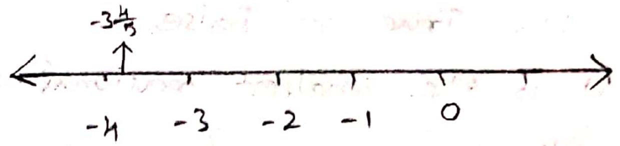
UNIT 1: NUMBERS

Exercise 1.1

1. Fill in the blanks

(i) $\frac{-19}{5}$ lies between the integers ____ and ____

$\frac{-19}{5} = -3\frac{4}{5}$



Ans : -4 and -3

(ii) The decimal form of the rational number $\frac{15}{-4}$ is ____.

3.75

$\frac{15}{-4} = -3.75$

Ans : -3.75

(iii) The rational numbers $\frac{-8}{3}$ and $\frac{8}{3}$ are equidistant from ____.

$\frac{8}{3} = 2\frac{2}{3}$

Ans : Zero.

(iv). The next rational number in the sequence $\frac{-15}{24}, \frac{20}{-32}, \frac{-25}{40}$ is ____.

$\frac{-\cancel{15}^{5}}{\cancel{24}_{8}}, \frac{\cancel{20}^{5}}{-\cancel{32}_{8}}, \frac{-\cancel{25}^{5}}{\cancel{40}_{8}}$

$\frac{-5}{8} \times \frac{3}{3}, \frac{-5}{8} \times \frac{-4}{-4}, \frac{-5}{8} \times \frac{5}{5}, \frac{-5}{8} \times \frac{-6}{-6}$

$\therefore$ Ans : $\frac{30}{-48}$

(v) The standard form of $\frac{58}{-78}$ is ____.

$$\frac{58 \div -2}{-78 \div -2} = \frac{-29}{39}$$

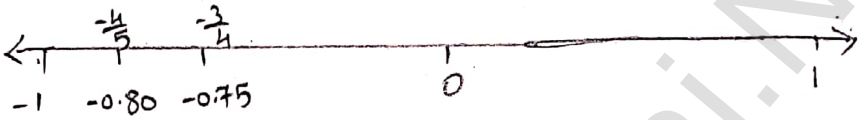
Ans: $\frac{-29}{39}$

2). Say True or False

(i) 0 is the smallest rational number. Ans: False

(ii) $\frac{-4}{5}$ lies to the left of $\frac{-3}{4}$

$\frac{-4}{5} = -0.80$; $\frac{-3}{4} = -0.75$



Ans: True.

(iii) $\frac{-19}{5}$ is greater than $\frac{15}{-4}$

L.C.M of 4 and 5 is 20.

$\frac{-19}{5} \times \frac{4}{4} = \frac{-76}{20}$ and $\frac{15}{-4} = \frac{-15}{4} \times \frac{5}{5} = \frac{-75}{20}$

$\frac{-76}{20} < \frac{-75}{20} \Rightarrow \frac{-19}{5} < \frac{15}{-4}$

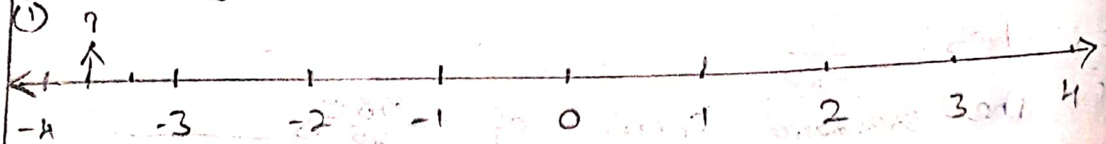
Ans: False

(iv) The average of two rational numbers lies between them. Ans: True.

(v) There are an unlimited number of rational numbers between 10 and 11. Ans: True.

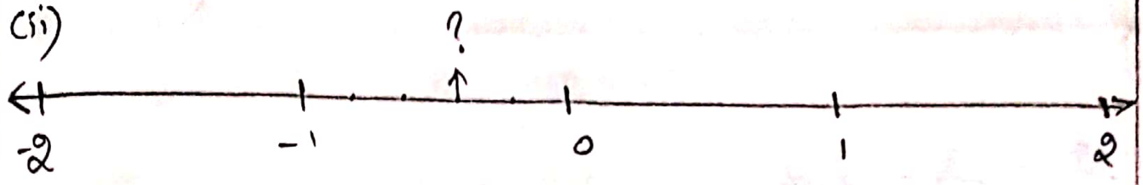
3). Find the rational numbers represented by each of the question marks marked on the following number lines.

(i)



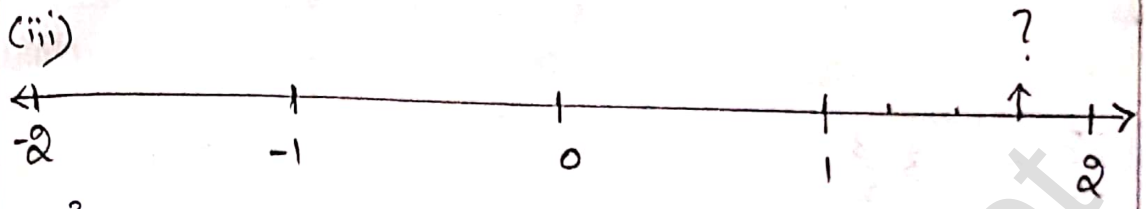
$-3\frac{2}{3} = \frac{-11}{3}$

(ii)



$-\frac{2}{5}$

(iii)



$1\frac{3}{4} = \frac{7}{4}$

4). The points S, Y, N, C, R, A, I, I and O on the number line are such that CN = NY = YS and RA = AT = TI = IO. Find the rational numbers represented by the letters Y, N, A, T and I.

$Y = -1\frac{2}{3} = -\frac{5}{3}$; $N = -1\frac{1}{3} = -\frac{4}{3}$

$A = 2\frac{1}{4} = \frac{9}{4}$; $T = 2\frac{2}{4} = \frac{10}{4}$; $I = 2\frac{3}{4} = \frac{11}{4}$

5). Draw a number line and represent the following rational numbers on it.

(i) $\frac{9}{4}$ (ii) $-\frac{8}{3}$ (iii) $\frac{-17}{-5}$ (iv) $\frac{15}{-4}$

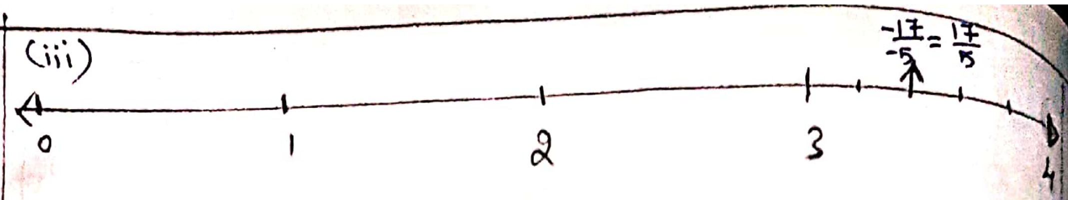
(i)

$\frac{9}{4} = 2\frac{1}{4}$

(ii)

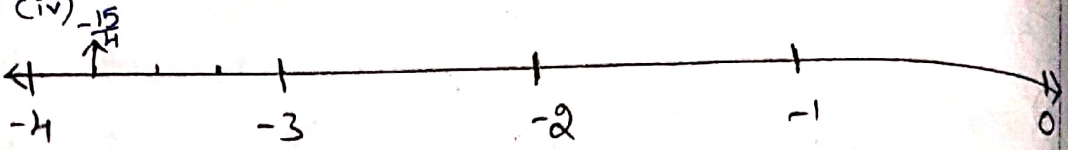
$-\frac{8}{3} = -2\frac{2}{3}$

(iii)



$\frac{-17}{-5} = \frac{17}{5} = 3\frac{2}{5}$

(iv)



$\frac{15}{-4} = -3\frac{3}{4}$

6) Write the decimal form of the following numbers (i) $\frac{1}{11}$ (ii) $\frac{13}{4}$ (iii) $\frac{-18}{7}$ (iv) $1\frac{2}{5}$ (v) $-3\frac{1}{2}$

(i) $\frac{1}{11}$

$11 \overline{)100}$ quotient $0.0909\ldots$
(-) 99
100
(-) 99
1

$\frac{1}{11} = 0.0909\ldots$

(ii) $\frac{13}{4}$

$4 \overline{)13}$ quotient 3.25
(-) 12
10
(-) 8
20
(-) 20
0

$\frac{13}{4} = 3.25$

(iii) $\frac{-18}{7}$

$7 \overline{)18}$ quotient 2.5714285
(-) 14
40
(-) 35
50
(-) 49
10
(-) 7
30
(-) 28
20
(-) 14
60
(-) 56
40
(-) 35
5

$\frac{-18}{7} = 2.571428$

(iv) $1\frac{2}{5} = \frac{7}{5}$

$5 \overline{)7}$ quotient 1.4
(-) 5
20
(-) 20
0

$1\frac{2}{5} = \frac{7}{5} = 1.4$

(v) $-3\frac{1}{2} = -\frac{7}{2}$

$2 \overline{)7}$ quotient 3.5
(-) 6
10
(-) 10
0

$-3\frac{1}{2} = -\frac{7}{2} = -3.5$

7). List any five rational numbers between the given rational numbers.

(i) -2 and 0 (ii) $\frac{-1}{2}$ and $\frac{3}{5}$ (iii) $\frac{1}{4}$ and $\frac{7}{20}$

(iv) $\frac{-6}{4}$ and $\frac{-23}{10}$.

(i) -2 and $0 \rightarrow \frac{-19}{10}, \frac{-18}{10}, \ldots\ldots, \frac{-2}{10}, \frac{-1}{10}$ (Any five)

(ii) $\frac{-1}{2}$ and $\frac{3}{5}$

L.C.M of $2, 5$

$$\begin{array}{r|l} 2 & 2, 5 \\ 5 & 1, 5 \\ \hline & 1, 1 \end{array}$$

$\frac{-1}{2} = \frac{-1}{2} \times \frac{5}{5} = \frac{-5}{10}$; $\frac{3}{5} = \frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$

L.C.M $= 2 \times 5 = 10$.

$\frac{-1}{2}$ and $\frac{3}{5} \rightarrow \frac{-4}{10}, \frac{-3}{10}, \ldots\ldots, \frac{4}{10}, \frac{5}{10}$ (Any five)

(iii) 0.25 and 0.35

$0.25 = \frac{25}{100}$ and $0.35 = \frac{35}{100}$

0.25 and $0.35 \rightarrow \frac{26}{100}, \frac{27}{100}, \ldots\ldots, \frac{33}{100}, \frac{34}{100}$ (Any five)

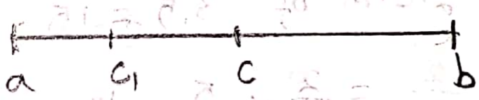
(iv) -1.2 and -2.3

$-1.2 = -\frac{12}{10}$ and $-2.3 = \frac{-23}{10}$ $[\because -23 < -12]$

-1.2 and $-2.3 \rightarrow \frac{-22}{10}, \frac{-21}{10}, \ldots\ldots, \frac{-14}{10}, \frac{-13}{10}$ (Any five)

8). Use the method of averages to write 2 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$.

Let $a = \frac{14}{5}$ and $b = \frac{16}{3}$



$$c = \frac{a+b}{2} = \frac{\frac{14}{5}+\frac{16}{3}}{2} = \frac{\frac{42+80}{15}}{2} = \frac{\frac{122}{15}}{2} = \frac{\overset{61}{\cancel{122}}}{15} \times \frac{1}{\cancel{2}_{1}} = \frac{61}{15}$$

$$c_1 = \frac{a+c}{2} = \frac{\frac{14}{5}+\frac{61}{15}}{2} = \frac{\frac{42+61}{15}}{2} = \frac{\frac{103}{15}}{2} = \frac{103}{15} \times \frac{1}{2} = \frac{103}{30}$$

$\therefore$ The 2 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$ are $\frac{103}{30}$ and $\frac{61}{15}$

9). compare the following pairs of rational numbers (i) $\frac{-11}{5}, \frac{-21}{8}$ (ii) $\frac{3}{-4}, \frac{-1}{2}$ (iii) $\frac{2}{3}, \frac{4}{5}$.

(i) $\frac{-11}{5}, \frac{-21}{8}$

L.C.M of $5, 8 = 40$

$$\begin{array}{r|l} 5 & 5, 8 \\ \hline 8 & 1, 8 \\ \hline & 1, 1 \end{array}$$

L.C.M $= 5 \times 8 = 40$

$\frac{-11}{5} = \frac{-11}{5} \times \frac{8}{8} = \frac{-88}{40}$; $\frac{-21}{8} = \frac{-21}{8} \times \frac{5}{5} = \frac{-105}{40}$

Now $\frac{-105}{40} < \frac{-88}{40} \Rightarrow \frac{-21}{8} < \frac{-11}{5}$ (or)

$\frac{-88}{40} > \frac{-105}{40} \Rightarrow \frac{-11}{5} > \frac{-21}{8}$.

(ii) $\frac{3}{-4}, \frac{-1}{2}$

L.C.M of $4, 2 = 4$

$$\begin{array}{r|l} 2 & 4, 2 \\ \hline 2 & 2, 1 \\ \hline & 1, 1 \end{array}$$

L.C.M $= 2 \times 2 = 4$

$\frac{3}{-4} = \frac{-3}{4} \times \frac{1}{1} = \frac{-3}{4}$; $\frac{-1}{2} = \frac{-1}{2} \times \frac{2}{2} = \frac{-2}{4}$

Now $\frac{-3}{4} < \frac{-2}{4} \Rightarrow \frac{3}{-4} < \frac{-1}{2}$ (or)

$\frac{-2}{4} > \frac{-3}{4} \Rightarrow \frac{-1}{2} > \frac{-3}{4}$

(iii) $\frac{2}{3}, \frac{4}{5}$

L.C.M of $3, 5 = 15$

$$\begin{array}{r|l} 3 & 3, 5 \\ \hline 5 & 1, 5 \\ \hline & 1, 1 \end{array}$$

L.C.M $= 3 \times 5 = 15$

$\frac{2}{3} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$; $\frac{4}{5} = \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$

Now $\frac{10}{15} < \frac{12}{15} \Rightarrow \frac{2}{3} < \frac{4}{5}$ (or)

$\frac{12}{15} > \frac{10}{15} \Rightarrow \frac{4}{5} > \frac{2}{3}$.

10) Arrange the following rational numbers in ascending and descending order

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$ (ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$

i.e., $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{7}{9}, \frac{12}{36}$

L.C.M of 12, 8, 24, 9, 36 is 72.

3	12, 8, 24, 9, 36
4	4, 8, 8, 3, 12
2	1, 2, 2, 3, 3
3	1, 1, 1, 3, 3
	1, 1, 1, 1, 1

L.C.M = 3 × 4 × 2 × 3
= 12 × 6
L.C.M = 72

$\frac{-5}{12} = \frac{-5}{12} \times \frac{6}{6} = \frac{-30}{72}$

$\frac{-11}{8} = \frac{-11}{8} \times \frac{9}{9} = \frac{-99}{72}$

$\frac{-15}{24} = \frac{-15}{24} \times \frac{3}{3} = \frac{-45}{72}$

$\frac{7}{9} = \frac{7}{9} \times \frac{8}{8} = \frac{56}{72}$

$\frac{12}{36} = \frac{12}{36} \times \frac{2}{2} = \frac{24}{72}$

Now $\frac{-99}{72} < \frac{-45}{72} < \frac{-30}{72} < \frac{24}{72} < \frac{56}{72}$

i.e., $\frac{-11}{8} < \frac{-15}{24} < \frac{-5}{12} < \frac{12}{36} < \frac{7}{9}$

i.e., $\frac{-11}{8} < \frac{-15}{24} < \frac{-5}{12} < \frac{12}{36} < \frac{-7}{-9}$

Ascending order is $\frac{-11}{8} < \frac{-15}{24} < \frac{-5}{12} < \frac{12}{36} < \frac{-7}{-9}$

Descending order is $\frac{-7}{-9} > \frac{12}{36} > \frac{-5}{12} > \frac{-15}{24} > \frac{-11}{8}$

(ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

L.C.M of 10, 5, 4, 20 is 20.

5	10, 5, 4, 20
2	2, 1, 4, 4
2	1, 1, 2, 2
	1, 1, 1, 2

L.C.M = 5 × 2 × 2
L.C.M = 20

$\frac{-17}{10} = \frac{-17}{10} \times \frac{2}{2} = \frac{-34}{20}$;

$\frac{-7}{5} = \frac{-7}{5} \times \frac{4}{4} = \frac{-28}{20}$

$\frac{-2}{4} = \frac{-2}{4} \times \frac{5}{5} = \frac{-10}{20}$; $\frac{-19}{20} = \frac{-19}{20} \times \frac{1}{1} = \frac{-19}{20}$

Now $\frac{-34}{20} < \frac{-28}{20} < \frac{-19}{20} < \frac{-10}{20} < 0$

i.e., $\frac{-17}{10} < \frac{-7}{5} < \frac{-19}{20} < \frac{-2}{4} < 0$.

Ascending order is $\frac{-17}{10} < \frac{-7}{5} < \frac{-19}{20} < \frac{-2}{4} < 0$ and

Descending order is $0 > \frac{-2}{4} > \frac{-19}{20} > \frac{-7}{5} > \frac{-17}{10}$.

Objective Type Questions:

11). The number which is subtracted from $-\frac{6}{11}$ to get $\frac{8}{9}$ is ______ (A) $\frac{34}{99}$ (B) $\frac{-142}{99}$ (C) $\frac{142}{99}$ (D) $\frac{-34}{99}$

$\frac{-6}{11} - x = \frac{8}{9} \Rightarrow \frac{-6}{11} - \frac{8}{9} = x \Rightarrow x = \frac{-54-88}{99} = \frac{-142}{99}$

$x = \frac{-142}{99}$ Ans: (B) $-\frac{142}{99}$.

12). which of the following Pairs is equivalent?

(A) $\frac{-20}{12}, \frac{5}{3}$ (B) $\frac{16}{-30}, \frac{-8}{15}$ (C) $\frac{-18}{36}, \frac{-20}{44}$ (D) $\frac{7}{-5}, \frac{-5}{7}$

(A) $\frac{-\cancel{20}^{5}}{\cancel{12}_{3}}, \frac{5}{3}$ (B) $\frac{\cancel{16}^{8}}{-\cancel{30}_{15}}, \frac{-8}{15}$ (C) $\frac{-\cancel{18}^{1}}{\cancel{36}_{2}}, \frac{-\cancel{20}^{5}}{\cancel{44}_{11}}$ (D) $\frac{7}{-5}, \frac{-5}{7}$

Ans: (B) $\frac{-8}{15}, \frac{-8}{15}$ i.e., $\frac{16}{-30}, \frac{-8}{15}$.

13). $\frac{-5}{4}$ is a rational number which lies between ______ (A) 0 and $\frac{-5}{4}$ (B) -1 and 0 (C) -1 and -2 (D) -4 and -5.

$-\frac{5}{4} = -1\frac{1}{4}$

Ans: (C) -1 and -2

14). Which of the following rational numbers is the greatest? (A) $\frac{-17}{24}$ (B) $\frac{-13}{16}$ (C) $\frac{7}{-8}$ (D) $\frac{-31}{32}$

L.C.M of 24, 16, 8, 32 is

$$\begin{array}{r|l} 8 & 24, 16, 8, 32 \\ 2 & 3, 2, 1, 4 \\ & 3, 1, 1, 2 \end{array}$$

L.C.M = 8×2×3×2 = 16×6

L.C.M = 96

$\frac{-17}{24} = \frac{-17}{24} \times \frac{4}{4} = \frac{-68}{96}$; $\frac{-13}{16} = \frac{-13}{16} \times \frac{6}{6} = \frac{-78}{96}$;

$\frac{7}{-8} = \frac{-7}{8} \times \frac{12}{12} = \frac{-84}{96}$; $\frac{-31}{32} = \frac{-31}{32} \times \frac{3}{3} = \frac{-93}{96}$

∴ The greatest rational number is $\frac{-68}{96} = \frac{-17}{24}$

Ans: (A) $\frac{-17}{24}$.

15). The sum of the digits of the denominator in the simplest form of $\frac{112}{528}$ is ______

(A) 4 (B) 5 (C) 6 (D) 7.

$$\frac{112}{528} = \frac{112 \div 8}{528 \div 8} = \frac{14 \div 2}{66 \div 2} = \frac{7}{33}$$

Sum of digits in the denominator = 3+3 = 6.

Ans: (C). 6.

Exercise 1.2

1). Fill in the blanks:

(i) The value of $\frac{-5}{12} + \frac{7}{15} =$ ______

$$\frac{-5}{12} + \frac{7}{15} = \frac{-5 \times 5}{12 \times 5} + \frac{7 \times 4}{15 \times 4} = \frac{-25+28}{60} = \frac{\cancel{3}^{1}}{\cancel{60}_{20}} = \frac{1}{20}$$

$$\begin{array}{r|l} 3 & 12, 15 \\ & 4, 5 \end{array}$$

L.C.M = 3×4×5

L.C.M = 60

Ans: $\frac{1}{20}$

(ii) The value of $\left(\frac{-3}{6}\right) \times \left(\frac{18}{-9}\right)$ is ______.

$$\left(\frac{-\cancel{3}^{1}}{\cancel{6}_{2}}\right) \times \left(\frac{\cancel{18}^{2}}{-9}\right) = 1$$

Ans: 1

(iii) The value of $\left(\frac{-15}{23}\right) \div \left(\frac{30}{-46}\right)$ is ______.

$$\left(\frac{-15}{23}\right) \div \left(\frac{30}{-46}\right) = \frac{-\cancel{15}^{1}}{\cancel{23}_{1}} \times \frac{-\cancel{46}^{2}}{\cancel{30}_{2}} = 1$$

Ans: 1

(iv) The rational number ______ does not have a reciprocal.

Ans: 0.

(v) The multiplicative inverse of -1 is ______ Ans: -1

(2). Say True or False :

(i) All rational numbers have an additive inverse.

Ans: True

(ii) The rational numbers that are equal to their additive inverses are 0 and -1.

Additive inverse of 0 is 0 and -1 is 1.

Ans: False.

(iii) The additive inverse of $\frac{-11}{-17}$ is $\frac{11}{17}$.

$\frac{-11}{-17} = \frac{11}{17}$ additive inverse is $\frac{-11}{17}$. Ans: False

(iv) The rational number which is its own reciprocal is -1.

Reciprocal of -1 is -1.

Ans: True.

(v) The multiplicative inverse exists for all rational numbers.

0 does have have a multiplicative inverse.

Ans: False.

3). Find the sum:

(i) $\frac{7}{5}+\frac{3}{5}$ (ii) $\frac{7}{5}+\frac{5}{7}$ (iii) $\frac{6}{5}+\left(\frac{-14}{15}\right)$

(iv) $-4\frac{2}{3}+7\frac{5}{12}$.

(i) $\frac{7}{5}+\frac{3}{5} = \frac{7+3}{5} = \frac{10}{5} = \frac{2}{1} = 2$.

(ii) $\frac{7}{5}+\frac{5}{7} = \frac{7\times7+5\times5}{5\times7} = \frac{49+25}{35} = \frac{74}{35}$

(iii) $\frac{6}{5} + \left(\frac{-14}{15}\right) = \frac{6}{5} - \frac{14}{15} = \frac{6\times3-14}{15} = \frac{18-14}{15}$

$\frac{6}{5} + \left(\frac{-14}{15}\right) = \frac{4}{15}$

(iv) $-4\frac{2}{3} + 7\frac{5}{12} = \frac{-14}{3} + \frac{89}{12} = \frac{-14\times4+89}{12}$

$= \frac{-56+89}{12}$

$= \frac{\cancel{33}^{11}}{\cancel{12}_{4}}$

$= 2\frac{3}{4}$

(4) Subtract : $\frac{-8}{44}$ from $\frac{-17}{11}$

$\frac{-17}{11} - \left(\frac{-8}{44}\right) = \frac{-17\times4+8}{44} = \frac{-68+8}{44} = \frac{\cancel{-60}^{15}}{\cancel{44}_{11}} = \frac{-15}{11}$

$= -1\frac{4}{11}$

(5) Evaluate : (i) $\frac{9}{132} \times \frac{-11}{3}$ (ii) $\frac{-7}{27} \times \frac{24}{-35}$

(i) $\frac{\cancel{9}^{\cancel{3}\,1}}{\cancel{132}_{\cancel{12}\,4}} \times \frac{\cancel{-11}^{1}}{\cancel{3}_{1}} = \frac{-1}{4}$ (ii) $\frac{\cancel{+7}^{1}}{\cancel{27}_{9}} \times \frac{\cancel{24}^{8}}{\cancel{+35}_{5}} = \frac{8}{45}$

(6) Divide : (i) $\frac{-21}{5}$ by $\frac{-7}{-10}$ (ii) $\frac{-3}{13}$ by -3 (iii) -2 by $\frac{-6}{15}$

(i) $\frac{-21}{5}$ by $\frac{-7}{-10}$

$\frac{-21}{5} \div \frac{-7}{-10} = \frac{\cancel{-21}^{3}}{\cancel{5}_{1}} \times \frac{\cancel{-10}^{2}}{\cancel{-7}_{1}} = -6$

(ii) $\frac{-3}{13}$ by -3

$\frac{-3}{13} \div -3 = \frac{\cancel{-3}}{13} \times \frac{1}{\cancel{-3}} = \frac{1}{13}$

(iii) -2 by $\frac{-6}{15}$

$$-2 \div \frac{-6}{15} = \frac{-\cancel{2}^{1}}{1} \times \frac{\cancel{15}^{5}}{-\cancel{6}_{\cancel{2}_1}} = 5$$

7). Find $(a+b) \div (a-b)$ if

(i) $a = \frac{1}{2}$, $b = \frac{2}{3}$ (ii) $a = \frac{-3}{5}$, $b = \frac{2}{15}$

(i) $(a+b) \div (a-b)$ if $a = \frac{1}{2}$, $b = \frac{2}{3}$

$$= \left(\frac{1}{2} + \frac{2}{3}\right) \div \left(\frac{1}{2} - \frac{2}{3}\right) = \left(\frac{3+4}{6}\right) \div \left(\frac{3-4}{6}\right)$$

$$= \left(\frac{7}{6}\right) \div \left(-\frac{1}{6}\right) = \frac{7}{\cancel{6}} \times \frac{\cancel{6}}{-1} = -7$$

(ii) $(a+b) \div (a-b)$ if $a = -\frac{3}{5}$, $b = \frac{2}{15}$

$$= \left(\frac{-3}{5} + \frac{2}{15}\right) \div \left(\frac{-3}{5} - \frac{2}{15}\right) = \left(\frac{-3\times3+2}{15}\right) \div \left(\frac{-3\times3-2}{15}\right)$$

$$= \left(\frac{-9+2}{15}\right) \div \left(\frac{-9-2}{15}\right) = \left(\frac{-7}{15}\right) \div \left(\frac{-11}{15}\right) = \frac{-7}{\cancel{15}} \times \frac{\cancel{15}}{-11}$$

$$= \frac{7}{11}$$

(8) simplify: $\frac{1}{2} + \left(\frac{3}{2} - \frac{2}{5}\right) \div \frac{3}{10} \times 3$ and show that it is a rational number between 11 and 12.

$$\frac{1}{2} + \left(\frac{3}{2} - \frac{2}{5}\right) \div \frac{3}{10} \times 3$$

$$= \frac{1}{2} + \left(\frac{15-4}{10}\right) \div \frac{3}{10} \times 3$$

$$= \frac{1}{2} + \frac{11}{10} \div \frac{3}{10} \times 3$$

$$= \frac{1}{2} + \frac{11}{\cancel{10}} \times \frac{\cancel{10}}{3} \times 3$$

$= \frac{1}{2} + \frac{11}{\cancel{3}} \times \cancel{3}^{1}$

$= \frac{1}{2} + 11$

$= \frac{1 + 11 \times 2}{2}$

$= \frac{1 + 22}{2}$

$= \frac{23}{2}$

$= 11\frac{1}{2}$

$\therefore 11\frac{1}{2}$ lies between 11 and 12.

9). Simplify:

(i) $\left[\frac{11}{8} \times \left(\frac{-6}{33}\right)\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20}\right)\right] - \left[\frac{4}{7} \times \frac{-7}{5}\right]$

(ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7}\right)\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4}{3} \times \left(\frac{-1}{4}\right)\right]$

(i) $\left[\frac{11}{8} \times \left(\frac{-6}{33}\right)\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20}\right)\right] - \left[\frac{4}{7} \times \frac{-7}{5}\right]$

$= \left[\frac{\cancel{11}^{1}}{\cancel{8}_{4}} \times -\frac{\cancel{6}^{3}}{\cancel{33}_{3}}\right] + \left[\frac{1}{3} + \frac{\cancel{3}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{20}^{4}}{\cancel{9}_{3}}\right] - \left[\frac{4}{\cancel{7}} \times \frac{-\cancel{7}}{5}\right]$

$= \left[-\frac{1}{4}\right] + \left[\frac{1}{3} + \frac{4}{3}\right] - \left[-\frac{4}{5}\right]$

$= -\frac{1}{4} + \frac{5}{3} + \frac{4}{5}$

$= \frac{-1 \times 15 + 5 \times 20 + 4 \times 12}{60}$

$= \frac{-15 + 100 + 48}{60} = \frac{148 - 15}{60} = \frac{133}{60}$ //

4	4, 3, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

L.C.M = 4 × 3 × 5

= 12 × 5

L.C.M = 60

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(ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7}\right)\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4}{3} \times \left(\frac{-1}{4}\right)\right]$

$= \left[\frac{\cancel{4}^{1}}{3} \times \frac{-7}{\cancel{8}_{2}}\right] - \left[\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}}\right] + \left[\frac{\cancel{4}^{1}}{3} \times \frac{-1}{\cancel{4}_{1}}\right]$

$= \left(-\frac{7}{6}\right) - (1) + \left(-\frac{1}{3}\right)$

$= \frac{-7-6-2}{6}$

$= -\frac{\cancel{15}^{5}}{\cancel{6}_{2}}$

$= -\frac{5}{2}$

10). A student had multiplied a number by $\frac{4}{3}$ instead of dividing by $\frac{4}{3}$ and get 70 more than the correct answer. Find the number.

Let the number be x.

$x \times \frac{4}{3} = \left(x \div \frac{4}{3}\right) + 70$

$x \times \frac{4}{3} = \left(x \times \frac{3}{4}\right) + 70$

$x \times \frac{4}{3} - x \times \frac{3}{4} = 70$

$x \times \left(\frac{4}{3} - \frac{3}{4}\right) = 70$

$x \times \left(\frac{16-9}{12}\right) = 70$

$x \times \left(\frac{7}{12}\right) = 70$

$x = \frac{\cancel{70}^{10} \times 12}{\cancel{7}_{1}} \Rightarrow \boxed{x = 120}$

Objective Type Questions:

11). The standard form of the sum $\frac{3}{4}+\frac{5}{6}+\left(\frac{-7}{12}\right)$ is ______ (A) 1 (B) $\frac{-1}{2}$ (C) $\frac{1}{12}$ (D) $\frac{1}{22}$

$\frac{3}{4}+\frac{5}{6}+\left(\frac{-7}{12}\right) = \frac{3\times3+5\times2+(-7)\times1}{12}$

$= \frac{9+10-7}{12}$

$= \frac{19-7}{12}$

$= \frac{\cancel{12}^{1}}{\cancel{12}_{1}}$

$= 1$

$$\begin{array}{r|l} 2 & 4,6,12 \\ \hline 2 & 2,3,6 \\ \hline 3 & 1,3,3 \\ \hline & 1,1,1 \end{array}$$

L.C.M = 2×2×3

L.C.M = 12

Ans : (A) 1

12). $\left(\frac{3}{4}-\frac{5}{8}\right)+\frac{1}{2}=$ ______

(A) $\frac{15}{64}$ (B) 1 (C) $\frac{5}{8}$ (D) $\frac{1}{16}$

$\left(\frac{3}{4}-\frac{5}{8}\right)+\frac{1}{2} = \frac{3\times2-5}{8}+\frac{1}{2} = \frac{6-5}{8}+\frac{1}{2} = \frac{1}{8}+\frac{1}{2}$

$= \frac{1+1\times4}{8} = \frac{1+4}{8} = \frac{5}{8}$ Ans: (C) $\frac{5}{8}$

13). $\frac{3}{4} \div \left(\frac{5}{8}+\frac{1}{2}\right) =$ ______

(A) $\frac{13}{10}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{8}$

$\frac{3}{4} \div \left(\frac{5}{8}+\frac{1}{2}\right) = \frac{3}{4} \div \left(\frac{5+4}{8}\right) = \frac{3}{4} \div \frac{9}{8} = \frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$

Ans: (B) $\frac{2}{3}$

14). $\frac{3}{4} \times \left(\frac{5}{8} \div \frac{1}{2}\right) =$ ______

(A) $\frac{5}{8}$ (B) $\frac{2}{3}$ (C) $\frac{15}{32}$ (D) $\frac{15}{16}$

$\frac{3}{4} \times \left(\frac{5}{8} \div \frac{1}{2}\right) = \frac{3}{4} \times \frac{5}{\cancel{8}_{4}} \times \frac{\cancel{2}^{1}}{1} = \frac{15}{16}$ Ans: (D) $\frac{15}{16}$

15). Which of these rational numbers which have additive inverse?

(A) 7 (B). $-\frac{5}{7}$ (C) 0 (D) All of these.

Additive inverse of 7 is -7, $-\frac{5}{7}$ is $\frac{5}{7}$ and 0 is 0.

Ans : (D) All of these

Exercise 1.3

1). Verify the closure property for addition and multiplication for the rational numbers $\frac{-5}{7}$ and $\frac{8}{9}$.

Closure property for addition:

If $a, b \in Q$ then $a+b \in Q$

Let $a = -\frac{5}{7}$ and $b = \frac{8}{9}$

$a+b = -\frac{5}{7} + \frac{8}{9} = \frac{-45+56}{63} = \frac{11}{63} \in Q$.

$\therefore$ Closure property for addition is true.

Closure property for multiplication:

If $a, b \in Q$ then $a \times b \in Q$

Let $a = \frac{-5}{7}$ and $b = \frac{8}{9}$.

$a \times b = \frac{-5}{7} \times \frac{8}{9} = -\frac{40}{63} \in Q$

$\therefore$ closure property for multiplication is true.

2). Verify the commutative property for addition and multiplication for the rational numbers $\frac{-10}{11}$ and $\frac{-8}{33}$

Commutative property for addition

$\boxed{a+b = b+a.}$

Let $a = \frac{-10}{11}$ and $b = \frac{-8}{33}$

$a + b = b + a$

$$\frac{-10}{11} + \left(-\frac{8}{33}\right) = \left(\frac{-8}{33}\right) + \left(\frac{-10}{11}\right)$$

$$\frac{-10\times3-8}{33} = \frac{-8-10\times3}{33}$$

$$\frac{-30-8}{33} = \frac{-8-30}{33}$$

$$\frac{-38}{33} = \frac{-38}{33}$$

$\therefore$ commutative property for addition is true.

Commutative property for multiplication:

$$\boxed{a \times b = b \times a}$$

$$\frac{-10}{11} \times \frac{-8}{33} = \frac{-8}{33} \times \frac{-10}{11}$$

$$\frac{80}{363} = \frac{80}{363}$$

$\therefore$ commutative property for multiplication is true

3). verify the associative property for addition and multiplication for the rational numbers $\frac{-7}{9}$, $\frac{5}{6}$ and $-\frac{4}{3}$.

Associative property for addition:

$$\boxed{(a+b)+c = a+(b+c)}$$

Let $a = \frac{-7}{9}$, $b = \frac{5}{6}$ and $c = \frac{-4}{3}$

$L.H.S = (a+b)+c$

$= \left[\left(-\frac{7}{9}\right)+\frac{5}{6}\right]+\left(-\frac{4}{3}\right)$

$= \left[\frac{-42+45}{54}\right]-\frac{4}{3}$

$= \frac{\cancel{3}^{1}}{\cancel{54}_{18}}-\frac{4}{3}$

$= \frac{1}{18}-\frac{4}{3}$

$= \frac{1-4\times 6}{18}$

$= \frac{1-24}{18}$

$= \frac{-23}{18}$

$R.H.S = a+(b+c)$

$= \left(-\frac{7}{9}\right)+\left[\frac{5}{6}+\left(-\frac{4}{3}\right)\right]$

$= \left(-\frac{7}{9}\right)+\left[\frac{5}{6}-\frac{4}{3}\right]$

$= \left(-\frac{7}{9}\right)+\left[\frac{5-4\times 2}{6}\right]$

$= \left(-\frac{7}{9}\right)+\left[\frac{5-8}{6}\right]$

$= -\frac{7}{9}+\left(\frac{-\cancel{3}^{1}}{\cancel{6}_{2}}\right)$

$= -\frac{7}{9}-\frac{1}{2}$

$= \frac{-14-9}{18}$

$= \frac{-23}{18}$

$\therefore L.HS = R.H.S$

Associative property for addition is true

Associative property for multiplication:

$(a\times b)\times c = a\times(b\times c)$

$= \left[-\frac{7}{9}\times\frac{5}{6}\right]\times\left(-\frac{4}{3}\right)$

$= \left(\frac{-35}{54}\right)\times\left(\frac{-4}{3}\right)$

$= \frac{-35}{\cancel{54}_{27}}\times\frac{-\cancel{4}^{2}}{3}$

$= \frac{70}{81}$

$= -\frac{7}{9}\times\left[\frac{5}{\cancel{6}_{3}}\times-\frac{\cancel{4}^{2}}{3}\right]$

$= -\frac{7}{9}\times\left[\frac{-10}{9}\right]$

$= -\frac{7}{9}\times\frac{-10}{9}$

$= \frac{70}{81}$

$\therefore$ Associative property for multiplication is true.

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4). Verify the distributive property $a \times (b+c) = (a \times b) + (a \times c)$ for the rational numbers $a = -\frac{1}{2}$, $b = \frac{2}{3}$ and $c = -\frac{5}{6}$.

L.H.S $= a \times (b+c)$

$= \left(-\frac{1}{2}\right) \times \left[\left(\frac{2}{3}\right) + \left(-\frac{5}{6}\right)\right]$

$= \left(-\frac{1}{2}\right) \times \left[\frac{2 \times 2 + (-5)}{6}\right]$

$= \left(-\frac{1}{2}\right) \times \left[\frac{4-5}{6}\right]$

$= \left(-\frac{1}{2}\right) \times \left[-\frac{1}{6}\right]$

$= \frac{1}{12}$

R.H.S $= (a \times b) + (a \times c)$

$= \left[\left(-\frac{1}{\cancel{2}}\right) \times \left(\frac{\cancel{2}}{3}\right)\right] + \left[\left(-\frac{1}{2}\right) \times \left(-\frac{5}{6}\right)\right]$

$= \left(-\frac{1}{3}\right) + \left(\frac{5}{12}\right)$

$= \frac{-1 \times 4 + 5}{12}$

$= \frac{-4+5}{12}$

$= \frac{1}{12}$

$\therefore$ L.H.S = R.H.S

$\therefore$ Distributive property is true.

5) verify the identity property for addition and multiplication for the rational numbers $\frac{15}{19}$ and $\frac{-18}{25}$

Identity Property for addition:

$\boxed{0 + a = a = a + 0}$

Let $a = \frac{15}{19}$

$0 + a = a = a + 0$

$0 + \frac{15}{19} = \frac{15}{19} = \frac{15}{19} + 0$

$\frac{15}{19} = \frac{15}{19} = \frac{15}{19}$

Let $b = -\frac{18}{25}$

$0 + b = b = b + 0$

$0 + \left(-\frac{18}{25}\right) = -\frac{18}{25} = \left(-\frac{18}{25}\right) + 0$

$\frac{-18}{25} = \frac{-18}{25} = \frac{-18}{25}$

$\therefore$ Identity property for addition is true.

Identity property for multiplication.

$\boxed{1 \times a = a = a \times 1}$

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Let $a = \frac{15}{19}$

$1 \times \frac{15}{19} = \frac{15}{19} = \frac{15}{19} \times 1$

$\frac{15}{19} = \frac{15}{19} = \frac{15}{19}$

Let $b = \frac{-18}{25}$

$1 \times \left(\frac{-18}{25}\right) = \frac{-18}{25} = \left(\frac{-18}{25}\right) \times 1$

$\frac{-18}{25} = \frac{-18}{25} = \frac{-18}{25}$

$\therefore$ Identity property for multiplication is true.

6). verify the additive and multiplicative inverse property for the rational numbers $\frac{-7}{17}$ and $\frac{17}{27}$.

Additive inverse property:

$$\boxed{a + (-a) = (-a) + a = 0}$$

Let $a = \frac{-7}{17}$ then $-a = \frac{7}{17}$

$a + (-a) = (-a) + a = 0$

$\frac{-7}{17} + \frac{7}{17} = \frac{7}{17} + \left(\frac{-7}{17}\right) = 0$

$\frac{-7+7}{17} = \frac{7-7}{17} = 0$

$\frac{0}{17} = \frac{0}{17} = 0$

$0 = 0 = 0$

Let $b = \frac{17}{27}$ then $-b = \frac{-17}{27}$

$b + (-b) = (-b) + b = 0$

$\frac{17}{27} + \left(\frac{-17}{27}\right) = \left(\frac{-17}{27}\right) + \frac{17}{27} = 0$

$\frac{17}{27} - \frac{17}{27} = \frac{-17}{27} + \frac{17}{27} = 0$

$\frac{17-17}{27} = \frac{-17+17}{27} = 0$

$\frac{0}{27} = \frac{0}{27} = 0$

$0 = 0 = 0.$

$\therefore$ Additive inverse property is true.

Multiplicative inverse property:

$$\boxed{a \times \frac{1}{a} = \frac{1}{a} \times a = 1}$$

Let $a = -\frac{7}{17}$ then $\frac{1}{a} = \frac{17}{-7}$

Let $b = \frac{17}{27}$ then $\frac{1}{b} = \frac{27}{17}$

$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$	$b \times \frac{1}{b} = \frac{1}{b} \times b = 1$
$\frac{-7}{17} \times \frac{17}{-7} = \frac{17}{-7} \times \frac{-7}{17} = 1$	$\frac{17}{27} \times \frac{27}{17} = \frac{27}{17} \times \frac{17}{27} = 1$
$1 = 1 = 1$	$1 = 1 = 1$

$\therefore$ Multiplicative inverse property is true.

Objective Type Questions:

7). Closure property is not true for division of rational numbers because of the number

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$ Ans: (C) 0.

8). $\frac{1}{2} - \left(\frac{3}{4} - \frac{5}{6}\right) \neq \left(\frac{1}{2} - \frac{3}{4}\right) - \frac{5}{6}$ illustrates that Subtraction does not satisfy the ——— Property for rational numbers.

(A) commutative (B) closure (C) distributive

(D) associative Ans: (D) associative

9). Which of the following illustrates the inverse property for addition?

(A) $\frac{1}{8} - \frac{1}{8} = 0$ (B) $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ (C) $\frac{1}{8} + 0 = \frac{1}{8}$ (D) $\frac{1}{8} - 0 = \frac{1}{8}$

Additive inverse Property : $a + (-a) = 0$.

Ans: (A) $\frac{1}{8} - \frac{1}{8} = 0$.

10). $\frac{3}{4} \times \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{3}{4} \times \frac{1}{2} - \frac{3}{4} \times \frac{1}{4}$ illustrates that multiplication is distributive over

(A) addition (B) subtraction (C) multiplication

(D) division.

Ans: (B) subtraction.

Exercise 1.4

1). Fill in the blanks :

(i) The ones digit in the square of 77 is ____.

$77^2 = 5929$ (or) $77^2 \rightarrow 7^2 = 49$

Ans: 9

(ii) The number of non-square numbers between 24^2 and 25^2 is ____.

$24^2 = 576$ and $25^2 = 625$

Non-square numbers (~~between~~) = $625 - 576 = 49 (-1) = 48$

∴ The number of non-square numbers is 48.

Ans: 48.

(iii) The number of perfect square numbers between 300 and 500 is ____.

The perfect square numbers between 300 and 500 ~~are~~

$18^2 = 324$, $19^2 = 361$, $20^2 = 400$, $21^2 = 441$, $22^2 = 484$.

Ans: 5.

(iv) If a number has 5 or 6 digits in it, then its square root will have ____ digits.

Ans: 3.

(v) The value of $\sqrt{180}$ lies between integers ____ and ____.

$13^2 = 169$ and $14^2 = 196$.

Ans: 13 and 14.

2. Say True or False :

(i) When a square number ends in 6, its square root will have 6 in unit's place.

$6^2 = 36$, $16^2 = 256$, $26^2 = 676$, ...

Ans: True.

(ii) A square number will not have odd number of zeros at the end.

Ans: True. $10^2 = 100$, $100^2 = 10000$, ... $20^2 = 400$, ...

(iii) The number of zeros in the square of 91000 is 9.

$91000^2 = 8281000000$, six zeros.

Ans: False

(iv) The square of 75 is 4925.

$75^2 = 5625$

Ans: False

(v) The square root of 225 is 15.

Ans: True

3). Find the square of the following numbers

(i) 17 (ii) 203 (iii) 1098

Sol:

(i) $17^2 = 289$

(ii) $203^2 = 203 \times 203 = 41209$

$$\begin{array}{r} 203 \\ \times 203 \\ \hline 41209 \end{array}$$

$$\begin{array}{r} 203 \times 203 \\ \hline 609 \\ 000 \\ 406 \\ \hline 41209 \end{array}$$

(iii) $1098^2 = 1098 \times 1098 = 1205604$

$$\begin{array}{r} 1098 \times 1098 \\ \hline 8784 \\ 9882 \\ 0000 \\ 1098 \\ \hline 1205604 \end{array}$$

4). Examine if each of the following is a perfect square

(i) 725 (ii) 190 (iii) 841 (iv) 1089

Sol:

(i) 725

$$\begin{array}{r|l} 5 & 725 \\ 5 & 145 \\ 29 & 29 \\ & 1 \end{array}$$

(ii) 190

$$\begin{array}{r|l} 5 & 190 \\ 2 & 38 \\ 19 & 19 \\ & 1 \end{array}$$

(iii) 841

$$\begin{array}{r|l} 29 & 841 \\ 29 & 29 \\ & 1 \end{array}$$

(iv) 1089

$$\begin{array}{r|l} 3 & 1089 \\ 3 & 363 \\ 11 & 121 \\ 11 & 11 \\ & 1 \end{array}$$

$725 = 5^2 \times 29$; $190 = 5 \times 2 \times 19$; $841 = 29^2$; $1089 = 3^2 \times 11^2$

725 is not a perfect square

190 is not a perfect square

841 is a perfect square

1089 is a perfect square

5). Find the square root by prime factorisation method

(i) 144 (ii) 256 (iii) 784 (iv) 1156 (v) 4761

(vi) 9025

Sol

(i) 144

$$\begin{array}{r|l} 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

(ii) 256

$$\begin{array}{r|l} 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

(iii) 784

$$\begin{array}{r|l} 2 & 784 \\ 2 & 392 \\ 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$144 = 2^2 \times 2^2 \times 3^2$

$144 = (2 \times 2 \times 3)^2$

$\sqrt{144} = 2 \times 2 \times 3$

$\sqrt{144} = 12$

$256 = 2^2 \times 2^2 \times 2^2 \times 2^2$

$256 = (2 \times 2 \times 2 \times 2)^2$

$\sqrt{256} = 2 \times 2 \times 2 \times 2$

$\sqrt{256} = 16$

$784 = 2^2 \times 2^2 \times 7^2$

$784 = (2 \times 2 \times 7)^2$

$\sqrt{784} = 2 \times 2 \times 7$

$\sqrt{784} = 28$

(iv) 1156 (v) 4761 (vi) 9025

$$\begin{array}{r|l} 2 & 1156 \\ \hline 2 & 578 \\ \hline 17 & 289 \\ \hline 17 & 17 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 4761 \\ \hline 3 & 1587 \\ \hline 23 & 529 \\ \hline 23 & 23 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 9025 \\ \hline 5 & 1805 \\ \hline 19 & 361 \\ \hline 19 & 19 \\ \hline & 1 \end{array}$$

$1156 = 2^2 \times 17^2$

$1156 = (2\times17)^2$

$\sqrt{1156} = 2\times17$

$\sqrt{1156} = 34$

$4761 = 3^2 \times 23^2$

$4761 = (3\times23)^2$

$\sqrt{4761} = 3\times23$

$\sqrt{4761} = 69$

$9025 = 5^2 \times 19^2$

$9025 = (5\times19)^2$

$\sqrt{9025} = 5\times19$

$\sqrt{9025} = 95$

6). Find the square root by long division method.

~~(i) 144 (ii) 256 (iii) 784 (iv) 1156 (v) 4761 (vi)~~

(i) 1764 (ii) 6889 (iii) 11025 (iv) 17956 (v) 418609

Sol

(i) 1764

$$\begin{array}{r|l} & \;\;42 \\ 4 & \overline{17}\,\overline{64} \\ & (-)16 \\ \hline 82 & \;\;164 \\ & (-)164 \\ \hline & \;\;\;0 \end{array}$$

$\therefore \sqrt{1764} = 42$

(ii) 6889

$$\begin{array}{r|l} & \;\;83 \\ 8 & \overline{68}\,\overline{89} \\ & (-)64 \\ \hline 163 & \;\;489 \\ & (-)489 \\ \hline & \;\;\;0 \end{array}$$

$\sqrt{6889} = 83$

(iii) 11025

$$\begin{array}{r|l} & \;\;105 \\ 1 & \overline{1}\,\overline{10}\,\overline{25} \\ & (-)1 \\ \hline 20 & \;\;010 \\ & (-)00 \\ \hline 205 & \;\;1025 \\ & (-)1025 \\ \hline & \;\;\;0 \end{array}$$

$\sqrt{11025} = 105$

iv). 17956

$$\begin{array}{r|l} & \;\;134 \\ 1 & \overline{1}\,\overline{79}\,\overline{56} \\ & -1 \\ \hline 23 & \;\;079 \\ & (-)69 \\ \hline 264 & \;\;1056 \\ & (-)1056 \\ \hline & \;\;\;0 \end{array}$$

$\therefore \sqrt{17956} = 134$

(v) 418609

$$\begin{array}{r|l} & \;\;647 \\ 6 & \overline{41}\,\overline{86}\,\overline{09} \\ & (-)36 \\ \hline 124 & \;\;586 \\ & (-)496 \\ \hline 1287 & \;\;9009 \\ & (-)9009 \\ \hline & \;\;\;0 \end{array}$$

$\therefore \sqrt{418609} = 647$

7. Estimate the value of the following square roots to the nearest whole number.

(i) $\sqrt{440}$ (ii) $\sqrt{800}$ (iii) $\sqrt{1020}$

Sol :

(i) $\sqrt{440}$

$20^2 = 400$

$21^2 = 441$

$\sqrt{440} \simeq 21$

(ii) $\sqrt{800}$

$28^2 = 784$

$29^2 = 841$

$\sqrt{800} \simeq 28$

(iii) $\sqrt{1020}$

$31^2 = 961$

$32^2 = 1024$

$\sqrt{1020} \simeq 32$

8. Find the square root of the following decimal numbers and fractions.

(i) 2.89 (ii) 67.24 (iii) 2.0164 (iv) $\frac{144}{225}$ (v) $7\frac{18}{49}$

Sol

(i) 2.89

1.7

1 | $\overline{2}.\overline{89}$

(-) 1

27 | 1 89

(-) 1 89

0

$\sqrt{2.89} = 1.7$

(ii) 67.24

8.2

8 | $\overline{67}.\overline{24}$

(-) 64

162 | 324

(-) 324

0

$\sqrt{67.24} = 8.2$

(iii) 2.0164

1.42

1 | $\overline{2}.\overline{01}\,\overline{64}$

(-) 1

24 | 1 01

(-) 96

282 | 564

(-) 564

0

(iv) $\frac{144}{225}$

$$\sqrt{\frac{144}{225}} = \sqrt{\left(\frac{12}{15}\right)^2} = \frac{12}{15} = \frac{4}{5}$$

(v) $7\frac{18}{49}$

$$\sqrt{7\frac{18}{49}} = \sqrt{\frac{343+18}{49}} = \sqrt{\frac{361}{49}} = \sqrt{\left(\frac{19}{7}\right)^2} = \frac{19}{7} = 2\frac{5}{7}$$

9. Find the least number that must be subtracted to 6666 so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

sol:

$$\begin{array}{r|l} & 81 \\ 8 & \overline{66}\,\overline{66} \\ & (-)64 \\ \hline 161 & \;\;266 \\ & (-)161 \\ \hline & \;\;105 \end{array}$$

The remainder in the last step is 105.

Is if 105 be subtracted from the given number the remainder will be zero and the new number will be a perfect square.

$\therefore$ The required number is 105.

The perfect square number is $6666 - 105 = 6561$

Also $\sqrt{6561} = 81$.

10. Find the least number by which 1800 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

$$\begin{array}{r|l} 5 & 1800 \\ 5 & 360 \\ 3 & 72 \\ 3 & 24 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$1800 = 5^2 \times 3^2 \times 2^2 \times 2$.

Here the last factor 2 has no pair. So if we multiply 1800 by 2, then the number becomes a perfect square.

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$$1800 \times 2 = (5^2 \times 3^2 \times 2^2 \times 2) \times 2$$
$$3600 = 5^2 \times 3^2 \times 2^2 \times 2^2$$
$$3600 = (5 \times 3 \times 2 \times 2)^2$$
$$3600 = 60^2$$
$$\therefore \sqrt{3600} = \sqrt{60^2} = 60.$$

Objective Type Questions:

11. The square of 43 ends with the digit ______. (A) 9 (B) 6 (C) 4 (D) 3.

$43^2 = 1849$

Ans : (A) 9

12. ______ is added to 24^2 to get 25^2

(A) 4^2 (B) 5^2 (C) 6^2 (D) 7^2.

$24^2 = 576$

$25^2 = (24)^2 + 24 + 25 = 576 + 49 = (24)^2 + 7^2 = 625$

Ans : (D) 7^2.

13. $\sqrt{48}$ is approximately equal to ______.

(A) 5 (B) 6 (C) 7 (D) 8.

$7^2 = 49$ $\quad \sqrt{48} \simeq 7$ $\quad$ Ans : (C) 7.

$6^2 = 36$

14. $\sqrt{128} - \sqrt{98} + \sqrt{18} =$ ______

(A) $\sqrt{2}$ (B) $\sqrt{8}$ (C) $\sqrt{48}$ (D) $\sqrt{32}$

2	128	2	98	2	18
2	64	7	49	3	9
2	32	7	7	3	3
2	16		1		1
2	8				
2	4				
2	2				
	1				

$\sqrt{128} - \sqrt{98} + \sqrt{18}$

$= \sqrt{2^2 \times 2^2 \times 2^2 \times 2} - \sqrt{7^2 \times 2} + \sqrt{3^2 \times 2}$

$= 2 \times 2 \times 2\sqrt{2} - 7\sqrt{2} + 3\sqrt{2}$

$= 8\sqrt{2} - 7\sqrt{2} + 3\sqrt{2}$

$= (8 - 7 + 3)\sqrt{2}$

$= (11 - 7)\sqrt{2}$

$= 4\sqrt{2}$

$= \sqrt{16}\sqrt{2}$

$= \sqrt{16 \times 2}$

$= \sqrt{32}$

Ans : (D). $\sqrt{32}$

15. The number of digits in the square root of 123454321 is ______.

(A) 4 (B) 5 (C) 6 (D) 7.

$\sqrt{\overline{1}\,\overline{23}\,\overline{45}\,\overline{43}\,\overline{21}}$; No. of bars = 5. Ans: (B). 5

Exercise 1.5

1. Fill in the blanks :

(i) The ones digits in the cube of 73 is ______.

If a natural number ends with 3 its cube ends with 7.

Ans : 7.

(ii) The maximum number of digits in the cube of a two digit number is ______.

The cube of a two digit number may have 4 or 5 or 6 digits in it.

Maximum number = 6.

Ans : 6

(iii) The smallest number to be added to 3333 to make it a perfect cube is ____.

$$\begin{array}{r|l} & 57 \\ 5 & \overline{33}\,\overline{33} \\ & 25 \\ 107 & 833 \\ & 749 \\ & 84 \end{array}$$

$57^2 = 3249$

$58^2 = 3364$

$58^2 = 57^2 + 58 + 57 = 3364$.

$3364 - 3333 = 31$.

$\overline{3}\,\overline{333} =$

$$\begin{array}{r} 225 \times 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \end{array}$$

$15^3 = 15 \times 15 \times 15 = 225 \times 15 = 3375$

$3375 - 3333 = 42$

Ans : 42.

(iv). The cube root of 540×50 is ____.

$$\begin{array}{r|l} 5 & 540 \\ 3 & 108 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \qquad \begin{array}{r|l} 5 & 50 \\ 5 & 10 \\ 2 & 2 \\ & 1 \end{array}$$

$\sqrt[3]{540 \times 50}$

$= \sqrt[3]{(5 \times 3 \times 2^3 \times 2 \times 2) \times (5^2 \times 2)}$

$= \sqrt[3]{5^3 \times 2^3 \times}$

$$\begin{array}{r|l} 5 & 540 \\ 3 & 108 \\ 3 & 36 \\ 3 & 12 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \qquad \begin{array}{r|l} 5 & 50 \\ 5 & 10 \\ 2 & 2 \\ & 1 \end{array}$$

$\sqrt[3]{540 \times 50}$

$= \sqrt[3]{(3^3 \times 2^2 \times 5) \times (5^2 \times 2)}$

$= \sqrt[3]{3^3 \times 2^3 \times 5^3}$

$= \sqrt[3]{(3 \times 2 \times 5)^3}$

$= 3 \times 2 \times 5$

$= 30$.

Ans : 30

(v) The cube root of 0.000004913 is ___.

$$\sqrt[3]{0.000004913} = \sqrt[3]{\frac{4913}{1000000000}} = \sqrt[3]{\left(\frac{17}{1000}\right)^3} = \frac{17}{1000}$$

$$= 0.017 \qquad \sqrt[3]{4913} = 17.$$

Ans: 0.017.

2. Say True or False:

(i) The cube of 24 ends with the digit 4.

If a natural number ends with 4 its cube also ends with 4.

Ans: True.

(ii) Subtracting 10^3 from 1729 gives 9^3.

$1729 - 10^3 = 1729 - 1000 = 729 = 9^3$

~~$10^3 - 1729 = 1000 - 1729 = -729 = 9^3$~~

$81 \times 9 = 729$

Ans: True

(iii) The cube of 0.0012 is 0.000001728.

$\sqrt[3]{1728} = 12.$

$$\sqrt[3]{0.000001728} = \sqrt[3]{\frac{1728}{1000000000}} = \sqrt[3]{\left(\frac{12}{1000}\right)^3} = \frac{12}{1000}$$

$$= 0.012$$

Ans: False.

(iv) 79570 is not a perfect cube.

79570 is not a perfect cube because it ends with only one zero.

Ans: True.

(v). The cube root of 250047 is 63.

$63^3 = 63^2 \times 63 = 3969 \times 63 = 250047$

3969×63

11907

23814

250047

Ans: True

3. Show that 1944 is not a perfect cube.

Sol:

$$\begin{array}{r|l} 2 & 1944 \\ \hline 2 & 972 \\ \hline 2 & 486 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$1944 = 2^3 \times 3^3 \times 3^2$

$\therefore$ 1944 is not a perfect cube.

4. Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube.

Sol:

$$\begin{array}{r|l} 5 & 10985 \\ \hline 13 & 2197 \\ \hline 13 & 169 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$10985 = 13^3 \times 5$

13	26	39
52	65	78
91	104	117

130

If we divide by 5, the new number will be a perfect cube.

$\therefore$ The required number is 5.

5. Find the smallest number by which 200 should be multiplied to make it a perfect cube.

Sol:

$$\begin{array}{r|l} 5 & 200 \\ \hline 5 & 40 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$200 = 2^3 \times 5^2$

We need one more 5 to make it a perfect cube.

$\therefore$ The required number is 5.

6. Find the cube root of $24 \times 36 \times 80 \times 25$

Sol

$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array} \quad \begin{array}{r|l} 5 & 80 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \quad \begin{array}{r|l} 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\sqrt[3]{24 \times 36 \times 80 \times 25}$$

$$= \sqrt[3]{(2^3 \times 3) \times (2^2 \times 3^2) \times (5 \times 2 \times 2^3) \times (5^2)}$$

$$= \sqrt[3]{2^3 \times 3 \times 2^2 \times 3^2 \times 5 \times 2 \times 2^3 \times 5^2}$$

$$= \sqrt[3]{(2^3) \times (3 \times 3^2) \times (2^2 \times 2) \times (2^3) \times (5 \times 5^2)}$$

$$= \sqrt[3]{2^3 \times 3^3 \times 2^3 \times 2^3 \times 5^3}$$

$$= \sqrt[3]{(2 \times 3 \times 2 \times 2 \times 5)^3}$$

$$= 2 \times 3 \times 2 \times 2 \times 5$$

$$= 6 \times 4 \times 5$$

$$= 24 \times 5$$

$$= 120$$

7. Find the cube root of 729 and 6859 by prime factorisation.

Sol

$$\begin{array}{r|l} 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\sqrt[3]{729} = \sqrt[3]{3^3 \times 3^3} = \sqrt[3]{(3 \times 3)^3} = 3 \times 3 = 9$$

19 | 6859
19 | 361
19 | 19
 | 1

$\sqrt[3]{6859} = \sqrt[3]{19^3} = 19$

1	9	19
2	18	38
3	27	57
4	36	76
5	45	95
6	54	114
7	63	133
8	72	152
9	81	171
10	90	190

8). What is the square root of cube root of 46656?

Sol Method I:

$\sqrt{\sqrt[3]{46656}} = \sqrt{36} = 6.$

$\overline{46}\,\overline{656} = 36$

Method II:

$\sqrt{\sqrt[3]{46656}}$

$= \sqrt{\sqrt[3]{3^3 \times 3^3 \times 2^3 \times 2^3}}$

$= \sqrt{\sqrt[3]{(3 \times 3 \times 2 \times 2)^3}}$

$= \sqrt{3 \times 3 \times 2 \times 2}$

$= \sqrt{3^2 \times 2^2}$

$= \sqrt{(3 \times 2)^2}$

$= \sqrt{6^2}$

$= 6.$

3 | 46656
3 | 15552
3 | 5184
3 | 1728
3 | 576
3 | 192
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2 | 2
 | 1

9. If the cube of a squared number is 729, find the square root of that number.

Sol. Let x be the number.

$$(x^3)^2 = 729$$

$$x^3 = \sqrt{729}$$

$$= \sqrt{3^2 \times 3^2 \times 3^2}$$

$$x^3 = \sqrt{(3\times3\times3)^2}$$

$$= 3\times3\times3$$

$$x^3 = 3^3$$

$$x = 3$$

3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore$ The square root of the number 3 is 1.732

	1.732
1	3.00 0000
	(−) 1
27	200
	(−) 189
343	1100
	(−) 1029
3462	7100
	(−) 6924
	176

$\sqrt{3} \simeq 1.732$

10. Find the two smallest perfect square numbers which when multiplied together gives a perfect cube number.

Sol.

Consider the numbers 4 and 16.

Their product $4\times16 = 64$

$64 = 4^3$

$\therefore$ The required square numbers are 4 and 16.

Exercise 1.6

1. Fill in the blanks:

(i) $(-1)^{\text{even integer}}$ is ____.

Ans : 1

(ii) For $a \neq 0$, a^0 is ____.

Ans: 1

(iii) $4^{-3} \times 5^{-3} = $ ____

$a^m \times b^m = (ab)^m \Rightarrow 4^{-3} \times 5^{-3} = (20)^{-3}$

Ans: 20^{-3}

(iv). $(-2)^{-7} = $ ____

$(-2)^{-7} = \frac{1}{(-2)^7} = \frac{1}{-128} = \frac{-1}{128}$

Ans : $\frac{-1}{128}$

(v) $\left(-\frac{1}{3}\right)^{-5} = $ ____

Ans : -243

$\left(-\frac{1}{3}\right)^{-5} = \left[-(3)^{-1}\right]^{-5} = \left[-3^5\right]$

$= -243$

Say True or false:

(i) If $8^x = \frac{1}{64}$, the value of x is -2.

$8^x = \frac{1}{64} \Rightarrow 8^x = (64)^{-1} \Rightarrow 8^x = (8^2)^{-1} \Rightarrow 8^x = 8^{-2}$

$\Rightarrow x = -2$

Ans: True.

(ii) The simplified form of $(256)^{-\frac{1}{4}} \times 4^2$ is $\frac{1}{4}$.

$(256)^{-\frac{1}{4}} \times 4^2 = (4^4)^{-\frac{1}{4}} \times 4^2 = 4^{-1} \times 4^2 = 4^{-1+2}$

Ans: False $= 4^1 = 4.$

(iii). Using the power rule, $(3^7)^{-2} = 3^5$.

$(3^7)^{-2} = 3^{-14}$ Ans: False

(iv). The standard form of 2×10^{-4} is 0.0002.

$2 \times 10^{-4} = 0.0002$ Ans: True

(v). The scientific form of 123.456 is 1.23456×10^{-2}.

$123.456 = 1.23456 \times 10^2$ Ans: ~~Ans~~ false.

3. Evaluate:

(i) $\left(\frac{1}{2}\right)^3$ (ii) $\left(\frac{1}{2}\right)^{-5}$ (iii) $\left(\frac{-5}{6}\right)^{-3}$ (iv) $(2^{-5} \times 2^7) \div 2^{-2}$

(v) $(2^{-1} \times 3^{-1}) \div 6^{-2}$.

Sol:

(i) $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(ii) $\left(\frac{1}{2}\right)^{-5} = (2^{-1})^{-5} = 2^5 = 32$ $a^{-m} = \frac{1}{a^m}; (a^m)^n = a^{mn}$

(iii) $\left(\frac{-5}{6}\right)^{-3} = \frac{(-5)^{-3}}{6^{-3}} = \frac{6^3}{(-5)^3} = \frac{216}{-125} = -\frac{216}{125}$

(iv) $(2^{-5} \times 2^7) \div 2^{-2}$

$= (2^{-5+7}) \div 2^{-2}$

$= 2^2 \div 2^{-2}$

$= \frac{2^2}{2^{-2}} = 2^{2-(-2)} = 2^{2+2} = 2^4 = 16.$

(v). $(2^{-1} \times 3^{-1}) \div 6^{-2}$

$= [(2\times 3)^{-1}] \div 6^{-2}$

$= [(6)^{-1}] \div 6^{-2}$

$= \dfrac{6^{-1}}{6^{-2}}$

$= (6)^{-1-(-2)}$

$= (6)^{-1+2}$

$= 6^{1}$

$= 6.$

4). Evaluate: (i) $\left(\frac{2}{5}\right)^{4} \times \left(\frac{5}{2}\right)^{-2}$ (ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$ (iii) $2^{7} \times \left(\frac{1}{2}\right)^{-3}$

Sol:

(i) $\left(\frac{2}{5}\right)^{4} \times \left(\frac{5}{2}\right)^{-2}$

$= \left(\frac{2}{5}\right)^{4} \times \left[\left(\frac{2}{5}\right)^{-1}\right]^{-2}$ $\left[\because a^{-m} = \frac{1}{a^m}\right]$

$= \left(\frac{2}{5}\right)^{4} \times \left(\frac{2}{5}\right)^{2}$ $\left[\because (a^m)^n = a^{mn}\right]$

$= \left(\frac{2}{5}\right)^{4+2}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= \left(\frac{2}{5}\right)^{6}$

$= \dfrac{2^6}{5^6}$ $\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= \dfrac{64}{15625}$

$5^2 = 25$
$5^4 = 625$
$5^6 = 5^4 \times 5^2 =$

625×25
3125
1250
15625

(ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$

$= \dfrac{\left(\frac{4}{5}\right)^{-2}}{\left(\frac{4}{5}\right)^{-3}}$

$= \left(\frac{4}{5}\right)^{-2-(-3)}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= \left(\frac{4}{5}\right)^{-2+3}$

$= \left(\frac{4}{5}\right)^{1}$

$= \frac{4}{5}$ $\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

(iii) $2^7 \times \left(\frac{1}{2}\right)^{-3}$

$= 2^7 \times (2^{-1})^{-3}$ $\left[\because \frac{1}{a^m} = a^{-m}\right]$

$= 2^7 \times 2^{-1 \times -3}$ $\left[\because (a^m)^n = a^{mn}\right]$

$= 2^7 \times 2^3$

$= 2^{7+3}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= 2^{10}$

$= 1024$

5. Evaluate : (i) $(5^0 + 6^{-1}) \times 3^2$ (ii) $(2^{-1} + 3^{-1}) \div 6^{-1}$
(iii) $(3^{-1} + 4^{-2} + 5^{-3})^0$

Sol:

$(5^0 + 6^{-1}) \times 3^2$

$= 5^0 \times 3^2 + 6^{-1} \times 3^2$

$= 1 \times 9 + (2 \times 3)^{-1} \times 3^2$ $[\because a^0 = 1]$

$= 9 + 2^{-1} \times 3^{-1} \times 3^2$ $[\because (ab)^m = a^m \times b^m]$

$= 9 + 2^{-1} \times 3^{-1+2}$ $[\because a^m \times a^n = a^{m+n}]$

$= 9 + 2^{-1} \times 3^1$

$= 9 + \frac{1}{2} \times 3$ $[\because a^{-m} = \frac{1}{a^m}]$

$= 9 + \frac{3}{2}$

$= \frac{18+3}{2}$

$= \frac{21}{2}$

(ii) $(2^{-1} + 3^{-1}) \div 6^{-1}$

$= \left(\frac{1}{2} + \frac{1}{3}\right) \div \left(\frac{1}{6}\right)$ $[\because a^{-m} = \frac{1}{a^m}]$

$= \left(\frac{3+2}{6}\right) \div \left(\frac{1}{6}\right)$

$= \frac{5}{6} \div \frac{1}{6}$

$= \frac{\frac{5}{6}}{\frac{1}{6}}$

$= \frac{5}{1}$

$= 5$

(iii) $(3^{-1} + 4^{-2} + 5^{-3})^0$

$= 1$ $[\because a^0 = 1 \text{ where } a \neq 0]$

(6). Simplify: (i) $(3^2)^3 \times (2\times3^5)^{-2} \times (18)^2$

(ii) $\dfrac{9^2\times7^3\times2^5}{84^3}$ (iii) $\dfrac{2^8\times2187}{3^5\times32}$

Sol.

(i) $(3^2)^3 \times (2\times3^5)^{-2} \times (18)^2$

$$\begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$= (3^2)^3 \times (2\times3^5)^{-2} \times (2\times3^2)^2$

$= (3^2)^3 \times (2^{-2}) \times (3^5)^{-2} \times (2)^2 \times (3^2)^2$

$= 3^6 \times 2^{-2} \times 3^{-10} \times 2^2 \times 3^4$

$= 3^{6-10+4} \times 2^{-2+2}$

$= 3^0 \times 2^0$

$= 1\times1$

$= 1.$

(ii) $\dfrac{9^2\times7^3\times2^5}{84^3}$

$$\begin{array}{r|l} 3 & 84 \\ 7 & 28 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \qquad \begin{array}{r|l} 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$= \dfrac{(3^2)^2\times7^3\times2^5}{(3\times7\times2^2)^3}$

$= \dfrac{(3^2)^2\times7^3\times2^5}{3^3\times7^3\times(2^2)^3}$

$= \dfrac{3^4\times7^3\times2^5}{3^3\times7^3\times2^6}$

$= 3^{4-3}\times7^{3-3}\times2^{5-6}$

$= 3^1\times7^0\times2^{-1}$

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$= 3 \times 1 \times \frac{1}{2}$

$= \frac{3}{2}$

(iii) $\frac{2^8 \times 2187}{3^5 \times 32}$

$3 \,|\, 2187$, $3 \,|\, 729$, $3 \,|\, 243$, $3 \,|\, 81$, $3 \,|\, 27$, $3 \,|\, 9$, $3 \,|\, 3$, 1

$2 \,|\, 32$, $2 \,|\, 16$, $2 \,|\, 8$, $2 \,|\, 4$, $2 \,|\, 2$, 1

$= \frac{2^8 \times 3^7}{3^5 \times 2^5}$

$= 2^{8-5} \times 3^{7-5}$

$= 2^3 \times 3^2$

$= 8 \times 9$

$= 72.$

7. Solve for x: (i) $\frac{2^{2x-1}}{2^{x+2}} = 4$

(ii) $\frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5}$

Sol

(i) $\frac{2^{2x-1}}{2^{x+2}} = 4$

$2^{(2x-1)-(x+2)} = 2^2$

$2x-1-(x+2) = 2.$

$2x-1-x-2 = 2$

$x-3 = 2$

$x = 2+3$

$x = 5 \Rightarrow \boxed{x=5}$

(ii) $\dfrac{5^{5} \times 5^{-4} \times 5^{x}}{5^{12}} = 5^{-5}$

$5^{5-4+x-12} = 5^{-5}$

$5-4+x-12 = -5$

$5+x-16 = -5$

$x-11 = -5$

$x = -5+11$

$x = 6$

$\therefore \boxed{x=6}$

8. Expand using exponents :

(i) 6054.321 (ii) 897.14

sol

(i) 6054.321

$= 6 \times 10^{3} + 0 \times 10^{2} + 5 \times 10^{1} + 4 \times 10^{0} + 3 \times 10^{-1} + 2 \times 10^{-2} + 1 \times 10^{-3}$

$= 6 \times 10^{3} + 5 \times 10^{1} + 4 \times 10^{0} + 3 \times 10^{-1} + 2 \times 10^{-2} + 1 \times 10^{-3}$

(ii) 897.14

$= 8 \times 10^{2} + 9 \times 10^{1} + 7 \times 10^{0} + 1 \times 10^{-1} + 4 \times 10^{-2}$

9. Find the number in standard form for the following expansions:

(i) $8 \times 10^{4} + 7 \times 10^{3} + 6 \times 10^{2} + 5 \times 10^{1} + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$

(ii) $5 \times 10^{3} + 5 \times 10^{1} + 5 \times 10^{-1} + 5 \times 10^{-3}$

(iii) The radius of a hydrogen atom is 2.5×10^{-11} m.

Sol

(i) $8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$

$= 8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 10^0 + 0 \times 10^{-1} + 4 \times 10^{-2} + 0 \times 10^{-3} + 7 \times 10^{-4}$

$= 87652.0407$

(ii) $5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$

$= 5 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 0 \times 10^0 + 5 \times 10^{-1} + 0 \times 10^{-2} + 5 \times 10^{-3}$

$= 5050.505$

(iii) 2.5×10^{-11} m

$= 0.000000000025$ m

10). Write the following numbers in scientific notation:

(i). 467800000000 (ii). 0.000001972

(iii). 1642.398

(iv). Earth's volume is about 1,083,000,000,000 cubic kilometers

(v). If you fill a bucket with dirt, the portion of the whole Earth that is in the bucket will be 0.00000000000000000000066 kg.

Sol.

(i). 467800000000 $= 4.678 \times 10^{11}$

(ii). 0.000001972 $= 1.972 \times 10^{-6}$

(iii). 1642.398 $= 1.642398 \times 10^{3}$

(iv). 1,083,000,000,000 $= 1.083 \times 10^{12}$ cu. km.

(v). $0.\underbrace{000}_{1}\,\underbrace{000}_{2}\,\underbrace{000}_{3}\,\underbrace{000}_{4}\,\underbrace{000}_{5}\,\underbrace{000}_{6}\,\underbrace{000}_{7}\,0016$ Kg.

$= 1.6 \times 10^{-24}$ Kg.

Objective Type Questions:

11. By what number should $(-4)^{-1}$ be multiplied so that the product becomes 10^{-1}? (A) $\frac{2}{3}$ (B) $\frac{-2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{-5}{2}$

$\cancel{12.\ (-2)^{-3} \times (-2)^{-2} = \_\_\_\_ \quad (A)\ \frac{-1}{32} \quad (B)\ \frac{1}{32}}$

$(-4)^{-1} \times x = 10^{-1}$

$x = \frac{10^{-1}}{(-4)^{-1}} = \left(\frac{10}{-4}\right)^{-1} = \left(\frac{-4}{10}\right) = -\frac{2}{5}$

Ans: (B). $\frac{-2}{5}$

12. $(-2)^{-3} \times (-2)^{-2} = \_\_\_\_$

(A) $\frac{-1}{32}$ (B) $\frac{1}{32}$ (C) 32 (D) -32.

$(-2)^{-3} \times (-2)^{-2} = (-2)^{-3+(-2)} = (-2)^{-3-2} = (-2)^{-5}$

$= \frac{1}{(-2)^5} = \frac{1}{-32} = -\frac{1}{32}$

Ans: (A). $\frac{-1}{32}$

13. Which is not correct?

(A) $\left(\frac{-1}{4}\right)^2 = 4^2$ (B) $\left(\frac{-1}{4}\right)^2 = \left(\frac{1}{2}\right)^4$ (C) $\left(\frac{-1}{4}\right)^2 = 16^{-1}$

(D). $-\left(\frac{1}{4}\right)^2 = 16^{-1}$

(A) ~~$\left(\frac{-1}{4}\right)^2 = 4^{-2} \Rightarrow \left[-(4)^{-1}\right]^2 = 4^{-2} \Rightarrow -(4)^{-2} \neq 4^{-2}$~~

(A) $\left(\frac{-1}{4}\right)^2 = 4^{-2} \Rightarrow \left(\frac{1}{4^2}\right) = \frac{1}{4^2}$

(B) $\left(-\frac{1}{4}\right)^2 = \left(\frac{1}{2}\right)^4 \Rightarrow \frac{1}{4^2} = \frac{1}{2^4} \Rightarrow \frac{1}{(2^2)^2} = \frac{1}{2^4}$

$\Rightarrow \frac{1}{2^4} = \frac{1}{2^4}$

(C) $\left(\frac{-1}{4}\right)^2 = 16^{-1} \Rightarrow \frac{1}{4^2} = \frac{1}{16} \Rightarrow \frac{1}{16} = \frac{1}{16}$

(D) $-\left(\frac{1}{4}\right)^2 = 16^{-1} \Rightarrow -\frac{1}{4^2} = \frac{1}{16} \Rightarrow -\frac{1}{16} \neq \frac{1}{16}$

Ans : (D). $\left(-\frac{1}{4}\right)^2 = 16^{-1}$

14. If $\frac{10^x}{10^{-3}} = 10^9$, then x is ____

(A) 4 (B) 5 (C) 6 (D) 7.

$\frac{10^x}{10^{-3}} = 10^9 \Rightarrow 10^{x-(-3)} = 10^9 \Rightarrow x-(-3) = 9$

$x+3 = 9 \Rightarrow x = 9-3 \Rightarrow \boxed{x=6}$

Ans : (C). 6.

15. 0. 0000000002020 in Scientific form is ____

(A) 2.02×10^9 (B) 2.02×10^{-9} (C) 2.02×10^{-8}

(D) 2.02×10^{-10}.

$0.0000000002020 = 2.02 \times 10^{-10}$

Ans : (D). 2.02×10^{-10}.

Exercise 1.7

1. If $\frac{3}{4}$ of a box of apples weighs 3 kg and 225 gm, how much does a full box of apples weigh?

Sol

Let the total weight of a box of apples $= x$ kg.

Weight of $\frac{3}{4}$ of a box apples $= 3$ kg 225 gm

$= 3.225$ kg

$\frac{3}{4} \times x = 3.225$ kg

$x = \frac{\overset{1.075}{\cancel{3.225}} \times 4}{\underset{1}{\cancel{3}}}$ kg

$= 4.300$ kg

$x = 4$ kg 300 gm

Total weight of a box of apples

$= 4$ kg 300 gm.

2. Mangalam buys a water jug of capacity $3\frac{4}{5}$ litre. If she buys another jug which is $2\frac{2}{3}$ times as large as the smaller jug, how many litre can the larger one hold?

Sol

Capacity of small water jug $= 3\frac{4}{5}$ litres.

Capacity of big water jug

$= 2\frac{2}{3}$ times of small one.

$= 2\frac{2}{3} \times 3\frac{4}{5}$

$= \frac{8}{3} \times \frac{19}{5}$

$= \frac{152}{15}$

$= 10\frac{2}{15}$ litres.

Capacity of the large jug = $10\frac{2}{15}$ litres.

3. Ravi multiplied $\frac{25}{8}$ and $\frac{16}{15}$ and he says that the simplest form of this Product is $\frac{10}{3}$ and chandru says the answer in the simplest form is $3\frac{1}{3}$. who is correct (or) Are they both correct? Explain.

Sol
Product of $\frac{25}{8}$ and $\frac{16}{15}$

$= \frac{\cancel{25}^{5}}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{\cancel{15}_{3}} = \frac{10}{3} = 3\frac{1}{3}$.

$\therefore$ Both are correct.

4. Find the length of a room whose area is $\frac{153}{10}$ sq.m and whose breadth is $2\frac{11}{20}$ m.

Sol
Given Area = $\frac{153}{10}$ sq.m and breadth = $2\frac{11}{20}$ m.

Area = length × breadth

$\frac{153}{10}$ sq.m = $l \times 2\frac{11}{20}$ m.

$\frac{153}{10} \div 2\frac{11}{20} = l$

$l = \frac{153/10}{51/20} = \frac{\cancel{153}^{3}}{\cancel{10}_{1}} \times \frac{\cancel{20}^{2}}{\cancel{51}_{1}} = 6$m.

Length of the room = 6 m.

5). There is a large square portrait of a leader that covers an area of 4489 cm². If each side has a 2 cm linear, what would be its area?

Sol

Area of the square = 4489 cm²

$(\text{Side})^2 = 4489 \text{ cm}^2$

$\text{side} = \sqrt{4489 \text{ cm}^2}$

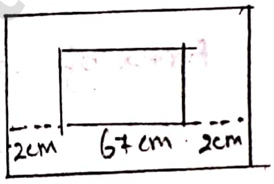
$= \sqrt{67^2} \text{ cm}$

side = 67 cm

$$\begin{array}{r|l} & 67 \\ 6 & 44\,89 \\ & (-)36 \\ 127 & 889 \\ & (-)889 \\ & 0 \end{array}$$

Length of the side which is linear

= 2 cm + 67 cm + 2 cm = 71 cm.



Area of the larger square = 71^2 sq.cm.

= 5041 cm²

Area of the linear = Area of large square − Area of small square.

$= (5041 - 4489) \text{ cm}^2 = 552 \text{ cm}^2$.

6). A greeting card has an area 90 cm². Between what two whole numbers is the length of its side?

Sol

Area = 90 cm²

$10^2 = 100$ and $9^2 = 81$

Length of its side lies between 9 cm and 10 cm.

7). 225 square shaped mosaic tiles, each of area 1 square decimetre exactly cover a square shaped veradah. How long is each side of the square shaped veradah?

Sol

Area of one tile = 1 sq. decimeter.

Area of 225 tiles = 225 sq. decimeter.

225 square tiles exactly covers the square shaped verandah.

∴ Area of 225 tiles = Area of the verandah

= 225 sq. decimeter

Area of square = 15 × 15 sq. decimeter

side = 15 ~~sq.~~ decimeters

Length of each side of verandah = 15 decimeters.

8). If $\sqrt[3]{1906624} \times \sqrt{x} = 3100$, find x.

Sol

$\sqrt[3]{1906624} \times \sqrt{x} = 3100$

$\sqrt[3]{2^3 \times 2^3 \times 31^3} \times \sqrt{x} = 3100$

$\sqrt[3]{(2\times2\times31)^3} \times \sqrt{x} = 3100$

$(2\times2\times31) \times \sqrt{x} = 3100$

$\sqrt{x} = \frac{3100}{2\times2\times31}$

$\sqrt{x} = 25$

$\sqrt{x} = \sqrt{625}$

$\boxed{x = 625}$

8	1906624
8	238328
31	29791
31	961
31	31
	1

31, 62, 93, 124, 155, 186, 217, 248, 279, 310

9). If $2^{m-1} + 2^{m+1} = 640$, then find m.

Sol

$$2^{m-1} + 2^{m+1} = 640.$$

$$2^{m-1} + 2^{m+1} = 128 + 512 \text{ [consecutive Powers of 2]}$$

$$2^{m-1} + 2^{m+1} = 2^{7} + 2^{9}$$

$$m-1 = 7 \Rightarrow m = 7+1 \Rightarrow \boxed{m=8}$$

10). Give the answer in scientific rotation:
A human heart beats at an average of 80 beats per minute. How many times does it beat in (i) an hour? (ii) a day? (iii) a year? (iv) 100 years?

Sol

Heart beat per minute = 80 beats.

(i) An hour:

1 hour = 60 minutes.

Heart beat per hour = $60 \times 80 = 4800$

$= 4.8 \times 10^{3}$

(ii) In a day:

1 day = 24 hours = 24×60 minutes.

24×48
1152

$\therefore$ Heart beat per day

$= 24 \times 60 \times 80 = 24 \times 4800 = 115200$

$= 1.152 \times 10^{5}$

(iii) A year?

1 year = 365 days = $365 \times 24 \times 60 \times 80$

$\therefore$ Heart beat per year = 365×115200

$= 42048000 = 4.2048 \times 10^7$

1152×365
5760
6912
3456
420480

(iv) 100 years

~~1 yea~~

Heart beats in 100 years

$= 100 \times$ Heart beat in 1 year

$= 100 \times 4.2048 \times 10^7$

$= 4.2048 \times 10^7 \times 10^2$

$= 4.2048 \times 10^9$

Challenging Problems.

11). In a map, if 1 inch refers to 120 km, then find the distance between two cities B and C which are $4\frac{1}{6}$ inches and $3\frac{1}{3}$ inches from the city A which lies between the cities B and C.

Sol

B $4\frac{1}{6}$ inch A $3\frac{1}{3}$ inch. C

Given 1 inch = 120 km.

Distance between B and C

$= \left(4\frac{1}{6} + 3\frac{1}{3}\right)$ inches

$= \frac{25}{6} + \frac{10}{3}$ inches

$= \frac{25}{6} + \frac{10 \times 2}{3 \times 2}$ inches

$= \frac{25+20}{6}$ inches

$= \frac{45}{6}$ inches

$= \frac{45}{\cancel{6}_1} \times \cancel{120}^{20}$ km

$= 900$ km.

Distance between B and C = 900 km.

12). Give an example and verify each of the following statements.

i) The collection of all non-zero rational numbers is closed under division.

ii). Subtraction is not commutative for rational numbers.

iii). Division is not associative for rational numbers.

iv). Distributive property of multiplication over subtraction is true for rational numbers. That is $a(b-c) = ab - ac$.

v). The mean of two rational numbers is rational and lies between them.

Sol

(i) Let $a = \frac{5}{4}$ and $b = -\frac{4}{3}$ be two non-zero rational numbers.

$a \div b = \frac{5}{6} \div \frac{-4}{3} = \frac{5}{\cancel{6}_2} \times \frac{\cancel{3}^1}{-4} = \frac{5}{-8} = \frac{-5}{8} \in Q$

$\therefore$ Collection of non-zero rational numbers are closed under division.

(ii) Let $a = \frac{1}{2}$ and $b = \frac{-5}{6}$ be two rational numbers.

$a-b = \frac{1}{2} - \left(-\frac{5}{6}\right) = \frac{1}{2} + \frac{5}{6} = \frac{3+5}{6} = \frac{8}{6} =$

$b-a = \frac{-5}{6} - \frac{1}{2} = \frac{-5-3}{6} = -\frac{8}{6}$

$a-b \neq b-a$

$\therefore$ Subtraction is not commutative for rational numbers.

(iii) Let $a = \frac{2}{5}$, $b = \frac{6}{5}$, $c = \frac{3}{5}$ be three rational numbers.

$a \div (b \div c) = \frac{2}{5} \div \left(\frac{6}{5} \div \frac{3}{5}\right) = \frac{2}{5} \div \left(\frac{\cancel{6}^{2}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{3}_{1}}\right)$

$= \frac{2}{5} \div \frac{2}{1} = \frac{\cancel{2}}{5} \times \frac{1}{\cancel{2}} = \frac{1}{5} \rightarrow ①$

$(a \div b) \div c = \left(\frac{2}{5} \div \frac{6}{5}\right) \div \frac{3}{5} = \left(\frac{\cancel{2}^{1}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}_{3}}\right) \div \frac{3}{5}$

$= \frac{1}{3} \div \frac{3}{5} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9} \rightarrow ②$

From ① and ② $\frac{1}{5} \neq \frac{5}{9}$

$\therefore a \div (b \div c) \neq (a \div b) \div c$

$\therefore$ Division is not associative for rational numbers.

(iv). Let $a = \frac{2}{9}$, $b = \frac{3}{6}$, $c = \frac{1}{3}$ be three rational numbers.

To prove : $a \times (b-c) = ab - ac$

$a \times (b-c) = \frac{2}{9} \times \left(\frac{3}{6} - \frac{1}{3}\right) = \frac{2}{9} \times \left(\frac{3-2}{6}\right) = \frac{2}{9} \times \frac{1}{\cancel{6}_{3}}$

$= \frac{1}{27} \rightarrow ①$

$ab - ac = \frac{2}{9} \times \frac{\cancel{3}}{\cancel{6}_{3}} - \frac{2}{9} \times \frac{1}{3} = \frac{3}{27} - \frac{2}{27} = \frac{1}{27} \rightarrow ②$

From ① and ②, $a \times (b-c) = ab - ac$

$\therefore$ Distributive property of multiplication over

Subtraction is true for rational numbers.

(v). Let $a = \frac{2}{11}$ and $b = \frac{5}{6}$ be two rational numbers.

Mean of a and b is c

ie., $c = \frac{a+b}{2} = \frac{\frac{2}{11} + \frac{5}{6}}{2} = \frac{\frac{12+55}{66}}{2} = \frac{\frac{67}{66}}{\frac{2}{1}}$

$= \frac{67}{66} \times \frac{1}{2} = \frac{67}{132} \in Q$

Also $\frac{2}{11} = \frac{2 \times 12}{11 \times 12} = \frac{24}{132}$

$\frac{5}{6} = \frac{5 \times 22}{6 \times 22} = \frac{110}{132}$

$\therefore \frac{24}{132} < \frac{67}{132} < \frac{110}{132}$

$\therefore$ The mean lies between the given rational numbers $\frac{2}{11}$ and $\frac{5}{6}$.

13). If $\frac{1}{4}$ of a ragi adai weighs 120 grams, what will be the weight of $\frac{2}{3}$ of the same ragi adai?

Sol

Let the weight of 1 ragi adai = x grams

Given: $\frac{1}{4}$ of x = 120 gm.

$\frac{1}{4} \times x = 120$

$x = 120 \times 4$

$x = 480$ gm

$\therefore \frac{2}{3}$ of the ragi adai $= \frac{2}{\cancel{3}} \times \overset{160}{\cancel{480}}$ gm

$\therefore \frac{2}{3}$ of the ragi adai = 320 gm.

14. If $p+2q=18$ and $pq=40$, find $\frac{2}{p}+\frac{1}{q}$.

Sol

$$\frac{2}{p}+\frac{1}{q} = \frac{2q+p}{pq} = \frac{\cancel{18}^{\,9}}{\cancel{40}_{\,20}} = \frac{9}{20}$$

15. Find x if $5\frac{x}{5} \times 3\frac{3}{4} = 21$.

Sol

$$\frac{x}{5} \times 3\frac{3}{4} = 21 \Rightarrow \frac{x}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{4} = 21 \Rightarrow x \times \frac{3}{4} = 21$$

$$\Rightarrow x = \frac{\cancel{21}^{7} \times 4}{\cancel{3}_{1}} \Rightarrow \boxed{x = 28}$$

$$5\frac{x}{5} \times 3\frac{3}{4} = 21 \Rightarrow \frac{25+x}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{4} = 21$$

$$25+x = \frac{\cancel{21}^{7} \times 4}{\cancel{3}_{1}} \Rightarrow 25+x = 28 \Rightarrow x = 28-25$$

$$\boxed{x = 3}$$

16. By how much does $\frac{1}{\left(\frac{10}{11}\right)}$ exceed $\frac{\left(\frac{1}{10}\right)}{11}$?

Sol

$$\text{Difference} = \frac{1}{\frac{10}{11}} - \frac{\frac{1}{10}}{11}$$

$$= \frac{\frac{1}{1}}{\frac{10}{11}} - \frac{\frac{1}{10}}{\frac{11}{1}}$$

$$= \frac{1}{1} \times \frac{11}{10} - \frac{1}{10} \times \frac{1}{11}$$

$$= \frac{11}{10} - \frac{1}{110}$$

$$= \frac{121}{110} - \frac{1}{110}$$

$$= \frac{120}{110}$$

$$= \frac{12}{11}$$

17). A group of 1536 cadets wanted to have a parade forming a square design. Is it possible? If it is not possible, how many more cadets would be required?

Sol

$1536 = 2^2 \times 2^2 \times 2^2 \times 2^2 \times 3 \times 2$

$$\begin{array}{r|l} 4 & 1536 \\ 4 & 384 \\ 4 & 96 \\ 4 & 24 \\ 3 & 6 \\ 2 & 2 \\ & 1 \end{array}$$

The numbers 2 and 3 are unpaired

It is impossible to have the parade forming square design with 1536 cadets.

$$\begin{array}{r|l} & \ \ 39 \\ 3 & 15\,36 \\ & (-)\,9 \\ 69 & \ \ 636 \\ & (-)\,621 \\ & \ \ \ 15 \end{array}$$

$39^2 = 1521$

$40^2 = 1600$

$\therefore 1600 - 1536 = 64$

$\therefore$ 64 more cadets would be required to form the square design.

18. Evaluate: $\sqrt{286225}$ and use it to compute $\sqrt{2862.25} + \sqrt{28.6225}$

Sol

$$\begin{array}{r|l} & \ \ 535 \\ 5 & 28\,62\,25 \\ & (-)\,25 \\ 103 & \ \ 362 \\ & (-)\,309 \\ 1065 & \ \ 5325 \\ & (-)\,5325 \\ & \ \ \ 0 \end{array}$$

$\therefore \sqrt{286225} = 535$

Now $\sqrt{2862.25} + \sqrt{28.6225}$

$= 53.5 + 5.35$

$= 58.85$

$$\begin{array}{r} 53.50 \\ 5.35 \\ \hline 58.85 \end{array}$$

19. Simplify: $(3.769 \times 10^5) + (4.21 \times 10^5)$

Sol

$(3.769 \times 10^5) + (4.21 \times 10^5)$

$= (3.769 + 4.21) \times 10^5$

$= 7.979 \times 10^5$

$$\begin{array}{r} 3.769 \\ 4.21 \\ \hline 7.979 \times 10^5 \end{array}$$

20). Order the following from the least to the greatest: 16^{25}, 8^{100}, 3^{500}, 4^{400}, 2^{600}.

Sol:

$16^{25} = (2^4)^{25} = 2^{100}$

$8^{100} = (2^3)^{100} = 2^{300}$

$3^{500} = 3^{500}$

$4^{400} = (2^2)^{400} = 2^{800}$

$2^{600} = 2^{600}$

$16^{25} = (2^4)^{25} = (2)^{100}$

$8^{100} = (8)^{100}$

$3^{500} = (3^5)^{100} = (243)^{100}$

$4^{400} = (4^4)^{100} = (256)^{100}$

$2^{600} = (2^6)^{100} = (64)^{100}$

$2^{100} < 8^{100} < 64^{100} < 243^{100} < 256^{100}$

ie, $16^{25} < 8^{100} < 2^{600} < 3^{500} < 4^{400}$