1. SET LANGUAGE

Exercise 1.1

1. Which of the following are sets?
(i) The collection of prime numbers upto 100.
(ii) The collection of rich people in India.
(iii) The collection of all rivers in India.
(iv) The collection of good Hockey players.

(i) $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$.
This is a set.

(ii) Rich people has no definition.
This is not a set.

(iii) A = { Cauvery, Ganga, Yamuna, sindhu, ... }
This is a set.

(iv) Hockey player's talent may vary from person to person.
This is not a set.

2. List the set of letters of the following words in Roster form.
(i) INDIA (ii) PARALLELOGRAM (iii) MISSISSIPPI
(iv) CZECHOSLOVAKIA.

(i) $A = \{I, N, D, A\}$ (ii) $B = \{P, A, R, L, E, O, G, M\}$
(iii) $C = \{M, I, S, P\}$ (iv) $D = \{C, Z, E, H, O, S, L, V, A, K, I\}$

3. Consider the following sets $A = \{0, 3, 5, 8\}$, $B = \{2, 4, 6, 10\}$, $C = \{12, 14, 18, 20\}$.
(a) State whether true or false:
(i) $18 \in C$ (ii) $6 \notin A$ (iii) $14 \notin C$ (iv) $10 \in B$
(v) $5 \in B$ (vi) $0 \in B$.
(b) Fill in the blanks:
(i) $3 \in$ ___ (ii) $14 \in$ ___ (iii) 18 ___ B (iv) 4 ___ B.

(a) (i) True (ii) True (iii) False (iv) True (v) False (vi) False
(b) (i) $3 \in A$ (ii) $14 \in C$ (iii) $18 \notin B$ (iv) $4 \in B$.

4. Represent the following sets in Roster form

(i) A = The set of all even natural numbers less than 20.

(ii) $B = \{y : y = \frac{1}{2n}, n \in N, n \leq 5\}$

(iii) $C = \{x : x$ is perfect cube, $27 < x < 216\}$

(iv) $D = \{x : x \in Z, -5 < x \leq 2\}$

(i) $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

(ii) $n = 1, 2, 3, 4, 5 \Rightarrow y = \frac{1}{2(1)} = \frac{1}{2}$; $y = \frac{1}{2(2)} = \frac{1}{4}$; $y = \frac{1}{2(3)} = \frac{1}{6}$.

$y = \frac{1}{2(4)} = \frac{1}{8}$; $y = \frac{1}{2(5)} = \frac{1}{10}$.

$B = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\}$

(iii) $C = \{64, 125\}$

$4^3 = 64$
$5^3 = 125$

(iv) $D = \{-4, -3, -2, -1, 0, 1, 2\}$

---

5. Represent the following sets in set builder form

(i) B = The set of all cricket players in India who scored double centuries in One Day Internationals

(ii) $C = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\}$

(iii) D = The set of all tamil months in a year.

(iv) E = The set of odd whole numbers less than 9

(i) $B = \{x : x$ is an Indian player who scored double centuries in one day internationals$\}$

(ii) $C = \{x : x = \frac{n}{n+1}, n \in N\}$

(iii) $D = \{x : x$ is a tamil month in a year$\}$

(iv) $E = \{x : x$ is odd number, $x \in W$, $x < 9$, where W is the set of whole number$\}$

6. Represent the following sets in descriptive form.

(i) $P = \{January, June, July\}$

(ii) $Q = \{7, 11, 13, 17, 19, 23, 29\}$

(iii) $R = \{x : x \in N, x < 5\}$

(iv) $S = \{x : x$ is a consonant in English alphabets$\}$.

(i) P is the set of English Months begining with J.

(ii) Q is the set of all prime numbers between 5 and 31.

(iii) R is the set of all natural numbers less than 5.

(iv) S is the set of all English consonants.

Exercise 1.2

1. Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

(ii) $P = \{x : x = 3n+2, n \in W$ and $x < 15\}$

(iii) $Q = \{y : y = \frac{4}{3n}, n \in N$ and $2 < n \le 5\}$

(iv) $R = \{x : x$ is an integers, $x \in Z$ and $-5 \le x < 5\}$

(v) S = The set of all leap years between 1882 and 1906.

(i) $n(M) = 6$

(ii) $W = \{0, 1, 2, 3, \ldots\}$

$n = 0,\ x = 3(0)+2 = 2$; $n = 1,\ x = 3(1)+2 = 3+2 = 5$.

$n = 2,\ x = 3(2)+2 = 6+2 = 8$; $n = 3$; $x = 3(3)+2 = 9+2 = 11$.

$n = 4,\ x = 3(4)+2 = 12+2 = 14$

$P = \{2, 5, 8, 11, 14\}$

$n(P) = 5$

(iii) $n = 3, 4, 5$; $y = \frac{4}{3(3)} = \frac{4}{9}$; $y = \frac{4}{3(4)} = \frac{4}{12}$; $y = \frac{4}{3(5)} = \frac{4}{15}$

$Q = \{\frac{4}{9}, \frac{4}{12}, \frac{4}{15}\}$

$n(Q) = 3$

(iv) $R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$n(R) = 10$.

(v) $S = \{1884, 1888, 1892, 1896, 1904\}$

$n(S) = 5$.

$\frac{1900}{400}$ $4 \overline{)19}$ = 4, 16, remainder 3   1900 is not a leap year

4\. 2. Identify the following sets as finite or infinite.

(i) X = The set of all districts in Tamilnadu.

(ii) Y = The set of all straight lines passing through a point.

(iii) $A = \{x : x \in Z \text{ and } x < 5\}$

(iv) $B = \{x : x^2 - 5x + 6 = 0, x \in N\}$

(i) Finite set. [∵ There are 38 districts in Tamilnadu]

(ii) Infinite set.

(iii) $A = \{\ldots\ldots, -2, -1, 0, 1, 2, 3, 4\}$ ∴ Infinite set

(iv) $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2, 3$

$B = \{2, 3\}$ ∴ Finite set.

3\. Which of the following sets are equivalent or unequal or equal sets?

(i) A = The set of vowels in English alphabets.

B = The set of all letters in the word "VOWEL"

(ii) $C = \{2, 3, 4, 5\}$

$D = \{x : x \in W, 1 < x < 5\}$

(iii) $X = \{x : x$ is a letter in the word "LIFE"$\}$

$Y = \{F, I, L, E\}$

(iv) $G = \{x : x$ is a prime number and $3 < x < 23\}$

$H = \{x : x$ is a divisor of $18\}$

(i) $A = \{a, e, i, o, u\}$; $B = \{V, O, W, E, L\}$

$n(A) = 5$; $n(B) = 5$

$A \approx B$ ∴ A and B are equivalent sets.

(ii) $C = \{2, 3, 4, 5\}$; $D = \{2, 3, 4\}$

Unequal sets

Scanned with CamScanner

(iii) $X = \{L, I, F, E\}$; $Y = \{F, I, L, E\}$

$X = Y$. X and Y are equal sets.

(iv) $G = \{5, 7, 11, 13, 17, 19\}$; $H = \{1, 2, 3, 6, 9, 18\}$.

$n(G) = 6 = n(H)$

$G \approx H$. G and H are equivalent sets.

4. Identify the following sets as null set or singleton set.

(i) $A = \{x : x \in N, 1 < x < 2\}$

(ii) B = The set of all even natural numbers which are not divisible by 2.

(iii) $C = \{0\}$.

(iv) D = The set of all triangles having four sides.

(i) $A = \{\}$. ∴ Null set.

(ii) $B = \{\}$ ∴ Null set.

(iii) $C = \{0\}$ ∴ Singleton set.

(iv) $D = \{\}$ ∴ Null set.

5. State which pairs of sets are disjoint or overlapping?

(i) $A = \{f, i, a, s\}$ and $B = \{a, n, f, h, s\}$

(ii) $C = \{x : x$ is a prime number, $x > 2\}$ and

$D = \{x : x$ is an even prime number$\}$

(iii) $E = \{x : x$ is a factor of $24\}$ and

$F = \{x : x$ is a multiple of $3, x < 30\}$.

(i) $A = \{f, i, a, s\}$; $B = \{a, n, f, h, s\}$

$A \cap B = \{f, a, s\}$

Since $A \cap B \neq \phi$, A and B are overlapping sets.

(ii) $C = \{3, 5, 7, 11, 13, \ldots\}$; $D = \{2\}$

$C \cap D = \phi$.

Since $C \cap D = \phi$, C and D are disjoint sets.

(iii) $E = \{1, 2, 3, 4, 6, 8, 12, 24\}$; $F = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

$E \cap F = \{3, 6, 12, 24\} \neq \phi$

∴ E and F are overlapping sets.

6. If S = {square, rectangle, circle, rhombus, triangle} list the elements of the following subset of S.

(i) The set of shapes which have 4 equal sides.

(ii) The set of shapes which have radius.

(iii) The set of shapes in which the sum of all interior angles is 180°.

(iv) The set of shapes which have 5 sides.

(i) {square, rhombus}

(ii) {circle}

(iii) {Triangle}

(iv) { }.

10.

7. If $A = \{a, \{a,b\}\}$, write all the subsets of A.

The subsets of A are { }, {a}, {a,b}, {a, {a,b}}

8. Write down the power set of the following sets.

(i) $A = \{a,b\}$ (ii) $B = \{1,2,3\}$ (iii) $D = \{p,q,r,s\}$ (iv) $E = \phi$

(i) $A = \{a,b\}$; $n(A) = 2$; $n[P(A)] = 2^2 = 4$.

$P(A) = \{\{\}, \{a\}, \{b\}, \{a,b\}\}$.

(ii) $B = \{1,2,3\}$; $n(B) = 3$; $n[P(A)] = 2^3 = 8$.

$P(B) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

(iii) $D = \{p,q,r,s\}$; $n(D) = 4$; $n[P(A)] = 2^4 = 16$

$P(D) = \{\{\}, \{p\}, \{q\}, \{r\}, \{s\}, \{p,q\}, \{p,r\}, \{p,s\}, \{q,r\}, \{q,s\}, \{r,s\}, \{p,q,r\}, \{p,q,s\}, \{p,r,s\}, \{q,r,s\}, \{p,q,r,s\}\}$

(iv) $E = \phi$; $n(E) = 0$; $n[P(E)] = 2^0 = 1$.

$P(E) = \{\ \}$.

9. Find the number of subsets and the number of proper subsets of the following sets.

(i) $W = \{red, blue, yellow\}$ (ii) $X = \{x^2 : x \in N, x^2 \leq 100\}$

(i) $W = \{red, blue, yellow\}$; $n(W) = 3$.

$\therefore$ No. of subsets $= n[P(W)] = 2^3 = 8$.

No. of Proper subsets $= n[P(W)] - 1 = 8 - 1 = 7$.

(ii) $X = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$; $n(X) = 10$.

$\therefore$ No. of Subsets $= n[P(X)] = 2^{10} = 1024$.

No. of Proper subsets $= n[P(X)] - 1 = 1024 - 1 = 1023$.

10. (i) If $n(A) = 4$, find $n[P(A)]$.

(ii) If $n(A) = 0$, find $n[P(A)]$.

(iii) If $n[P(A)] = 256$, find $n(A)$.

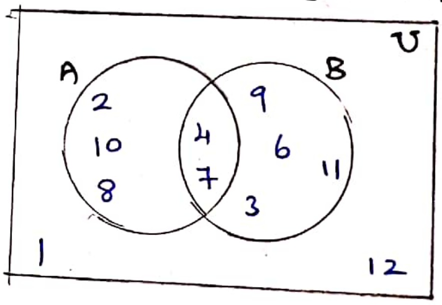
(i) $n(A) = 4$ then $n[P(A)] = 2^4 = 16$.

(ii) $n(A) = 0$ then $n[P(A)] = 2^0 = 1$.

(iii) $n[P(A)] = 256 = 2^8$ then $n(A) = 8$.

Exercise 1.3

1) Using the given venn diagram, write the elements of (i) A (ii) B (iii) $A \cup B$ (iv) $A \cap B$ (v) $A - B$ (vi) $B - A$ (vii) A' (viii) B' (ix) U.



(i) $A = \{2, 4, 7, 8, 10\}$

(ii) $B = \{3, 4, 6, 7, 9, 11\}$

(iii) $A \cup B = \{2, 3, 4, 6, 7, 8, 9, 10, 11\}$

(iv) $A \cap B = \{4, 7\}$

(v) $A - B = \{2, 8, 10\}$

(vi) $B - A = \{3, 6, 9, 11\}$

(vii) $A' = \{1, 3, 6, 9, 11, 12\}$

(viii) $B' = \{1, 2, 8, 10, 12\}$

(ix) $U = \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12\}$

2. Find $A \cup B$, $A \cap B$, $A-B$ and $B-A$ for the follo[wing] sets.

(i) $A = \{2, 6, 10, 14\}$ and $B = \{2, 5, 14, 16\}$

(ii) $A = \{a, b, c, e, u\}$ and $B = \{a, e, i, o, u\}$

(iii) $A = \{x : x \in N, x \leq 10\}$ and $B = \{x : x \in W, x$ [cut off]

(iv) A = Set of all letters in the word "mathematics"

B = Set of all letters in the word "geometry"

(i) $A = \{2, 6, 10, 14\}$; $B = \{2, 5, 14, 16\}$

$A \cup B = \{2, 6, 10, 14\} \cup \{2, 5, 14, 16\} = \{2, 5, 6, 10, 14,$ [cut off]

$A \cap B = \{\underline{2}, 6, 10, \underline{14}\} \cap \{\underline{2}, 5, \underline{14}, 16\} = \{2, 14\}$

$A - B = \{\not{2}, 6, 10, \not{14}\} - \{2, 5, 14, 16\} = \{6, 10\}$

$B - A = \{\not{2}, 5, \not{14}, 16\} - \{2, 6, 10, 14\} = \{5, 16\}$

(ii) $A = \{a, b, c, e, u\}$; $B = \{a, e, i, o, u\}$

$A \cup B = \{a, b, c, e, u\} \cup \{a, e, i, o, u\} = \{a, b, c, e, i, o, u\}$

$A \cap B = \{\underline{a}, b, c, \underline{e}, \underline{u}\} \cap \{\underline{a}, \underline{e}, i, o, \underline{u}\} = \{a, e, u\}$

$A - B = \{\not{a}, b, c, \not{e}, \not{u}\} - \{a, e, i, o, u\} = \{b, c\}$

$B - A = \{\not{a}, \not{e}, i, o, \not{u}\} - \{a, b, c, e, u\} = \{i, o\}$

(iii) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; $B = \{0, 1, 2, 3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{0, 1, 2, 3, 4, 5\}$

$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cap B = \{\underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, 6, 7, 8, 9, 10\} \cap \{0, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}\} = \{1, 2, 3, 4, 5$ [cut off]

$A - B = \{\not{1}, \not{2}, \not{3}, \not{4}, \not{5}, 6, 7, 8, 9, 10\} - \{0, 1, 2, 3, 4, 5\} = \{6, 7, 8, 9, 10\}$

$B - A = \{0, \not{1}, \not{2}, \not{3}, \not{4}, \not{5}\} - \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = \{0\}$

(iv). $A = \{m, a, t, h, e, i, c, s\}$; $B = \{g, e, o, m, t, r, y\}$

$A \cup B = \{m, a, t, h, e, i, c, s\} \cup \{g, e, o, m, t, r, y\}$

$A \cup B = \{m, a, t, h, e, i, c, s, g, o, r, y\}$

$A \cap B = \{m, a, t, h, e, i, c, s\} \cap \{g, e, o, m, t, r, y\} = \{m, t, e\}$

$A - B = \{m, a, t, h, e, i, c, s\} - \{g, e, o, m, t, r, y\} = \{a, h, i, c, s\}$

$B - A = \{g, e, o, m, t, r, y\} - \{m, a, t, h, e, i, c, s\} = \{g, o, r, y\}$

3. If $U = \{a, b, c, d, e, f, g, h\}$, $A = \{b, d, f, h\}$ and $B = \{a, d, e, h\}$, find the following sets.

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$

(v) $(A \cup B)'$ (vi) $(A \cap B)'$ (vii) $(A')'$ (viii) $(B')'$

(i) $A' = U - A = \{a, b, c, d, e, f, g, h\} - \{b, d, f, h\} = \{a, c, e, g\}$

(ii) $B' = U - B = \{a, b, c, d, e, f, g, h\} - \{a, d, e, h\} = \{b, c, f, g\}$

(iii) $A' \cup B' = \{a, c, e, g\} \cup \{b, c, f, g\} = \{a, b, c, e, f, g\}$

(iv) $A' \cap B' = \{a, c, e, g\} \cap \{b, c, f, g\} = \{c, g\}$

(v) $(A \cup B)'$

$A \cup B = \{b, d, f, h\} \cup \{a, d, e, h\} = \{a, b, d, e, f, h\}$

$(A \cup B)' = U - (A \cup B) = \{a, b, c, d, e, f, g, h\} - \{a, b, d, e, f, h\}$

$(A \cup B)' = \{c, g\}$

(vi) $(A \cap B)'$

$A \cap B = \{b, d, f, h\} \cap \{a, d, e, h\} = \{d, h\}$

$(A \cap B)' = U - (A \cap B) = \{a, b, c, d, e, f, g, h\} - \{d, h\}$

$\therefore (A \cap B)' = \{a, b, c, e, f, g\}$

(vii) $(A')' = U - A' = \{a, b, c, d, e, f, g, h\} - \{a, c, e, g\}$

$(A')' = \{b, d, f, h\} = A.$

(viii) $(B')' = U - B' = \{a, b, c, d, e, f, g, h\} - \{b, c, f, g\}$

$(B')' = \{a, d, e, h\} = B.$

4. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$ and $B = \{0, 2, 3, 5, 7\}$ find the following sets.

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$ (v) $(A \cup B)'$ (vi) $(A \cap B)'$

(vii) $(A')'$ (viii) $(B')'$

(i) $A' = U - A = \{0,1,2,3,4,5,6,7\} - \{1,3,5,7\} = \{0,2,4,6\}$

(ii) $B' = U - B = \{0,1,2,3,4,5,6,7\} - \{0,2,3,5,7\} = \{1,4,6\}$

(iii) $A' \cup B' = \{0,2,4,6\} \cup \{1,4,6\} = \{0,1,2,4,6\}$

(iv) $A' \cap B' = \{0,2,4,6\} \cap \{1,4,6\} = \{4,6\}$

(v) $(A \cup B)'$

$A \cup B = \{1,3,5,7\} \cup \{0,2,3,5,7\} = \{0,1,2,3,5,7\}$

$(A \cup B)' = U - (A \cup B) = \{0,1,2,3,4,5,6,7\} - \{0,1,2,3,5,7\}$

$(A \cup B)' = \{4,6\}$

(vi) $(A \cap B)'$

$A \cap B = \{1,3,5,7\} \cap \{0,2,3,5,7\} = \{3,5,7\}$

$(A \cap B)' = U - (A \cap B) = \{0,1,2,3,4,5,6,7\} - \{3,5,7\}$

$(A \cap B)' = \{0,1,2,4,6\}$

(vii) $(A')' = U - A' = \{0,1,2,3,4,5,6,7\} - \{0,2,4,6\}$

$(A')' = \{1,3,5,7\} = A$

(viii) $(B')' = U - B' = \{0,1,2,3,4,5,6,7\} - \{1,4,6\}$

$(B')' = \{0,2,3,5,7\}$

5. Find the symmetric difference between the following sets.

(i) $P = \{2,3,5,7,11\}$ and $Q = \{1,3,5,11\}$

(ii) $R = \{l,m,n,o,p\}$ and $S = \{j,l,n,q\}$

(iii) $X = \{5,6,7\}$ and $Y = \{5,7,9,10\}$

(i) $P \Delta Q = (P-Q) \cup (Q-P)$

$P = \{2,3,5,7,11\}$ and $Q = \{1,3,5,11\}$

$P - Q = \{2,3,5,7,11\} - \{1,3,5,11\} = \{2,7\}$

$Q - P = \{1,3,5,11\} - \{2,3,5,7,11\} = \{1\}$

$P \Delta Q = (P-Q) \cup (Q-P) = \{2,7\} \cup \{1\} = \{1,2,7\}$

(ii) $R = \{l,m,n,o,p\}$ and $S = \{j,l,n,q\}$

$R - S = \{l,m,n,o,p\} - \{j,l,n,q\} = \{m,o,p\}$

$S - R = \{j,l,n,q\} - \{l,m,n,o,p\} = \{j,q\}$

$R \Delta S = (R-S) \cup (S-R) = \{m, o, p\} \cup \{j, q\} = \{j, m, o, p, q\}$.

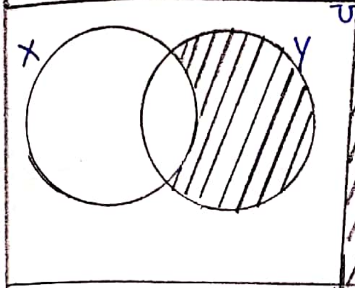
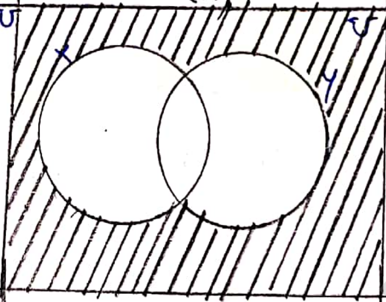
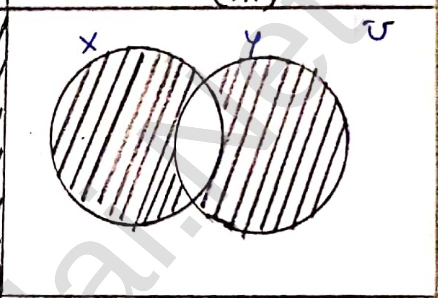
(iii) $X = \{5, 6, 7\}$ and $Y = \{5, 7, 9, 10\}$

$X - Y = \{5, 6, 7\} - \{5, 7, 9, 10\} = \{6\}$

$Y - X = \{5, 7, 9, 10\} - \{5, 6, 7\} = \{9, 10\}$

$X \Delta Y = (X-Y) \cup (Y-X) = \{6\} \cup \{9, 10\} = \{6, 9, 10\}$.

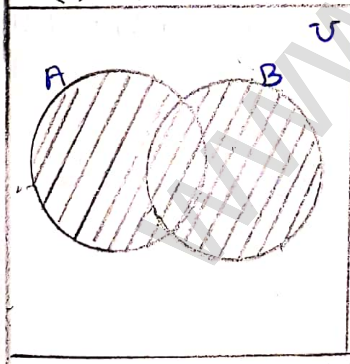
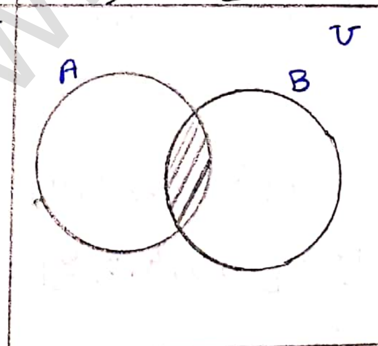
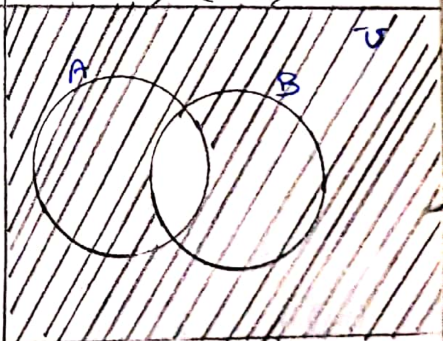
6. Using the set symbols, write down the expressions for the shaded region in the following

(i)	(ii)	(iii)
		
$X-Y$	$(X \cup Y)'$	$(X-Y) \cup (Y-X)$

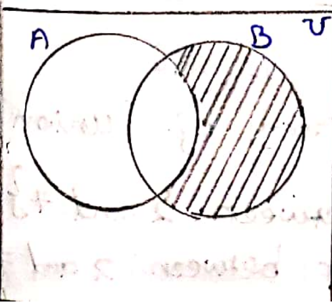
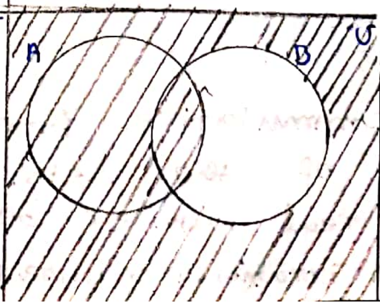
7. Let A and B be two overlapping sets and the universal set U. Draw appropriate venn diagram for each of the following.

(i) $A \cup B$ (ii) $A \cap B$ (iii) $(A \cap B)'$ (iv) $(B-A)'$ (v) $A' \cup B'$

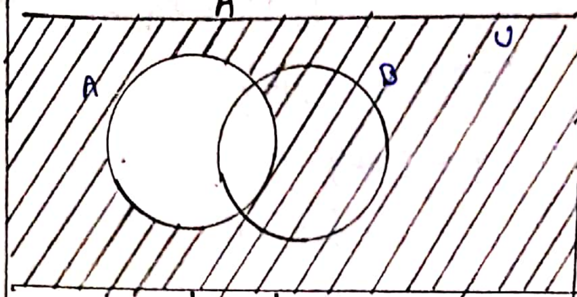
(vi) $A' \cap B'$. (vii) What do you observe from the diagram (iii) and (v)?

(i) $A \cup B$	(ii) $A \cap B$	(iii) $(A \cap B)'$
		

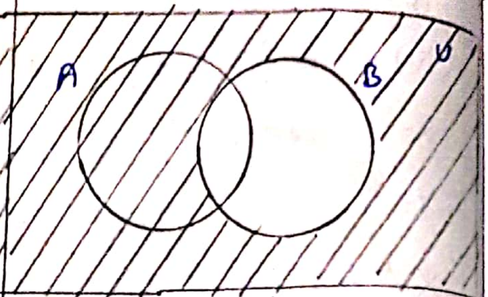
(iv) $(B-A)'$

$B-A$	$(B-A)'$
	

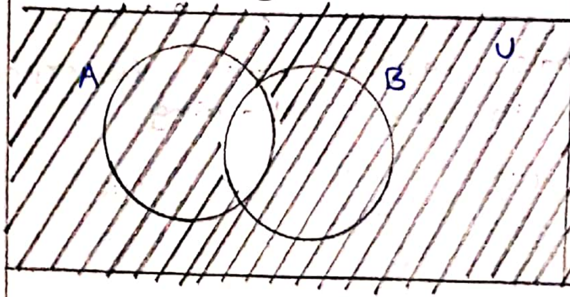
A'



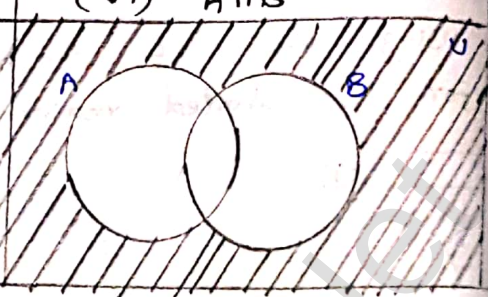
B'



(v) $A' \cup B'$



(vi) $A' \cap B'$



(vii) From the diagrams (iii) and (v) we observe that $(A \cap B)' = A' \cup B'$

Exercise 1.4

1. If $P = \{1, 2, 5, 7, 9\}$, $Q = \{2, 3, 5, 9, 11\}$, $R = \{3, 4, 5, 7, 9\}$ and $S = \{2, 3, 4, 5, 8\}$ then find
(i) $(P \cup Q) \cup R$ (ii) $(P \cap Q) \cap S$ (iii) $(Q \cap S) \cap R$.

(i) $(P \cup Q) \cup R$

$P \cup Q = \{1, 2, 5, 7, 9\} \cup \{2, 3, 5, 9, 11\} = \{1, 2, 3, 5, 7, 9, 11\}$

$(P \cup Q) \cup R = \{1, 2, 3, 5, 7, 9, 11\} \cup \{3, 4, 5, 7, 9\} = \{1, 2, 3, 4, 5, 7, 9, 11\}$

(ii) $(P \cap Q) \cap S$

$P \cap Q = \{1, 2, 5, 7, 9\} \cap \{2, 3, 5, 9, 11\} = \{2, 5, 9\}$

$(P \cap Q) \cap S = \{2, 5, 9\} \cap \{2, 3, 4, 5, 8\} = \{2, 5\}$

(iii) $(Q \cap S) \cap R$.

$Q \cap S = \{2, 3, 5, 9, 11\} \cap \{2, 3, 4, 5, 8\} = \{2, 3, 5\}$

$(Q \cap S) \cap R = \{2, 3, 5\} \cap \{3, 4, 5, 7, 9\} = \{3, 5\}$

2. Test for the commutative property of union and intersection of the sets.
$P = \{x : x$ is a real number between 2 and 7$\}$ and
$Q = \{x : x$ is an irrational number between 2 and 7$\}$

3.

i) Commutative property of union of sets, $P \cup Q = Q \cup P$

$P = \{3, 4, \sqrt{5}, 5, 6, 7, \sqrt{7}, \sqrt{8}\}$; $Q = \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\}$

$P \cup Q = \{3, 4, \sqrt{5}, 5, 6, 7, \sqrt{7}, \sqrt{8}\} \cup \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\}$

$P \cup Q = \{3, 4, 5, 6, 7, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \rightarrow (1)$

$Q \cup P = \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \cup \{3, 4, \sqrt{5}, 5, 6, \sqrt{7}, \sqrt{8}\}$

$Q \cup P = \{3, 4, 5, 6, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \rightarrow (2)$

From (1) and (2), we get $P \cup Q = Q \cup P$

Hence it is verified

ii) Commutative property of intersection of sets, $P \cap Q = Q \cap P$.

$P = \{3, 4, 5, 6, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\}$; $Q = \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\}$

$P \cap Q = \{3, 4, 5, 6, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \cap \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\}$

$P \cap Q = \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \rightarrow (3)$

$Q \cap P = \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \cap \{3, 4, 5, 6, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\}$

$Q \cap P = \{\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}\} \rightarrow (4)$

From (3) and (4), we get $P \cap Q = Q \cap P$.

Hence it is verified

3. If $A = \{p, q, r, s\}$, $B = \{m, n, q, s, t\}$ and $C = \{m, n, p, q, s\}$ then verify the associative property of union of sets.

Associative property of union of sets,

$$A \cup (B \cup C) = (A \cup B) \cup C$$

LHS : $A \cup (B \cup C)$

$B \cup C = \{m, n, q, s, t\} \cup \{m, n, p, q, s\} = \{m, n, p, q, s, t\}$

$A \cup (B \cup C) = \{p, q, r, s\} \cup \{m, n, p, q, s, t\} = \{m, n, p, q, r, s, t\} \rightarrow (1)$

RHS: $(A \cup B) \cup C$

$A \cup B = \{p, q, r, s\} \cup \{m, n, q, s, t\} = \{m, n, p, q, r, s, t\}$

$(A \cup B) \cup C = \{m, n, p, q, r, s, t\} \cup \{m, n, p, q, s\} = \{m, n, p, q, r, s, t\} \rightarrow (2)$

From (1) and (2), we have

$A \cup (B \cup C) = (A \cup B) \cup C$

Hence it is verified.

4. verify the associative property of intersection of sets for $A=\{-11, \sqrt{2}, \sqrt{5}, 7\}$, $B=\{\sqrt{3}, \sqrt{5}, 6, 13\}$ and $C=\{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$.

Associative property of intersection of sets

$$A\cap(B\cap C) = (A\cap B)\cap C$$

LHS : $A\cap(B\cap C)$

$B\cap C = \{\sqrt{3}, \sqrt{5}, 6, 13\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\} = \{\sqrt{3}, \sqrt{5}\}$

$A\cap(B\cap C) = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}\} = \{\sqrt{5}\} \rightarrow (1)$

RHS : $(A\cap B)\cap C$

$A\cap B = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}, 6, 13\} = \{\sqrt{5}\}$

$(A\cap B)\cap C = \{\sqrt{5}\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\} = \{\sqrt{5}\} \rightarrow (2)$

From (1) and (2), we get $A\cap(B\cap C) = (A\cap B)\cap C$

Hence it is verified.

5. If $A=\{x : x=2^n, n\in W \text{ and } n<4\}$; $B=\{x : x=2n, n\in N \text{ and } n\leq 4\}$ and $C=\{0,1,2,5,6\}$ then verify the associative property of intersection of sets.

$A=\{x : x=2^n, n\in W \text{ and } n<4\}$

$n = 0, 1, 2, 3$ $\quad x = 2^0, 2^1, 2^2, 2^3$

$x = 1, 2, 4, 8.$

$A=\{1, 2, 4, 8\}$

$B=\{x : x=2n, n\in N, \text{ and } n\leq 4\}$

$n = 1, 2, 3, 4$ $\quad x = 2(1), 2(2), 2(3), 2(4)$

$x = 2, 4, 6, 8$

$B=\{2, 4, 6, 8\}$. and $C=\{0, 1, 2, 5, 6\}$

Associative property of intersection of sets.

$$A\cap(B\cap C) = (A\cap B)\cap C$$

LHS : $A\cap(B\cap C)$

$B\cap C = \{2,4,6,8\}\cap\{0,1,2,5,6\} = \{2,6\}$

$A\cap(B\cap C) = \{1,2,4,8\}\cap\{2,6\} = \{2\} \rightarrow (1)$

RHS : $(A\cap B)\cap C$

$A\cap B = \{1,3,4,8\}\cap\{2,4,6,8\} = \{2,4,8\}$

$(A\cap B)\cap C = \{2,4,8\}\cap\{0,1,2,5,6\} = \{2\} \rightarrow (2)$

From (1) and (2), we get $A\cap(B\cap C) = (A\cap B)\cap C$

Hence it is verified.

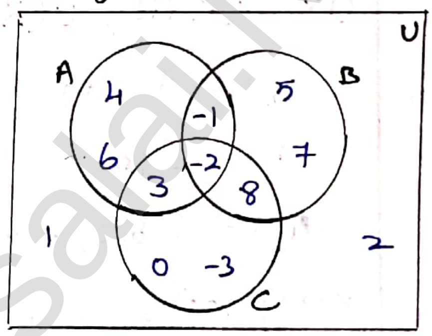
Exercise 1.5

1). Using the adjacent venn diagram, find the following sets.

(i) $A-B$ (ii) $B-C$ (iii) $A'\cup B'$

(iv) $A'\cap B'$ (v) $(B\cup C)'$

(vi) $A-(B\cup C)$ (vii) $A-(B\cap C)$



(i) $A-B = \{3,4,6\}$

(ii) $B-C = \{-1,5,7\}$

(iii) $A'\cup B'$

$A' = \{-3,0,1,2,5,7,8\}$

$B' = \{-3,0,1,2,3,4,6\}$

$A'\cup B' = \{-3,0,1,2,5,7,8\}\cup\{-3,0,1,2,3,4,6\}$

$A'\cup B' = \{-3,0,1,2,3,4,5,6,7,8\}$

(iv) $A'\cap B' = \{-3,0,1,2,5,7,8\}\cap\{-3,0,1,2,3,4,6\}$

$A'\cap B' = \{-3,0,1,2\}$

(v) $(B\cup C)'$

$B\cup C = \{-2,-1,5,7,8\}\cup\{-3,-2,0,3,8\}$

$B\cup C = \{-3,-2,-1,0,3,5,7,8\}$

$(B\cup C)' = U-(B\cup C) = \{-3,-2,-1,0,1,2,3,4,5,6,7,8\} - \{-3,-2,-1,0,3,5,7,8\}$

$(B\cup C)' = \{1,2,4,6\}$

(vi) $A-(B\cup C) = \{-2, 1, 3, 4, 6\} - \{-3, -2, -1, 0, 3, 5, 7, 8\}$

$A-(B\cup C) = \{4, 6\}$

(vii) $A-(B\cap C)$

$B\cap C = \{-2, -1, 5, 7, 8\} \cap \{-3, -2, 0, 3, 8\} = \{-2, 8\}$

$A-(B\cap C) = \{-2, -1, 3, 4, 6\} - \{-2, 8\} = \{-1, 3, 4, 6\}$

2. If $K=\{a, b, d, e, f\}$, $L=\{b, c, d, g\}$ and $M=\{a, b, c, d, h\}$, then find the following

(i) $K\cup(L\cap M)$ (ii) $K\cap(L\cup M)$ (iii) $(K\cup L)\cap(K\cup M)$

(iv) $(K\cap L)\cup(K\cap M)$ and verify distributive laws.

$K=\{a, b, d, e, f\}$, $L=\{b, c, d, g\}$, $M=\{a, b, c, d, h\}$

(i) $K\cup(L\cap M)$

$L\cap M = \{b, c, d, g\} \cap \{a, b, c, d, h\} = \{b, c, d\}$

$K\cup(L\cap M) = \{a, b, d, e, f\} \cup \{b, c, d\} = \{a, b, c, d, e, f\}$

(ii) $K\cap(L\cup M)$

$L\cup M = \{b, c, d, g\} \cup \{a, b, c, d, h\} = \{a, b, c, d, g, h\}$

$K\cap(L\cup M) = \{a, b, d, e, f\} \cap \{a, b, c, d, g, h\} = \{a, b, d\}$

(iii) $(K\cup L)\cap(K\cup M)$

$K\cup L = \{a, b, d, e, f\} \cup \{b, c, d, g\} = \{a, b, c, d, e, f, g\}$

$K\cup M = \{a, b, d, e, f\} \cup \{a, b, c, d, h\} = \{a, b, c, d, e, f, h\}$

$(K\cup L)\cap(K\cup M) = \{a, b, c, d, e, f, g\} \cap \{a, b, c, d, e, f, h\}$

$(K\cup L)\cap(K\cup M) = \{a, b, c, d, e, f\}$

(iv) $(K\cap L)\cup(K\cap M)$

$K\cap L = \{a, b, d, e, f\} \cap \{b, c, d, g\} = \{b, d\}$

$K\cap M = \{a, b, d, e, f\} \cap \{a, b, c, d, h\} = \{a, b, d\}$

$(K\cap L)\cup(K\cap M) = \{b, d\} \cup \{a, b, d\} = \{a, b, d\}$

Distributive Property : Intersection over union

$K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$

LHS : $K \cap (L \cup M)$

$K \cap (L \cup M) = \{a, b, d\} \rightarrow$ ①

RHS : $(K \cap L) \cup (K \cap M)$

$(K \cap L) \cup (K \cap M) = \{a, b, d\} \rightarrow$ ②

From ① and ②, we get $K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$

Hence it is verified.

Distributive Property : Union over intersection.

$K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$

LHS : $K \cup (L \cap M)$

$K \cup (L \cap M) = \{a, b, c, d, e, f\} \rightarrow$ ③

RHS : $(K \cup L) \cap (K \cup M)$

$(K \cup L) \cap (K \cup M) = \{a, b, c, d, e, f\} \rightarrow$ ④

From ③ and ④, we get $K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$

Hence it is verified.

3. If $A = \{x : x \in Z, -2 < x \le 4\}$, $B = \{x : x \in W, x \le 5\}$, $C = \{-4, -1, 0, 2, 3, 4\}$ then verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A = \{-1, 0, 1, 2, 3, 4\}$; $B = \{0, 1, 2, 3, 4, 5\}$

$C = \{-4, -1, 0, 2, 3, 4\}$

LHS : $A \cup (B \cap C)$

$B \cap C = \{0, 1, 2, 3, 4, 5\} \cap \{-4, -1, 0, 2, 3, 4\} = \{0, 2, 3, 4\}$

$A \cup (B \cap C) = \{-1, 0, 1, 2, 3, 4\} \cup \{0, 2, 3, 4\} = \{-1, 0, 1, 2, 3, 4\} \rightarrow$ ①

RHS : $(A \cup B) \cap (A \cup C)$

$A \cup B = \{-1, 0, 1, 2, 3, 4\} \cup \{0, 1, 2, 3, 4, 5\} = \{-1, 0, 1, 2, 3, 4, 5\}$

$A \cup C = \{-1, 0, 1, 2, 3, 4\} \cup \{-4, -1, 0, 2, 3, 4\} = \{-4, -1, 0, 1, 2, 3, 4\}$

$(A \cup B) \cap (A \cup C) = \{-1, 0, 1, 2, 3, 4, 5\} \cap \{-4, -1, 0, 1, 2, 3, 4\}$

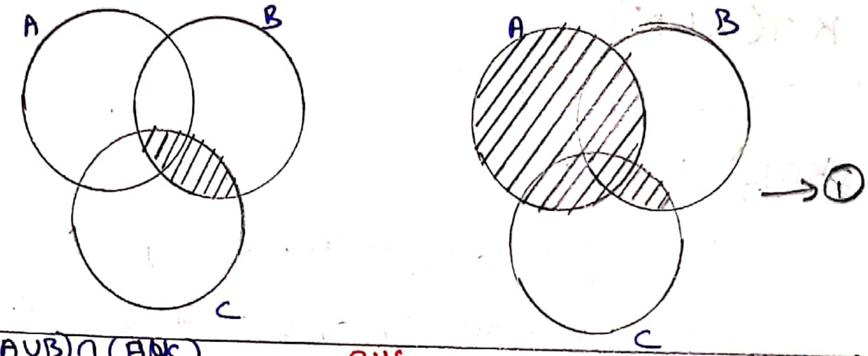
$(A \cup B) \cap (A \cup C) = \{-1, 0, 1, 2, 3, 4\} \rightarrow$ ②

From ① and ②, we get $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

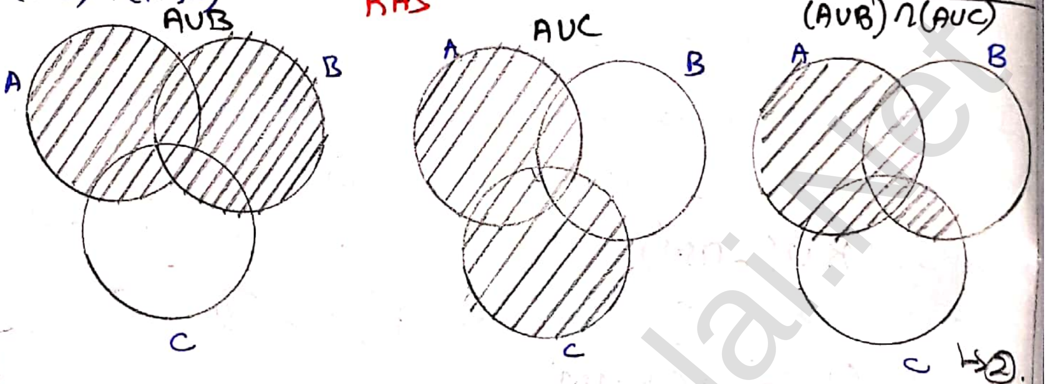
Hence it is verified.

4. verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using venn diagram

$A \cup (B \cap C)$ $B \cap C$ LHS $A \cup (B \cap C)$



$(A \cup B) \cap (A \cup C)$ RHS



From ① and ②, we get $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Hence it is verified.

5. If $A = \{b, c, e, g, h\}$, $B = \{a, c, d, g\}$ and $C = \{a, d, e, g, h\}$ then show that $A - (B \cap C) = (A - B) \cup (A - C)$.

$A = \{b, c, e, g, h\}$, $B = \{a, c, d, g\}$, $C = \{a, d, e, g, h\}$

LHS: $A - (B \cap C)$

$B \cap C = \{a, c, d, g\} \cap \{a, d, e, g, h\} = \{a, d, g\}$

$A - (B \cap C) = \{b, c, e, g, h\} - \{a, d, g\} = \{b, c, e, h\}$ → ①

RHS: $(A - B) \cup (A - C)$

$A - B = \{b, c, e, g, h\} - \{a, c, d, g\} = \{b, e, h\}$

$A - C = \{b, c, e, g, h\} - \{a, d, e, g, h\} = \{b, c\}$

$(A - B) \cup (A - C) = \{b, e, h\} \cup \{b, c\} = \{b, c, e, h\}$ → ②

From ① and ②, we get $A - (B \cap C) = (A - B) \cup (A - C)$

Hence it is verified.

6. If $A=\{x : x=6n, n\in W \text{ and } n<6\}$, $B=\{x : x=2n, n\in N \text{ and } 2<n\le 9\}$ and $C=\{x : x=3n, n\in N \text{ and } 4\le n<10\}$ then show that $A-(B\cap C)=(A-B)\cup(A-C)$

$A=\{x : x=6n, n\in W \text{ and } n<6\}$

$n=0,1,2,3,4,5$; $x=6(0), 6(1), 6(2), 6(3), 6(4), 6(5)$

$x=0, 6, 12, 18, 24, 30.$

$A=\{0,6,12,18,24,30\}$

$B=\{x : x=2n, n\in N \text{ and } 2<n\le 9\}$

$n=3,4,5,6,7,8,9$; $x=2(3), 2(4), 2(5), 2(6), 2(7), 2(8), 2(9)$

$x=6, 8, 10, 12, 14, 16, 18.$

$B=\{6,8,10,12,14,16,18\}$

$C=\{x : x=3n, n\in N \text{ and } 4\le n<10\}$

$n=4,5,6,7,8,9$; $x=3(4), 3(5), 3(6), 3(7), 3(8), 3(9)$

$x=12, 15, 18, 21, 24, 27.$

$C=\{12,15,18,21,24,27\}$

LHS: $A-(B\cap C)$

$B\cap C=\{6,8,10,12,14,16,18\}\cap\{12,15,18,21,24,27\}=\{12,18\}$

$A-(B\cap C)=\{0,6,12,18,24,30\}-\{12,18\}=\{0,6,24,30\}\rightarrow (1)$

RHS: $(A-B)\cup(A-C)$

$A-B=\{0,6,12,18,24,30\}-\{6,8,10,12,14,16,18\}=\{0,24,30\}$

$A-C=\{0,6,12,18,24,30\}-\{12,15,18,21,24,27\}=\{0,6,30\}$

$(A-B)\cup(A-C)=\{0,24,30\}\cup\{0,6,30\}=\{0,6,24,30\}\rightarrow (2)$

From (1) and (2), we get $A-(B\cap C)=(A-B)\cup(A-C)$

Hence it is verified.

7. If $A=\{-2,0,1,3,5\}$, $B=\{-1,0,2,5,6\}$ and $C=\{-1,2,5,6,7\}$ then show that $A-(B\cup C)=(A-B)\cap(A-C)$.

$A=\{-2,0,1,3,5\}$, $B=\{-1,0,2,5,6\}$, $C=\{-1,2,5,6,7\}$

LHS: $A-(B\cup C)$

$B\cup C = \{-1,0,2,5,6\} \cup \{-1,2,5,6,7\} = \{-1,0,2,5,6,7\}$

$A-(B\cup C) = \{-2,0,1,3,5\} - \{-1,0,2,5,6,7\} = \{-2,1,3\} \rightarrow (1)$

RHS: $(A-B)\cap(A-C)$

$A-B = \{-2,0,1,3,5\} - \{-1,0,2,5,6\} = \{-2,1,3\}$

$A-C = \{-2,0,1,3,5\} - \{-1,2,5,6,7\} = \{-2,0,1,3\}$

$(A-B)\cap(A-C) = \{-2,1,3\} \cap \{-2,0,1,3\} = \{-2,1,3\} \rightarrow (2)$

From (1) and (2), we get $A-(B\cup C) = (A-B)\cap(A-C)$.

Hence it is verified.

8. If $A = \{y : y = \frac{a+1}{2}, a\in W \text{ and } a\leq 5\}$, $B = \{y : y = \frac{2n-1}{2}, n\in W \text{ and } n<5\}$ and $C = \{-1, \frac{-1}{2}, 1, \frac{3}{2}, 2\}$ then show that $A-(B\cup C) = (A-B)\cap(A-C)$.

$A = \{y : y = \frac{a+1}{2}, a\in W \text{ and } a\leq 5\}$

$a = 0,1,2,3,4,5;\quad y = \frac{0+1}{2}, \frac{1+1}{2}, \frac{2+1}{2}, \frac{3+1}{2}, \frac{4+1}{2}, \frac{5+1}{2}$

$y = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}$

$y = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3.$

$A = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\}$

$B = \{y : y = \frac{2n-1}{2}, n\in W \text{ and } n<5\}$

$n = 0,1,2,3,4;\quad y = \frac{2(0)-1}{2}, \frac{2(1)-1}{2}, \frac{2(2)-1}{2}, \frac{2(3)-1}{2}, \frac{2(4)-1}{2}$

$y = \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

$B = \{\frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\}$

$C = \{-1, \frac{-1}{2}, 1, \frac{3}{2}, 2\}$

LHS: $A-(B\cup C)$

$B\cup C = \{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\} \cup \{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\}$

$B\cup C = \{-1, -\frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \frac{7}{2}\}$

$A-(B\cup C) = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\} - \{-1, -\frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \frac{7}{2}\}$

$A-(B\cup C) = \{3\} \rightarrow (1)$

RHS : $(A-B)\cap(A-C)$

$A-B=\{\not{\tfrac{1}{2}},1,\not{\tfrac{3}{2}},2,\not{\tfrac{5}{2}},3\}-\{-\tfrac{1}{2},\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\tfrac{7}{2}\}$

$A-B=\{1,2,3\}$

$A-C=\{\tfrac{1}{2},\not{1},\not{\tfrac{3}{2}},\not{2},\tfrac{5}{2},3\}-\{-1,-\tfrac{1}{2},1,\tfrac{3}{2},2\}$

$A-C=\{\tfrac{1}{2},\tfrac{5}{2},3\}$

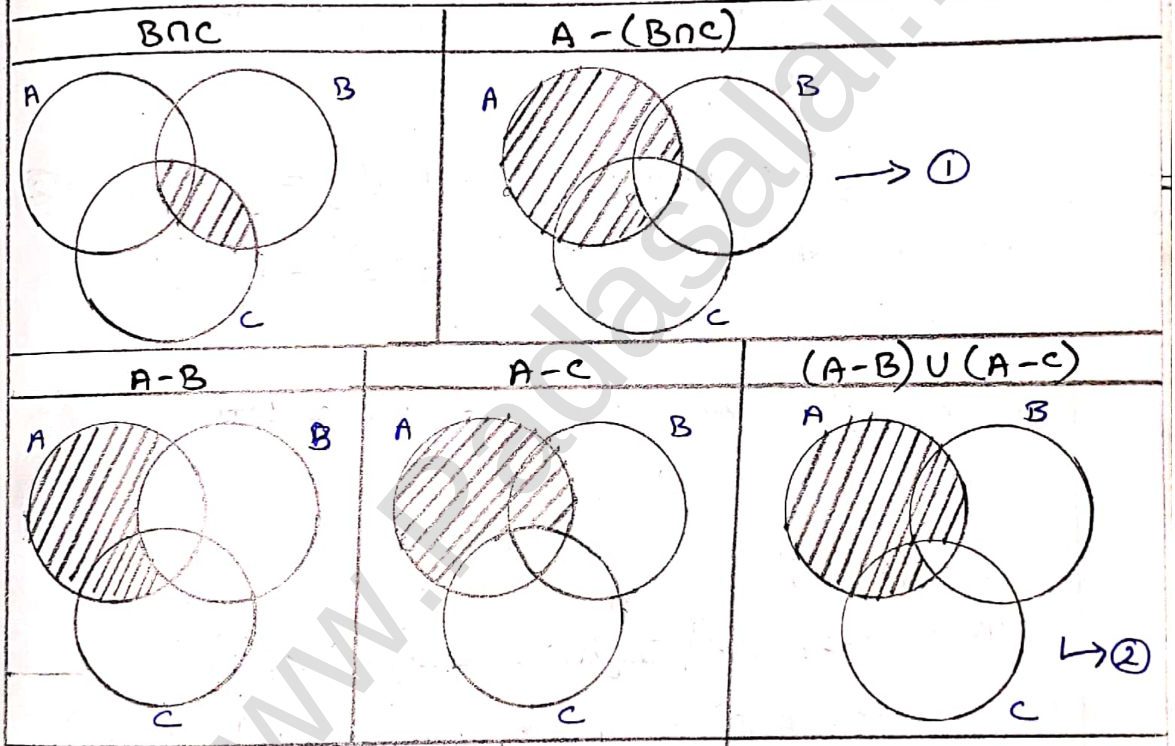
$(A-B)\cap(A-C)=\{1,2,3\}\cap\{\tfrac{1}{2},\tfrac{5}{2},3\}=\{3\}\rightarrow$ ②

From ① and ②, we get $A-(B\cup C)=(A-B)\cap(A-C)$.

Hence it is verified.

9. verify $A-(B\cap C)=(A-B)\cup(A-C)$ using venn diagram.

LHS: $A-(B\cap C)$



RHS: $(A-B)\cup(A-C)$

From ① and ②, we get $A-(B\cap C)=(A-B)\cup(A-C)$.

Hence it is verified.

10. If $U=\{4,7,8,10,11,12,15,16\}$, $A=\{7,8,11,12\}$ and $B=\{4,8,12,15\}$ then verify De Morgan's Law for complementation.

$U=\{4,7,8,10,11,12,15,16\}$, $A=\{7,8,11,12\}$,

$B=\{4,8,12,15\}$

Demorgan's law for complementation

(i) $(A\cup B)' = A'\cap B'$

LHS: $(A\cup B)'$

$A\cup B = \{7,8,11,12\}\cup\{4,8,12,15\} = \{4,7,8,11,12,15\}$

$(A\cup B)' = U-(A\cup B) = \{4,7,8,10,11,12,15,16\} - \{4,7,8,11,12,15\}$

$(A\cup B)' = \{10,16\} \rightarrow (1)$

RHS: $A'\cap B'$

$A' = U-A = \{4,7,8,10,11,12,15,16\} - \{7,8,11,12\}$

$A' = \{4,10,15,16\}$

$B' = U-B = \{4,7,8,10,11,12,15,16\} - \{4,8,12,15\}$

$B' = \{7,10,11,16\}$

$A'\cap B' = \{4,10,15,16\}\cap\{7,10,11,16\} = \{10,16\} \rightarrow (2)$

From (1) and (2), we get $(A\cup B)' = A'\cap B'$

Hence it is verified.

Demorgan's law for complementation

(ii) $(A\cap B)' = A'\cup B'$

LHS: $(A\cap B)'$

$A\cap B = \{7,8,11,12\}\cap\{4,8,12,15\} = \{8,12\}$

$(A\cap B)' = U-(A\cap B) = \{4,7,8,10,11,12,15,16\} - \{8,12\}$

$(A\cap B)' = \{4,7,10,11,15,16\} \rightarrow (3)$

RHS: $A'\cup B'$

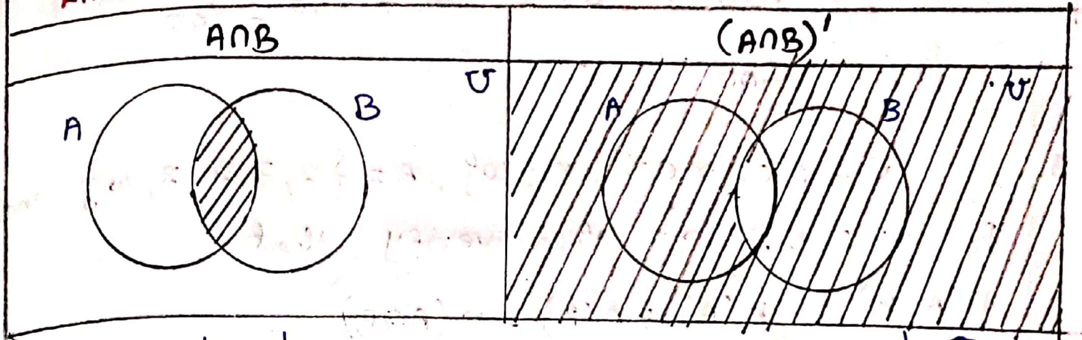
$A'\cup B' = \{4,10,15,16\}\cup\{7,10,11,16\} = \{4,7,10,11,15,16\} \rightarrow (4)$

From (3) and (4), we get $(A\cap B)' = A'\cup B'$.

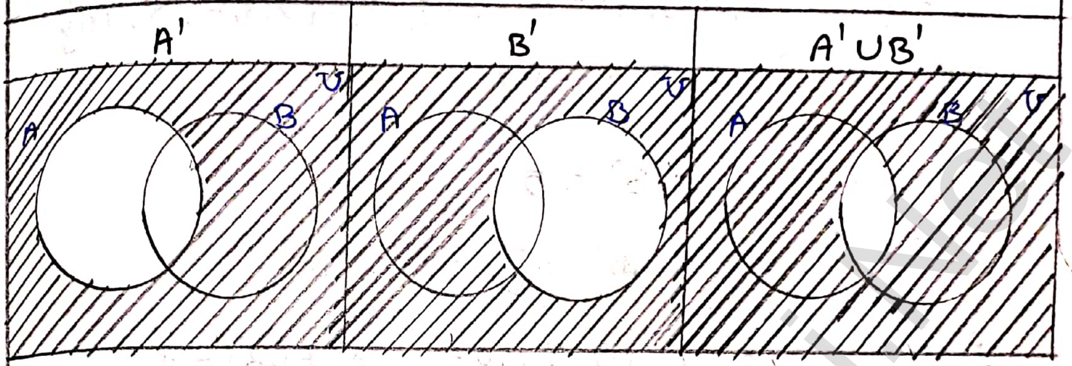
Hence it is verified.

11. verify $(A\cap B)' = A'\cup B'$ using venn diagram.

LHS: $(A \cap B)'$



RHS: $A' \cup B'$



From ① and ②, we get $(A \cap B)' = A' \cup B'$

Hence it is verified.

Exercise 1.6

1). (i). If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$.

(ii) If $n(A) = 300$, $n(A \cup B) = 500$, $n(A \cap B) = 50$ and $n(B') = 350$, find $n(B)$ and $n(U)$.

(i) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 25 + 40 - 50$

$n(A \cap B) = 65 - 50 = 15$

$\boxed{n(A \cap B) = 15}$

$n(U) = n(B) + n(B') = 40 + 25 = 65$

$\boxed{n(U) = 65}$

(ii) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$50 = 300 + n(B) - 500$

$50 = n(B) - 200$

$200 + 50 = n(B)$

$\boxed{n(B) = 250}$

$n(U) = n(B) + n(B') = 250 + 350 = 600$

$\boxed{n(U) = 600}$

2. If $U = \{x : x \in N, x \leq 10\}$, $A = \{2, 3, 4, 8, 10\}$ and $B = \{1, 2, 5, 8, 10\}$, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 3, 4, 8, 10\}$

and $B = \{1, 2, 5, 8, 10\}$.

$n(A) = 5$; $n(B) = 5$

$A \cup B = \{2, 3, 4, 8, 10\} \cup \{1, 2, 5, 8, 10\}$

$A \cup B = \{1, 2, 3, 4, 5, 8, 10\}$. $n(A \cup B) = 7$.

$A \cap B = \{2, 3, 4, 8, 10\} \cap \{1, 2, 5, 8, 10\} = \{2, 8, 10\}$

$n(A \cap B) = 3$.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$7 = 5 + 5 - 3$

$7 = 10 - 3$

$7 = 7$

Hence it is verified.

3. Verify $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$ for the following sets.

(i) $A = \{a, c, e, f, h\}$, $B = \{c, d, e, f\}$ and $C = \{a, b, c, f\}$

(ii) $A = \{1, 3, 5\}$, $B = \{2, 3, 5, 6\}$ and $C = \{1, 5, 6, 7\}$.

(i) $A = \{a, c, e, f, h\}$, $B = \{c, d, e, f\}$ and $C = \{a, b, c, f\}$

$n(A) = 5$, $n(B) = 4$, $n(C) = 4$.

$A \cap B = \{c, e, f\}$; $B \cap C = \{c, f\}$; $A \cap C = \{a, c, f\}$

$n(A \cap B) = 3$; $n(B \cap C) = 2$; $n(A \cap C) = 3$.

$A \cup B \cup C = \{a, b, c, d, e, f, h\}$; $A \cap B \cap C = \{c, f\}$

$n(A\cup B\cup C) = 7$; $n(A\cap B\cap C) = 2$.

$n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$

$7 = 5+4+4-3-2-3+2$

$7 = 15-8$

$7 = 7$

Hence it is verified.

(ii) $A=\{1,3,5\}$, $B=\{2,3,5,6\}$ and $C=\{1,5,6,7\}$

$n(A)=3$; $n(B)=4$; $n(C)=4$.

$A\cap B = \{3,5\}$; $B\cap C=\{5,6\}$; $A\cap C=\{1,5\}$

$n(A\cap B)=2$; $n(B\cap C)=2$; $n(A\cap C)=2$.

$A\cup B\cup C = \{1,2,3,5,6,7\}$; $A\cap B\cap C=\{5\}$

$n(A\cup B\cup C)=6$; $n(A\cap B\cap C)=1$.

$n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$

$6 = 3+4+4-2-2-2+1$

$6 = 12-6$

$6 = 6$

Hence it is verified.

4. In a class, all students take part in either music or drama or both. 25 students take part in music, 30 students take part in drama and 8 students take part in both music and drama. Find

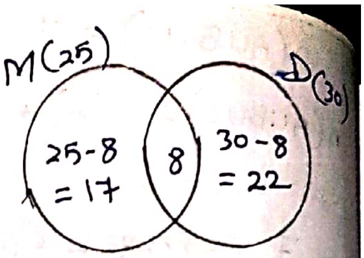
(i) The number of students who take part in only music.

(ii) The number of students who take part in only drama.

(iii) The total number of students in the class.

Let the number of students take part in music and drama is M and D respectively.

$n(M)=25$; $n(D)=30$; $n(M\cap D)=8$



(i) The number of students take part in only music is 17.

(ii) The number of students take part in only drama is 22.

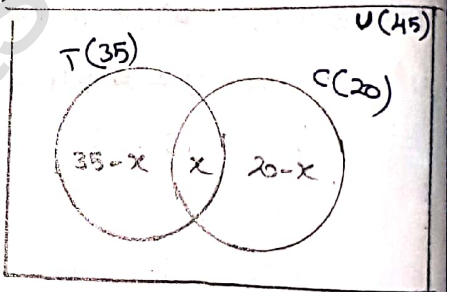
(iii) The total number of students in the class is = 17 + 8 + 22 = 47.

5) In a party of 45 people, each one likes tea or coffee or both. 35 people like tea and 20 people like coffee. Find the number of people who

(i) like both tea and coffee.

(ii) do not like tea.

(iii) do not like coffee.



Let the people who like tea and coffee be T and C respectively.

(i) The number of people who like both tea and coffee be x.

$$35 - x + x + 20 - x = 45$$
$$55 - x = 45$$
$$55 - 45 = x$$
$$\boxed{x = 10}$$

∴ The number of people who like both tea and coffee is 10.

(ii) The number of people who do not like tea = 45 - 35 = 10.

(iii) The number of people who do not like coffee = 45 - 20 = 25.

6. In an examination 50% of the students passed in Mathematics and 70% of students passed in Science while 10% students failed in both subjects. 300 students passed in atleast one subjects. Find the total number of students who appeared in the examination, if they took examination in only two subjects.

Let the students who appeared in the examination be 100%.

Let the percentage of students who failed in mathematics be M.

Let the percentage of students who failed in science be S.

Failed in Maths = 100% − Pass% = 100% − 50% = 50%

Failed in science = 100% − Pass% = 100% − 70% = 30%

Failed in both = 10%.

$n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$= 50\% + 30\% - 10\%$

$= 80\% - 10\%$

$n(M \cup S) = 70\%$

% of students failed in atleast one subject = 70%

∴ The % of students who have passed in atleast one subject = 100% − 70% = 30%

30% = 300

$\therefore 100\% = \frac{100 \times 300}{30} = 1000$.

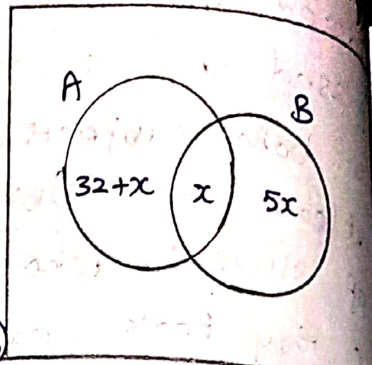
∴ The total number of students who appeared in the examination = 1000 students.

7). A and B two sets such that $n(A-B) = 32 + x$, $n(B-A) = 5x$ and $n(A \cap B) = x$. Illustrate the information by means of a venn diagram.

Given $n(A) = n(B)$, calculate the value.

$n(A-B) = 32 + x$, $n(B-A) = 5x$

$n(A \cap B) = x$.



$$n(A) = n(B)$$

$$n(A-B) + n(A \cap B) = n(B-A) + n(A \cap B)$$

$$32 + x + x = 5x + x$$

$$32 + 2x = 6x$$

$$32 = 6x - 2x$$

$$32 = 4x$$

$$\frac{32}{4} = x$$

$$\boxed{x = 8}$$

8). Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars, Is this data correct?

Given $n(A \cup B) = 500$, $n(A) = 400$, $n(B) = 200$, $n(A \cap B) = 50$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$500 = 400 + 200 - 50$$

$$= 600 - 50$$

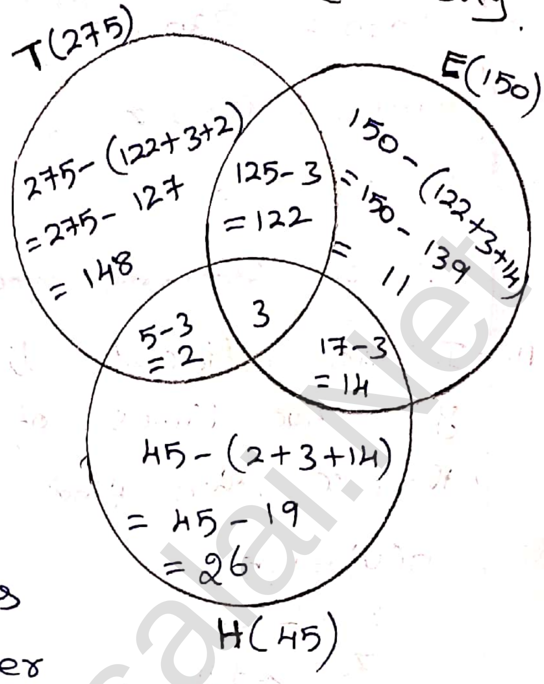
$$500 \neq 550$$

$\therefore$ The given data is wrong.

9). In a colony, 275 families buy Tamil newspaper, 150 families buy English newspaper, 45 families buy Hindi newspaper, 125 families buy Tamil and English newspapers, 17 families buy English and Hindi newspapers, 5 families buy Tamil and Hindi newspapers and 3 families buy all the

three newspapers. If each family buy atleast one of these newspapers then find
(i) Number of families buy only one newspaper
(ii) Number of families buy atleast two newspapers
(iii) Total number of families in the colony.

$n(T) = 275$,
$n(E) = 150$,
$n(H) = 45$
$n(T \cap E) = 125$
$n(E \cap H) = 17$
$n(T \cap H) = 5$
$n(T \cap E \cap H) = 3$.



(i) Number of families buy only one newspaper
$= 148 + 11 + 26 = 185$

(ii) Number of families buy at least two newspapers
$= 122 + 14 + 2 + 3 = 141$

(iii) Total number of families in the colony
$= 148 + 122 + 11 + 14 + 26 + 2 + 3 = 326$

10). A survey of 1000 farmers found that 600 grew paddy, 350 grew ragi, 280 grew corn, 120 grew paddy and ragi, 100 grew ragi and corn, 80 grew paddy and corn. If each farmer grew atleast any one of the above three, then find the number of farmers who grew all the three.

Given $n(P) = 600$, $n(R) = 350$, $n(C) = 280$,
$n(P \cap R) = 120$, $n(R \cap C) = 100$, $n(P \cap C) = 80$
$n(P \cup R \cup C) = 1000$; $n(P \cap R \cap C) = x$.

$n(P\cup R\cup C) = n(P)+n(R)+n(C)-n(P\cap R)-n(R\cap C)-n(P\cap C)+n(P\cap R\cap C)$

$1000 = 600+350+280-120-100-80+x$

$1000 = 1230-300+x$

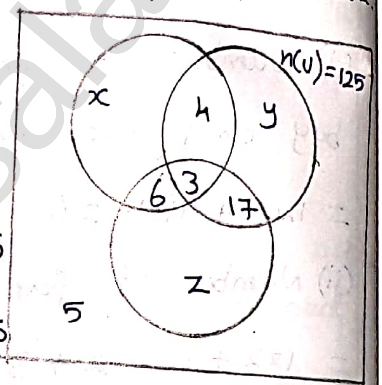
$1000 = 930+x$

$1000-930 = x$

$\therefore \boxed{x = 70}$

$\therefore$ 70 farmers grew all the three crops.

11. In the adjacent diagram, if $n(U) = 125$, y is two times of x and z is 10 more than x, then find the value of x, y and z.



$n(U) = 125$.

$y = 2x$

$z = x+10$

$\therefore x+4+y+17+z+6+3+5 = 125$

$x+4+2x+17+x+10+6+3+5 = 125$

$4x+45 = 125$

$4x = 125-45$

$4x = 80 \Rightarrow x = \frac{80}{4}$

$\boxed{x = 20}$

$y = 2x = 2(20) = 40 \Rightarrow \boxed{y=40}$

$z = x+10 = 20+10 = 30 \Rightarrow \boxed{z=30}$

$\therefore (x, y, z) = (20, 40, 30)$

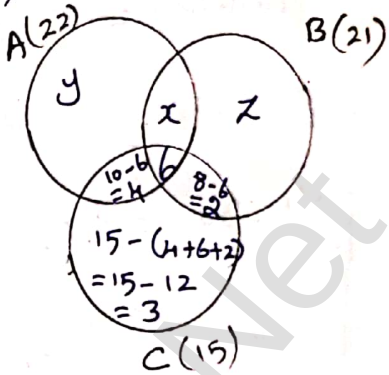
12. Each student in a class of 35 plays atleast one game among, chess, carrom and table tennis, 22 play chess, 21 play carrom, 15 play table tennis, 10 play chess and table tennis, 8 play carrom and table tennis and 6 play all the three games. Find the number of students who play

(i) chess and carrom but not table tennis
(ii) only chess
(iii) only carrom.

Let A - chess, B - carrom and C - table tennis.

$n(A) = 22$, $n(B) = 21$, $n(C) = 15$

$n(A \cap C) = 10$, $n(B \cap C) = 8$, $n(A \cap B \cap C) = 6$.



(i) chess and carrom but not table tennis.

$y = 22 - (x + 6 + 4)$

$= 22 - (x + 10)$

$= 22 - x - 10$

$y = 12 - x$

$z = 21 - (x + 6 + 2) = 21 - (x + 8) = 21 - x - 8$

$z = 13 - x$

$y + x + z + 2 + 3 + 4 + 6 = 35$

$12 - x + x + 13 - x + 2 + 3 + 4 + 6 = 35$

$40 - x = 35$

$40 - 35 = x$

$\boxed{x = 5}$

∴ The number of students who play chess and carrom but not table tennis = 5.

(ii) Number of students who play only chess

$\Rightarrow y = 12 - x = 12 - 5 = 7 \Rightarrow \boxed{y = 7}$

∴ The number of students who play only chess = 7.

(iii) Number of students who play only carrom

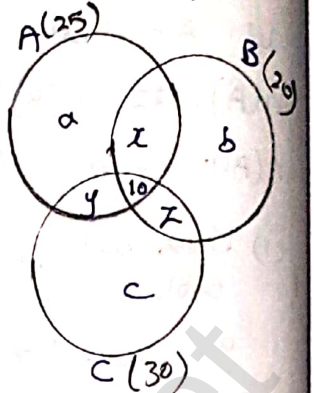
$\Rightarrow z = 13 - x = 13 - 5 = 8 \Rightarrow \boxed{z = 8}$

∴ The number of students who play only carrom = 8.

13. In a class of 50 students, each one come to school by bus or by bicycle or on foot. 25 by bus, 20 by bicycle, 30 on

Scanned with CamScanner

foot and 10 students by all the three. Now how many students come to school exactly by two modes of transport?



A - by bus, B - by bicycle, C - by foot.

$n(A) = 25$, $n(B) = 20$, $n(C) = 30$.

$n(A \cap B \cap C) = 10$. and $n(A \cup B \cup C) = 50$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$50 = 25 + 20 + 30 - (x+10) - (z+10) - (y+10) + 10$

$= 85 - x - 10 - z - 10 - y - 10$

$= 85 - 30 - (x+y+z)$

$50 = 55 - (x+y+z)$

$x+y+z = 55 - 50 = 5$.

$\therefore x+y+z = 5$.

$\therefore$ The number of students who come to school exactly by two modes of transport $= 5$.

Exercise 1.7

Multiple choice Questions:

1) Which of the following is correct?

(a) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$ (b) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$

(c) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$ (d) $\{7\} \not\subseteq \{1,2,3,4,5,6,7,8,9,10\}$

Ans: (b). $7 \in \{1,2,3,4,5,6,7,8,9,10\}$

2) The set $P = \{x \mid x \in Z, -1 < x < 1\}$ is a ______

(a) singleton set (b) Power set (c) Null set (d) Subset

$P = \{0\}$

Ans: (a) Singleton set.

3) If $U = \{x \mid x \in N, x < 10\}$ and $A = \{x \mid x \in N, 2 \le x < 6\}$ then $(A')'$ is ____

(a) $\{1, 6, 7, 8, 9\}$ (b) $\{1, 2, 3, 4\}$ (c) $\{2, 3, 4, 5\}$ (d) $\{\ \}$

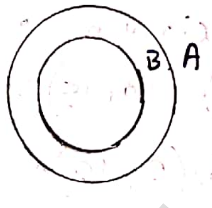
$(A')' = A = \{2, 3, 4, 5\}$

Ans: (c) $\{2, 3, 4, 5\}$.

4) If $B \subseteq A$ then $n(A \cap B)$ is ____

(a) $n(B-A)$ (b) $n(B)$ (c) $n(B-A)$ (d) $n(A)$.

$B \subseteq A \Rightarrow A \cap B = B$



Ans: (b) $n(B)$

5) If $A = \{x, y, z\}$ then the number of non-empty subsets of A is ____.

(a) 8 (b) 5 (c) 6 (d) 7.

Number of non-empty subsets $= 2^3 - 1 = 8 - 1 = 7$.

Ans: (d) 7.

6) Which of the following is correct?

(a) $\phi \subseteq \{a, b\}$ (b) $\phi \in \{a, b\}$ (c) $\{a\} \in \{a, b\}$

(d) $a \subseteq \{a, b\}$.

Ans: (a) $\phi \subseteq \{a, b\}$.

7) If $A \cup B = A \cap B$ then ____

(a) $A \neq B$ (b) $A = B$ (c) $A \subset B$ (d) $B \subset A$.

Ans: (b) $A = B$.

8) If $B - A$ is B, then $A \cap B$ is ____

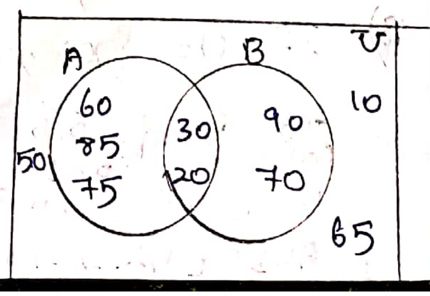
(a) A (b) B (c) U (d) ϕ.

$B - A = B \Rightarrow$ A and B are disjonit sets.

Ans: (d) ϕ.

9) From the adjacent diagram $n[P(A \Delta B)]$ is ____

(a) 8 (b) 16 (c) 32 (d) 64.



$A \Delta B = (A-B) \cup (B-A) = \{60, 85, 75\} \cup \{70, 90\}$

$A \Delta B = \{60, 70, 75, 85, 90\}$

$n(A \Delta B) = 5.$

$n[P(A \Delta B)] = 2^5 = 32.$

Ans : (c) 32.

10). If $n(A) = 10$ and $n(B) = 15$ then the minimum and maximum number of elements in $A \cap B$ is ____. (a) (10, 15) (b) (15, 10) (c) (10, 0) (d) (0, 10)

Ans : (d) (0, 10)

11). Let $A = \{\phi\}$ and $B = P(A)$ then $A \cap B$ is ____.
(a) $\{\phi, \{\phi\}\}$ (b) $\{\phi\}$ (c) ϕ (d) $\{0\}$.

$B = P(A) = \{\phi, \{\phi\}\}$

$A \cap B = \{\phi\}.$

Ans : (b) $\{\phi\}$.

12). In a class of 50 boys, 35 boys play carrom and 20 play chess then the number of boys play both games is ____.
(a) 5 (b) 30 (c) 15 (d) 10.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$50 = 35 + 20 - n(A \cap B)$

$50 = 55 - n(A \cap B)$

$n(A \cap B) = 55 - 50 = 5.$

Ans : (a) 5.

13) If $U = \{x : x \in N$ and $x < 10\}$, $A = \{1, 2, 3, 5, 8\}$ and $B = \{2, 5, 6, 7, 9\}$ then $n[(A \cup B)']$ is ____.
(a) 1 (b) 2 (c) 4 (d) 8.

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cup B = \{1, 2, 3, 5, 6, 7, 8, 9\}$

$(A\cup B)' = U - (A\cup B) = \{4\}$

$n[(A\cup B)'] = 1$

Ans : (a) 1.

14) For any three sets P, Q and R, $P-(Q\cap R)$ is ___

(a) $P-(Q\cup R)$ (b) $(P\cap Q)-R$ (c) $(P-Q)\cup(P-R)$

(d) $(P-Q)\cap(P-R)$.

Ans : (c) $(P-Q)\cup(P-R)$

15) Which of the following is true?

(a) $A-B = A\cap B$ (b) $A-B = B-A$ (c) $(A\cup B)' = A'\cup B'$

(d) $(A\cap B)' = A'\cup B'$

Ans : (d) $(A\cap B)' = A'\cup B'$

16) If $n(A\cup B\cup C)=100$, $n(A)=4x$, $n(B)=6x$, $n(C)=5x$, $n(A\cap B)=20$, $n(B\cap C)=15$, $n(A\cap C)=25$ and $n(A\cap B\cap C)=10$ then the value of x is ___.

(a) 10 (b) 15 (c) 25 (d) 30.

$n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$

$100 = 4x+6x+5x-20-25-15+10$

$100 = 15x+10-60$

$100 = 15x-50$

$100+50 = 15x \Rightarrow 15x = 150 \Rightarrow \boxed{x=10}$

Ans : (a) 10.

17) For any three sets A, B and C, $(A-B)\cap(B-C)$ is equal to ___.

(a) A only (b) B only (c) C only (d) ϕ.

Ans: (d) ϕ.

18) If J = set of three sided shapes, K = set of shapes with two equal sides and L = set of shapes with right angles then $J\cap K\cap L$ is ___.

(a) set of isoceles triangles (b) set of equilateral triangles

(c) set of isoceles right triangles (d) set of right angled triangles

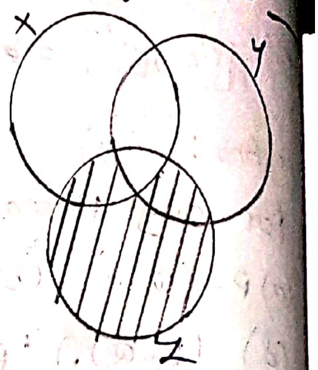
$J = \{\triangle, ◺, ▭\}$, $K = \{◺\}$, $L = \{◺\}$

$J\cap K\cap L = \{◺\}$ Ans: (c) set of isoceles right triangles

19). The shaded region in the venn diagram is __

(a) $Z-(X\cup Y)$ (b) $(X\cup Y)\cap Z$

(c) $Z-(X\cap Y)$ (d) $Z\cup(X\cap Y)$



Ans: (c) $Z-(X\cap Y)$.

20). In a city, 40% people like only one fruit, 35% people like only two fruits, 20% people like all the three fruits. How many percentage of people do not like any one of the above three fruits?

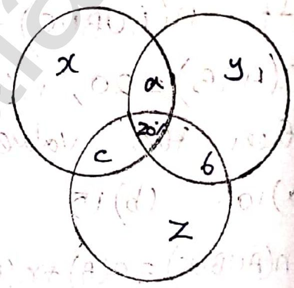
(a) 5 (b) 8 (c) 10 (d) 15.

$40+35+20+x = 100\%$

$95\% + x = 100\%$

$x = 100\% - 95\%$

$x = 5\%$



Ans: (a) 5