

12th PHYSICS

VOLUME - I

UNSOLVED PROBLEMS

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1. ELECTROSTATICS

1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

GIVEN: $q = 50 \text{ nC} = 50 \times 10^{-9} \text{ C}$.

SOLUTION: $e = 1.6 \times 10^{-19} \text{ C}$.

$$n = \frac{q}{e}$$

$$= \frac{50 \times 10^{-9}}{1.6 \times 10^{-19}} = \frac{50 \times 10^{-9} \times 10^{19}}{1.6}$$

$$= \frac{50 \times 10^{10}}{1.6} = \frac{25 \times 10^{10}}{8 \times 10^{-1}}$$

$$n = 3.125 \times 10^{11} \text{ electrons (or) } 31.25 \times 10^{10} \text{ electrons}$$

2. The total number of electrons in the human body is typically in the order of 10^{28} . Suppose, due to some reason, you and your friend lost 1% of this number of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

GIVEN: $n = 10^{28}$, $r = 1 \text{ m}$; $m = 60 \text{ kg}$

$n' = 1\% \times 10^{28} = \frac{1}{100} \times 10^{28} = 10^{26}$

n also loses 1% of e^- 's \Rightarrow
 $n = 10^{26}$

$F_e = ?$; $W = ?$

SOLUTION:

$$i) F_e = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2}$$

$$q = ne = 10^{26} \times 1.6 \times 10^{-19} = 1.6 \times 10^7 \text{ C}$$

$$q' = ne = 10^{26} \times 1.6 \times 10^{-19} = 1.6 \times 10^7 \text{ C}$$

$$\therefore F_e = \frac{9 \times 10^9 \times 1.6 \times 10^7 \times 1.6 \times 10^7}{1^2} = 9 \times 2.56 \times 10^{23}$$

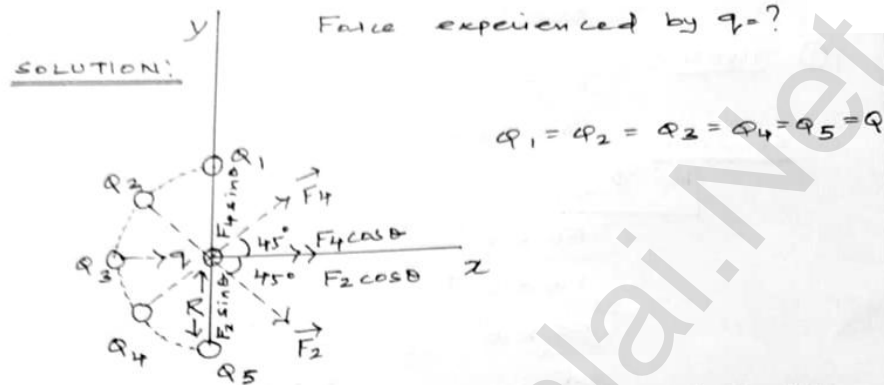
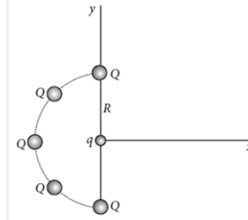
$$F_e = 23.04 \times 10^{23} \text{ N} \quad \therefore F_e = 23 \times 10^{23} \text{ N}$$

ii) Weight, $W = mg = 60 \times 9.8$

$$W = 588 \text{ N}$$

$$\therefore \frac{F_e}{W} = \frac{23 \times 10^{23}}{588} = 0.039 \times 10^{23} = 3.9 \times 10^{21}$$

3. Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the centre of the circle of radius R . Calculate the electrostatic force experienced by the charge q .



1. Forces acting on q due to Q_1 and Q_5 are equal and opposite. Hence \vec{F}_1 & \vec{F}_5 get cancelled. Net force is zero.

2. Forces acting on q due to Q_2 [\vec{F}_2] and Q_4 [\vec{F}_4] is resolved into two components.

$F_2 \sin \theta$ and $F_4 \sin \theta$ are horizontal components acts in same direction. gets added

$F_2 \sin \theta$ and $F_4 \sin \theta$ are equal in magnitude but opposite in direction, get cancelled.

$F_2 \cos \theta$ and $F_4 \cos \theta$ are horizontal components acts in same direction, gets added

3. Force acting on q due to Q_3 is \vec{F}_3

\therefore total force acting on q is

$$\vec{F} = \vec{F}_3 + F_2 \cos \theta \hat{i} + F_4 \cos \theta \hat{i} \quad [\because \theta = 45^\circ]$$

$$\vec{F} = k \frac{qQ}{R^2} \hat{i} + k \frac{qQ}{R^2} \cos 45^\circ \hat{i} + k \frac{qQ}{R^2} \cos 45^\circ \hat{i}$$

$$\begin{aligned}
 &= \frac{k q q}{R^2} \hat{i} \left[1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right] \\
 &= \frac{k q q}{R^2} \hat{i} \left[1 + \frac{2}{\sqrt{2}} \right] = \frac{k q q}{R^2} \hat{i} \left[1 + \frac{\sqrt{2} \sqrt{2}}{\sqrt{2}} \right] \\
 \therefore \vec{F} &= \frac{1}{4\pi\epsilon_0} \frac{q q}{R^2} [1 + \sqrt{2}] \hat{i} \text{ N.}
 \end{aligned}$$

4. Suppose a charge $+q$ on Earth's surface and another $+q$ charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change?
(Take $m_E = 5.9 \times 10^{24}$ kg, $m_M = 7.9 \times 10^{22}$ kg)

GIVEN: $m_E = 5.9 \times 10^{24}$ kg ; $m_M = 7.9 \times 10^{22}$ kg.

i) $q = ?$ ii) If $r = r/2$, $q = ?$

SOLUTION:

i) $F_e = F_g$

$$\frac{1}{4\pi\epsilon_0} \frac{q \cdot q}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{m_E m_M}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} \frac{q \cdot q}{r^2} = G \frac{m_E m_M}{r^2}$$

$$9 \times 10^9 q^2 = 6.67 \times 10^{-11} \times 5.9 \times 10^{24} \times 7.9 \times 10^{22}$$

$$q^2 = \frac{6.67 \times 5.9 \times 7.9 \times 10^{35}}{9 \times 10^9}$$

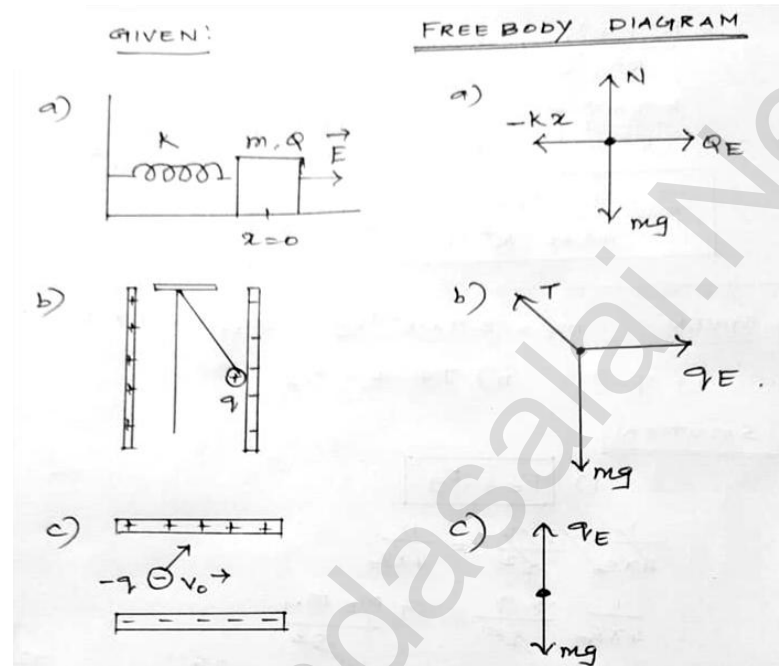
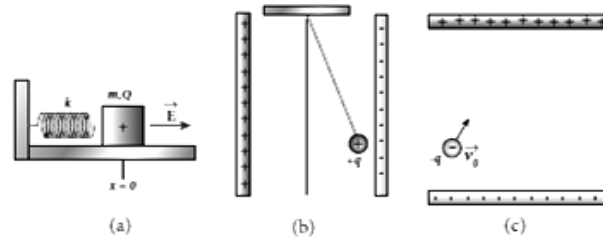
$$q = \sqrt{\frac{6.67 \times 5.9 \times 7.9 \times 10^{26}}{9}} = 10^{13} \sqrt{\frac{6.67 \times 5.9 \times 7.9}{9}}$$

$$\therefore q = 5.87 \times 10^{13} \text{ C.}$$

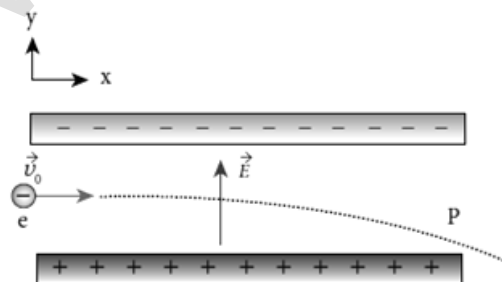
ii) $k q^2 = G m_E m_M$.

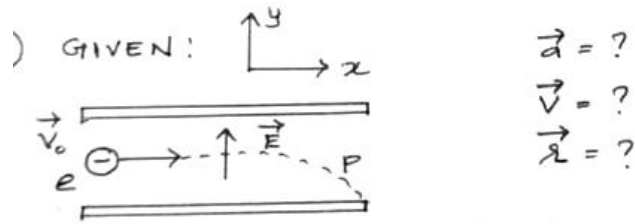
If $r = r/2$ \therefore The charge 'q' will not change.

5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).



6. Consider an electron travelling with a speed v_0 and entering into a uniform electric field \vec{E} which is perpendicular to \vec{v}_0 as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.





SOLUTION: $F = ma$, $a = F/m$.

(i) acceleration: (\vec{a})

$$F = eE$$

$$\vec{a} = \frac{eE}{m} (-\hat{j}) = -\frac{eE}{m} \hat{j}$$

(ii) Velocity: (\vec{v})

$$\vec{v} = \vec{u} + \vec{a}t, \quad u = v_0 \hat{i}, \quad \vec{a} = -\frac{eE}{m} \hat{j}$$

$$\vec{v} = v_0 \hat{i} - \frac{eE}{m} t \hat{j}$$

(iii) Position Vector (\vec{r}):

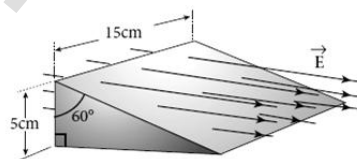
$$\vec{s} = \vec{u}t + \frac{1}{2} \vec{a}t^2$$

~~$$\vec{r} = v_0 t \hat{i} - \frac{eE}{m} t^2 \hat{j}$$~~

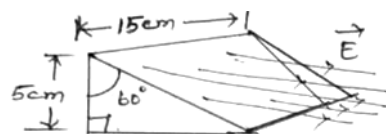
$$\vec{r} = v_0 \hat{i} + \frac{1}{2} \left[-\frac{eE}{m} \hat{j} \right] t^2$$

$$\vec{r} = v_0 \hat{i} - \frac{1}{2} \frac{eE}{m} t^2 \hat{j}$$

7. A closed triangular box is kept in an electric field of magnitude $E = 2 \times 10^3 \text{ N C}^{-1}$ as shown in the figure.



Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface.



$$E = 2 \times 10^3 \text{ Nm}^{-1}$$

$$\Phi_E = ?$$

a) Vertical rectangle surface:

$$\Phi_E = EA \cos \theta, \quad A = 15 \text{ cm} \times 5 \text{ cm}.$$

$$\Phi_E = 2 \times 10^3 \times 75 \times 10^{-4} \times \cos 0^\circ.$$

$$= 150 \times 10^{-1}$$

$$\Phi_E = 15 \text{ N m}^2 \text{ C}^{-1}.$$

b) slanted surface:

$$\Phi_E = EA \cos \theta$$

$$A = 15 \times 10^{-2} \times 10 \times 10^{-2} = 150 \times 10^{-4} \text{ m}^2$$

$$\Phi_E = 2 \times 10^3 \times 150 \times 10^{-4} \times \cos 60^\circ$$

$$= 300 \times 10^{-1} \times \frac{1}{2}.$$

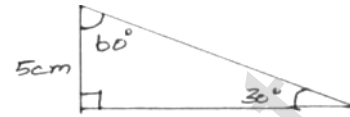
$$\Phi_E = 15 \text{ N m}^2 \text{ C}^{-1}$$

c) Entire surface:

Inward flux = outward flux.

$$\Phi_E = 0.$$

\therefore Net flux is zero.



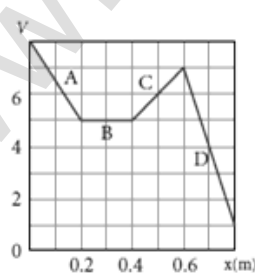
$$\sin \theta = \frac{\text{OPP.}}{\text{hyp.}}$$

$$\sin 30^\circ = \frac{5 \times 10^{-2}}{\text{hyp}}$$

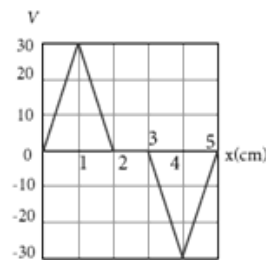
$$\text{hyp} = \frac{5 \times 10^{-2}}{\frac{1}{2}}$$

$$\text{hyp} = 10 \times 10^{-2} \text{ m}$$

8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).



(a)



(b)

$$E_x = ?$$

(i) Region A:

$$E_x = - \frac{dv}{dx}$$

From 0 to 0.2 m ; slope is -ve.

$$\therefore E_x = - \left[- \frac{dv}{dx} \right] = \frac{dv}{dx} = \frac{3}{0.2} = 15 \text{ Vm}^{-1}$$

(ii) Region B:

From 0.2 m to 0.4 m ; Potential is constant
[V = constant].

$$V = 5V$$

$$E_x = - \frac{dv}{dx} = 0$$

(iii) Region C:

From 0.4 m to 0.6 m ; slope is +ve.

$$E_x = - \frac{dv}{dx} = - \frac{2}{0.2} = -10 \text{ Vm}^{-1}$$

(iv) Region D:

From 0.6 m to 0.8 m , slope is -ve.

$$E_x = - \left[- \frac{dv}{dx} \right] = \frac{dv}{dx} = \frac{6}{0.2} = 30 \text{ Vm}^{-1}$$

- For Fig(b)

(i) From 0 to 1m.

slope +ve

$$E_x = - \frac{dv}{dx} = - \frac{30}{1}$$

$$E_x = -30 \text{ Vm}^{-1}$$

(ii) From 1 to 2m.

slope -ve.

$$E_x = - \left[\frac{dv}{dx} \right] = \frac{dv}{dx} = \frac{30}{1}$$

$$E_x = 30 \text{ Vm}^{-1}$$

(iii) From 2 to 3m. V = 0

$$\therefore E_x = - \frac{dv}{dx} = 0$$

(iv) From 3 to 4cm.

slope -ve.

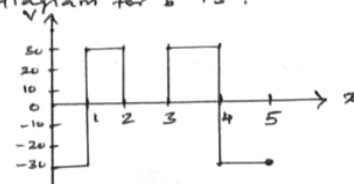
$$E_x = - \left[- \frac{dv}{dx} \right] = \frac{dv}{dx} = \frac{30}{1}$$

$$E_x = 30 \text{ Vm}^{-1}$$

(v) From 4 to 5cm ; slope +ve.

$$E_x = - \frac{dv}{dx} = - \frac{30}{1} = -30 \text{ Vm}^{-1}$$

Electric field as a function of 'x'
Diagram for b' is.



9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure.



To create the spark, an electric field of magnitude $3 \times 10^6 \text{ Vm}^{-1}$ is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same? (c) find the potential difference if the gap is 1 mm.

1) Given: $E = 3 \times 10^6 \text{ V m}^{-1}$; $d = 0.6 \text{ mm} = 6 \times 10^{-4} \text{ m}$.
 $V = ?$

Solution:

a) Potential difference, $V = E \times d$

$$V = 3 \times 10^6 \times 6 \times 10^{-4} = 18 \times 10^2 \text{ V} = 1800 \text{ Volt.}$$

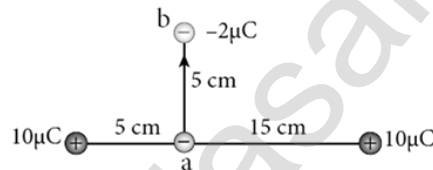
b) $V \propto d$.

If gap increases, then potential also will increase.

c) If $d = 1 \text{ mm}$, $V = E \times d$.

$$V = 3 \times 10^6 \times 1 \times 10^{-3} = 3 \times 10^3 \text{ V} = 3000 \text{ Volt.}$$

10. A point charge of $+10 \mu\text{C}$ is placed at a distance of 20 cm from another identical point charge of $+10 \mu\text{C}$. A point charge of $-2 \mu\text{C}$ is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.



GIVEN: $q_1 = q_2 = 10 \mu\text{C}$; $q = -2 \mu\text{C}$.

$$r_1 = 5 \text{ cm}, r_2 = 15 \text{ cm}.$$

$$r_1' = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2} \text{ cm}.$$

$$r_2' = \sqrt{15^2 + 5^2} = \sqrt{250} = \sqrt{25 \times 10} = 5\sqrt{10} \text{ cm}.$$

\therefore change in P.E $\Delta U = ?$

SOLUTION:

(i) Initial potential energy (U_i) when $-2 \mu\text{C}$ charge at 'a'.

$$U_i = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q}{r_2}$$

$$= \frac{q q_1}{4\pi\epsilon_0} \left[\frac{1}{r_1} + \frac{1}{r_2} \right] \quad \because q_1 = q_2$$

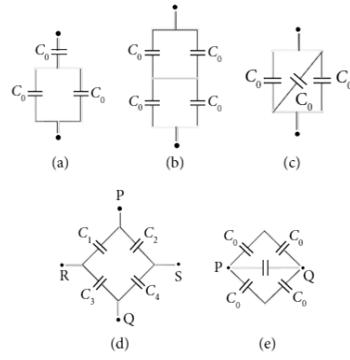
$$U_i = 9 \times 10^9 \times (-2 \times 10^{-6}) \times 10 \times 10^{-6} \times \left[\frac{1}{5} + \frac{1}{15} \right] \times \frac{1}{182}$$

$$= -18 \times 10^2 \times \left[\frac{20}{75} \right] \times \frac{1}{10^2}$$

$$= -18 \times 10^2 \times \frac{4}{15} \times \frac{1}{10^2} = -\frac{24}{5}$$

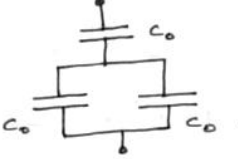
$$\boxed{U_i = -4.85}$$

11. Calculate the resultant capacitances for each of the following combinations of capacitors.



SOLUTION:

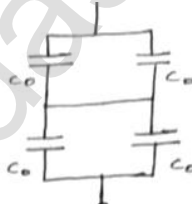
FIG (A):

$C_p = C_0 + C_0 = 2C_0$


 $\frac{1}{C_s} = \frac{1}{C_0} + \frac{1}{2C_0} = \frac{2+1}{2C_0} = \frac{3}{2C_0}$

 $C_s = \frac{2C_0}{3}$

FIG (B):

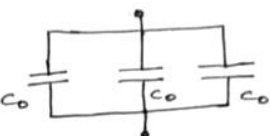
$C_p = C_0 + C_0 = 2C_0$


 $\frac{1}{C_s} = \frac{1}{2C_0} + \frac{1}{2C_0}$

 $= \frac{2}{2C_0} = \frac{1}{C_0}$

 $C_s = C_0$

FIG (C):

$C_p = C_0 + C_0 + C_0$


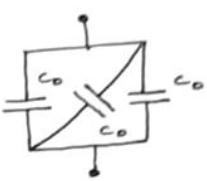
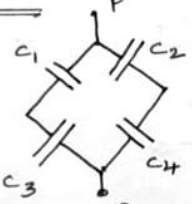
 $C_p = 3C_0$


FIG (D):

Equivalent capacitance b/w P and Q:
 

 C_1 and C_3 in series,
 $\frac{1}{C_{S1}} = \frac{1}{C_1} + \frac{1}{C_3} = \frac{C_3 + C_1}{C_1 C_3}$

 $C_{S1} = \frac{C_1 C_3}{C_1 + C_3}$

C_2 and C_4 in series,

$$\frac{1}{C_{S2}} = \frac{1}{C_2} + \frac{1}{C_4} = \frac{C_2 + C_4}{C_2 C_4} \Rightarrow \boxed{C_{S2} = \frac{C_2 C_4}{C_2 + C_4}}$$

C_{S1} and C_{S2} are in parallel.

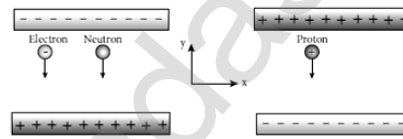
$$C_{PQ} = C_{S1} + C_{S2}$$

$$= \frac{C_1 C_3}{C_1 + C_3} + \frac{C_2 C_4}{C_2 + C_4}$$

$$C_{PQ} = \frac{C_1 C_3 [C_2 + C_4] + C_2 C_4 [C_1 + C_3]}{(C_1 + C_3) \cdot (C_2 + C_4)}$$

$$\boxed{C_{PQ} = \frac{C_1 C_2 C_3 + C_1 C_3 C_4 + C_1 C_2 C_4 + C_2 C_3 C_4}{(C_1 + C_3) (C_2 + C_4)}}$$

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance $h = 1$ mm as shown in the figure.



- (a) Calculate the time of flight for both electron and proton (b) Suppose if a neutron is allowed to fall, what is the time of flight? (c) Among the three, which one will reach the bottom first? (Take $m_p = 1.6 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg and $g = 10$ ms^{-2})

12) Given: $m_p = 1.6 \times 10^{-27}$ kg ; $m_e = 9.1 \times 10^{-31}$ kg ☺
 $g = 10 \text{ ms}^{-2}$

Solution:

$$h = 1 \text{ mm} = 1 \times 10^{-3} \text{ m} ; \quad V = 5 \text{ V}$$

(i) Time of flight (t_e):

$$E = \frac{V}{h} = \frac{5}{10^{-3}} = 5 \times 10^3 \text{ N C}^{-1}$$

For electron:

$$s = ut + \frac{1}{2} at^2$$

$$s = h ; \quad u = 0 ; \quad a = \frac{F}{m} = \frac{eE}{m} ; \quad t = t_e$$

$$h = 0 + \frac{1}{2} \left[\frac{eE}{m} \right] t_e^2 = \frac{1}{2} \left(\frac{eE}{m} \right) t_e^2$$

$$t_e = \sqrt{\frac{2mh}{eE}} = \sqrt{\frac{2 \times 9.1 \times 10^{-31} \times 1 \times 10^{-3}}{1.6 \times 10^{-19} \times 5 \times 10^3}}$$

$$t_e = \sqrt{\frac{18.2 \times 10^{-18}}{84}} = \frac{10^{-9}}{2} \sqrt{9.1} = \frac{3 \times 10^{-9}}{2} \quad \sqrt{9.1} \approx 3$$

$$t_e = 1.5 \times 10^{-9} \text{ s} \quad ; \quad \boxed{t_e = 1.5 \text{ ns}}$$

(ii) Time of flight for proton (t_p):

$$s = ut + \frac{1}{2}at^2 \quad ; \quad s = h; \quad u = 0; \quad t = t_p; \quad a = \frac{Ee}{m_p}$$

$$\therefore h = 0 + \frac{1}{2} \left[\frac{eE}{m_p} \right] t_p^2$$

$$t_p = \sqrt{\frac{2m_p h}{eE}} = \sqrt{\frac{2 \times 1.6 \times 10^{-27} \times 1 \times 10^{-3}}{1.6 \times 10^{-19} \times 5 \times 10^3}}$$

$$t_p = \sqrt{0.4 \times 10^{-14}} = 10^{-7} \sqrt{0.4} = 10^{-7} \times 0.632$$

$$= 10^{-7} \times 6.3 \times 10^{-1} = 6.3 \times 10^{-8}$$

$$\boxed{t_p = 63 \text{ ns}}$$

(iii) Time of flight for neutron (t_n):

$$t_n = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1 \times 10^{-3}}{10}} \quad \begin{array}{l} s = ut + \frac{1}{2}at^2 \\ u = 0; \quad a = g; \quad s = h \\ t = t_n. \end{array}$$

$$t_n = 10^{-2} \sqrt{2} = 1.414 \times 10^{-2} \text{ s}$$

$$\boxed{t_n = 14.14 \text{ ms}}$$

$$\therefore t_e < t_p < t_n$$

\therefore Electrons will reach the bottom first.

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air ($3 \times 10^6 \text{ Vm}^{-1}$), lightning will occur.

(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25 C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

GIVEN:

$$E = 3 \times 10^6 \text{ Vm}^{-1} ; d = 1000 \text{ m} ; V = ? , u = ? , q = 25 \text{ C.}$$

SOLUTION:

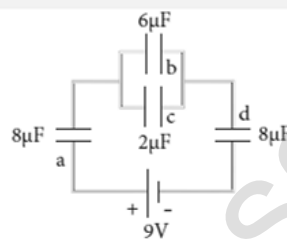
(i) Electric potential difference $V = E \times d$.

$$V = 3 \times 10^6 \times 1000 = \underline{3 \times 10^9 \text{ Volt.}}$$

(ii) Electro static potential energy (U):

$$U = V \times q = 3 \times 10^9 \times 25 = \underline{75 \times 10^9 \text{ J.}}$$

14. For the given capacitor configuration (a) Find the charges on each capacitor (b) potential difference across them (c) energy stored in each capacitor

SOLUTION: $V = 9 \text{ V}$

$$C_a = 8 \mu\text{F}, C_b = 6 \mu\text{F.}$$

$$C_c = 2 \mu\text{F}, C_d = 8 \mu\text{F.}$$

C_b and C_c in parallel

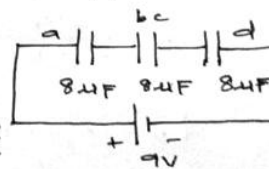
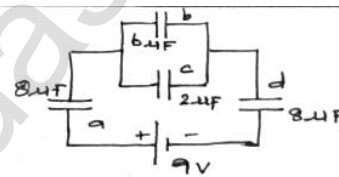
$$C_{bc} = C_b + C_c = 6 + 2 = 8 \mu\text{F.}$$

Potential difference in each capacitor:

$$V_a = V/3 = 9/3 = 3 \text{ Volt.}$$

$$V_b = V_c = V_d = V/3 = 9/3 = 3 \text{ Volt.}$$

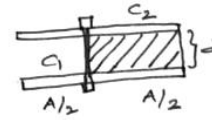
$$\boxed{V_a = V_b = V_c = V_d = 3 \text{ Volt.}}$$



15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ϵ_r as shown in the figure. Calculate the capacitance of capacitors P and Q.

capacitor 'P'

(i) capacitance of a parallel plate capacitor, $C = \frac{\epsilon_0 A}{d}$.



with dielectric, $C = \frac{\epsilon_r \epsilon_0 A}{d}$.

In capacitor 'P' C_1 and C_2 parallel.

$$C_p = C_1 + C_2$$

$$C_1 = \frac{\epsilon_0 A}{d} \quad \text{here } A = A/2$$

$$C_1 = \frac{\epsilon_0 (A/2)}{d} = \frac{\epsilon_0 A}{2d}$$

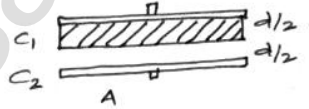
$$\therefore C_2 = \frac{\epsilon_r \epsilon_0 (A/2)}{d} = \frac{\epsilon_r \epsilon_0 A}{2d}$$

$$\therefore C_p = \frac{\epsilon_0 A}{2d} + \frac{\epsilon_r \epsilon_0 A}{2d}$$

$$C_p = \frac{\epsilon_0 A}{2d} (1 + \epsilon_r)$$

In capacitor Q:

$$A = A ; d = d/2.$$



C_1, C_2 are in series

$$\frac{1}{C_q} = \frac{1}{C_1} + \frac{1}{C_2} ; C_1 = \frac{\epsilon_r \epsilon_0 A}{d/2}$$

$$C_1 = \frac{2\epsilon_r \epsilon_0 A}{d}$$

$$C_2 = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d}$$

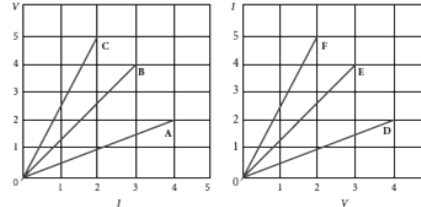
$$\frac{1}{C_q} = \frac{d}{2\epsilon_r \epsilon_0 A} + \frac{d}{2\epsilon_0 A} = \frac{d}{2\epsilon_0 A} \left[\frac{1}{\epsilon_r} + 1 \right]$$

$$\frac{1}{C_q} = \frac{d}{2\epsilon_0 A} \left[\frac{1 + \epsilon_r}{\epsilon_r} \right]$$

$$C_q = \frac{2\epsilon_0 A}{d} \left[\frac{\epsilon_r}{1 + \epsilon_r} \right]$$

2. CURRENT ELECTRICITY

1. The following graphs represent the current versus voltage and voltage versus current for the six conductors A, B, C, D, E and F. Which conductor has least resistance and which has maximum resistance?



Solution

According to ohm's law,

$$V = IR; \quad R = \frac{V}{I}$$

GRAPH 1:

$$A : \text{slope} = R_A = \frac{\Delta V}{\Delta I} = \frac{2}{4} = 0.5 \Omega$$

$$B : \text{slope} = R_B = \frac{\Delta V}{\Delta I} = \frac{4}{3} = 1.33 \Omega$$

$$C : \text{slope} = R_C = \frac{\Delta V}{\Delta I} = \frac{5}{2} = 2.5 \Omega$$

GRAPH 2

$$D : \text{slope} = R_D = \frac{\Delta V}{\Delta I} = \frac{4}{2} = 2 \Omega$$

$$E : \frac{1}{\text{slope}} = R_E = \frac{\Delta V}{\Delta I} = \frac{3}{4} = 0.75 \Omega$$

$$F : \frac{1}{\text{slope}} = R_F = \frac{\Delta V}{\Delta I} = \frac{2}{5} = 0.4 \Omega$$

Least Resistance, $R_F = 0.4 \Omega$

Maximum Resistance, $R_C = 2.5 \Omega$

2. Lightning is very good example of natural current. In typical lightning, there is 10^9 J energy transfer across the potential difference of 5×10^7 V during a time interval of 0.2 s.

Solution

$$N = 10^9 \text{ J}; V = 5 \times 10^7 \text{ V}; t = 0.2 \text{ s} = 2 \times 10^{-1} \text{ s}.$$

1. Total amount of charge:

$$W = V \cdot q$$

$$q = \frac{W}{V} = \frac{10^9}{5 \times 10^7} = 0.2 \times 10^2$$

$$\boxed{q = 20 \text{ C}}$$

2. CURRENT : $I = \frac{q}{t}$

$$= \frac{20}{2 \times 10^{-1}} = 100 \text{ A}$$

3. POWER : $P = VI$

$$= 5 \times 10^7 \times 10^2 = 5 \times 10^9 \text{ W}$$

$$\boxed{P = 5 \text{ GW}}$$

3. A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of free electrons per cubic meter in the wire is 8×10^{28} , calculate the current density and average drift velocity of electrons.

Solution

SOLUTION:

$$J = I/A = \frac{2}{10^{-6}} = 2 \times 10^6 \text{ A m}^{-2}$$

$$J = neV_d$$

$$V_d = \frac{J}{ne} = \frac{2 \times 10^6}{8 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= \frac{10^{-3}}{6.4} = 1.56 \times 10^{-4} \text{ m/s}$$

$$\boxed{V_d = 15.6 \times 10^{-5} \text{ m s}^{-1}}$$

4. The resistance of a nichrome wire at 20°C is $10\ \Omega$. If its temperature coefficient of resistivity of nichrome is $0.004/^{\circ}\text{C}$, find the resistance of the wire at boiling point of water. Comment on the result.

Solution

$$\alpha = \frac{R_T - R_0}{R_0 [T - T_0]}$$

$$0.004 = \frac{R_{100} - 10}{10 [100 - 20]}$$

$$= \frac{R_{100} - 10}{10 [80]}$$

$$[0.004 \times 800] = R_{100} - 10$$

$$3.2 + 10 = R_{100}$$

$$\boxed{R_{100} = 13.2\ \Omega}$$

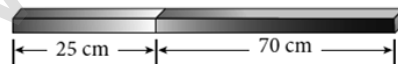
[OR]

$$R_t = R_0 [1 + \alpha (T - T_0)]$$

$$R_{100} = 10 [1 + 0.004 [100 - 20]]$$

$$R_{100} = 10 [1 + 0.32] = 13.2\ \Omega$$

5. The rod given in the figure is made up of two different materials.



Both have square cross sections of 3 mm side. The resistivity of the first material is $4 \times 10^{-3}\ \Omega\text{m}$ and that of second material has resistivity of $5 \times 10^{-3}\ \Omega\text{m}$. What is the resistance of rod between its ends?

Solution

$$P_1 = 4 \times 10^{-3} \text{ m} ; \rho_1 = 25 \text{ cm.}$$

$$P_2 = 5 \times 10^{-3} \text{ m} ; \rho_2 = 70 \text{ cm.}$$

side length = 3 mm.

$$\text{Area } A = [3 \times 10^{-3}]^2$$

$$A = 9 \times 10^{-6} \text{ m}^2.$$

SOLUTION:

$$R_s = R_1 + R_2$$

$$R_1 = \frac{\rho_1 P_1}{A} = \frac{4 \times 10^{-3} \times 25 \times 10^{-2}}{9 \times 10^{-6}}$$

$$= \frac{4 \times 25 \times 10^{-5}}{9 \times 10^{-6}} = \frac{1000}{9} \Omega$$

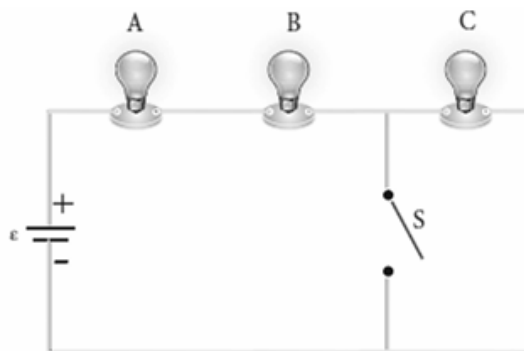
$$R_2 = \frac{\rho_2 P_2}{A} = \frac{5 \times 10^{-3} \times 70 \times 10^{-2}}{9 \times 10^{-6}}$$

$$= \frac{5 \times 70 \times 10^{-5}}{9 \times 10^{-6}} = \frac{3500}{9} \Omega$$

$$R_s = \frac{1000}{9} + \frac{3500}{9} = \frac{4500}{9}$$

$$R_s = 500 \Omega$$

6. Three identical lamps each having a resistance R are connected to the battery of emf ε as shown in the figure.



Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A, B and C . (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decrease, increase or remain same?

Solution

a) CURRENT:

ohm's law $\frac{V}{R} = I$ $V = IR$

$\mathcal{E} = V$.

$$R_1 = R_2 = R_3 = R$$

Three lamps are connected in series, $R_s = R_1 + R_2 + R_3$.

$$R_s = R + R + R = 3R.$$

Switch is open:

$$I = \frac{V}{R_s} = \frac{\mathcal{E}}{3R}.$$

Switch is closed:

No current flow through lamp 'c'.

$$I = \frac{\mathcal{E}}{2R}.$$

b) INTENSITIES:

switch is open:

All bulbs having equal intensities.

Switch is closed:

A and B equal intensiti
But 'c' will not glow.
because no current.

c) VOITAGE:

switch is open:

$$I = \frac{\mathcal{E}}{3R}$$

$$V_A = IR = \frac{\mathcal{E}}{3R} \times R = \frac{\mathcal{E}}{3}$$

$$V_B = IR = \frac{\mathcal{E}}{3R} \times R = \frac{\mathcal{E}}{3}$$

$$V_C = IR = \frac{\mathcal{E}}{3R} \times R = \frac{\mathcal{E}}{3}$$

switch is closed:

$$V_A = IR = \frac{\mathcal{E}}{2R} \times R = \frac{\mathcal{E}}{2}$$

$$V_B = IR = \frac{\mathcal{E}}{2R} \times R = \frac{\mathcal{E}}{2}$$

$$V_C = IR = 0 \times R = 0.$$

[Bulb 'c' is in parallel]

d) POWER:

switch is open:

$$P_A = V_A I_A = \frac{\mathcal{E}}{3} \times \frac{\mathcal{E}}{3R}$$

$$= \frac{\mathcal{E}^2}{9R}.$$

$$P_A = P_B = P_C. \text{ [power is same]}$$

Switch is closed:

$$P_A = V_A I_A = \frac{\mathcal{E}}{2} \times \frac{\mathcal{E}}{2R}$$

$$P_A = \frac{\mathcal{E}^2}{4R}.$$

$$P_B = V_B I_B = \frac{\mathcal{E}^2}{4R}.$$

$$P_C = V_C I_C = 0.$$

\therefore Total power will increase.

7. An electronics hobbyist is building a radio which requires 150Ω in her circuit.

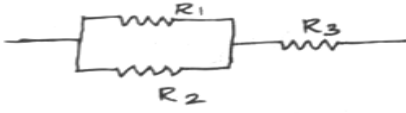
But she has only 220Ω , 79Ω and 92Ω resistors available. How can she connect the available resistors to get the desired value of resistance?

Solution

Resistance required $R = 150 \Omega$

$R_1 = 220 \Omega$, $R_2 = 79 \Omega$


$R_3 = 92 \Omega$



$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{220 \times 79}{220 + 79}$

$R_p = \frac{17380}{299} = 58 \Omega$

Now, R_p and R_3



$R_s = R_p + R_3 = 58 + 92$

$\therefore R_s = 150 \Omega$

Therefore , parallel combination of 230Ω and 79Ω in series with 92Ω

8. A cell supplies a current of 0.9 A through a 2Ω resistor and a current of 0.3 A through a 7Ω resistor. Calculate the internal resistance of the cell.

Solution

GIVEN: $I_1 = 0.9 \text{ A}$, $R_1 = 2 \Omega$
 $I_2 = 0.3 \text{ A}$, $R_2 = 7 \Omega$
 $r = ?$

SOLUTION:

$I_1 = \frac{E}{R_1 + r}$; $E = I_1 [R_1 + r]$

$I_2 = \frac{E}{R_2 + r}$; $E = I_2 [R_2 + r]$

$I_1 R_1 + I_1 r = I_2 R_2 + I_2 r$

$I_1 r - I_2 r = I_2 R_2 - I_1 R_1$

$r [I_1 - I_2] = I_2 R_2 - I_1 R_1$

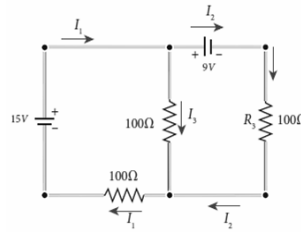
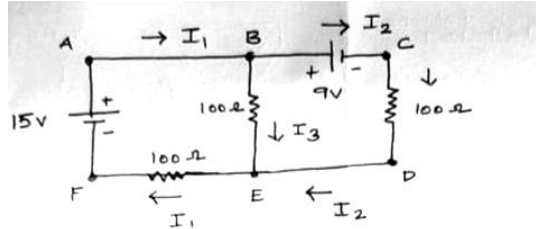
$r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}$

$= \frac{0.3 \times 7 - 0.9 \times 2}{0.9 - 0.3}$

$= \frac{2.1 - 1.8}{0.6} = \frac{0.3}{0.6} = \frac{1}{2}$

$r = 0.5 \Omega$

9. Calculate the currents in the following circuit.

Solution

$$I_1 = ? \quad ; \quad I_2 = ? \quad ; \quad I_3 = ?$$

Applying Kirchhoff's current rule at junction B.

$$I_1 - I_2 - I_3 = 0 \quad ; \quad I_3 = I_1 - I_2 \quad \rightarrow (1)$$

Applying Kirchhoff's Voltage rule for closed path ABFEA, $100I_1 + 100I_3 = 15$

$$100I_1 + 100[I_1 - I_2] = 15$$

$$100I_1 + 100I_1 - 100I_2 = 15.$$

$$200I_1 - 100I_2 = 15 \quad \rightarrow (2)$$

Applying Kirchhoff's Voltage rule for closed path BCDEB,

$$100I_2 - 100I_3 = -9$$

$$100I_2 - 100[I_1 - I_2] = -9$$

$$100I_2 - 100I_1 + 100I_2 = -9$$

$$200I_2 - 100I_1 = -9$$

$$-100I_1 + 200I_2 = -9 \quad \rightarrow (3)$$

$$(2) \Rightarrow 200I_1 - 100I_2 = 15.$$

$$(3) \times 2 \Rightarrow -200I_1 + 400I_2 = -18.$$

$$\frac{300I_2 = -3}{100}$$

$$I_2 = \frac{-1}{10^2} = -1 \times 10^{-2} \Rightarrow I_2 = -0.01 \text{ A}$$

$$(2) \Rightarrow 200I_1 - 100 \times \frac{-1}{100} = 15$$

$$200I_1 + 1 = 15$$

$$200I_1 = 14 \quad ; \quad I_1 = \frac{14}{200}$$

$$I_1 = 0.07 \text{ A}$$

$$(1) \Rightarrow I_3 = I_1 - I_2$$

$$= 0.07 - [-0.01]$$

$$I_3 = 0.08 \text{ A}$$

10. A potentiometer wire has a length of 4 m and resistance of 20Ω . It is connected in series with resistance of 2980Ω and a cell of emf 4 V. Calculate the potential gradient along the wire.

GIVEN:

$$l = 4 \text{ m}, R = 20 \Omega$$

$$E = 4 \text{ V}, R' = 2980 \Omega$$

$$E = ?$$

SOLUTION:

Effective resistance for two resistors in series combination. $R_s = R + R'$

$$R_s = 20 + 2980$$

$$R_s = 3000 \Omega$$

$$I = \frac{E}{R} = \frac{4}{3000} \text{ A.}$$

Potential drop across the wire,

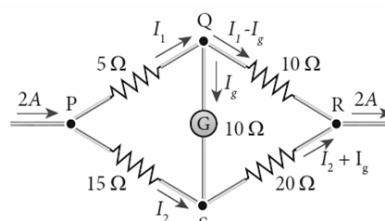
$$V = IR; V = I R = \frac{4}{3000} \times 20 = \frac{4}{150} \text{ Volt}$$

Potential gradient, $E = \frac{V}{l}$

$$= \frac{4}{150} \times \frac{1}{4} = \frac{1}{150} = \frac{1}{15 \times 10} = 0.066 \times 10^{-1}$$

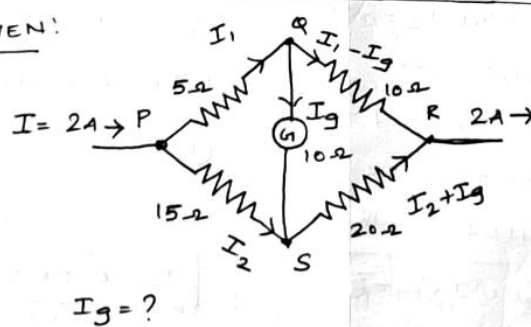
$$E = 0.66 \times 10^{-2} \text{ V m}^{-1}$$

11. Determine the current flowing through the galvanometer (G) as shown in the figure.



Solution

GIVEN:

 $I_g = ?$ SOLUTION:

Applying Kirchhoff's Current rule at junction P.

$$I - I_1 - I_2 = 0$$

$$I - I_1 = I_2 \rightarrow \textcircled{1}$$

Apply Kirchhoff's Voltage rule for closed path PQSP.

$$5I_1 + 10I_g - 15I_2 = 0$$

$$\textcircled{1} \Rightarrow 5I_1 + 10I_g - 15[I - I_1] = 0$$

$$5I_1 + 10I_g - 15I + 15I_1 = 0$$

$$I = 2A, \quad 20I_1 + 10I_g - 15 \times 2 = 0$$

$$20I_1 + 10I_g = 30 \rightarrow \textcircled{2}$$

Apply Kirchhoff's Voltage rule for closed path QRSP.

$$10[I_1 - I_g] - 20[I_2 + I_g] - 10I_g = 0$$

$$10I_1 - 10I_g - 20I_2 - 20I_g - 10I_g = 0$$

$$\textcircled{1} \Rightarrow 10I_1 - 40I_g - 20[I - I_1] = 0$$

$$10I_1 - 40I_g - 20I + 20I_1 = 0$$

$$I = 2A, \quad 30I_1 - 40I_g - 20[2] = 0$$

$$30I_1 - 40I_g = 40 \rightarrow \textcircled{3}$$

$$\textcircled{2} \times 3 \Rightarrow 60I_1 + 30I_g = 90$$

$$\textcircled{3} \times 2 \Rightarrow \begin{array}{r} 60I_1 - 80I_g = 80 \\ \hline [-] \quad [+]\quad [-] \end{array}$$

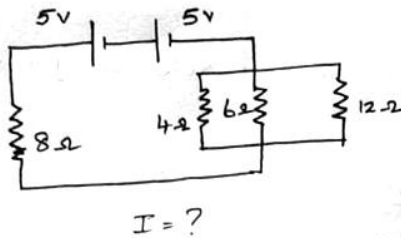
$$110I_g = 10$$

$$I_g = \frac{10}{110} = \frac{1}{11}$$

$$I_g = \frac{1}{11} A$$

12. Two cells each of 5V are connected in series with a $8\ \Omega$ resistor and three parallel resistors of $4\ \Omega$, $6\ \Omega$ and $12\ \Omega$. Draw a circuit diagram for the above arrangement. Calculate i) the current drawn from the cells (ii) current through each resistor

Solution



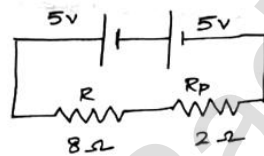
SOLUTION:

$$R_1 = 4\ \Omega ; R_2 = 6\ \Omega ; R_3 = 12\ \Omega$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{3+2+1}{12}$$

$$\frac{1}{R_P} = \frac{6}{12} \Rightarrow R_P = 2\ \Omega$$

$$R_P = 2\ \Omega$$



$$R_S = R + R_P$$

$$R_S = 8 + 2 = 10\ \Omega$$

$$R_S = 10\ \Omega$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = 5 + 5 = 10\text{V}$$

\therefore current drawn from each cell,

$$I = \frac{\mathcal{E}}{R_S} = \frac{10}{10} = 1\text{A}$$

Potential through $8\ \Omega$ resistor is $V = IR$

$$V = 1 \times 8 = 8\text{V}$$

\therefore Potential through $2\ \Omega$; $V = IR = 1 \times 2 = 2\text{V}$

(i) CURRENT through $8\ \Omega$; $I = \frac{V}{R} = \frac{8}{8}$; $I = 1\text{A}$

(ii) CURRENT through $4\ \Omega, 6\ \Omega, 12\ \Omega$;

$$R = 4\ \Omega ; I = \frac{V}{R} = \frac{2}{4} ; R = 6\ \Omega ; I = \frac{V}{R} = \frac{2}{6} ; R = 12\ \Omega ; I = \frac{V}{R} = \frac{2}{12}$$

$$I = 0.5\text{A}$$

$$I = 0.33\text{A}$$

$$I = 0.166\text{A}$$

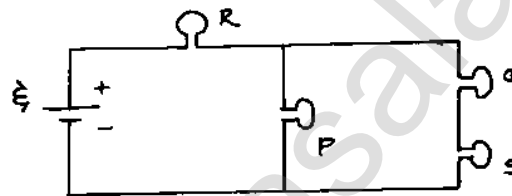
13. Four bulbs P, Q, R, S are connected in a circuit of unknown arrangement.

When each bulb is removed one at a time and replaced, the following behaviour is observed.

	P	Q	R	S
P removed	*	on	on	on
Q removed	on	*	on	off
R removed	off	off	*	off
S removed	on	off	on	*

Draw the circuit diagram for these bulbs.

Solution



14. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?

Solution

GIVEN: $\mathcal{E}_1 = 1.25 \text{ V}$; $\mathcal{E}_2 = ?$ $\mathcal{E}_1 = 5/4 \text{ V}$

$r_1 = 35 \times 10^{-2} \text{ m}$; $r_2 = 63 \times 10^{-2} \text{ m}$

SOLUTION:

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{r_2}{r_1} ; \mathcal{E}_2 = \mathcal{E}_1 \times \frac{r_2}{r_1}$$

$$\mathcal{E}_2 = \frac{5}{4} \times \frac{63 \times 10^{-2}}{35 \times 10^{-2}} = \frac{9}{4} = 2.25$$

$$\mathcal{E}_2 = 2.25 \text{ Volt}$$

3. MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

1. A bar magnet having a magnetic moment \vec{p}_m is cut into four pieces i.e., first cut into two pieces along the axis of the magnet and each piece is further cut along the axis into two pieces. Compute the magnetic moment of each piece.

Magnetic moment \vec{p}_m .

When a bar magnet is first cut into two pieces along the axis, their magnetic moment is

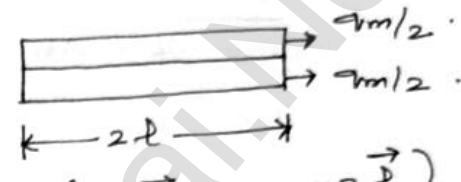
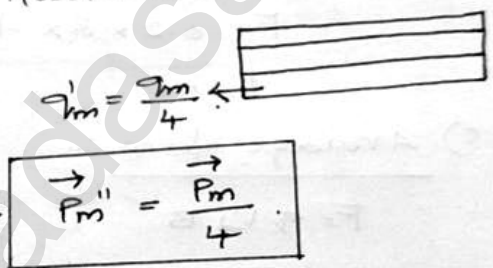
$$\vec{p}_m' = \frac{\vec{p}_m}{2}$$

$$\vec{p}_m' = \vec{q}_m' \times 2l$$

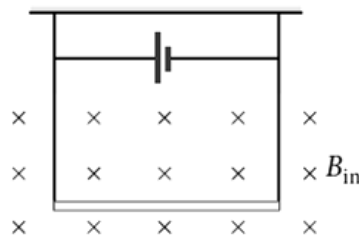
$$\vec{p}_m' = \frac{q_m}{2} \times 2l \quad \vec{p}_m' = \frac{\vec{p}_m}{2} \quad (\because \vec{p}_m = q_m \times 2l)$$

Each piece is further cut into pieces.

$$\therefore \vec{p}_m'' = q_m'' \times 2l$$

$$= \frac{q_m}{4} \times 2l \quad \therefore \vec{p}_m'' = \frac{\vec{p}_m}{4}$$



2. A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume $g = 10 \text{ m s}^{-2}$



Magnetic field, $B = 1 \text{ T}$.

Downward force, $F = mg$.

Linear mass density,

$$= m/l = 0.2 \times 10^{-3} \text{ kg m}^{-1}$$

$m = \text{Linear mass density} \times l$.

$$m = 0.2 \times 10^{-3} \times l$$

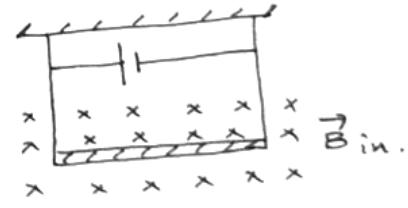
$$F = mg = 0.2 \times 10^{-3} \times l \times 10$$

$$F = 2 \times 10^{-3} l \rightarrow \textcircled{1}$$

$$F = BIl = 1 \times I \times l \quad \left[\begin{array}{l} \text{This is upward magnetic} \\ \text{force acting on the wire} \end{array} \right]$$

From $\textcircled{1}$ and $\textcircled{2}$

$$I \times l = 2 \times 10^{-3} l \quad ; \quad \boxed{I = 2 \text{ mA}}$$



3. A circular coil with cross-sectional area 0.1 cm^2 is kept in a uniform magnetic field of strength 0.2 T . If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate (a) total torque on the coil (b) total force on the coil (c) average force on each electron in the coil due to the magnetic field. (The free electron density for the material of the wire is 10^{28} m^{-3}).

cross-sectional area of coil

$$A = 0.1 \text{ cm}^2 = 0.1 \times (10^{-2} \text{ m})^2$$

$$A = 0.1 \times 10^{-4} \text{ m}^2 \quad ; \quad B = 0.2 \text{ T} \quad ; \quad I = 3 \text{ A}$$

\therefore Angle between the magnetic field and normal to the coil, $\theta = 0^\circ$

a) Total torque on the coil :-

$$\tau = ABI \sin \theta$$

$$= 0.1 \times 10^{-4} \times 0.2 \times 3 \times \sin 0^\circ$$

$$\tau = 0.$$

[$\because \sin 0^\circ = 0$].

b) Total force on the coil :-

$$F = BIl \sin \theta$$

$$F = 0.2 \times 3 \times l \times \sin 0^\circ$$

$$F = 0.$$

c) Average force :-

$$F = q V_d B$$

$$V_d = \frac{I}{neA}$$

$$F = q \times \frac{I}{n \cancel{q} A} \times B$$

[$\because q = e$]

$$F = \frac{IB}{nA} = \frac{3 \times 0.2}{10^{28} \times 0.1 \times 10^{-4}} = 6 \times 10^{-24}.$$

$$F = 0.6 \times 10^{-23} \text{ N}$$

4. A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T. If the bar magnet is oriented at an angle 30° with the external field experiences a torque of 0.2 Nm. Calculate: (i) the magnetic moment of the magnet (ii) the work done by the magnetic field in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

$$B = 0.8 \text{ T}; \theta = 30^\circ; \tau = 0.2 \text{ Nm}$$

SOLUTION:-

$$(i) \tau = P_m B \sin \theta.$$

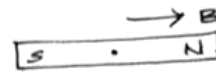
$$P_m = \frac{\tau}{B \sin \theta} = \frac{0.2}{0.8 \times \sin 30^\circ} = \frac{1}{4} \times \frac{1}{2}$$

$$P_m = 0.5 \text{ Am}^2$$

(ii) In stable configuration work done by an applied force,

$$U_i = -P_m B \cos \theta$$

$$U_i = -P_m B$$



$$\theta = 0^\circ ; \cos 0^\circ = 1.$$

(iii) In unstable configuration work done by an applied force,

$$U_f = -P_m B \cos \theta$$

$$U_f = P_m B$$



$$\theta = 180^\circ$$

$$\cos 180^\circ = -1.$$

Work done by applied magnetic field,

$$W = U_f - U_i$$

$$= P_m B - [-P_m B]$$

$$= 2 P_m B$$

$$= 2 \times 0.5 \times 0.8$$

$$W = 0.8 \text{ J.}$$

5. A non-conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turn concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current I required to rest the sphere in equilibrium.

GIVEN:-

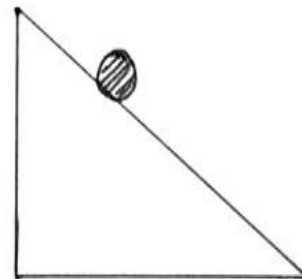
Mass of the sphere (m)

$$m = 100 \text{ g} = 100 \times 10^{-3} \text{ kg}$$

$$\text{Radius } R = 20 \text{ cm} = 20 \times 10^{-2} \text{ m.}$$

$$n = 5, \quad B = 0.5 \text{ T} = \frac{1}{2} \text{ T.}$$

$$I = ?$$



SOLUTION:- At equilibrium,

$$\int_3 R - P_m B \sin \theta = 0.$$

$$\int_3 R = P_m B \sin \theta \quad [\theta = 90^\circ]$$

$$MGR = NIAB$$

$$I = \frac{MGR}{NBA} = \frac{MGR}{NBTR^2} = \frac{mg}{NBTR} = \frac{20 \times 10^{-3} \times 10}{5 \times \frac{1}{2} \times \pi \times 20 \times 10^{-2}}$$

$$I = \frac{20}{\pi} \times 10^{-1} \Rightarrow \boxed{I = \frac{2}{\pi} \text{ A}}$$

$$\left[\begin{array}{l} P_m = IA \\ \text{For 'n' turns} \\ P_m = N \times IA \end{array} \right]$$

6. Calculate the magnetic field at the centre of a square loop which carries a current of 1.5 A, length of each side being 50 cm.

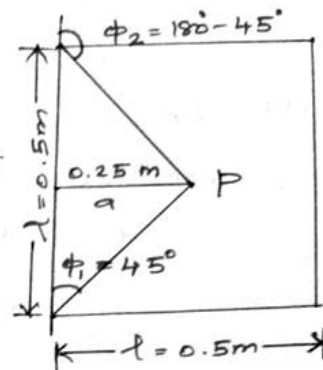
$$I = 1.5 \text{ A} ; \quad l = 50 \text{ cm} = 0.5 \text{ m}$$

Magnetic field at the centre of current carrying square loop, $B' = ?$

SOLUTION:

According to Biot - Savart law,
Magnetic field due to a current carrying straight line

$$B' = \frac{\mu_0 I}{4\pi a} (\cos \phi_1 - \cos \phi_2)$$



$$= \frac{4\pi \times 10^{-7} \times 1.5}{4\pi \times 0.25} [\cos 45^\circ - \cos(180-45^\circ)]$$

$$= 6 \times 10^{-7} [\cos 45^\circ + \cos 45^\circ] \quad (\because \cos(180^\circ - \theta) = -\cos \theta)$$

$$= 6 \times 10^{-7} \times \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= 6 \times 10^{-7} \times \frac{2}{\sqrt{2}} = 6 \times 10^{-7} \times \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}}$$

$$B' = 6 \times 10^{-7} \times 1.414$$

$$B' = 8.484 \times 10^{-7} \text{ T.}$$

\(\therefore\) For Four sides,

$$B' = 4 \times 8.484 \times 10^{-7}$$

$$B' = 33.9 \times 10^{-7}$$

$$B' = 3.4 \times 10^{-6} \text{ T.}$$

4. ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil.

GIVEN:

$$A = 30 \times 10^{-2} \times 30 \times 10^{-2} \text{ m}^2.$$

$$A = 900 \times 10^{-4} = 9 \times 10^{-2} \text{ m}^2$$

$B = 0.4 \text{ T}$, Plane inclined to the field, $\theta = 30^\circ$.

$$\therefore \theta = 90^\circ - 30^\circ = 60^\circ ; N = 500 \text{ turns}, \phi_B = ?$$

SOLUTION:

$$\phi_B = NBA \cos \theta = 500 \times 0.4 \times 9 \times 10^{-2} \times \cos 60^\circ.$$

$$= 5 \times 10^2 \times \frac{1}{2} \times 10^{-1} \times 9 \times 10^{-2} \times \frac{1}{2}$$

$$\phi_B = 10 \times 10^{-1} \times 9$$

$$\boxed{\phi_B = 9 \text{ Wb.}}$$

2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire.

GIVEN:

$$d\phi = 4 \text{ mWb} = 4 \times 10^{-3} \text{ Wb.}$$

$$dt = 0.4 \text{ s} = 4 \times 10^{-1} \text{ s.}$$

$$E = ?$$

SOLUTION:

$$E = \frac{d\phi}{dt} = \frac{4 \times 10^{-3}}{4 \times 10^{-1}} = 10 \times 10^{-3} \text{ V}$$

$$\boxed{E = 10 \text{ mV}}$$

3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\phi_B = (2t^3 + 4t^2 + 8t + 8) \text{ Wb}$. If the resistance of the coil is 5Ω , determine the induced current through the coil at a time $t = 3$ second.

GIVEN:

$$\phi_B = (2t^3 + 4t^2 + 8t + 8) \text{ Wb} ; R = 5 \Omega, t = 3 \text{ s}$$

$$i = ?$$

SOLUTION:

$$E = \frac{d\phi_B}{dt}$$

$$E = \frac{d(2t^3 + 4t^2 + 8t + 8)}{dt}$$

$$E = 6t^2 + 8t + 8$$

$$t = 3s, \quad E = 6 \times (3)^2 + 8 \times 3 + 8$$

$$= 6 \times 9 + 24 + 8 = 54 + 32$$

$$E = 86V.$$

$$i = E/R = 86/5 \quad \boxed{i = 17.2A}$$

4. A closely wound circular coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced in it. Calculate the number of turns in the coil.

GIVEN:

$$r = 0.02m = 2 \times 10^{-2}m; \quad B_1 = 8000T; \quad B_2 = 2000T.$$

$$\Delta t = 6s; \quad E = 44V; \quad \theta = 0^\circ, \quad N = ?$$

SOLUTION:

$$E = \frac{d\phi_B}{dt} = NA \frac{(B_1 - B_2)}{t} \times \cos\theta.$$

$$A = \pi r^2 = 3.14 \times (2 \times 10^{-2})^2;$$

$$\frac{44}{6} = \frac{N \times 3.14 \times 4 \times 10^{-4} \times (8000 - 2000) \times \cos 0^\circ}{6}$$

$$11 = \frac{N \times 3.14 \times 10^{-4} \times 6000 \times 1}{6}$$

$$11 = N \times 3.14 \times 10^{-1}$$

$$N = \frac{110}{3.14} = \frac{110^5}{22/7} = 5 \times 7 = 35$$

$$\boxed{N = 35 \text{ turns.}}$$

5. A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T . Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180° . If the resistance of the coil is 35Ω , find the amount of charge flowing through the coil.

SOLUTION:

$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{+NBA(\cos\theta_1 - \cos\theta_2)}{t}$$

(only magnitude)

$$\mathcal{E} = \frac{35 \times 10^2 \times 4 \times 10^{-1} \times 6 \times 10^{-4} \times [\cos 0^\circ - \cos 180^\circ]}{1}$$

$$\mathcal{E} = 840 \times 10^{-3} \times (1+1) = 1680 \times 10^{-3} \text{ V.}$$

$$i = \mathcal{E}/R = \frac{1680 \times 10^{-3}}{35} = 48 \times 10^{-3} \text{ A}$$

$$q = it = 48 \times 10^{-3} \times 1 \text{ C}$$

$$\boxed{q = 48 \text{ mC}}$$

6. An induced current of 2.5 mA flows through a single conductor of resistance 100Ω . Find out the rate at which the magnetic flux is cut by the conductor.

GIVEN:

$$i = 2.5 \text{ mA} \quad ; \quad R = 100 \Omega$$

$$\frac{d\phi_B}{dt} = ?$$

SOLUTION:

$$\frac{d\phi_B}{dt} = \mathcal{E} \quad ; \quad \mathcal{E} = iR$$

$$\frac{d\phi_B}{dt} = 2.5 \times 10^{-3} \times 100 = 250 \times 10^{-3}$$

\therefore Rate of change of flux;

$$\boxed{\frac{d\phi}{dt} = 250 \text{ m wb/s}}$$

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of $4 \times 10^{-3} \text{ T}$. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

GIVEN:

$$r = 0.4 \text{ m} ; B = 4 \times 10^{-3} \text{ T} ; E = 0.02 \text{ V} = 2 \times 10^{-2} \text{ V}$$

$$W = ? \quad A = \pi r^2 = 3.14 \times (0.4)^2$$

SOLUTION:

$$E = NBAW \sin \theta \quad [N=1, \theta=90^\circ, \sin 90^\circ=1]$$

$$W = \frac{E}{NBA \sin \theta}$$

$$= \frac{2 \times 10^{-2}}{1 \times 4 \times 10^{-3} \times 3.14 \times (0.4)^2 \times 1} = \frac{10}{2 \times 3.14 \times 0.16}$$

$$= 9.95 \text{ revolutions per second.}$$

$$W = 9.95 \text{ rps}$$

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of $4 \times 10^{-5} \text{ T}$. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.

GIVEN:

$$r = 1 \text{ m} ; B = 4 \times 10^{-5} \text{ T} ; A = \pi r^2 = 3.14 \times (1)^2 = 3.14 \text{ m}^2$$

$$E = 31.4 \text{ mV} = 31.4 \times 10^{-3} \text{ V} = 3.14 \times 10^{-2} \text{ V} ; W = ?$$

SOLUTION:

$$E = NBAW \sin \theta \quad [N=1 ; \theta=90^\circ ; \sin 90^\circ=1]$$

$$W = \frac{E}{NBA \sin \theta}$$

$$= \frac{3.14 \times 10^{-2}}{1 \times 4 \times 10^{-5} \times 3.14 \times \sin 90^\circ} = \frac{1000}{4 \times 1}$$

$$W = 250 \text{ rps}$$

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m.

GIVEN:
 $N = 4000 \text{ turns}$; $l = 2 \text{ m}$; $d = 0.04 \text{ m}$; $r = 0.02 \text{ m}$
 $r = 2 \times 10^{-2} \text{ m}$

$A = \pi r^2$; $L = ?$

$L = \mu_0 n^2 A l$; $n = N/l$

$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (4000)^2 \times 3.14 \times (2 \times 10^{-2})^2}{2}$

$L = 2 \times 3.14 \times 10^{-7} \times 16 \times 10^6 \times 3.14 \times 4 \times 10^{-4}$

$L = 3.14 \times 3.14 \times 128 \times 10^{-5}$

$L = 1262.02 \times 10^{-5} \text{ H} = 12.62 \times 10^{-3} \text{ H}$

$L = 12.62 \text{ mH}$

10. A coil of 200 turns carries a current of 4 A. If the magnetic flux through the coil is $6 \times 10^{-5} \text{ Wb}$, find the magnetic energy stored in the medium surrounding the coil.

GIVEN:
 $N = 200 = 2 \times 10^2 \text{ turns}$; $I = 4 \text{ A}$, $\phi_B = 6 \times 10^{-5} \text{ Wb}$

$U_B = ?$

SOLUTION:

$U_B = \frac{1}{2} L I^2$; $L I = N \phi_B$
 $L = N \phi_B / I$

$= \frac{1}{2} \times \frac{N \phi_B}{I} \times I^2$

$= \frac{1}{2} N \phi_B \times I = \frac{1}{2} \times 2 \times 10^2 \times 6 \times 10^{-5} \times 4$

$U_B = 24 \times 10^{-3} \text{ J}$; $U_B = 0.024 \text{ J}$

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux linked with each turn when it carries a current of 1 A.

GIVEN:

$$l = 50 \text{ cm} = 5 \times 10^{-1} \text{ m}; n = 400 \text{ turns per cm} = \frac{400}{10^{-2} \text{ m}}$$

$$n = 4 \times 10^4 \text{ turns.}$$

$$d = 0.04 \text{ m}; r = 0.02 \text{ m} = 2 \times 10^{-2} \text{ m.}$$

$$A = \pi r^2 = 3.14 \times (2 \times 10^{-2})^2 = 3.14 \times 4 \times 10^{-4} \text{ m}^2$$

$$A = 12.56 \times 10^{-4} \text{ m}^2; I = 1 \text{ A}$$

$$\phi_B = ?$$

SOLUTION:

$$\phi_B = LI; L = \mu_0 n^2 A l$$

$$\phi_B = \mu_0 n^2 A l \times I$$

$$= 4\pi \times 10^{-7} \times (4 \times 10^4)^2 \times 12.56 \times 10^{-4} \times 5 \times 10^{-1} \times 1$$

$$= 12.56 \times 10^{-12} \times 16 \times 10^8 \times 12.56 \times 5$$

$$= 12.56 \times 12.56 \times 80 \times 10^{-4}$$

$$= 1.262 \times 10^4 \times 10^{-4} \text{ wb}$$

$$\phi_B = 1.262 \text{ wb}$$

Then, the total flux linked with the coil

$$N \phi_B = LI$$

$$N = n l = \frac{400}{10^{-2}} \times 5 \times 10^{-1} = 2 \times 10^4 \text{ turns.}$$

$$2 \times 10^4 \phi_B = 1.262$$

$$\phi_B = 6.31 \times 10^{-4} \text{ wb.}$$

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with each turn of the coil, find the inductance of the coil.

GIVEN:

$$N = 200 \text{ turns}; I = 0.4 \text{ A} = 4 \times 10^{-1}; \phi_B = 4 \text{ mwb} = 4 \times 10^{-3} \text{ wb.}$$

$$L = ?$$

SOLUTION:

$$N \phi_B = LI; L = \frac{N \phi_B}{I}$$

$$L = \frac{200 \times 4 \times 10^{-3}}{4 \times 10^{-1}}$$

$$L = 2 \text{ H}$$

GIVEN:

$$l = 80 \text{ cm} = 8 \times 10^{-1} ; A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

$$N_1 = 1200 \text{ turns} = 12 \times 10^2 ; N_2 = 400 = 4 \times 10^2$$

M = ?

SOLUTION:

$$M = \frac{\mu_0 N_1 N_2 A}{l} ; n_1 = \frac{N_1}{l} ; n_2 = \frac{N_2}{l}$$

$$[M = \mu_0 n_1 n_2 A l]$$

$$= \frac{4\pi \times 10^{-7} \times 12 \times 10^2 \times 4 \times 10^2 \times 5 \times 10^{-4}}{8 \times 10^{-1}} = \frac{4 \times 3.14 \times 30 \times 10^{-6}}{8}$$

$$= 12.56 \times 3 \times 10^{-5} = 37.68 \times 10^{-5} \text{ H}$$

$$M = 0.3768 \times 10^{-3} \text{ H} = 0.38 \text{ mH.}$$

13. Two air core solenoids have the same length of 80 cm and same cross-sectional area 5 cm². Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns.

GIVEN:

$$l = 80 \text{ cm} = 8 \times 10^{-1} ; A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

$$N_1 = 1200 \text{ turns} = 12 \times 10^2 ; N_2 = 400 = 4 \times 10^2$$

M = ?

SOLUTION:

$$M = \frac{\mu_0 N_1 N_2 A}{l} ; n_1 = \frac{N_1}{l} ; n_2 = \frac{N_2}{l}$$

$$[M = \mu_0 n_1 n_2 A l]$$

$$= \frac{4\pi \times 10^{-7} \times 12 \times 10^2 \times 4 \times 10^2 \times 5 \times 10^{-4}}{8 \times 10^{-1}} = \frac{4 \times 3.14 \times 30 \times 10^{-6}}{8}$$

$$= 12.56 \times 3 \times 10^{-5} = 37.68 \times 10^{-5} \text{ H}$$

$$M = 0.3768 \times 10^{-3} \text{ H} = 0.38 \text{ mH.}$$

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm² is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

GIVEN: $N_1 = \frac{400}{10^2} = 400 \times 10^2 = 4 \times 10^4$
 $N_2 = 100 \Rightarrow 10^2$; $A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$; $I = 2 \text{ A}$
 $t = 0.04 \text{ s} \Rightarrow 4 \times 10^{-2} \text{ s}$; $E = ?$

SOLUTION:

$$E = -M \frac{dI}{dt} \quad ; \quad M = \mu_0 n_1 n_2 A l = \frac{\mu_0 N_1 N_2 A}{l}$$

$$[n_1 = N_1/l \quad ; \quad n_2 = N_2/l]$$

$$M = \frac{4\pi \times 10^{-7} \times 4 \times 10^4 \times 10^2 \times 4 \times 10^{-4}}{1} = 4 \times 3.14 \times 16 \times 10^{-5}$$

$$= 12.56 \times 16 \times 10^{-5} = 200.96 \times 10^{-5} = 2 \times 10^{-3}$$

$$M = 2 \text{ mH}$$

$$E = M \cdot \frac{dI}{dt} = 2 \times 10^{-3} \times \frac{2}{4 \times 10^{-2}} = 10^{-1} \text{ V}$$

$$E = 0.1 \text{ V}$$

15. A 200 turn circular coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil and the solenoid.

GIVEN:

$$N_2 = 200 = 2 \times 10^2 \quad ; \quad r = 2 \times 10^{-2} \text{ m} \quad ; \quad A = \pi r^2$$

$$N_1 = 90 \text{ turns/cm} = \frac{90}{10} = 90 \times 10^2 \quad ; \quad M = ?$$

SOLUTION:

$$M = \frac{\mu_0 N_1 N_2 A}{l} \Rightarrow \frac{4\pi \times 10^{-7} \times 9 \times 10^3 \times 2 \times 10^2 \times 3.14 \times (2 \times 10^{-2})^2}{1}$$

$$M = 4 \times 3.14 \times 18 \times 3.14 \times 4 \times 10^{-4} \times 10^{-2}$$

$$= 12.56 \times 18 \times 12.56 \times 10^{-6}$$

$$M = 2.84 \times 10^{-3} \text{ H}$$

$$M = 2.84 \text{ mH}$$

16. The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. Their areas of their cross-section and their lengths are the same and are 4 cm^2 and 0.04 m respectively. If the number of turns in S_1 is 200 and that in S_2 is 800, calculate the mutual inductance between the solenoids. If the current in solenoid 1 is increased from 2 A to 8 A in 0.04 second , calculate the induced emf in solenoid 2.

GIVEN:

$$\mu_r = 900 = 9 \times 10^2 ; \ell = 0.04 \text{ m} ; A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

$$N_1 = 200 = 2 \times 10^2 \text{ turns} ; N_2 = 800 = 8 \times 10^2 ; I_1 = 2 \text{ A} ; I_2 = 8 \text{ A}$$

$$dI = I_2 - I_1 = 8 - 2 = 6 \text{ A} ; t = 0.04 \text{ s} = 4 \times 10^{-2} \text{ s}.$$

$$E_2 = ?$$

SOLUTION:

$$E_2 = -M \frac{dI}{dt} ; M = \frac{\mu_0 \mu_r N_1 N_2 A}{\ell}$$

$$M = \frac{4\pi \times 10^{-7} \times 2 \times 10^2 \times 8 \times 10^2 \times 4 \times 10^{-4} \times 9 \times 10^2}{4 \times 10^{-2}}$$

$$= 4 \times 3.14 \times 16 \times 9 \times 10^{-3}$$

$$= 12.56 \times 144 \times 10^{-3}$$

$$= 1.81 \times 10^{-3} \times 10^3$$

$$M = 1.81 \text{ H}$$

$$E_2 = -M \cdot \frac{dI}{dt} = -1.81 \times \frac{6}{4 \times 10^{-2}} = -2.715 \times 10^2 \text{ V}$$

$$\boxed{E = 271.5 \text{ V}} \text{ (Magnitude).}$$

17. A step-down transformer connected to main supply of 220 V is used to operate $11 \text{ V}, 88 \text{ W}$ lamp. Calculate (i) Voltage transformation ratio and (ii) Current in the primary.

GIVEN:

$$V_p = 220V ; V_s = 11V ; \text{output power} = V_s I_s = 88W$$

$$K = ? ; I_p = ?$$

SOLUTION:

$$(i) \text{ Transformer ratio } (K) = \frac{V_s}{V_p} = \frac{11}{220}$$

$$K = \frac{1}{20}$$

$$(ii) V_s I_s = 88 ; 11 \times I_s = 88 ; I_s = 8A$$

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} ; I_p = \frac{V_s}{V_p} \times I_s$$

$$I_p = \frac{11}{220} \times 8 = 0.4A$$

$$\therefore I_p = 0.4A \quad [\text{OR}]$$

$$I_p / I_s = K ; I_p = K I_s = \frac{1}{20} \times 8$$

$$I_p = 0.4A$$

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω . Find the current drawn by the primary of the transformer.

GIVEN:

$$V_p = 200V ; V_s = 120V ; \eta = 90\% = \frac{90}{100}$$

$$R_s = 40\Omega ; I_p = ?$$

SOLUTION:

$$V_s = I_s R_s$$

$$I_s = \frac{V_s}{R_s} = \frac{120}{40} = 3A \quad I_s = 3A$$

$$\therefore \eta = \frac{V_s I_s}{V_p I_p} \Rightarrow \frac{90}{100} = \frac{120 \times 3}{200 \times I_p}$$

$$I_p = 2A$$

19. The 300 turn primary of a transformer has resistance 0.82Ω and the resistance of its secondary of 1200 turns is 6.2Ω . Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

GIVEN :

$$N_p = 300 ; R_p = 0.82 \Omega ; N_s = 1200 ; R_s = 6.2 \Omega$$

$$V_p = ? \quad V_s = 1600 \text{ V}$$

$$\text{output power} = V_s I_s = 32 \text{ kW} = 32 \times 10^3 \text{ W}$$

$$I_p^2 R_p = ? \quad \eta = 80\% = \frac{80}{100}$$

$$I_s^2 R_s = ?$$

SOLUTION :

$$(i) \frac{V_p}{V_s} = \frac{N_p}{N_s} ; V_p = \frac{N_p}{N_s} \times V_s = \frac{300}{1200} \times 1600$$

$$\boxed{V_p = 400 \text{ V}}$$

$$(ii) \text{ output power, } V_s I_s = 32 \times 10^3 \text{ W}$$

$$I_s = \frac{32 \times 10^3}{16 \times 10^2} = 20 \text{ A}$$

$$\boxed{I_s = 20 \text{ A}}$$

$$(iii) \eta = \frac{V_s I_s}{V_p I_p} = \frac{80}{100} = \frac{1600 \times 20}{400 \times I_p}$$

$$\therefore \boxed{I_p = 100 \text{ A}}$$

$$\therefore \text{ Power loss in primary coil} = I_p^2 R_p$$

$$\therefore I_p^2 R_p = (100)^2 \times 0.82 = 8200 \text{ W}$$

$$\boxed{I_p^2 R_p = 8.2 \text{ kW}}$$

$$\therefore \text{ Power loss in secondary coil} = I_s^2 R_s$$

$$\therefore I_s^2 R_s = (20)^2 \times 6.2 = 4 \times 10^2 \times 6.2 = 24.8 \times 10^2$$

$$\therefore \boxed{I_s^2 R_s = 2.48 \text{ kW}}$$

20. Calculate the instantaneous value at 60° , average value and RMS value of an alternating current whose peak value is 20 A.

$$\theta = 60^\circ ; i = ? ; I_{avg} = ? ; I_{rms} = ?$$

$$I_m = 20 \text{ A}$$

SOLUTION:

(i) Instantaneous current, $i = I_m \sin \omega t$
 $\theta = \omega t ; i = 20 \sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} = 10 \times 1.732$
 $i = 17.32 \text{ A}$

(ii) $I_{avg} = \frac{2 I_m}{\pi} = \frac{2 \times 20}{3.14} = \frac{40}{3.14} = 12.75 \text{ A}$
 $\therefore I_{avg} = 12.75 \text{ A}$

(iii) $I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m = 0.707 \times 20$
 $I_{rms} = 14.14 \text{ A}$

5. ELECTROMAGNETIC WAVES

1. Consider a parallel plate capacitor whose plates are closely spaced. Let R be the radius of the plates and the current in the wire connected to the plates is 5 A , calculate the displacement current through the surface passing between the plates by directly calculating the rate of change of flux of electric field through the surface.

GIVEN:

$$I_c = 5\text{ A}; I_d = ?$$

SOLUTION:

$$\text{Electric flux } \phi_E = q/\epsilon_0.$$

$$\therefore I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left[\frac{q}{\epsilon_0} \right]$$

$$I_d = dq/dt = \frac{d(I_c t)}{dt}$$

$$I_d = I_c dt/dt$$

$$\boxed{I_d = I_c = 5\text{ A}}$$

$$I_c = q/t$$

$$q = I_c t.$$

2. A transmitter consists of LC circuit with an inductance of $1\ \mu\text{H}$ and a capacitance of $1\ \mu\text{F}$. What is the wavelength of the electromagnetic waves it emits?

GIVEN:

$$L = 1\ \mu\text{H} = 10^{-6}\text{ H}$$

$$C = 1\ \mu\text{F} = 10^{-6}\text{ F}$$

$$\lambda = ?$$

SOLUTION:

$$\lambda = c/f.$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$c = 3 \times 10^8\text{ m/s}.$$

$$f = \frac{1}{2 \times 3.14 \sqrt{10^{-6} \times 10^{-6}}} = \frac{1}{6.28 \times 10^{-6}} = \frac{10^6}{6.28}\text{ Hz}.$$

$$\lambda = c/f = \frac{3 \times 10^8 \times 6.28}{10^6} = 18.84 \times 10^2.$$

$$\boxed{\lambda = 18.84 \times 10^2\text{ m}}$$

3. A pulse of light of duration 10^{-6} s is absorbed completely by a small object initially at rest. If the power of the pulse is 60×10^{-3} W, calculate the final momentum of the object.

GIVEN: $t = 10^{-6}$ s ; Power = 60×10^{-3} W
Momentum $P = ?$

SOLUTION:

$$P = U/c \quad ; \quad U = \text{Power} \times \text{time.}$$

$$= \frac{60 \times 10^{-3} \times 10^{-6}}{3 \times 10^8} \Rightarrow \boxed{P = 20 \times 10^{-17} \text{ kg m s}^{-1}}$$

4. Let an electromagnetic wave propagate along the x - direction, the magnetic field oscillates at a frequency of 10^{10} Hz and has an amplitude of 10^{-5} T, acting along the y - direction. Then, compute the wavelength of the wave. Also write down the expression for electric field in this case.

GIVEN: $f = 10^{10}$ Hz , $B_0 = 10^{-5}$ T , $c = 3 \times 10^8$ m/s.

$\lambda = ?$ $\vec{E} = ?$

SOLUTION:

(i) Wavelength, $\lambda = c/f = \frac{3 \times 10^8}{10^{10}} = 3 \times 10^{-2}$ m.

$$\boxed{\lambda = 3 \times 10^{-2} \text{ m.}}$$

(ii) Amplitude of oscillating electric field.

$$E_0 = c B_0 = 3 \times 10^8 \times 10^{-5} = 3 \times 10^3 \text{ N C}^{-1}$$

Expression for Electric field in EM wave,

$$E = E_0 \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{3 \times 10^{-2}} = \frac{6.28 \times 10^2}{3} = 2.09 \times 10^2$$

$$\omega = 2\pi f = 2 \times 3.14 \times 10^{10} = 6.28 \times 10^{10}$$

$$\boxed{\vec{E} = 3 \times 10^3 \sin[2.09 \times 10^2 x - 6.28 \times 10^{10} t] \hat{i} \text{ N C}^{-1}}$$

5. If the relative permeability and relative permittivity of a medium are 1.0 and 2.25 respectively, find the speed of the electromagnetic wave in this medium.

GIVEN:

$$\mu_r = 1.0 \quad ; \quad \epsilon_r = 2.25 \quad V = ?$$

SOLUTION:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad ; \quad v = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\mu = \mu_0 \epsilon_r \quad ; \quad \epsilon = \epsilon_0 \epsilon_r$$

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \times \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{1 \times 2.25}}$$

$$v = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s.}$$

$$v = 2 \times 10^8 \text{ m s}^{-1}$$