

## NUMBER SYSTEM

## INTRODUCTION

Everywhere, except for computer related operation the main system of mathematical notation today is the decimal system, which is a base 10 system. As in the other number systems, the position of a symbol in terms of exponential values of the base. That is in the decimal system, the quantity represented by any of the ten symbols used $0,1,2,3,4,5,6,7,8$, and 9 depends on its position in the number.

Unlike the decimal system, only two digits 0,1 suffice is representing the number in the binary system. The binary system plays a crucial role in computer science and technology.

## DECIMAL SYSTEM

The base or the radix of a number system is defined as the number of digits it uses to represent the numbers in the system. Since, the decimal number system uses 10 digits 0 to 9 its base or radix is 10 , the decimal number system is also called base 10 number system. The weight of each digit of a decimal number system depends on its relative position within the number.

Example:

$$
2763=2000+700+60+3
$$

in other words,

$$
2763=2 \times 10^{3}+7 \times 10^{2}+6 \times 10^{1}+3 \times 10^{\circ}
$$

## BINARY SYSTEM

The binary system is a great help in the Nim-like games plainim, Nimble, Turning, Turtle, Scoring, Northcott's game etc. More importantly, the binary system underlies modern technology of electronic digital computers. Computer memory comprises small elements that may only be in two states - OFF / ON that are associated with digits 0 and 1 . Such an element is said to represent one bit - binary digit.

For example, 1101
To represent numbers the decimal system uses the powers of 10 , whereas the binary system uses in a similar manner the power of 2 .

$$
(1101) 2=1.2^{3}+1.2^{2}+0.2+1
$$

The numbers are different, In fact

$$
(1101)_{2}=8+4+0+1=(13)_{10}
$$

Table:

| Decimal | Binary |
| :---: | :---: |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |
| 10 | 1010 |
| 11 | 1011 |
| 12 | 1100 |
| 13 | 1101 |
| 14 | 1110 |
| 15 | 1111 |
| 16 | 10000 |
| 17 | 10001 |
| 18 | 10010 |
| 19 | 10011 |
| 20 | 10100 |

## BINARY ADDITION AND SUBTRACTION

## Binary Addition

Binary addition is performed in the same manner as decimal addition. Actually, binary arithmetic is much simpler to learn. The complete table for binary addition as follows,

$$
0+0=0
$$

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$$
\begin{aligned}
& 0+1=1 \\
& 1+0=1 \\
& 1+1=0 \text { plus a carry over of } 1
\end{aligned}
$$

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Examples of binary addition:

1. $7+2$

| Decimal | Binary |
| :---: | :---: |
| 7 | 111 |
| 2 | 010 |
| 9 | 1001 |

2. $4+8$

| Decimal | Binary |
| :---: | :---: |
| 4 | 100 |
| 8 | 1000 |
| 12 | 1100 |

3. $12+15$

| Decimal | Binary |
| :---: | :---: |
| 12 | 1100 |
| 15 | 1111 |
| 27 | 11011 |

4. $17+18$

| Decimal | Binary |
| :---: | :---: |
| 17 | 10001 |
| 18 | 10010 |
| 35 | $\mathbf{1 0 0 0 1 1}$ |

## Binary Subtraction

binary numbers is when 1 is subtracted from 0 . The remainder is 1 , but it is necessary to borrow Binary subtraction is the inverse operation of addition. The only case in which this occurs with

1 from the next column to the left. This is the binary subtract table.
$0-0=0$
$1-0=1$
$1-1=0$
$0-1=1$ with a borrow of 1 .

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Examples of binary Subtraction:

1. $7-4$

| Decimal | Binary |
| :---: | :---: |
| 7 | 111 |
| -4 | 100 |
| 3 | 011 |

2. $18-6$

| Decimal | Binary |
| :---: | :---: |
| 18 | 10010 |
| -6 | 110 |
| 12 | 1100 |

3. $16-4$

| Decimal | Binary |
| :---: | :---: |
| 16 | 10000 |
| -4 | 00100 |
| 12 | 1100 |

4. $12-2$

| Decimal | Binary |
| :---: | :---: |
| 12 | 1100 |
| -2 | 0010 |
| $\mathbf{1 0}$ | $\mathbf{1 0 1 0}$ |
| .- |  |

## BINARY MULTIPLICATION AND DIVISION

## Binary Multiplication

The table for binary multiplication is very short, with only four entries instead of the necessary for decimal multiplication.

$$
\begin{aligned}
0 * 0 & =0 \\
0 * 1 & =0 \\
1 * 0 & =0 \\
1 * 1 & =1
\end{aligned}
$$

Examples for binary Multiplication:

1. $4 \times 3$

2. $10 \times 4$

| Decimal | Binary |
| :---: | :---: |
| $10 \times 4$ | $1010 \times 100$ |
|  | 0000 |
|  | 0000 |
|  | 1010 |
| 40 | 101000 |

3. $102 \times 8$

| Decimal | Binary |
| :---: | :---: |
|  | $1100110 \times 1000$ |
|  | 0000000 |
| $102 \times 8$ | 0000000 |
|  | 0000000 |
|  | 1100110 |
| 816 | 1100110000 |

## Binary Division

It is a very simple. As in the decimal system (or in any other), division by zero is meaningless. The complete table as,

$$
\begin{aligned}
& 0 / 1=0 \\
& 1 / 0=1
\end{aligned}
$$

Examples for binary division:

1. $25 \div 5$

2. $20 \div 4$
Decimal Binary

3. $18 \div 6$
Decimal
Binary


011 | 110 |
| :---: | :---: |
| 10010 |
| 011 |
| 011 |
| 011 |
| 0 |

4. $29 \div 12$
Decimal
Binary

|  | 2.416 |  |
| :---: | :---: | :---: |
| 12 | 29 | 1100 |
|  | 24 |  |
|  | 50 |  |
|  | 48 |  |
|  | 20 |  |
|  | 12 |  |
|  | 80 |  |
|  | 72 |  |


| 10.011010101 |
| :---: |
| 11101.00 |
| 1100 |
| 10100 |
| 1100 |
| 10000 |
| 1100 |
| 10000 |
| 1100 |

## CONVERTING DECIMAL NUMBER TO BINARY

There are several methods for converting a decimal number to a binary number for instance, suppose that you want to convert decimal into corresponding binary number.

For examples of Decimal to Binary:
1.

| 2 | 74 |  |
| :--- | :--- | :--- |
| 2 | 37 | -0 |
| 2 | 18 | -1 |
| 2 | 9 | -0 |
| 2 | 4 | -1 |
| 2 | 2 | -0 |
|  | 1 | -0 |
|  |  | $(74)_{10}=(\mathbf{1 0 0 1 0 1 0})_{\mathbf{2}}$ |

2. 

| 2 | 93 |  |
| :--- | :--- | :--- |
| 2 | 46 | -1 |
| 2 | 23 | -0 |
| 2 | 11 | -1 |
| 2 | 5 | -1 |
| 2 | 2 | -1 |
|  | 1 | -0 |
|  |  | $(93)_{\mathbf{1 0}}$ |$=(\mathbf{( 1 0 1 1 1 0 1})_{2}$

3. 

| 2 | 121 |  |
| :--- | :--- | :--- |
| 2 | 60 | -1 |
| 2 | 30 | -0 |
| 2 | 15 | -0 |
| 2 | 7 | -1 |
| 2 | 3 | -1 |
|  | 1 | -1 |

$$
(121)_{10}=(1111001)_{2}
$$

## CONVERTING BINARY NUMBER TO DECIMAL

The conversion of binary to decimal may be accomplished by using several techniques.
Example of Binary to decimal:

$(101)_{2}=(5)_{10}$

$(1011)_{2}=(11)_{10}$

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## NEGATIVE NUMBER REPRESENTATION

There are three methods usually employed to represent negative integer in binary system. These methods are known as
(i) Signed magnitude method
(ii) One's complement method
(iii) Two's complement method

## The Signed Magnitude Method

In this method the signed magnitude method, the sign bit is assigned the value of 1 to denote that it is a negative number.

Examples of signed magnitude method:
Example 1:

$$
\begin{aligned}
(24)_{10} & =11000 \\
(-24)_{10} & =111000
\end{aligned}
$$

| 2 | 24 |  |
| :--- | :--- | :--- |
| 2 | 12 | -0 |
| 2 | 6 | -0 |
| 2 | 3 | -0 |
|  | 1 | -1 |

Example 2:

$$
\begin{aligned}
(147) & =10010111 \\
(-147) & =110010111
\end{aligned}
$$

| 2 | 147 |  |
| :--- | :--- | :--- |
| 2 | 73 | -1 |
| 2 | 36 | -1 |
| 2 | 18 | -0 |
| 2 | 9 | -0 |
| 2 | 4 | -1 |
| 2 | 2 | -0 |
|  | 1 | -1 |

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## One's Complement Method

Subtracting using the 1's complement system is also straight forward. The 1's complement of a number is formed by changing each 1 is the number to a 0 and each 0 is the number to 1 .

When subtracting is performed in the 1s complement system, any end-around carry is added to the least significant.


## Two's Complement

The 2s complement of a binary number is formed by simply subtracting each digit (bit) of the number from the radix minus one and adding a 1 to the least significant bit. Since, the radix in the binary number system is 2 , each bit of the binary number is subtracted from 1 . The application of this rule is actually very simple; every 1 in the number is changed to a 0 and every 0 to a 1 . Then, a 1 is added to the least significant bit of the number formed.


## DECIMAL COMPONENTS

In the decimal system the two types are referred to as the 10's complement and 9's complement.

## 9's Complement

This complement of a decimal number is formed by subtracting each digit of the number from 9 .
For examples of 9's complement:
(i) $23-9$ 's

99
(ii) $48-9$ 's

| 23 |
| ---: |
| 76 |
| 99 |
| 48 |
| 51 |

## 10's Complement

The 10's complement of any number may be formed by subtracting each digit of the number from 9 and then adding 1 to the least significant digit of the number thus formed.

For example of 10's complement:
Normal subtracts:
10's complement:
1.

$$
\begin{array}{r}
78 \\
-24 \\
\hline 54 \\
\hline
\end{array}
$$

$$
\begin{array}{cc|c} 
& \begin{array}{c}
78-78 \\
24-76 \\
\text { carry is dropped } \\
\hline
\end{array} \begin{array}{c}
1 \\
\hline
\end{array} \\
\hline
\end{array}
$$

Where,

$$
\begin{array}{r}
99 \\
24 \\
\hline 75 \\
1 \\
\hline 76 \\
\hline
\end{array}
$$

## OCTAL NUMBER SYSTEM

The octal number system has a base or radix; of 8 , eight different symbols are used to represent the numbers. There are commonly $0,1,2,3,4,5,6$ and 7 . An octal number can be converted easily into the binary number with a group of three binary digits replacing each digits of octal system.

The relation between the decimal system, octal system, Hexadecimal and binary system is shown in table.

| Decimal | Binary | Octal | HexaDecimal |
| :---: | :---: | :---: | :---: |
| 0 | 00000 | 00 | 0 |
| 1 | 00001 | 01 | 1 |
| 2 | 00010 | 02 | 2 |
| 3 | 00011 | 03 | 3 |
| 4 | 00100 | 04 | 4 |
| 5 | 00101 | 05 | 5 |
| 6 | 00110 | 06 | 6 |
| 7 | 00111 | 07 | 7 |
| 8 | 01000 | 10 | 8 |
| 9 | 01001 | 11 | 9 |
| 10 | 01010 | 12 | A |
| 11 | 01011 | 13 | B |
| 12 | 01100 | 14 | C |
| 13 | 01101 | 15 | D |
| 14 | 01110 | 16 | E |
| 15 | 01111 | 17 | F |
| 16 | 10000 | 20 | 10 |
| 17 | 10001 | 21 | 11 |
| 18 | 10010 | 22 | 12 |
| 19 | 10011 | 23 | 13 |
| 20 | 10100 | 24 | 14 |

## CONVERT BINARY TO OCTAL

There is a simple trick for converting a binary number to an octal number simply group the binary digits into groups of 3 , starting at the octal point and read each set of three binary digits.

For example of binary to octal:

$$
\begin{aligned}
(011100)_{2} & \rightarrow(34)_{8} \\
(100111000)_{2} & \rightarrow(470)_{8} \\
(10101)_{2} & \rightarrow(25)_{8} \\
(1001)_{2} & \rightarrow(11)_{8} \\
(1011.110)_{2} & \rightarrow(13.6)_{8} \\
(10101.11)_{2} & \rightarrow(25.6)_{8}
\end{aligned}
$$

CONVERT DECIMAL TO OCTAL
1.

| 8 | 487 |  |
| :--- | :--- | :--- |
| 8 | $60-7$ |  |
|  | 7 | -4 |

$$
(487)_{10}=(747)_{8}
$$



$$
(200)_{10}=(310)_{8}
$$

## HEXADECIMAL

Most mini computers and microcomputers have their memories organized into sets of bytes, each consisting of eight binary digits. Each byte either is used as single entities to represent a single alpha numeric character or is broken into two 4 bit pieces. When the bytes are handles in two 4 bit pieces the programmer is given the option of declaring each 4 bit character as a piece of a binary number or as two, BCD numbers. For instance, the byte 00011000 can be declared a binary number in which case it is equal to 24 decimal, or as two BCD characters in which case it represents the decimal number 18.

When the machine is handling numbers in binary but in groups of four digits it is convenient to have a code for representing each of these sets of four digits. Since, 16 possible different numbers can be represented, the digits 0 through 9 will not suffice, so the letters A, B, C, D, E and F are used.

## CONVERT BINARY TO HEXADECIMAL

Example:

$$
\begin{aligned}
1010 / 1110 / 1101 & \rightarrow(\mathrm{AED})_{16} \\
010 / 1010_{2} & =2 \mathrm{~A}_{16} \\
1010 / 0010 & =\mathrm{A2}_{16}
\end{aligned}
$$

## CONVERT HEXADECIMAL TO DECIMAL

1. 



$$
\mathrm{AB6}_{16}=(2742)_{10}
$$

2. 


$(1 \mathrm{~A} 2 \mathrm{E})_{16}=(6702)_{10}$

## EXCESS - 3 CODE

The excess 3 code is another important BCD code. This code has been derived from the standard BCD code. The excess -3 code for the decimal numbers is obtained by adding 0011 to the code in standard BCD is 0110 . We get the excess 3 code for 6 by adding 0011 to 0110 that is the excess 3 code for 6 is 1001. The excess 3 code is given in table

| Decimal | Excess-3 Code |
| :---: | :--- |
| 0 | 0001 |
| 1 | 0100 |
| 2 | 0101 |
| 3 | 0110 |
| 4 | 0111 |
| 5 | 1000 |
| 6 | 1001 |
| 7 | 1010 |
| 8 | 1011 |
| 9 | 1100 |
| 20 | 01000011 |
| 99 | 01010011 |
| 100 | 11001100 |

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## GRAY CODE

This code is a unit distance code and a non-weighted code. It is not suitable for arithmetic operations but it is very useful for transducers line shaft position encoders, Input output devices, analog to digital converters and other peripheral equipments.

The relation between the gray code and the natural binary is shown in the table.

| Decimal | Binary | Gray Code |
| :---: | :--- | :--- |
| 1 | 0000 | 0000 |
| 2 | 0001 | 0001 |
| 3 | 0010 | 0011 |
| 4 | 0100 | 0110 |
| 5 | 0101 | 0111 |
| 6 | 0110 | 0101 |
| 7 | 0111 | 0100 |
| 8 | 1000 | 1100 |
| 9 | 1001 | 1101 |
| 10 | 1010 | 1111 |
| 11 | 1011 | 1110 |
| 12 | 1100 | 1010 |
| 13 | 1101 | 1011 |
| 14 | 1110 | 1001 |
| 15 | 1111 | 1000 |

## QUESTIONS

## Fill up the blanks

1. Binary 1010 in decimal system is equivalent to $\qquad$ -
2. The decimal equivalent of the binary number 10110.0101011101 is $\qquad$ .
3. Binary 101010 is equivalent to decimal number is $\qquad$ .
4. The hexadecimal number A492 is equivalent to decimal number is $\qquad$ .
5. The binary number 0111011011 is equivalent to decimal number $\qquad$
6. The digit 0 with carry of 1 is the sum of binary addition is $\qquad$ .
7. In BCD code is represented as $\qquad$
8. Octal 16 is equal to decimal $\qquad$ .
9. The sum of $(111010)_{2}$ and $(11011)_{2}$ in decimal form will be $\qquad$
10. $(100101)_{2}$ is $\qquad$

## True or false:

1. As compared to digital computers, micro computers have high cost and big size.
2. The number of binary digits that make up the word is the word length.
3. A byte is an 8 binary digit word length.
4. BCD numbers express each decimal digit as a byte.
5. For the binary number 101101110 the equivalent octal number is 556 .
