil is maintained at very high potential, air between the electrodes gets ionized and spark is produced. This discharge of electricity affects another electrode which is kept at far distance. This implies that the energy is transmitted from electrode to the receiver in the form of waves, known as electromagnetic waves.

If the receiver is rotated by 90°, then no spark is observed by the receiver. This confirms that electromagnetic waves are transverse waves as predicted by Maxwell. Hertz detected radio waves and also computed the speed of radio waves which is equal to the speed of light (3×10^8 m s^{-1}).

3. *SOURCES OF ELECTROMAGNETIC WAVES*

The electromagnetic waves are transverse waves, the direction of propagation of electromagnetic waves is perpendicular to the planes containing electric and magnetic field vectors.



Any oscillatory motion is also an accelerated motion. So, when the charge oscillates about their mean position , it produces electromagnetic waves.

Suppose the electromagnetic field in free space propagates along z-direction and if the electric field vector points along x-axis, then the magnetic field vector will be mutually perpendicular to both electric field and the direction of wave propagation.

$$E_x = E_O \sin[kz - \omega t]$$

$$B_y = B_O \sin[kz - \omega t]$$

E_O and B_O are amplitudes of oscillating electric and magnetic field, k is a wave number, ω is the angular frequency of the wave and $\hat{k}$ denotes the direction of propagation of electromagnetic wave.

In free space or in vacuum, the ratio between E_O and B_O is equal to the speed of electromagnetic wave and is equal to speed of light c.

$$C = {}^{E_o}/_{B_o}$$

In any medium,

$$V = {}^{E_o}/_{B_o} < C$$

The energy of electromagnetic waves comes from the energy of the oscillating charge.

PART -D (5 MARKS)

1. DISPLACEMENT CURRENT AND MAXWELL'S CORRECTION TO AMPERE'S CIRCUTIAL LAW

INDUCED MAGNETIC FIELD

Faraday's law of electromagnetic induction states that the change in magnetic field produces an electric field. Mathematically,

$$\underbrace{\oint_l \vec{E}.\vec{dl}}_{\text{Electric field induced along a closed loop}} = \underbrace{-\frac{d}{dt}\phi_B}_{\text{Variation of Magnetic flux With time}} = \underbrace{-\frac{d}{dt}\oint_s \vec{B}.\vec{dA}}_{\text{Charging magnetic flux } \phi_B \text{ in the region enclosed by the loop}}$$

$\emptyset_B$ is the magnetic flux and $\frac{d}{dt}$ is the total derivative with respect to time.

From symmetry considerations, James Clerk Maxwell showed that the change in electric field also produces a magnetic field.

$$\underbrace{\oint_l \vec{B}.\vec{dl}}_{\text{Magnetic field induced along a closed loop}} = \underbrace{-\frac{d}{dt}\emptyset_E}_{\text{Variation of Electric flux With time}} = \underbrace{-\frac{d}{dt}\oint_s \vec{E}.\vec{dA}}_{\text{Charging electric flux } \emptyset_E \text{ in the region enclosed by the loop}}$$

$\emptyset_B$ is the Electric flux.

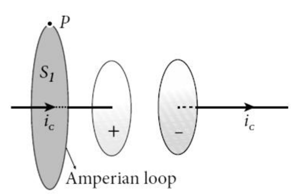
*This is known as **Maxwell's law of induction.***

This symmetry between electric and magnetic fields explains the existence of electromagnetic waves such as radio waves, gamma rays, infrared rays etc.

DISPLACEMENT CURRENT - MAXWELL'S CORRECTION

Let us consider a situation of charging a parallel plate capacitor which contains non−conducting medium between the plates.

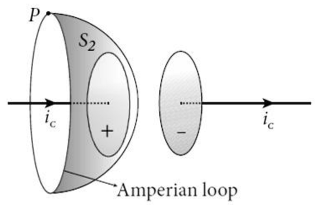
Let a time−dependent current ic, called conduction current be passed through the wire to charge the capacitor. Ampere's circuital law can be used to find the magnetic field produced around the current carrying wire.



To calculate the magnetic field at a point P near the wire and outside the capacitor, let us draw a circular Amperian loop which encloses the circular surface S_1 .Using Ampere's circuital law for this loop,

$$\oint_{enclosing\ s_1} \vec{B}.\vec{dl} = h_o i_c$$

μ_o is the permeability of free space.

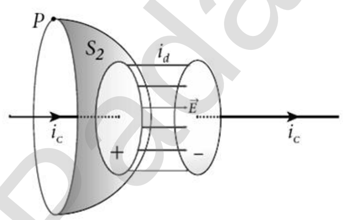


Now, the same loop is enclosed by balloon shaped surface S_2 such that boundaries of two surfaces S_1 and S_2 are same but the shape of the surfaces is different .As Ampere's law applied

$$\oint_{enclosing\ s_2} \vec{B}.\vec{dl} = 0$$

RHS of equation is zero because the surface S_2 nowhere touches the wire carrying conduction current and further, there is no current flowing between the plates of the capacitor . So the magnetic field at a point P is zero.

Maxwell resolved this inconsistency as follows: While the capacitor is being charged up, varying electric field is produced between capacitor plates. There must be a current associated with the changing electric field between capacitor plates. In other words, time−varying electric field produces a current. This is known as displacement current flowing between the plates of the capacitor .



From Gauss's law of electrostatics, the electric flux ,

$$\emptyset_E = \oint_s \vec{E}.\vec{dA} = EA = \frac{q}{\varepsilon_0}$$

A is the area of the plates of capacitor.

The change in electric flux

$$\frac{d\emptyset_E}{dt} = \frac{1}{\varepsilon_0}\frac{dq}{dt}$$

$$\frac{dq}{dt} = \varepsilon_0 \frac{d\emptyset_E}{dt}$$

$$i_d = \varepsilon_0 \frac{d\emptyset_E}{dt}$$

i_d is known as displacement current or Maxwell's displacement current.

DISPLACEMENT CURRENT

The current which comes into play in the region in which the electric field is changing with time.

Maxwell's modified Ampere's law as

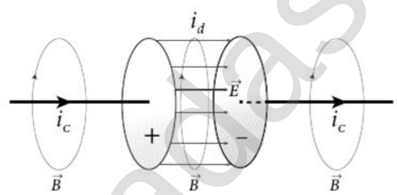
$$\oint_l \vec{B}.\vec{dl} = \mu_0 i = \mu_0 [i_c + i_d]$$

$$\oint_l \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\therefore \quad i = i_c + i_d$$

This is known as Ampere –Maxwell law, when the current is constant , the $i_d = 0$

Between the plates, $i_c = 0$ *while the displacement current is non−zero. This displacement current or time−varying electric field can also produce a magnetic field between the plates of the capacitor. The magnetic field at a point inside the capacitor is perpendicular to the electric field*



This magnetic field can be determined using ,

$$\oint_l \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

IMPORTANCE OF MAXWELL'S CORRECTION

Displacement current can also produce a magnetic field. Though conduction current is zero in an empty space,

$$\oint_l \vec{B}.\vec{dl} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

In stars, due to thermal excitation of atoms, time−varying electric field is produced which in turn, produces time− varying magnetic field. According to Faraday's law, this time−varying magnetic field produces again time−varying electric field and so on. The coupled time−varying electric and magnetic fields travel through empty space with the speed of light and is called electromagnetic wave.

2. PROPERTIES OF ELECTROMAGNETIC WAVES

1. *Electromagnetic waves are produced by any accelerated charge.*
2. *Electromagnetic waves do not require any medium for propagation. So electromagnetic wave is a non-mechanical wave.*
3. *Electromagnetic waves are transverse in nature.*
4. *Electromagnetic waves travel with speed which is equal to the speed of light in vacuum or free space.*

$$C = \frac{1}{\sqrt{\mu_O \varepsilon_O}} = 3 \text{ X } 10^8 \text{ m/s}$$

ε_O -> *Permitivity of free space or Vacuum.*

μ_O -> *Permeability of free space or Vacuum.*

5. *In a medium of refractive index,*

$$n = C/_V = \frac{1}{\sqrt{\varepsilon_O \mu_O}} \Big/ \frac{1}{\sqrt{\mu\varepsilon}} \qquad \therefore n = \sqrt{\varepsilon_r \mu_r}$$

ε-> *Permitivity of the medium.*

μ -> *Permeability of the medium.*

ε_r -> *Relative Permitivity of the medium.*

μ_r -> *Relative Permeability of the medium.*

6. *Electromagnetic waves are not deflected by electric field or magnetic field.*
7. *Electromagnetic waves can exhibit interference, diffraction and polarization.*
8. *Like other waves, electromagnetic waves also carry energy, linear momentum and angular momentum.*

3. ELECTROMAGNETIC SPECTRUM

An orderly distribution of electromagnetic waves in terms of wavelength or frequency

RADIOWAVES:

- *They are produced by accelerated motion of charges in conducting wires.*

- *Frequency range is from a few Hz 10^9 to 10^{11} Hz.*
- *It show reflection and diffraction.*

USES

1. *Radio and television communication systems*
2. *Cellular phones to transmit voice communication in the ultra high frequency UHF) band.*

MICROWAVES:

- *It is produced by special vacuum tubes such as klystron, magnetron and gunndiode.*
- *Frequency range of microwaves is 10^9 Hz to 10^{11} Hz.*
- *Undergo reflection and can be polarised.*

USES

1. *Radar system for aircraft navigation, speed of the vehicle.*
2. *Microwave oven for cooking .*
3. *Very long distance wireless communication through satellites.*

INFRARED RADIATION:

- *It is produced by hot bodies and also by when the molecules undergoing rotational and vibrational transitions.*
- *The frequency range is 10^{11} Hz to 4×10^{14} Hz.*

USES

1. *It provides electrical energy to satellites by means of solar cells.*
2. *To produces dehydrated fruits, in green houses to keep the plants warm.*
3. *Heat therapy for muscular pain or sprain.*
4. *TV remote as a signal carrier.*
5. *To look through haze fog or mist and used in night vision or infrared photography.*

VISIBLE LIGHT:

- *It is produced by incandescent bodies and also it is radiated by excited atoms in gases.*
- *Frequency range is from 4×10^{14} Hz to 8×10^{14} Hz.*
- *obeys the laws of reflection and refraction.*
- *It undergoes interference, diffraction and can be polarised.*
- *It exhibits photo-electric effect*

USES

1. *Study the structure of molecules, arrangement of electrons in external shells of atoms.*
2. *It causes sensation of vision.*

ULTRAVIOLET RADIATION:

- *It is produced by Sun, arc and ionized gases.*
- *Its frequency range is from 8×10^{14} Hz to 10^{17} Hz.*
- *Less penetrating power.*
- *It can be absorbed by atmospheric ozone and is harmful to human body.*

USES

1. *Destroy bacteria in sterilizing the surgical instruments.*
2. *Burglar alarm.*
3. *To detect the invisible writing, finger prints*
4. *Study of atomic structure.*

X-RAYS:

- *There is sudden stopping of high speed electrons at high-atomic number target, and also by electronic transitions among the innermost orbits of atoms.*
- *Frequency range of X-rays is from 10^{17} Hz to 10^{19} Hz.*
- *More penetrating power than ultraviolet radiation.*

USES

1. *Study the structures of inner atomic electron shells and crystal structures.*

2. *Detecting fractures, diseased organs, formation of bones and stones,*
3. *observing the progress of healing bones.*
4. *Detect faults, cracks, flaws and holes in metal*

GAMMA RAYS:

- *It is produced by transitions of radioactive nuclei and decay of certain elementary particles.*
- *They produce chemical reactions on photographic plates, fluorescence, ionisation, diffraction.*
- *Frequency range is 10^{18} Hz and above.*
- *Higher penetrating power than X-rays and ultraviolet radiations.*
- *No charge but harmful to human body.*

USES

1. *Study the structure of atomic nuclei.*
2. *Radio therapy for the treatment of cancer and tumour.*
3. *In food industry to kill pathogenic microorganism*

4. TYPES OF SPECTRUM

TYPES OF SPECTRUM

- ***EMISSION SPECTRA***
  - *Continuous Emission Spectrum*
  - *Line Emission Spectrum*
  - *Band Emission Spectrum*
- ***ABSORPTION SPECTRA***
  - *Continuous Absorption Spectrum*
  - *Line Absorption Spectrum*
  - *Band Absorption Spectrum*

EMISSION SPECTRA

When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic

(i) CONTINUOUS EMISSION SPECTRUM

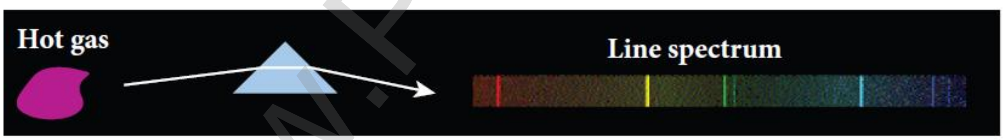
If the light from incandescent lamp is allowed to pass through prism, it splits up into seven colours. Thus, it consists of wavelengths containing all the visible colours ranging from violet to red.

***Examples**: Carbon arc and incandescent solids.*

(ii) LINE EMISSION SPECTRUM

Light from hot gas is allowed to pass through prism, line spectrum is observed Line spectra are also known as discontinuous spectra. The line spectra consists of sharp lines of definite wavelengths or frequencies. Such spectra arise due to excited atoms of elements. These lines are the characteristics of the element and are different for different elements.

***Examples**: Spectra of atomic hydrogen, helium, etc.*



(iii) BAND EMISSION SPECTRUM

It consists of several number of very closely spaced spectral lines which overlap together forming specific bands which are separated by dark spaces. This spectrum has a sharp edge at one end and fades out at the other end. Such spectra arise when the molecules are excited.

Band spectrum is study the structure of the molecules

***Examples**: Spectra of ammonia gas in the discharge tube .*

ABSORPTION SPECTRA

When light is allowed to pass through a medium or an absorbing substance then the spectrum obtained is known as absorption spectrum. It is the characteristic of absorbing substance.

(i) CONTINUOUS ABSORPTION SPECTRUM

A white light through a blue glass plate, it absorbs all the colours except blue and gives continuous absorption spectrum.

(ii) LINE ABSORPTION SPECTRUM

Light from the incandescent lamp is passed through cold gas ,the spectrum obtained through the dispersion due to prism is line absorption spectrum . Similarly, if the light from the carbon arc is made to pass through sodium vapour, a continuous spectrum of carbon arc with two dark lines in the yellow region are obtained.

(iii) BAND ABSORPTION SPECTRUM

When white light is passed through the iodine vapour, dark bands on continuous bright background is obtained and white light is passed through diluted solution of blood or chlorophyll or through certain solutions of organic and inorganic compounds.

UNSOLVED PROBLEMS

1. ***Consider a parallel plate capacitor whose plates are closely spaced. Let R be the radius of the plates and the current in the wire connected to the plates is 5 A, calculate the displacement current through the surface passing between the plates by directly calculating the rate of change of flux of electric field through the surface.***

Solution

The conduction current $I_C = 5A$

According to Gauss's Law,

Electric flux, $\theta_E = \frac{q}{\varepsilon 0}$

$$I_d = \varepsilon_0 \cdot \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{\varepsilon_0}{\varepsilon_0}\left(\frac{dq}{dt}\right)$$

$$I_c = \frac{q}{t} \Rightarrow q = I_c t$$

$$I_d = \frac{d}{dt}(I_c t) = I_c \frac{d(t)}{dt} = 5\text{ A}$$

$$I_d = I_c = 5\text{ A}$$

2. ***A transmitter consists of LC circuit with an inductance of 1 μH and a capacitance of 1 μF. What is the wavelength of the electromagnetic waves it emits?***

Solution

Inductance of LC circuit, $L = 1\ \mu H = 1 \times 10^{-6}$ H

Capacitance of LC circuit, $C = 1\ \mu F = 1 \times 10^{-6}$ F

Wave length of the electromagnetic wave $X = \lambda = \frac{C}{f}$

Velocity of light $C = 3 \times 10^8\ ms^{-1}$

Frequency of electromagnetic wave, $f = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\times 3.14\sqrt{1\times 10^{-6}\times 10^{-6}}} = \frac{1}{6.28\times 10^{-6}} \Rightarrow f = 15.92\times 10^4\text{ Hz}$$

$$\text{Wave Length } \lambda = \frac{C}{f} = \frac{3\times 10^8}{15.92\times 10^4}$$

$= 0.1884 \times 10^4$

$\lambda = 18.84 \times 10^2$ m

3. ***A pulse of light of duration 10^{-6} s is absorbed completely by a small object initially at rest. If the power of the pulse is 60×10^{-3} W, calculate the final momentum of the object.***

Solution

Duration of the absorption of light pulse, t = 10^{-6} s

Power of the pulse P = 60 x 10^{-3} W

Final momentum of the object, P = $\frac{U}{c}$

Velocity of light, C = 3 x 10^8

Energy U = power x time

Momentum, P = $\frac{60\times10^{-3}\times10^{-6}}{3\times10^8}$

P = 20 × 10^{-17} kg m s^{-1}

4. ***Let an electromagnetic wave propagate along the x - direction, the magnetic field oscillates at a frequency of 10^{10} Hz and has an amplitude of 10^{-5} T, acting along the y - direction. Then, compute the wavelength of the wave. Also write down the expression for electric field in this case.***

<u>Solution</u>

Frequency of electromagnetic wave, v = 10^{10} Hz

Amplitude of Oscillating magnetic field, B_0 = 10^{-5} T

Wave length of the wave, λ = $\frac{C}{f}$ = $\frac{3\times10^8}{10^{10}}$ = 3x 10^{-2} m

Amplitude of oscillating electric field, E_0 = B_0 C

C = $\frac{E_0}{B_0}$

E_0 = 10^{-5} × 3 × 10^8

E_0 = 3 × 10^3 = NC^{-1}

Experession for electric field in Oscillataing wave

E = E_0 sin (kx -wt)

K = $\frac{2\pi}{\lambda}$ = $\frac{2\times3.14}{3\times10^{-2}}$ = 209 x 10^2

W = 2πf = 2 × 3.14 x 10^{10} = 6.28 × 10^{10}

$\vec{E}$ = 3 x 10^3 sin (2.09 × 10^2 x – 6.28 × 10^{10} t) $\hat{i}$ NC^{-1}.

5. ***If the relative permeability and relative permittivity of a medium are 1.0 and 2.25 respectively, find the speed of the electromagnetic wave in this medium.***

<u>Solution</u>

Relative permeability of the medium, μ_r = 1

Relative permitivity of the medium, ε_r = 2.25

$$\left(\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \,\&\, \mu_r = \frac{\mu}{\mu_0}\right)$$

Speed of electromagnetic wave, v = $\frac{1}{\sqrt{\mu\varepsilon}}$

$$= \frac{1}{\sqrt{\mu_r\mu_\circ\,\varepsilon_r\varepsilon_\circ}} = \frac{C}{\sqrt{\mu_r\varepsilon_r}} \qquad \left[\text{Where, } C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

$$= \frac{3\times10^8}{\sqrt{1\times2.25}} = \frac{3\times10^8}{1.5}$$

$v = 2 \times 10^8\ \text{ms}^{-1}$

MODEL QUESTION PAPER [VOLUME 1]

HOURS *: 3 hrs* ***TOTAL MARKS*** *: 70*

PART - A

15 x 1 =15

1. *Which charge configuration produces a uniform electric field?*
   (a) point charge *(b) uniformly charged infinite line*
   (c) uniformly charged infinite plane *(d) uniformly charged spherical shell*
2. *Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is*
   (a) 8.80×10^{-17} J (b) -8.80×10^{-17} J (c) 4.40×10^{-17} J (d) 5.80×10^{-17} J
3. *A toaster operating at 240 V has a resistance of 120 Ω. Its power is*
   a) 400 W b) 2 W c) 480 W d) 240 W
4. *The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is*
   a) 0.2 Ω b) 0.5 Ω c) 0.8 Ω d) 1.0 Ω
5. *The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?*
   (a) 30° (b) 45° (c) 60° (d) 90°
6. *The potential energy of magnetic dipole whose dipole moment is* $\vec{P_m}$ =(-0.5 $\hat{\imath}$ + 0.4 $\hat{\jmath}$) Am^2 *kept in uniform magnetic field* $\vec{B}$ =0.5 $\hat{\imath} T$
   (a) –0.1 J (b) –0.8 J (c) 0.1 J (d) 0.8 J
7. *When the current changes from +2A to −2A in 0.05 s, an emf of 8 V is induced in a coil. The co-efficient of self-induction of the coil is*
   (a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H
8. *The flux linked with a coil at any instant t is given by* $\emptyset_B = 10t^2 - 50t + 250$ *.The induced emf at t = 3 s is*
   (a) –190 V (b) –10 V (c) 10 V (d) 190 V
9. *What is the unit of electric field ?*
   (a) NC^{-1} (b) Vm^{-1} (c) NC (d) a and b
10. *What is the input of van de graff generator ?*
   (a) 10^7 V (b) 10^4 V (c) 10^6 V (d) 100 V
11. *What is the critical temperature of mercury ?*
   (a) 4.2 k (b) 4.2°C (c) -268.8°C (d) a and c
12. *What is the melting point of tungsten ?*
   a) 3380°C b) 4380°C c) 5380°C d) 2830°C
13. *Unit of magnetic flux*
   a) Weber b) 10^8 Maxwell c) Tesla d) a and b
14. *Bohr magneton value _________*
   (a) 9.27×10^{-24} Am^2 (b) 9.27×10^{24} c/kg (c) 8.78×10^{10} c/kg (d) 9.27×10^{-34} JT^{-1}
15. *Unit of inductance _____________*
   (a) H (b) Wb A^2 (c) VsA^{-1} (d) All of these

PART – B

6 X 2 = 12

(Question No: 18 is compulsory)

16. *State Coulomb's law.*
17. *What is corona discharge?*
18. *Write a short note on superconductors? **
19. *State Joule's law of heating.*
20. *State Ampere's circuital law.*
21. *Define Q- factor.*
22. *What is meant by Wattless Current*
23. *State Fraunhofer lines*
24. *Write the uses of Ultraviolet waves.*

PART – C

6 X 3 = 18

***(Question** No: 31 is compulsory)*

25. *Derive an expression for electrostatic potential due to a point charge.*
26. *Explain the process of electrostatic induction.*
27. *State and explain Kirchoff's rule .*
28. *Derive the expression for power P=VI in electrical circuit.*
29. *State the application of seebeck effect.*
30. *Show that the mutual inductance between a pair of coils is same.(M_{12}= M_{21})*
31. *Is an ammeter connected in series or parallel in a circuit ? why? **
32. *Define Production of electromagnetic waves.*
33. *What is source of electromagnetic waves.*

PART – D

5 X 5 = 25

34. *Derive an expression for electrostatic potential due to an electric dipole.*

    [or]

    Explain in detail the construction and working of a Van de graaff generator.

35. *Obtain the condition for bridge balance in wheatstone's bridge.*

    [or]

Describe the microscopic model of current and Obtain general form of ohm's law.

36. *Calculate the magnetic field at a point on the equatorial line of a bar magnet.*

[or]

Derive the expression for the force on a current-carrying conductor in a magnetic field.

37. *Explain the construction and working of transformer.*

[or]

Prove that the total energy is conserved during LC oscillations.

38. *Explain Spectrum .Write the types of spectrum in detail.*

[or]

Write the properties of electromagnetic waves.

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ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

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