XI MATHS

VOL -I

QUESTION BANK

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An equation means nothing to me unless it expresses a thought of God.

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Chapter – 1 SETS, RELATIONS AND FUNCTIONS

2 & 3 MARKS IMPORTANT

Example 1.1 Find the number of subsets of A if $A = \{x : x = 4n+1, 2 \le n \le 5, n \in \mathbb{N}\}$.

Example 1.3 Prove that $((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'$.

Example 1.4 If $X = \{1, 2, 3, \ldots 10\}$ and $A = \{1, 2, 3, 4, 5\}$, find the number of sets $B \subseteq X$ such that $A - B = \{4\}$

Example 1.7 If $n(A) = 10$ and $n(A \cap B) = 3$, find $n((A \cap B)' \cap A)$.

Example 1.8 If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$.

Example 1.9 If $P(A)$ denotes the power set of A, then find $n(P(P(P(\phi))))$.

EXERCISE 1.1

5. Justify the trueness of the statement:
   "An element of a set can never be a subset of itself."
6. If $n(P(A)) = 1024$, $n(A \cup B) = 15$ and $n(P(B)) = 32$, then find $n(A \cap B)$.
7. If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n(P(A \Delta B))$.

Example 1.10 Check the relation $R = \{(1,1), (2,2), (3,3) \ldots\ldots (n,n)\}$ defined on the set $S = \{1,2,3,\ldots..n\}$ for the three basic relations.

5 MARKS IMPORTANT

Example 1.2 In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C, and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A.

Example 1.5 If A and B are two sets so that $n(B - A) = 2n(A - B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$, then find $n(P(A))$.

Example 1.6 Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k.

Example 1.13 In the set Z of integers, define mRn if $m - n$ is a multiple of 12. Prove that R is an equivalence relation.

Example 1.30
If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.

EXERCISE 1.3

2. Write the values of f at $-4, 1, -2, 7, 0$ if

$$f(x) = \begin{cases} -x+4 & if \quad -\infty < x \le -3 \\ x+4 & if \quad -3 < x < -2 \\ x^2 - x & if \quad -2 \le x < 1 \\ x - x^2 & if \quad 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

3. Write the values of f at $-3, 5, 2, -1, 0$ if

$$f(x) = \begin{cases} x^2 + x - 5 & if \quad x \in (-\infty, 0) \\ x^2 + 3x - 2 & if \quad x \in (3, \infty) \\ x^2 & if \quad x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

EXERCISE 1.2

2. Let $X = \{a, b, c, d\}$ and $R = \{(a, a), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it

   i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

3. Let $A = \{a, b, c\}$ and $R = \{(a, a), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it

   i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

6. Prove that the relation "friendship" is not an equivalence relation on the set of all people in Chennai.

9. In the set Z of integers, define mRn if $m - n$ is divisible by 7. Prove that R is an equivalence relation.

EXERCISE 1.3

12. If $f : R \to R$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse.

19. The formula for converting from Fahrenheit to Celsius temperatures is $y = \frac{5x}{9} - \frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function.

20. A simple cipher takes a number and codes it, using the function $f(x) = 3x - 4$. Find the inverse this function, determine whether the inverse is also a function and verify the symmetrical property about the line $y = x$ (by drawing the lines).

2. BASIC ALGEBRA

2 & 3 MARKS IMPORTANT

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Example 2.5 Solve : $\left|\frac{2}{x-4}\right| > 1$, $x \neq 4$

Example 2.8 Solve the following system of linear inequalities : $3x - 9 \geq 0$, $4x - 10 \leq 6$.

Example 2.7 Solve $3x - 5 \leq x + 1$ for x.

Example 2.9 A girl A is reading a book having 446 pages and she has already finished reading 271 pages. She wants to finish reading this book within a week. What is the minimum number of pages she should read per day to complete reading the book within a week?

Example 2.6 Our monthly electricity bill contains a basic charge, which does not change with number of units used, and a charge that depends only on how many units we use. Let us say Electricity Board charges Rs.110 as basic charge and charges Rs. 4 for each unit we use. If a person wants to keep his electricity bill below Rs.250, then what should be his electricity usage?

EXERCISE 2.3

7. Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

5. To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84, 87, 95, 91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A grade in the course?

10. A and B are working on similar jobs but their annual salaries differ by more than Rs 6000. If B earns rupees 27000 per month, then what are the possibilities of A's salary per month?

Example 2.12 Find the number of solutions of $x^2 + |x - 1| = 1$.

Example 2.10 If a and b are the roots of the equation $x^2 - px + q = 0$, find the value of $\frac{1}{a} + \frac{1}{b}$.

EXERCISE 2.4

1. Construct a quadratic equation with roots 7 and -3.

10. Write $f(x) = x^2 + 5x + 4$ in completed square form.

Example 2.14 Solve : $\sqrt{x+14} < x + 2$ **Example 2.24** Solve : $\frac{x+1}{x+3} < 3$

EXERCISE 2.8

1. Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$

2. Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$

3. Solve : $\frac{x^2-4}{x^2-2x-15} \leq 0$. **Example 2.25** Resolve into partial fractions: $\frac{x}{(x+3)(x-4)}$.

EXERCISE 2.9

1. Resolve into partial fractions: $\frac{1}{x^2-a^2}$ 2. Resolve into partial fractions: $\frac{3x+1}{(x-2)(x+1)}$

Example 2.32 Rationalize the denominator of $\frac{\sqrt{5}}{(\sqrt{6}+\sqrt{2})}$.

Example 2.33 Find the square root of $7 - 4\sqrt{3}$.

EXERCISE 2.11

2. Evaluate : $(((256)^{-1/2})^{-1/4})^3$

4. Simplify and hence find the value of n : $3^{2n} 9^2 3^{-n} / 3^{3n} = 27$.

5. Find the radius of the spherical tank whose volume is $32\pi/3$ units.

8. If $x = \sqrt{2} + \sqrt{3}$ find $\frac{x^2+1}{x^2-2}$.

Example 2.38 Solve : $x^{\log_3 x} = 9$. **Example 2.39** Compute : $\log_3 5 \log_{25} 27$.

EXERCISE 2.12

5. If $a^2+b^2=7ab$, show that $\log\frac{a+b}{3}=\frac{1}{2}(\log a+\log b)$

6. Prove : $\log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}=0$ 8. Prove: $\log_{a^2} a\ \log_{b^2} b\ \log_{c^2} c=\frac{1}{8}$.

9. Prove: $\log a+\log a^2+\log a^3+\ldots\ldots\ldots+\log a^n=\frac{n(n+1)}{2}\log a$.

12. Solve: $\log_{5-x}(x^2-6x+65)=2$

5 MARKS IMPORTANT

EXERCISE 2.3

6. A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

EXERCISE 2.4

3. If α and β are the roots of the quadratic equation $x^2+\sqrt{2}x+3=0$, form a quadratic polynomial with zeros $\frac{1}{\alpha},\frac{1}{\beta}$.

4. If one root of $k(x-1)^2=5x-7$ is double the other root , show that $k=2$ or -25.

Example 2.26 Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$.

Example 2.27 Resolve into partial fractions: $\frac{x+1}{x^2(x-1)}$.

EXERCISE 2.9

5. Resolve into partial fractions: $\frac{1}{x^4-1}$

EXERCISE 2.11

7. Simplify : $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$.

Example 2.36 Prove: $\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$.

Example 2.37 If $\log_2 x+\log_4 x+\log_{16} x=\frac{7}{2}$ find the value of x.

EXERCISE 2.12

3. Solve : $\log_8 x+\log_4 x+\log_2 x=11$

7. Prove that $\log 2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}=1$.

10. If $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$, then prove that $xyz=1$

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3. TRIGONOMETRY

2 & 3 MARKS IMPORTANT

Example 3.5 Convert (i) $\frac{\pi}{5}$ radians to degrees (ii) 6 radians to degrees

Example 3.6 Find the length of an arc of a circle of radius 5 *cm* subtending a central angle measuring 15°

Example 3.7

If the arcs of same lengths in two circles subtend central angles 30° and 80°, find the ratio of their radii.

EXERCISE 3.2

1. Express each of the following angles in radian measure:

   i) 30° (ii) 135° (iii) −205° (iv) 150° (v) 330°

2. Find the degree measure corresponding to the following radian measures :

   iii) $\frac{2\pi}{5}$ iv) $\frac{7\pi}{3}$ v) $\frac{10\pi}{9}$

3. What must be the radius of a circular running path, around which an athlete must run 5 times in order to describe 1 km?

Example 3.11 Find the value of (i) sin 150° (ii) cos 135° (iii) tan 120°

Example 3.12 Find the value of: (ii) cosec (−1410°) (iii) $\cot\left(\frac{-15\pi}{4}\right)$

Example 3.13 Prove that tan 315° cot (−405°) + cot 495° tan (−585°) = 2

EXERCISE 3.3

1. Find the values of (ii) sin (−1110°) vi) $\tan\left(\frac{19\pi}{3}\right)$

4. Prove that $\frac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)}=\cos^2\theta\cot\theta$

5. Find all the angles between 0° and 360° which satisfy the equation $\sin^2\theta=\frac{3}{4}$

6. Show that $\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}=2$.

EXERCISE 3.4

5. Find the value of (i) cos 105° (ii) sin 105° (iii) $\tan\frac{7\pi}{12}$

10. If $a\cos(x+y)=b\cos(x-y)$, show that $(a+b)\tan x=(a-b)\cot y$.

15. Prove that $\sin(n+1)\theta\sin(n-1)\theta+\cos(n+1)\theta\cos(n-1)\theta=\cos 2\theta,\ n\in\mathbb{Z}$.

20. Show that (i) $\tan(45^\circ+A)=\frac{1+\tan A}{1-\tan A}$

Example 3.22 Find the value of $\sin\left(22\frac{1}{2}^\circ\right)$.

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EXERCISE 3.5

8. Prove that $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=2\tan 2\theta$.

3. If $\cos\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)$, show that $\cos 3\theta=\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$

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5. Prove that $\sin 4\alpha=4\tan\alpha\frac{1-\tan^2\alpha}{(1+\tan^2\alpha)^2}$

Example 3.33

Express each of the following product as a sum or difference, (i) $\sin 40°\cos 30°$

Example 3.36 Show that $\cos 36°\ \cos 72°\ \cos 108°\ \cos 144°=\frac{1}{16}$

Example 3.38 Show that $\cos 10°\cos 30°\ \cos 50°\cos 70°=\frac{3}{16}$

EXERCISE 3.6

4. Show that $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}=\frac{1}{128}$

5 MARKS IMPORTANT

EXERCISE 3.4

7. Find a quadratic equation whose roots are $\sin 15°$ and $\cos 15°$.

8. Expand cos (A+B+C) . Hence prove that cosAcosB cosC = sin A sinB cosC + sinB sinC cosA+sinCsinAcosB, if A+B+C= $\frac{\pi}{2}$.

19. If $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$, then prove that $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$.

25. If $\theta+\varphi=\alpha$ and $\tan\theta=k\tan\varphi$, then prove that $\sin(\theta-\varphi)=\frac{k-1}{k+1}\sin\alpha$

EXERCISE 3.5

6. If $A+B=45°$, show that $(1+\tan A)(1+\tan B)=2$.

7. Prove that $(1+\tan 1°)(1+\tan 2°)(1+\tan 3°)\ldots(1+\tan 44°)$ is a multiple of 4.

9. Show that $\cot\left(7\frac{1}{2}^{\circ}\right)=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

11. Prove that $32(\sqrt{3})\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6}=3$.

EXERCISE 3.4

7. Find a quadratic equation whose roots are $\sin 15°$ and $\cos 15°$.

8. Expand cos (A+B+C) . Hence prove that cosAcosB cosC = sin A sinB cosC + sinB sinC cosA+sinCsinAcosB, if A+B+C= $\frac{\pi}{2}$.

19. If $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$, then prove that $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$.

25. If $\theta+\varphi=\alpha$ and $\tan\theta=k\tan\varphi$, then prove that $\sin(\theta-\varphi)=\frac{k-1}{k+1}\sin\alpha$

Example 3.25 Prove that $\sin x=2^{10}\sin\left(\frac{x}{2^{10}}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\ldots\ldots\cos\left(\frac{x}{2^{10}}\right)$

Example 3.32 Prove that $\cos A\cos 2A\ \cos 2^2A\ \cos 2^3A\ldots\cos 2^{n-1}A=\frac{\sin 2^n A}{2^n\sin A}$

4. COMBINATORICS AND MATHEMATICAL INDUCTION

2 & 3 MARKS IMPORTANT

Example 4.1

Suppose one girl or one boy has to be selected for a competition from a class comprising 17 boys and 29 girls. In how many different ways can this selection be made?

Example 4.3

A School library has 75 books on Mathematics, 35 books on Physics. A student can choose only one book. In how many ways a student can choose a book on Mathematics or Physics?

Example 4.8

(i) Find the number of strings of length 4, which can be formed using the letters of the word BIRD, without repetition of the letters.

(ii) How many strings of length 5 can be formed out of the letters of the word PRIME taking all the letters at a time without repetition.

Example 4.13

Find the total number of outcomes when 5 coins are tossed once.

Example 4.15 There are 10 bulbs in a room. Each one of them can be operated independently. Find the number of ways in which the room can be illuminated.

Example 4.16 Find the value of (ii) $6!-5!$ (iii) $\frac{8!}{5!\times 2!}$ **Example 4.17** Simplify : $\frac{7!}{2!}$

Example 4.20 If $\frac{6!}{n!} = 6$, then find the value of n.

Example 4.21 If $n! + (n-1)! = 30$, then find the value of n.

Example 4.22 What is the unit digit of the sum $2! + 3! + 4! + ... + 22!$?

Example 4.23 If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ then find the value of A.

Exercise 4.1

2. (i) A mobile phone has a passcode of 6 distinct digits. What is the maximum number of attempts one makes to retrieve the passcode?

Exercise 4.2

14. Find the value of (v) $\frac{12!}{9!\text{x}3!}$ 15. Evaluate $\frac{n!}{r!(n-r)!}$ when i) $n = 6$, r = 2

16. Find the value of n if i) $(n+1)! = 20(n-1)!$ ii) $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$

Example 4.25 Evaluate: (iii) ${}^{8}P_{4}$ (iv) ${}^{6}P_{5}$

Example 4.26 If ${}^{(n+2)}P_4 = 42 \text{ x } {}^{n}P_2$, find n. **Example 4.27** If ${}^{10}P_r = {}^{7}P_{r+2}$ find r.

Example 4.35

If the letters of the word TABLE are permuted in all possible ways and the words thus formed are arranged in the dictionary order (alphabetical order), find the ranks of the words (i) TABLE, (ii) BLEAT

Example 4.36 Find the number of ways of arranging the letters of the word BANANA.

Example 4.37 Find the number of ways of arranging the letters of the word RAMANUJAN so that the relative positions of vowels and consonants are not changed.

Example 4.41

If the different permutations of all letters of the word BHASKARA are listed as in a dictionary, how many strings are there in this list before the first word starting with B?

Exercise 4.2

2. If ${}^{10}P_{r-1} = 2 \text{ x } {}^{6}P_r$, find r.
4. Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time?
7. How many strings can be formed from the letters of the word ARTICLE, so that vowels occupy the even places?
9. Find the distinct permutations of the letters of the word MISSISSIPPI?
10. How many ways can the product $a^2b^3c^4$ be expressed without exponents?

Exercise 4.2

12. In how many ways can the letters of the word SUCCESS be arranged so that all Ss are together?

16. If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words

(i) GARDEN (ii) DANGER.

18. If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY.

Example 4.46 If ${}^{n}C_4 = 495$, What is n?.

Example 4.48 Prove that ${}^{24}C_4 + \sum_{r=0}^{4} {}^{(28-r)}C_3 = {}^{29}C_4$

Example 4.49 Prove that ${}^{10}C_2 + 2 \text{ x } {}^{10}C_3 + {}^{10}C_4 = {}^{12}C_4$

Example 4.51

A salad at a certain restaurant consists of 4 of the following fruits: apple, banana, guava, pomegranate, grapes, papaya and pineapple. Find the total possible number of fruit salads.

Example 4.60 How many diagonals are there in a polygon with n sides?

EXERCISE 4.3

1. If ${}^{n}C_{12} = {}^{n}C_9$ find ${}^{21}C_n$.
4. Prove that ${}^{15}C_3 + 2 \text{ x } {}^{15}C_4 + {}^{15}C_5 = {}^{17}C_5$.
7. Prove that ${}^{2n}C_n = \frac{2^n \text{x}1\text{x}3\text{x}........(2\text{n}-1)}{n!}$
25. A polygon has 90 diagonals. Find the number of its sides?

5 MARKS IMPORTANT

Example 4.61 By the principle of mathematical induction, prove that, for all integers $n \geq 1$, $1 + 2 + 3 + \cdot \cdot \cdot + n = \frac{n(n+1)}{2}$

Example 4.63 By the principle of mathematical induction, prove that, for all integers $n \geq 1$, $1^2 + 2^2 + 3^2 + \cdot \cdot \cdot + n^2 = \frac{n(n+1)(2n+1)}{6}$

Example 4.65 Prove that for any natural number n, $a^n - b^n$ is divisible by $a - b$,

Example 4.66 Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

EXERCISE 4.4

1. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \dots\dots\dots\dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

4. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1.2 + 2.3 + 3.4 + \dots\dots\dots + n.(n+1) = \frac{n(n+1)(n+2)}{3}.$$

10. Using the mathematical induction, show that for any natural number n, $x^{2n} - y^{2n}$ is divisible by $x+y$.

12. Use induction to prove that $n^3 - 7n + 3$, is divisible by 3, for all natural numbers n.

13. Use induction to prove that $5^{n+1} + 4 \times 6^n$ when divided by 20 leaves a remainder 9, for all natural numbers n.

14. Use induction to prove that $10^n + 3 \times 4^{n+2} + 5$, is divisible by 9, for all natural numbers n.

Exercise 4.1

12. How many strings can be formed using the letters of the word LOTUS if the word (i) either starts with L or ends with S? (ii) neither starts with L nor ends with S?

Example 4.42

If the letters of the word IITJEE are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word IITJEE

Example 4.43

Find the sum of all 4-digit numbers that can be formed using the digits 1, 2, 4, 6, 8.

Exercise 4.2

17. Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary, what will be the 85^{th} string?

19. Find the sum of all 4-digit numbers that can be formed using digits 1, 2, 3, 4, and 5 repetitions not allowed?

20. Find the sum of all 4-digit numbers that can be formed using digits 0, 2, 5, 7, 8 without repetition?

Example 4.58

Find the number of strings of 5 letters that can be formed with the letters of the word PROPOSITION.

5. BINOMIAL THEOREM, SEQUENCES AND SERIES

2 & 3 MARKS IMPORTANT

EXERCISE 5.2

5. Write the n[th] term of the sequence $\frac{3}{1^2 2^2}, \frac{5}{2^2 3^2}, \frac{7}{3^2 4^2}, \dots\dots\dots$ as a difference of two terms.

7. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression.

9. If the roots of the equation $(q-r)x^2 + (r-p)x + p - q = 0$ are equal, then show that p, q and r are in AP.

EXERCISE 5.4

1. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid i) $\frac{1}{5+x}$ ii) $\frac{2}{(3+4x)^2}$

2. Find $\sqrt[3]{1001}$ approximately (two decimal places)

Example 5.21 Expand $(1+x)^{\frac{2}{3}}$ up to four terms for $|x| < 1$.

Example 5.23 Expand $\frac{1}{(3+2x)^2}$ in powers of x. Find a condition on x for which the expansion is valid.

Example 5.24 Find $\sqrt[3]{65}$

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5 MARKS IMPORTANT

EXERCISE 5.4

3. Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

4. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.

8. If $p-q$ is small compared to either p or q, then show that $\sqrt[n]{\frac{p}{q}} \simeq \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$.

Hence find $\sqrt[8]{\frac{15}{16}}$.

Example 5.25 Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

EXERCISE 5.2

10. If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a GP, show that $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$.

6. TWO DIMENSIONAL ANALYTICAL GEOMETRY

2 & 3 MARKS IMPORTANT

Example 6.7

Find the slope of the straight line passing through the points (5,7) and (7,5). Also find the angle of inclination of the line with the x-axis.

Example 6.8

Find the equation of a straight line cutting an intercept of 5 from the negative direction of the y-axis and is inclined at an angle $150°$ to the x-axis.

Example 6.9

Show the points $\left(0,-\frac{3}{2}\right)$, $(-1,1)$ and $\left(2,-\frac{1}{2}\right)$ are collinear

Example 6.14

Find the equation of the straight line passing through $(-1, 1)$ and cutting off equal intercepts, but opposite in signs with the two coordinate axes.

Example 6.16

The length of the perpendicular drawn from the origin to a line is 12 and makes an angle $150°$ with positive direction of the x-axis. Find the equation of the line.

Example 6.20 Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form.

EXERCISE 6.2

1. Find the equation of the lines passing through the point (1,1)

   (iv) and the perpendicular from the origin makes an angle $60°$ with x- axis.

2. If $P(r, c)$ is mid point of a line segment between the axes, then show that $\frac{x}{r}+\frac{y}{c}=2$

4. If p is length of perpendicular from origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

8. Find the equation of the line, if the perpendicular drawn from the origin makes an angle $30°$ with x-axis and its length is 12.

Example 6.22

Find the equations of a parallel line and a perpendicular line passing through the point (1, 2) to the line $3x+4y=7$.

Example 6.23

Find the distance (ii) from a point (1, 2) to the line $5x + 12y - 3 = 0$
(iii) between two parallel lines $3x + 4y = 12$ and $6x+8y+1=0$.

Example 6.24 Find the nearest point on the line $2x + y = 5$ from the origin.

EXERCISE 6.3

1. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.
2. Find the equation of the straight line parallel to $5x - 4y + 3 = 0$ and having x-intercept 3.
3. Find the distance between the line $4x + 3y + 4 = 0$, and a point $(-2, 4)$
4. Write the equation of the lines through the point $(1,-1)$ (i) parallel to $x + 3y - 4 = 0$
   (ii) perpendicular to $3x + 4y = 6$

12. Find the distance between the parallel lines
   (i) $12x + 5y = 7$ and $12x+5y+7=0$ (ii) $3x - 4y+5 = 0$ and $6x-8y-15=0$.
13. Find the family of straight lines (i) Perpendicular (ii) Parallel to $3x + 4y - 12 = 0$.

EXERCISE 6.4

1. Find the combined equation of the straight lines whose separate equations are $x - 2y - 3 = 0$ and $x + y+5 = 0$.
2. Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.

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5 MARKS IMPORTANT

EXERCISE 6.2

3. Find the equation of the line passing through the point (1, 5) and also divides the co-ordinate axes in the ratio 3:10.

Example 6.29

Find the equation of the line through the intersection of the lines $3x+2y+5 = 0$ and $3x - 4y+6=0$ and the point (1,1).

Example 6.25

Find the equation of the bisector of the acute angle between the lines $3x+4y+2=0$ and $5x+12y-5=0$.

Example 6.26

Find the points on the line $x + y = 5$, that lie at a distance 2 units from the line $4x + 3y - 12 = 0$

EXERCISE 6.3

5. If $(-4, 7)$ is one vertex of a rhombus and if the equation of one diagonal is $5x - y + 7 = 0$, then find the equation of another diagonal.
7. Find the equations of two straight lines which are parallel to the line $12x + 5y + 2 = 0$ and at a unit distance from the point $(1, -1)$.
8. Find the equations of straight lines which are perpendicular to the line $3x+4y-6=0$ and are at a distance of 4 units from (2, 1).

17. Find the image of the point $(-2, 3)$ about the line $x + 2y - 9 = 0$.

Example 6.33 Separate the equations $5x^2 + 6xy + y^2 = 0$

Example 6.36

Show that the straight lines $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle.

Example 6.41

Show that the straight lines joining the origin to the points of intersection of $3x - 2y + 2 = 0$ and $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$ are at right angles

EXERCISE 6.4

6. Find the equation of the pair of straight lines passing through the point $(1, 3)$ and perpendicular to the lines $2x - 3y + 1 = 0$ and $5x + y - 3 = 0$

8. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.

9. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

11. Find p and q, if the following equation represents a pair of perpendicular lines $6x^2 + 5xy - py^2 + 7x + qy - 5 = 0$

14. Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

15. Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them.

16. Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a + b)^2 = 4h^2$

18. Prove that the straight lines joining the origin to the points of intersection of $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and $3x - 2y - 1 = 0$ are at right angles.

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Exercise - 1.5

Choose the correct or the most suitable answer.

1. If $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$ then $n(A \cap B)$ is
   (1) Infinity (2) 0 (3) 1 (4) 2

2. If $A = \{(x, y) : y = \sin x, x \in R\}$ and $B = \{(x, y) : y = \cos x, x \in R\}$ then $A \cap B$ contains
   (1) no element (2) infinitely many elements
   (3) only one element (4) cannot be determined.

3. The relation R defined on a set $A = \{0, -1, 1, 2\}$ by xRy if $|x^2 + y^2| \le 2$, then which one of the following is true?
   (1) $R = \{(0, 0), (0, -1), (0, 1), (-1, 0), (-1, 1), (1, 2), (1, 0)\}$
   (2) $R^{-1} = \{(0, 0), (0, -1), (0, 1), (-1, 0), (1, 0)\}$
   (3) Domain of R is $\{0, -1, 1, 2\}$
   (4) Range of R is $\{0, -1, 1\}$

4. If $f(x) = |x - 2| + |x + 2|, x \in \mathbb{R}$, then
   (1) $f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ 4 & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$
   (2) $f(x) = \begin{cases} 2x & \text{if } x \in (-\infty, -2] \\ 4x & \text{if } x \in (-2, 2] \\ -2x & \text{if } x \in (2, \infty) \end{cases}$
   (3) $f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ -4x & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$
   (4) $f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ 2x & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$

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5. Let $\mathbb{R}$ be the set of all real numbers. Consider the following subsets of the plane $\mathbb{R} \times \mathbb{R}$:
   $S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$ and $T = \{(x, y) : x - y \text{ is an integer }\}$
   Then which of the following is true?
   (1) T is an equivalence relation but S is not an equivalence relation.
   (2) Neither S nor T is an equivalence relation
   (3) Both S and T are equivalence relation
   (4) S is an equivalence relation but T is not an equivalence relation.

6. Let A and B be subsets of the universal set $\mathbb{N}$, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is
   (1) A (2) A' (3) B (4) $\mathbb{N}$

7. The number of students who take both the subjects Mathematics and Chemistry is 70. This represents 10% of the enrollment in Mathematics and 14% of the enrollment in Chemistry. The number of students take at least one of these two subjects, is
   (1) 1120 (2) 1130 (3) 1100 (4) insufficient data

12. The number of relations on a set containing 3 elements is
(1) 9 (2) 81 (3) 512 (4) 1024

13. Let R be the universal relation on a set X with more than one element. Then R is
(1) not reflexive (2) not symmetric (3) transitive (4) none of the above

14. Let $X = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (2, 1), (3, 1), (1, 4), (4, 1)\}$. Then R is
(1) reflexive (2) symmetric (3) transitive (4) equivalence

21. Let $X = \{1, 2, 3, 4\}, Y = \{a, b, c, d\}$ and $f = \{(1, a), (4, b), (2, c), (3, d), (2, d)\}$. Then f is
(1) an one-to-one function (2) an onto function
(3) a function which is not one-to-one (4) not a function

22. The inverse of $f(x) = \begin{cases} x & \text{if } x < 1 \\ x^2 & \text{if } 1 \le x \le 4 \\ 8\sqrt{x} & \text{if } x > 4 \end{cases}$ is

(1) $f^{-1}(x) = \begin{cases} x & \text{if } x < 1 \\ \sqrt{x} & \text{if } 1 \le x \le 16 \\ \frac{x^2}{64} & \text{if } x > 16 \end{cases}$ (2) $f^{-1}(x) = \begin{cases} -x & \text{if } x < 1 \\ \sqrt{x} & \text{if } 1 \le x \le 16 \\ \frac{x^2}{64} & \text{if } x > 16 \end{cases}$

(3) $f^{-1}(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } 1 \le x \le 16 \\ \frac{x^2}{64} & \text{if } x > 16 \end{cases}$ (4) $f^{-1}(x) = \begin{cases} 2x & \text{if } x < 1 \\ \sqrt{x} & \text{if } 1 \le x \le 16 \\ \frac{x^2}{8} & \text{if } x > 16 \end{cases}$

24. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \sin x + \cos x$ is
(1) an odd function
(2) neither an odd function nor an even function
(3) an even function
(4) both odd function and even function.

25. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \dfrac{(x^2 + \cos x)(1 + x^4)}{(x - \sin x)(2x - x^3)} + e^{-|x|}$ is
(1) an odd function (2) neither an odd function nor an even function
(3) an even function (4) both odd function and even function.

Exercise - 2.13

Choose the correct or the most suitable answer.

1. If $|x + 2| \le 9$, then x belongs to
(1) $(-\infty, -7)$ (2) $[-11, 7]$ (3) $(-\infty, -7) \cup [11, \infty)$ (4) $(-11, 7)$

2. Given that x, y and b are real numbers $x < y, b > 0$, then
(1) $xb < yb$ (2) $xb > yb$ (3) $xb \le yb$ (4) $\frac{x}{b} \ge \frac{y}{b}$

3. If $\dfrac{|x - 2|}{x - 2} \ge 0$, then x belongs to
(1) $[2, \infty)$ (2) $(2, \infty)$ (3) $(-\infty, 2)$ (4) $(-2, \infty)$

4. The solution of $5x - 1 < 24$ and $5x + 1 > -24$ is
(1) $(4, 5)$ (2) $(-5, -4)$ (3) $(-5, 5)$ (4) $(-5, 4)$

5. The solution set of the following inequality $|x - 1| \ge |x - 3|$ is
(1) $[0, 2]$ (2) $[2, \infty)$ (3) $(0, 2)$ (4) $(-\infty, 2)$

6. The value of $\log_{\sqrt{2}} 512$ is
(1) 16 (2) 18 (3) 9 (4) 12

7. The value of $\log_3 \frac{1}{81}$ is
(1) -2 (2) -8 (3) -4 (4) -9

8. If $\log_{\sqrt{x}} 0.25 = 4$, then the value of x is
(1) 0.5 (2) 2.5 (3) 1.5 (4) 1.25

9. The value of $\log_a b \; \log_b c \; \log_c a$ is
(1) 2 (2) 1 (3) 3 (4) 4

10. If 3 is the logarithm of 343, then the base is
(1) 5 (2) 7 (3) 6 (4) 9

11. Find a so that the sum and product of the roots of the equation $2x^2 + (a - 3)x + 3a - 5 = 0$ are equal is
(1) 1 (2) 2 (3) 0 (4) 4

12. If a and b are the roots of the equation $x^2 - kx + 16 = 0$ and satisfy $a^2 + b^2 = 32$, then the value of k is
(1) 10 (2) -8 (3) $-8, 8$ (4) 6

13. The number of solutions of $x^2 + |x - 1| = 1$ is
(1) 1 (2) 0 (3) 2 (4) 3

14. The equation whose roots are numerically equal but opposite in sign to the roots of $3x^2 - 5x - 7 = 0$ is
(1) $3x^2 - 5x - 7 = 0$ (2) $3x^2 + 5x - 7 = 0$ (3) $3x^2 - 5x + 7 = 0$ (4) $3x^2 + x - 7$

15. If 8 and 2 are the roots of $x^2 + ax + c = 0$ and 3, 3 are the roots of $x^2 + dx + b = 0$, then the roots of the equation $x^2 + ax + b = 0$ are
(1) $1, 2$ (2) $-1, 1$ (3) $9, 1$ (4) $-1, 2$

16. If a and b are the real roots of the equation $x^2 - kx + c = 0$, then the distance between the points $(a, 0)$ and $(b, 0)$ is
(1) $\sqrt{k^2 - 4c}$ (2) $\sqrt{4k^2 - c}$ (3) $\sqrt{4c - k^2}$ (4) $\sqrt{k - 8c}$

17. If $\dfrac{kx}{(x + 2)(x - 1)} = \dfrac{2}{x + 2} + \dfrac{1}{x - 1}$, then the value of k is (1) 1 (2) 2 (3) 3 (4) 4

18. If $\frac{1-2x}{3+2x-x^2} = \frac{A}{3-x} + \frac{B}{x+1}$, then the value of $A + B$ is (1) $\frac{-1}{2}$ (2) $\frac{-2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

19. The number of roots of $(x + 3)^4 + (x + 5)^4 = 16$ is (1) 4 (2) 2 (3) 3 (4) 0

20. The value of $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$ is
(1) 1 (2) 2 (3) 3 (4) 4

Exercise - 3.12

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Choose the correct answer or the most suitable answer:

1. $\dfrac{1}{\cos 80^\circ} - \dfrac{\sqrt{3}}{\sin 80^\circ} =$

   (1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) 2 (4) 4

2. If $\cos 28^\circ + \sin 28^\circ = k^3$, then $\cos 17^\circ$ is equal to

   (1) $\dfrac{k^3}{\sqrt{2}}$ (2) $-\dfrac{k^3}{\sqrt{2}}$ (3) $\pm\dfrac{k^3}{\sqrt{2}}$ (4) $-\dfrac{k^3}{\sqrt{3}}$

3. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\dfrac{x}{2} + \cos\dfrac{x}{2}$ is

   (1) $4+\sqrt{2}$ (2) $3+\sqrt{2}$ (3) 9 (4) 4

4. $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\dfrac{5\pi}{8}\right)\left(1+\cos\dfrac{7\pi}{8}\right) =$

   (1) $\dfrac{1}{8}$ (2) $\dfrac{1}{2}$ (3) $\dfrac{1}{\sqrt{3}}$ (4) $\dfrac{1}{\sqrt{2}}$

5. If $\pi < 2\theta < \dfrac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals to

   (1) $-2\cos\theta$ (2) $-2\sin\theta$ (3) $2\cos\theta$ (4) $2\sin\theta$

6. If $\tan 40^\circ = \lambda$, then $\dfrac{\tan 140^\circ - \tan 130^\circ}{1+\tan 140^\circ \tan 130^\circ} =$

   (1) $\dfrac{1-\lambda^2}{\lambda}$ (2) $\dfrac{1+\lambda^2}{\lambda}$ (3) $\dfrac{1+\lambda^2}{2\lambda}$ (4) $\dfrac{1-\lambda^2}{2\lambda}$

7. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots + \cos 179^\circ =$

   (1) 0 (2) 1 (3) -1 (4) 89

8. Let $f_k(x) = \dfrac{1}{k}\left[\sin^k x + \cos^k x\right]$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x) =$

   (1) $\dfrac{1}{4}$ (2) $\dfrac{1}{12}$ (3) $\dfrac{1}{6}$ (4) $\dfrac{1}{3}$

9. Which of the following is not true?

   (1) $\sin\theta = -\dfrac{3}{4}$ (2) $\cos\theta = -1$ (3) $\tan\theta = 25$ (4) $\sec\theta = \dfrac{1}{4}$

10. $\cos 2\theta \cos 2\phi + \sin^2(\theta-\phi) - \sin^2(\theta+\phi)$ is equal to

    (1) $\sin 2(\theta+\phi)$ (2) $\cos 2(\theta+\phi)$ (3) $\sin 2(\theta-\phi)$ (4) $\cos 2(\theta-\phi)$

11. $\dfrac{\sin(A-B)}{\cos A\cos B} + \dfrac{\sin(B-C)}{\cos B\cos C} + \dfrac{\sin(C-A)}{\cos C\cos A}$ is

    (1) $\sin A + \sin B + \sin C$ (2) 1 (3) 0 (4) $\cos A + \cos B + \cos C$

13. If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 + ax + b = 0$, then $\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$ is equal to

    (1) $\dfrac{b}{a}$ (2) $\dfrac{a}{b}$ (3) $-\dfrac{a}{b}$ (4) $-\dfrac{b}{a}$

16. $\dfrac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

(1) $\cos 2x$ (2) $\cos x$ (3) $\cos 3x$ (4) $2\cos x$

18. A wheel is spinning at 2 radians/second. How many seconds will it take to make 10 complete rotations?

    (1) 10π seconds (2) 20π seconds (3) 5π seconds (4) 15π seconds

19. If $\sin\alpha + \cos\alpha = b$, then $\sin 2\alpha$ is equal to

    (1) b^2-1, if $b \leq \sqrt{2}$ (2) b^2-1, if $b > \sqrt{2}$ (3) b^2-1, if $b \geq 1$ (4) b^2-1, if $b \geq \sqrt{2}$

Exercise - 4.5

Choose the correct or the most suitable answer

1. The sum of the digits at the 10^{th} place of all numbers formed with the help of 2, 4, 5, 7 taken all at a time is

   (1) 432 (2) 108 (3) 36 (4) 18

2. In an examination there are three multiple choice questions and each question has 5 choices . Number of ways in which a student can fail to get all answer correct is

   (1) 125 (2) 124 (3) 64 (4) 63

3. The number of ways in which the following prize be given to a class of 30 boys first and second in mathematics, first and second in physics, first in chemistry and first in English is

   (1) $30^4 \times 29^2$ (2) $30^3 \times 29^3$ (3) $30^2 \times 29^4$ (4) 30×29^5.

4. The number of 5 digit numbers all digits of which are odd is

   (1) 25 (2) 5^5 (3) 5^6 (4) 625.

5. In 3 fingers, the number of ways four rings can be worn is $\cdots\cdots\cdots$ ways.

   (1) $4^3 - 1$ (2) 3^4 (3) 68 (4) 64

6. If ${}^{(n+5)}P_{(n+1)} = \left(\frac{11(n-1)}{2}\right){}^{(n+3)}P_n$, then the value of n are

   (1) 7 and 11 (2) 6 and 7 (3) 2 and 11 (4) 2 and 6.

7. The product of r consecutive positive integers is divisible by

   (1) $r!$ (2) $(r-1)!$ (3) $(r+1)!$ (4) r^r.

8. The number of five digit telephone numbers having at least one of their digits repeated is

   (1) 90000 (2) 10000 (3) 30240 (4) 69760.

9. If ${}^{a^2-a}C_2 = {}^{a^2-a}C_4$ then the value of 'a' is

   (1) 2 (2) 3 (3) 4 (4) 5

10. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is

    (1) 45 (2) 40 (3) 39 (4) 38.

11. The number of ways in which a host lady invite 8 people for a party of 8 out of 12 people of whom two do not want to attend the party together is

    (1) $2 \times {}^{11}C_7 + {}^{10}C_8$ (2) ${}^{11}C_7 + {}^{10}C_8$ (3) ${}^{12}C_8 - {}^{10}C_6$ (4) ${}^{10}C_6 + 2!$.

12. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines.

    (1) 6 (2) 9 (3) 12 (4) 18

13. Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is $\cdots\cdots\cdots\cdots$

(1) 11 (2) 12 (3) 10 (4) 6

14. Number of sides of a polygon having 44 diagonals is $\cdots\cdots$

(1) 4 (2) 4! (3) 11 (4) 22

15. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, then the total number of points of intersection are

(1) 45 (2) 40 (3) 10! (4) 2^{10}

16. In a plane there are 10 points are there out of which 4 points are collinear, then the number of triangles formed is

(1) 110 (2) ${}^{10}C_3$ (3) 120 (4) 116

17. In ${}^{2n}C_3 :{}^{n}C_3 = 11 : 1$ then n is

(1) 5 (2) 6 (3) 11 (4) 7

18. ${}^{(n-1)}C_r + {}^{(n-1)}C_{(r-1)}$ is

(1) ${}^{(n+1)}C_r$ (2) ${}^{(n-1)}C_r$ (3) ${}^{n}C_r$ (4) ${}^{n}C_{r-1}$.

19. The number of ways of choosing 5 cards out of a deck of 52 cards which include at least one king is

(1) ${}^{52}C_5$ (2) ${}^{48}C_5$ (3) ${}^{52}C_5 + {}^{48}C_5$ (4) ${}^{52}C_5 - {}^{48}C_5$.

20. The number of rectangles that a chessboard has $\cdots$

(1) 81 (2) 9^9 (3) 1296 (4) 6561

21. The number of 10 digit number that can be written by using the digits 2 and 3 is

(1) ${}^{10}C_2 + {}^{9}C_2$ (2) 2^{10} (3) $2^{10} - 2$ (4) 10!

22. If P_r stands for ${}^{r}P_r$ then the sum of the series $1 + P_1 + 2P_2 + 3P_3 + \cdots + nP_n$ is

(1) P_{n+1} (2) $P_{n+1} - 1$ (3) $P_{n-1} + 1$ (4) ${}^{(n+1)}P_{(n-1)}$

23. The product of first n odd natural numbers equals

(1) ${}^{2n}C_n \times {}^{n}P_n$ (2) $(\frac{1}{2})^n \times {}^{2n}C_n \times {}^{n}P_n$ (3) $(\frac{1}{4})^n \times {}^{2n}C_n \times {}^{2n}P_n$ (4) ${}^{n}C_n \times {}^{n}P_n$

24. If ${}^{n}C_4$, ${}^{n}C_5$, ${}^{n}C_6$ are in AP the value of n can be

(1) 14 (2) 11 (3) 9 (4) 5

25. $1 + 3 + 5 + 7 + \cdots + 17$ is equal to

(1) 101 (2) 81 (3) 71 (4) 61

Exercise - 5.5

Choose the correct or the most suitable answer.

1. The value of $2 + 4 + 6 + \cdots + 2n$ is

(1) $\frac{n(n-1)}{2}$ (2) $\frac{n(n+1)}{2}$ (3) $\frac{2n(2n+1)}{2}$ (4) $n(n+1)$

4. If ${}^{n}\mathrm{C}_{10} > {}^{n}\mathrm{C}_r$ for all possible r, then a value of n is

(1) 10 (2) 21 (3) 19 (4) 20.

12. The n^{th} term of the sequence $1, 2, 4, 7, 11, \cdots$ is

(1) $n^3 + 3n^2 + 2n$ (2) $n^3 - 3n^2 + 3n$ (3) $n(n+1)(n+2)$ / [illegible] (4) $\frac{n^2-n+2}{2}$.

13. The sum up to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \cdots$ is

(1) $\sqrt{2n+1}$ (2) $\frac{\sqrt{2n+1}}{2}$ (3) $\sqrt{2n+1} - 1$ (4) $\frac{\sqrt{2n+1}-1}{2}$.

14. The n^{th} term of the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \cdots$ is

(1) $2^n - n - 1$ (2) $1 - 2^{-n}$ (3) $2^{-n} + n - 1$ (4) 2^{n-1}.

15. The sum up to n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \cdots$ is

(1) $\frac{n(n+1)}{2}$ (2) $2n(n+1)$ (3) $\frac{n(n+1)}{2}$ (4) 1.

Exercise - 6.5

Choose the correct or more suitable answer

1. The equation of the locus of the point whose distance from y-axis is half the distance from origin is

(1) $x^2 + 3y^2 = 0$ (2) $x^2 - 3y^2 = 0$ (3) $3x^2 + y^2 = 0$ (4) $3x^2 - y^2 = 0$

2. Which of the following equation is the locus of $(at^2, 2at)$

(1) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (3) $x^2 + y^2 = a^2$ (4) $y^2 = 4ax$

3. Which of the following point lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$

(1) $(0, 0)$ (2) $(-2, 3)$ (3) $(1, 2)$ (4) $(0, -1)$

4. If the point $(8, -5)$ lies on the locus $\frac{x^2}{16} - \frac{y^2}{25} = k$, then the value of k is

(1) 0 (2) 1 (3) 2 (4) 3

5. Straight line joining the points $(2, 3)$ and $(-1, 4)$ passes through the point (α, β) if

(1) $\alpha + 2\beta = 7$ (2) $3\alpha + \beta = 9$ (3) $\alpha + 3\beta = 11$ (4) $3\alpha + \beta = 11$

6. The slope of the line which makes an angle $45°$ with the line $3x - y = -5$ are

(1) $1, -1$ (2) $\frac{1}{2}, -2$ (3) $1, \frac{1}{2}$ (4) $2, -\frac{1}{2}$

7. Equation of the straight line that forms an isosceles triangle with coordinate axes in the I-quadrant with perimeter $4 + 2\sqrt{2}$ is

(1) $x + y + 2 = 0$ (2) $x + y - 2 = 0$ (3) $x + y - \sqrt{2} = 0$ (4) $x + y + \sqrt{2} = 0$

8. The coordinates of the four vertices of a quadrilateral are (−2,4), (−1,2), (1,2) and (2,4) taken in order.The equation of the line passing through the vertex (−1,2) and dividing the quadrilateral in the equal areas is

(1) $x + 1 = 0$ (2) $x + y = 1$ (3) $x + y + 3 = 0$ (4) $x - y + 3 = 0$

9. The intercepts of the perpendicular bisector of the line segment joining (1, 2) and (3,4) with coordinate axes are

(1) $5, -5$ (2) $5, 5$ (3) $5, 3$ (4) $5, -4$

10. The equation of the line with slope 2 and the length of the perpendicular from the origin equal to $\sqrt{5}$ is

(1) $x + 2y = \sqrt{5}$ (2) $2x + y = \sqrt{5}$ (3) $2x + y = 5$ (4) $x + 2y - 5 = 0$

11. A line perpendicular to the line $5x - y = 0$ forms a triangle with the coordinate axes. If the area of the triangle is 5 sq. units, then its equation is

(1) $x + 5y \pm 5\sqrt{2} = 0$ (2) $x - 5y \pm 5\sqrt{2} = 0$ (3) $5x + y \pm 5\sqrt{2} = 0$ (4) $5x - y \pm 5\sqrt{2} = 0$

12. Equation of the straight line perpendicular to the line $x-y+5=0$, through the point of intersection the y-axis and the given line

(1) $x-y-5=0$ (2) $x+y-5=0$ (3) $x+y+5=0$ (4) $x+y+10=0$

13. If the equation of the base opposite to the vertex (2, 3) of an equilateral triangle is $x+y=2$, then the length of a side is

(1) $\sqrt{\frac{3}{2}}$ (2) 6 (3) $\sqrt{6}$ (4) $3\sqrt{2}$

14. The line $(p+2q)x + (p-3q)y = p-q$ for different values of p and q passes through the point

(1) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (2) $\left(\frac{2}{5}, \frac{2}{5}\right)$ (3) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (4) $\left(\frac{2}{5}, \frac{3}{5}\right)$

15. The point on the line $2x-3y=5$ is equidistance from (1,2) and $(3,4)$ is

(1) $(7,3)$ (2) $(4,1)$ (3) $(1,-1)$ (4) $(-2,3)$

16. The image of the point (2, 3) in the line $y=-x$ is

(1) $(-3,-2)$ (2) $(-3,2)$ (3) $(-2,-3)$ (4) $(3,2)$

17. The length of $\perp$ from the origin to the line $\frac{x}{3}-\frac{y}{4}=1$, is

(1) $\frac{11}{5}$ (2) $\frac{5}{12}$ (3) $\frac{12}{5}$ (4) $-\frac{5}{12}$

18. The y-intercept of the straight line passing through (1,3) and perpendicular to $2x-3y+1=0$ is

(1) $\frac{3}{2}$ (2) $\frac{9}{2}$ (3) $\frac{2}{3}$ (4) $\frac{2}{9}$

19. If the two straight lines $x+(2k-7)y+3=0$ and $3kx+9y-5=0$ are perpendicular then the value of k is

(1) $k=3$ (2) $k=\frac{1}{3}$ (3) $k=\frac{2}{3}$ (4) $k=\frac{3}{2}$

20. If a vertex of a square is at the origin and its one side lies along the line $4x+3y-20=0$, then the area of the square is

(1) 20 sq. units (2) 16 sq. units (3) 25 sq. units (4) 4 sq.units

21. If the lines represented by the equation $6x^2+41xy-7y^2=0$ make angles α and β with x- axis, then $\tan\alpha\tan\beta =$

(1) $-\frac{6}{7}$ (2) $\frac{6}{7}$ (3) $-\frac{7}{6}$ (4) $\frac{7}{6}$

22. The area of the triangle formed by the lines $x^2-4y^2=0$ and $x=a$ is

(1) $2a^2$ (2) $\frac{\sqrt{3}}{2}a^2$ (3) $\frac{1}{2}a^2$ (4) $\frac{2}{\sqrt{3}}a^2$

23. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$,, then c equals to

(1) -3 (2) -1 (3) 3 (4) 1

24. θ is acute angle between the lines $x^2-xy-6y^2=0$, then $\frac{2\cos\theta+3\sin\theta}{4\sin\theta+5\cos\theta}$ is

(1) 1 (2) $-\frac{1}{9}$ (3) $\frac{5}{9}$ (4) $\frac{1}{9}$

25. The equation of one the line represented by the equation $x^2+2xy\cot\theta-y^2=0$ is

(1) $x-y\cot\theta=0$ (2) $x+y\tan\theta=0$ (3) $x\cos\theta + y(\sin\theta+1)=0$
(4) $x\sin\theta + y(\cos\theta+1)=0$