ANANTH.S

(PG ASST IN MATHS)

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FIVE MARKS FOR

I-REVISION TEST

(CHAP-I-IV)

XII-STD

XII-MATHEMATICS

1. Applications of Matrices And Determinants

1. Example 1.1 If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ verify that A (adj A) = (adj A)A = $|A|I_3$.

Solu:

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$$

$$= 8(21-16) + 6(-18+8) + 2(24-14)$$

$$= 40 - 60 + 20$$

$$= 0$$

$$\therefore |A|\, I_3 = 0 \quad ...(1)$$

$$\text{adj } A = \begin{bmatrix} (21-16) & -(-18+8) & (24-14) \\ -(-18+8) & (24-4) & -(-32+12) \\ (24-14) & -(-32+12) & (56-36) \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$$

$$A(\text{adj} A) = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 40-60+20 & 80-120+40 & 80-120+40 \\ -30+70-40 & -60+140-80 & -60+140-80 \\ 10-40+30 & 20-80+60 & 20-80+60 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0 \quad ...(2)$$

$$(\text{adj} A) = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 40-60+20 & -30+70-40 & 10-40+30 \\ 80-120+40 & -60+140-80 & 20-80+60 \\ 80-120+40 & -60+140-80 & 20-80+60 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0 \quad ...(3)$$

Hence $A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\, I_3$ is verified.

2. **Example 1.12** If A= $\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}.

Solu:

If A is orthogonal then $AA^T = A^T A = I_3$

Let $AA^T = I_3$

$$\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix} \frac{1}{7}\begin{bmatrix} 6 & b & 2 \\ -3 & -2 & c \\ a & 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{49}\begin{bmatrix} 36+9+a^2 & 6b+6+6a & 12-3c+3a \\ 6b+6+6a & b^2+4+36 & 2b-2c+18 \\ 12-3c+3a & 2b-2c+18 & c^2+4+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 45+a^2 & 6b+6+6a & 12-3c+3a \\ 6b+6+6a & b^2+40 & 2b-2c+18 \\ 12-3c+3a & 2b-2c+18 & c^2+13 \end{bmatrix} = 49\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$45+a^2=49$	$b^2+40=49$	$c^2+13=49$
$a^2=49-45$	$b^2=49-40$	$c^2=49-13$
$a^2=4$	$b^2=9$	$c^2=36$

$6b+6+6a=0$	$12-3c+3a=0$	$2b-2c+18=0$
$6a+6b=-6$	$3a-3c=-12$	$2b-2c=-18$
$\div 6 \quad a+b=-1$	$\div 3 \quad a-c=-4$	$\div 2 \quad b-c=-9$

$a^2 = 4$	$b^2 = 9$	$C^2 = 36$
$a = \pm 2$	$b = \pm 3$	$C = \pm 6$
$\boldsymbol{a = 2}$	$\boldsymbol{b = -3}$	$\boldsymbol{C = 6}$

$$\therefore \quad A = \frac{1}{7}\begin{bmatrix} 6 & -3 & 2 \\ -3 & -2 & 6 \\ 2 & 6 & 3 \end{bmatrix}$$

$$AA^T = I$$

$$\therefore \quad A^{-1} = A^T$$

$$= \frac{1}{7}\begin{bmatrix} 6 & -3 & 2 \\ -3 & -2 & 6 \\ 2 & 6 & 3 \end{bmatrix}$$

3.**Er:1.1(3)** If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

Solu:

$$F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$|F(\alpha)| = \cos\alpha(\cos\alpha - 0) + \sin\alpha(0 + \sin\alpha)$$

$$= \cos^2\alpha + \sin^2\alpha$$

$$= 1 \neq 0$$

$$\therefore [F(\alpha)]^{-1} \text{ exists.}$$

$$\text{adj } F(\alpha) = \begin{bmatrix} (\cos\alpha - 0) & -(0-0) & (0+\sin\alpha) \\ -(0-0) & (\cos^2\alpha + \sin^2\alpha) & (0+0) \\ (0-\sin\alpha) & -(0-0) & (\cos\alpha - 0) \end{bmatrix}^T$$

$$\text{adj } F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}^T$$

$$\text{adj } F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$[F(\alpha)]^{-1} = \frac{1}{|F(\alpha)|}\text{adj} F(\alpha)$$

$$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \ldots(1)$$

$$F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \ldots(2)$$

From (1) and (2) $[F(\alpha)]^{-1} = F(-\alpha)$ is verified.

4. **Example 1.23** Solve the following system of equations, using matrix inversion method: $2x_1+3x_2+3x_3 = 5, x_1-2x_2+x_3 = -4, 3x_1-x_2-2x_3 = 3$.

Solu:

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \quad \ldots(1)$$

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

$$|A| = 2(4+1) - 3(-2-3) + 3(-1+6)$$

$$= 10 + 15 + 15$$

$$= 40 \neq 0$$

A^{-1} exists

$$\text{adj } A = \begin{bmatrix} (4+1) & -(-2-3) & (-1+6) \\ -(-6+3) & (-4-9) & -(-2-9) \\ (3+6) & -(2-3) & (-4-3) \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$(1) \Rightarrow \quad X = A^{-1} B$

$$= \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

solution is $x_1 = 1, x_2 = 2, x_3 = -1$

5. Example 1.24 If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x-y+z = 4, x-2y-2z = 9, 2x+y+3z = 1.

Solu:

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$$

$$BA = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4+7+5 & 4-1-3 & 4-3-1 \\ -4+14-10 & 4-2+6 & 4-6+2 \\ -8-7+15 & 8+1-9 & 8+3-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3.$$

we get $AB = BA = 8I_3$.

$$\left(\frac{1}{8}A\right)B = B\left(\frac{1}{8}A\right) = I_3. \text{Hence, } B^{-1} = \frac{1}{8}A.$$

Writing the given system of equations in matrix form, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

That is, $B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \left(\frac{1}{8}A\right)\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}-16+36+4\\-28+9+3\\20-27-1\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}24\\-16\\-8\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\-2\\-1\end{bmatrix}$$

the solution is $(x=3, y=-2, z=-1)$.

6. Er:1.3(1-iv) Solve the following system of linear equations by matrix inversion method: x+y+z-2 = 0, 6x-4y+5z-31 = 0, 5x+2y+2z = 13.

Solu:

$$\begin{bmatrix}1 & 1 & 1\\6 & -4 & 5\\5 & 2 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\31\\13\end{bmatrix}$$

$$AX=B$$

$$X=A^{-1}B \quad ...(1)$$

$$A=\begin{bmatrix}1 & 1 & 1\\6 & -4 & 5\\5 & 2 & 2\end{bmatrix}$$

$$|A|=1(-8-10)-1(12-25)+1(12+20)$$

$$=-18+13+32$$

$$=27\neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$\text{adj } A=\begin{bmatrix}(-8-10) & -(12-25) & (12+20)\\-(2-2) & (2-5) & -(2-5)\\(5+4) & -(5-6) & (-4-6)\end{bmatrix}^T$$

$$=\begin{bmatrix}-18 & 13 & 32\\0 & -3 & 3\\9 & 1 & -10\end{bmatrix}^T$$

$$=\begin{bmatrix}-18 & 0 & 9\\13 & -3 & 1\\32 & 3 & -10\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{adj } A$$

$$=\frac{1}{27}\begin{bmatrix}-18 & 0 & 9\\13 & -3 & 1\\32 & 3 & -10\end{bmatrix}$$

$$(1)\Rightarrow \quad X=\frac{1}{27}\begin{bmatrix}-18 & 0 & 9\\13 & -3 & 1\\32 & 3 & -10\end{bmatrix}\begin{bmatrix}2\\31\\13\end{bmatrix}$$

$$=\frac{1}{27}\begin{bmatrix}-36+0+117\\26-93+13\\64+93-130\end{bmatrix}$$

$$=\frac{1}{27}\begin{bmatrix}81\\-54\\27\end{bmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-2\\1\end{pmatrix}$$

solution is $x=3$; $y=-2$ and $z=1$

7. Er:1.3(2) If A = $\begin{bmatrix}-5 & 1 & 3\\7 & 1 & -5\\1 & -1 & 1\end{bmatrix}$ and

B = $\begin{bmatrix}1 & 1 & 2\\3 & 2 & 1\\2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations x+y+2z = 1,3x+2y+z = 7, 2x+y+3z = 2.

Solu:

$$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$AB = BA = 4I_3 \quad \Rightarrow \quad B^{-1} = \tfrac{1}{4}A \rightarrow (1)$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow BX = C \Rightarrow X = B^{-1}C \rightarrow (2)$$

From (1) $\Rightarrow B^{-1} = \frac{1}{4}A = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

From (2) $\Rightarrow X = B^{-1}C \Rightarrow$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Solution : $(x, y, z) = (2,1,-1)$

8. Er:1.3(5) The prices of three commodities A,B and C are` x,y and z per units respectively. A person P purchases 4 units ofB and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B . Person R purchases one unit of A and sells 3 unit of B and one unit of C . In the process, P, Qand R earn 15,000, `1,000 and `4,000 respectively. Find the prices per unit of A,B and C . (Use matrix inversion method to solve the problem.)

Solu:

From the given data,

$$2x - 4y + 5z = 15000 \rightarrow (1)$$

$$3x + y - 2z = 1000 \rightarrow (2)$$

$$-x + 3y + z = 4000 \rightarrow (3)$$

$$\begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B$$

$$|A| = 2(1+6) - (-4)(3-2) + 5(9+1)$$

$$= 14 + 4 + 50 = 68$$

A^{-1} exist

$$\text{adj}A = \begin{bmatrix} 1+6 & 15+4 & 8-5 \\ 2-3 & 2+5 & 15+4 \\ 9+1 & 4-6 & 2+12 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}adjA$$

$$= \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1000}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

$$= \frac{1000}{68}\begin{bmatrix} 105+19+12 \\ -15+7+76 \\ 150-2+56 \end{bmatrix}$$

$$= \frac{1000}{68}\begin{bmatrix} 136 \\ 68 \\ 204 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

The price of one unit of A, B and C are Rs.2000, 1000 and 3000 respectively.

9. Example 1.26 In a T20 match, a team needed just 6 runs to win with

Solu:

$$y = ax^2 + bx + c$$

$(10,8) \Rightarrow 100a + 10b + c = 8 \rightarrow (1)$

$(20,16) \Rightarrow 400a + 20b + c = 16 \rightarrow (2)$

$(40,22) \Rightarrow 1600a + 40b + c = 22 \rightarrow (3)$

$$\Delta = \begin{vmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 1600 & 40 & 1 \end{vmatrix}$$

$$= 100 \times 10 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 16 & 4 & 1 \end{vmatrix}$$

$$= 1000[1(2-4) - 1(4-16) + 1(16-32)]$$

$$= 1000[-2 + 12 - 16]$$

$$= -6000$$

$$\Delta_a = \begin{vmatrix} 8 & 10 & 1 \\ 16 & 20 & 1 \\ 22 & 40 & 1 \end{vmatrix}$$

$$= 2 \times 10 \begin{vmatrix} 4 & 1 & 1 \\ 8 & 2 & 1 \\ 11 & 4 & 1 \end{vmatrix}$$

$$= 20[4(2-4) - 1(8-11) + 1(32-22)]$$

$$= 20[-8 + 3 + 10]$$

$$= 100$$

$$\Delta_b = \begin{vmatrix} 100 & 8 & 1 \\ 400 & 16 & 1 \\ 1600 & 22 & 1 \end{vmatrix}$$

$$= 100 \times 2 \begin{vmatrix} 1 & 4 & 1 \\ 4 & 8 & 1 \\ 16 & 11 & 1 \end{vmatrix}$$

$$= 200[1(8-11) - 4(4-16) + 1(44-128)]$$

$$= 200[-3 + 48 - 84]$$

$$= -7800$$

$$\Delta_c = \begin{vmatrix} 100 & 10 & 8 \\ 400 & 20 & 16 \\ 1600 & 40 & 22 \end{vmatrix}$$

$$= 100 \times 10 \times 2 \begin{vmatrix} 1 & 1 & 4 \\ 4 & 2 & 8 \\ 16 & 4 & 11 \end{vmatrix}$$

$$= 2000[1(22-32) - 1(44-128) + 4(16-32)]$$

$$= 2000[-10 + 84 - 64]$$

$$= 20000$$

$$a = \frac{\Delta_a}{\Delta} = \frac{100}{-6000} = -\frac{1}{60};$$

$$b = \frac{\Delta_b}{\Delta} = \frac{-7800}{-6000} = \frac{13}{10};$$

$$c = \frac{c}{\Delta} = \frac{20000}{-6000} = -\frac{10}{3}$$

∴ The equation of the path is $y = -\frac{1}{60}x^2 + \frac{13}{10}x - \frac{10}{3}$

Sub. $x = 70$, we get

$$y = -\frac{1}{60}(70)^2 + \frac{13}{10}(70) - \frac{10}{3}:$$

$$= -\frac{4900}{60} + \frac{910}{10} - \frac{10}{3} = 6$$

Hence the ball went for six and the Chennai Super Kings won the match.

10.Er:1.4(1-iii) Solve the following system of linearequations by matrix inversion method: 3x+3y-z = 11, 2x-y+2z = 9, 4x+3y+2z = 25.

Solu:

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$$

$$= 3(-2-6) - 3(4-8) - 1(6+4)$$

$$= -24 + 12 - 10$$

$$= -22$$

$$\Delta_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$$

$$= 11(-2-6) - 3(18-50) - 1(27+25)$$

$$= -88 + 96 - 52$$

$$= -44$$

$$\Delta_y = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$$

$$= 3(18-50) - 11(4-8) - 1(50-36)$$

$$= -96 + 44 - 14$$

$$= -66$$

$$\Delta_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$$

$$= 3(-25-27) - 3(50-36) + 11(6+4)$$

$$= -156 - 42 + 110$$

$$= -88$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-44}{-22} = 2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-66}{-22} = 3$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-88}{-22} = 4$$

Solution : $(x, y, z) = (2,3,4)$

11. Er:1.4(1-iv) Solve the following systems of linear equations by Cramer's rule:

$$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0,$$
$$\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0.$$

Solu:

Let $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$

$3a - 4b - 2c = 1 \rightarrow (1)$

$a + 2b + c = 2 \rightarrow (2)$

$2a - 5b - 4c = -1 \rightarrow (3)$

$$\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix}$$

$= 3(-8+5) - (-4)(-4-2) - 2(-5-4)$

$= -9 - 24 + 18$

$= -15$

$$\Delta_a = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix}$$

$= 1(-8+5) - (-4)(-8+1) - 2(-10+2)$

$= -3 - 28 + 16$

$= -15$

$$\Delta_b = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix}$$

$= 3(-8+1) - 1(-4-2) - 2(-1-4)$

$= -21 + 6 + 10$

$= -5$

$$\Delta_c = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix}$$

$= 3(-2+10) - (-4)(-1-4) + 1(-5-4)$

$= 24 - 20 - 9$

$= -5$

$a = \frac{\Delta_a}{\Delta} = \frac{-15}{-15} = 1 \Rightarrow x = \frac{1}{a} = 1$

$b = \frac{\Delta_b}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Rightarrow y = \frac{1}{b} = 3$

$c = \frac{\Delta_c}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Rightarrow z = \frac{1}{c} = 3$

Solution : $(x, y, z) = (1,3,3)$

12. **Er:1.4(5) Er:1.4(5)**A family of 3 people went out for dinner in a restaurant. The cost of two dosai,three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is 200. The cost of five dosai, four idlies and two vadais is Rs. 250. The family has Rs .350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

Solu:

Let Rs.x, y and z be the cost of I dosai, 1 idly and I vadai respectively.

From the given data,

$2x + 3y + 2z = 150 \rightarrow (1)$

$2x + 2y + 4z = 200,$

$\div 2$

$x + y + 2z = 100 \rightarrow (2)$

$5x + 4y + 2z = 250 \rightarrow (3)$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 1 & 2 \\ 5 & 4 & 2 \end{vmatrix}$$

$= 2(2-8) - 3(2-10) + 2(4-5)$

$= -12 + 24 - 2$

$= 10$

$$\Delta_x = \begin{vmatrix} 150 & 3 & 2 \\ 100 & 1 & 2 \\ 250 & 4 & 2 \end{vmatrix}$$

$= 150(2-8) - 3(200-500) + 2(400-250)$

$= -900 + 900 + 300$

$= 300$

$$\Delta_y = \begin{vmatrix} 2 & 150 & 2 \\ 1 & 100 & 2 \\ 5 & 250 & 2 \end{vmatrix}$$

$$= 2(200-500) - 150(2-10) + 2(250-500)$$
$$= -600 + 1200 - 500$$
$$= 100$$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 150 \\ 1 & 1 & 100 \\ 5 & 4 & 250 \end{vmatrix}$$

$$= 2(250-400) - 3(250-500) + 150(4-5)$$
$$-300 + 750 - 150$$
$$= 300$$

$$x = \frac{\Delta_x}{\Delta} = \frac{300}{10} = 30$$

$$y = \frac{\Delta_y}{\Delta} = \frac{100}{10} = 10$$

$$z = \frac{\Delta_z}{\Delta} = \frac{300}{10} = 30$$

The cost of 3 dosai , 6 idlies and 6 vadais

$$= 3x + 6y + 6z$$
$$= 3(30) + 6(10) + 6(30)$$
$$= 90 + 60 + 180$$
$$= 330 < 350$$

Hence they can manage to pay the bill.

13. Example 1.28 The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a,b and c are constants. It has been found that the speed at times t= 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t =15 seconds. (Use Gaussian elimination method.)

Solu:

$$v(t) = at^2 + bt + c$$
$$v(3) = 64 \Rightarrow 9a + 3b + c = 64 \rightarrow (1)$$
$$v(6) = 133 \Rightarrow 36a + 6b + c = 133 \rightarrow (2)$$
$$v(9) = 208 \Rightarrow 81a + 9b + c = 208 \rightarrow (3)$$

$$[A|B] = \left(\begin{array}{ccc|c} 9 & 3 & & 64 \\ 36 & 6 & 1 & 133 \\ 81 & 9 & & 208 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 9 & 3 & & 64 \\ 0 & -6 & -3 & -123 \\ 0 & -18 & -8 & -368 \end{array}\right) \begin{array}{l} R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - 9R_1 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 9 & 3 & & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 18 & 8 & 368 \end{array}\right) \begin{array}{l} R_2 \rightarrow \left(-\frac{1}{3}\right) R_2 \\ R_3 \rightarrow (-) R_3 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 9 & 3 & & 64 \\ 0 & 2 & & 41 \\ 0 & 0 & -1 & -1 \end{array}\right) R_3 \rightarrow R_3 - 9R_2$$

$$9a + 3b + c = 64 \rightarrow (1)$$
$$2b + c = 41 \rightarrow (2)$$
$$-c = -1 \quad c = 1$$
$$(2) \Rightarrow 2b + 1 = 41$$
$$2b = 40$$
$$b = 20$$

When $b = 20$, c=1

$$(1) \Rightarrow 9a + 60 + 1 = 64$$
$$9a = 3$$
$$a = \frac{1}{3}$$
$$\therefore v(t) = \frac{1}{3}t^2 + 20t + 1$$
$$v(15) = \frac{1}{3}(15)^2 + 20(15) + 1$$
$$= 75 + 300 + 1 = 376 \ miles/sec$$

14. Er:1.5(1) Solve the following systems of linear equations by Gaussian elimination method: 2x+4y+6z = 22, 3x+8y+5z = 27, -x+y+2z = 2.

Solu:

$$2x+4y+6z=22,$$
$$\div 2 \Rightarrow x+2y+3z=11$$
$$-x+y+2z=2$$

$$[A|B]=\left(\begin{array}{ccc|c}1&2&3&11\\3&8&5&27\\-1&1&2&2\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c}1&2&3&11\\0&2&-4&-6\\0&3&5&13\end{array}\right)\begin{array}{l}R_2\to R_2-3R_1\\R_3\to R_3+R_1\end{array}$$

$$\sim\left(\begin{array}{ccc|c}1&2&3&11\\0&2&-4&-6\\0&0&22&44\end{array}\right)R_3\to 2R_3-3R_2$$

$$x+2y+3z=11\to(1)$$
$$2y-4z=-6\to(2)$$
$$22z=44$$
$$z=2$$

Put $z=2$ in (2)

$$2y-4(2)=-6$$
$$2y=-6+8$$
$$2y=2$$
$$y=1$$

Put $y=1$ $z=2$ in (1)

$$x+2(1)+3(2)=11$$
$$x+8=11$$
$$x=3$$

Solution: $(x,y,z)=(3,1,2)$

15. Er:1.5(2) If ax^2+bx+c is divided by $x+3$, $x-5$, and $x-1$, the remainders are 21, 61 and 9 respectively. Find a, b andc. (Use Gaussian elimination method.) **PTA-3**

Solu:

Let $p(x)=ax^2+bx+c$

$p(-3)=21$

$$a(-3)^2+b(-3)+c=21$$
$$9a-3b+c=21\to(1)$$

$p(5)=61$

$$a(5)^2+b(5)+c=61$$
$$25a+5b+c=61\to(2)$$

$p(1)=9$

$$a(1)^2+b(1)+c=9$$
$$a+b+c=9\to(3)$$

$$[A|B]=\left(\begin{array}{ccc|c}9&-3&1&21\\25&5&1&61\\1&1&1&9\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c}1&1&1&9\\25&5&1&61\\9&-3&1&21\end{array}\right)R_1\leftrightarrow R_3$$

$$\sim\left(\begin{array}{ccc|c}1&1&1&9\\0&-20&-24&-164\\0&-12&-8&-60\end{array}\right)\begin{array}{l}R_2\to R_2-25R_1\\R_3\to R_3-9R_1\end{array}$$

$$\sim\left(\begin{array}{ccc|c}1&1&1&9\\0&5&6&41\\0&3&2&-15\end{array}\right)\begin{array}{l}R_2\to\left(-\frac{1}{4}\right)R_2\\R_3\to\left(-\frac{1}{4}\right)R_3\end{array}$$

$$\sim\left(\begin{array}{ccc|c}1&1&1&9\\0&5&6&41\\0&0&-8&-48\end{array}\right)R_3\to 5R_3-3R_2$$

$$a+b+c=9\to(1)$$
$$5b+6c=41\to(2)$$
$$-8c=-48$$
$$c=6$$

Put $c=6$ in (2)

$$5b+6(6)=41$$
$$5b=41-36$$
$$5b=5$$
$$b=1$$

Put $b=1$, $c=6$ in (1)

$$a+1+6=9$$
$$a=2$$

Solution: $(a,b,c)=(2,1,6)$

16. Er:1.5(3) An amount of ` 65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is ` 4,800. The income from the third bond is 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

Solu:

Let the price of three bond be x, y and z respectively.

From the given data,

$$x+y+z=65000\to(1)$$

Total annual income = Rs.5,000 $\Rightarrow$

$$6\%\ of\ x+8\%\ of\ y+10\%\ of\ z=5000$$
$$\Rightarrow\frac{6}{100}x+\frac{8}{100}y+\frac{10}{100}z=5000$$

$\Rightarrow 6x + 8y + 10z = 500000$

$\Rightarrow \div 2, 3x + 4y + 5z = 250000 \rightarrow (2)$

Income from third bond = income from second bond + Rs.800

$\Rightarrow 10\% \, of \, z = 8\% \, of \, y + 800$

$\Rightarrow \frac{10}{100} z = \frac{8}{100} y + 800$

$\Rightarrow \frac{10}{100} z = \frac{8y+80000}{100}$

$\Rightarrow 10z = 8y + 80000$

$\Rightarrow -8y + 10z = 80000$

$\Rightarrow \div 2, -4y + 5z = 40000 \rightarrow (3)$

$$[A|B] = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 3 & 4 & 5 & 250000 \\ 0 & -4 & 5 & 40000 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & -4 & 5 & 40000 \end{array}\right) R_2 \rightarrow R_2 - 3R_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 0 & 13 & 260000 \end{array}\right) R_3 \rightarrow R_3 + 4R_2$$

$\Rightarrow x + y + z = 65000 \rightarrow (1)$

$y + 2z = 55000 \rightarrow (2);$

$13z = 260000$

$z = 20000$

Put $z = 20000 \, in \, (2)$

$y + 2(20000) = 55000$

$y = 55000 - 40000$

$y = 15000$

Put $y = 15000, z = 20000 \, in \, (1)$

$x + 5000 + 20000 = 65000$

$x + 35000 = 65000$

$x = 30000$

Solution: The price of 6%, 8% and 10% are Rs.30000, Rs.15000 and Rs.20000 respectively.

17. Er:1.5(4) A boy is walking along the path $y = ax^2+bx+c$ through the points (-6,8), (-2,-12) and (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian elimination method.)

Solu:

$y = ax^2 + bx + c$

$(-6,8)$ $\quad 8 = a(-6)^2 + b(-6) + c$

$36a - 6b + c = 8 \rightarrow (1)$

$(-2,-12)$ $\quad -12 = a(-2)^2 + b(-2) + c$

$4a - 2b + c = -12 \rightarrow (2)$

$(3,8)$ $\quad 8 = a(3)^2 + b(3) + c$

$9a + 3b + c = 8 \rightarrow (3)$

$$[A|B] = \left(\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -12 & 8 & -116 \\ 0 & 18 & 3 & 24 \end{array}\right) \begin{matrix} R_2 \rightarrow 9R_2 - R_1 \\ R_3 \rightarrow 4R_3 - R_1 \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -6 & 4 & -58 \\ 0 & 6 & 1 & 8 \end{array}\right) \begin{matrix} R_2 \rightarrow \left(\frac{1}{2}\right) R_2 \\ R_3 \rightarrow \left(\frac{1}{3}\right) R_3 \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -6 & 4 & -58 \\ 0 & 0 & 5 & -50 \end{array}\right) R_3 \rightarrow R_3 + R_2$$

$36a - 6b + c = 8 \rightarrow (1)$

$-6b + 4c = -58 \rightarrow (2)$

$5c = -50$

$c = -10$

Put $c = -10 \, in \, (2)$

$-6b + 4(-10) = -58$

$-6b - 40 = -58$

$-6b = -58 + 40$

$-6b = -18$

$b = 3$

Put $b = 3, c = -10 \, in \, (1)$

$36a - 6(3) - 10 = 8$

$36a - 18 - 10 = 8$

$36a = 8 + 28$

$36a = 36$

$a = 1$

Solution: $(a, b, c) = (1, 3, -10) \Rightarrow \boxed{y = x^2 + 3x - 10}$

Put $x = 7 \Rightarrow y = (7)^2 + 3(7) - 10$

$= 49 + 21 - 10$

$= 60$

$y = 60$

The point $P(7,60)$ satisfies the equation $y = x^2 + 3x - 10$, hence the boy will meet friend at $P(7,60)$.

2. Complex Numbers

1. Example 2.8 Show that

(i) $(2+i\sqrt{3})^{10}+(2-i\sqrt{3})^{10}$ is real and

(ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

Solu:

$$\frac{19-7i}{9+i}=\frac{19-7i}{9+i}\times\frac{9-i}{9-i}$$
$$=\frac{164-82i}{82}$$
$$=2-i$$
$$\frac{20-5i}{7-6i}=\frac{20-5i}{7-6i}\times\frac{7+6i}{7+6i}$$
$$=\frac{170+85i}{85}$$
$$=2+i$$
$$z=\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$$
$$z=(2-i)^{12}+(2+i)^{12}$$
$$\bar{z}=(2+i)^{12}+(2-i)^{12}$$
$$\bar{z}=z\Rightarrow z\text{ is real.}$$

(ii)

$$\frac{19+9i}{5-3i}=\frac{19+9i}{5-3i}\times\frac{5+3i}{5+3i}$$
$$=\frac{68+102i}{34}$$
$$=2+3i$$
$$\frac{8+i}{1+2i}=\frac{8+i}{1+2i}\times\frac{1-2i}{1-2i}$$
$$=\frac{10-15i}{5}$$
$$=2-3i$$
$$z=\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$$
$$z=(2+3i)^{15}-(2-3i)^{15}$$
$$\bar{z}=-(2+3i)^{15}+(2-3i)^{15}$$
$$\bar{z}=-z\ \Rightarrow z\text{ purely imaginary.}$$

2. Example 2.14 Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and, $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

Solu:

Let $z_1=1, z_2=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ and $z_3=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

The length of the sides of the triangle are

$$|z_1-z_2|=\left|1-\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right|$$
$$=\left|\frac{3}{2}-i\frac{\sqrt{3}}{2}\right|$$
$$=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$
$$=\sqrt{\frac{9}{4}+\frac{3}{4}}=\sqrt{\frac{12}{4}}=\sqrt{3}$$
$$|z_2-z_3|=\left|\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)-\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\right|$$
$$=\left|-\frac{1}{2}+i\frac{\sqrt{3}}{2}+\frac{1}{2}+i\frac{\sqrt{3}}{2}\right|$$
$$=\sqrt{\left(\frac{2\sqrt{3}}{2}\right)^2}$$
$$=\sqrt{3}$$
$$|z_3-z_1|=\left|\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)-1\right|$$
$$=\left|-\frac{3}{2}-i\frac{\sqrt{3}}{2}\right|$$
$$=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$
$$=\sqrt{\frac{9}{4}+\frac{3}{4}}$$
$$=\sqrt{3}$$

Since all the three sides are equal,

the given points form an equilateral triangle.

3. Example 2.15 Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

Solu:

$$|z_1|=r$$
$$|z_1|^2=r^2;$$
$$\therefore z_1\ \overline{z_1}=r^2$$
$$z_1=\frac{r^2}{\overline{z_1}}$$

Similarly, $z_2 = \frac{r^2}{\overline{z_2}}$; $z_3 = \frac{r^2}{\overline{z_3}}$

$$z_1 + z_2 + z_3 = \frac{r^2}{\overline{z_1}} + \frac{r^2}{\overline{z_2}} + \frac{r^2}{\overline{z_3}} = r^2\left(\frac{\overline{z_2 z_3} + \overline{z_1 z_3} + \overline{z_1 z_2}}{\overline{z_1 z_2 z_3}}\right)$$

$$\Rightarrow |z_1 + z_2 + z_3| = |r^2|\left|\frac{\overline{z_2 z_3} + \overline{z_1 z_3} + \overline{z_1 z_2}}{\overline{z_1 z_2 z_3}}\right|$$

$$\Rightarrow |z_1 + z_2 + z_3| = r^2 \frac{|\overline{z_2 z_3 + z_1 z_3 + z_1 z_2}|}{|\overline{z_1 z_2 z_3}|}$$

$$\Rightarrow |z_1 + z_2 + z_3| = r^2 \frac{|z_1 z_2 + z_2 z_3 + z_3 z_1|}{|z_1||z_2||z_3|}$$

$$\Rightarrow |z_1 + z_2 + z_3| = r^2 \frac{|z_1 z_2 + z_2 z_3 + z_3 z_1|}{r.r.r}$$

$$\Rightarrow |z_1 + z_2 + z_3| = \frac{|z_1 z_2 + z_2 z_3 + z_3 z_1|}{r}$$

$$\Rightarrow \left|\frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3}\right| = r$$

4. Er:2.5(7) If z_1, z_2 and z_3 are three complex numbers such that $|z_1|$1, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$.

Solu:

$$|z_1| = 1 \Rightarrow$$

$$|z_1|^2 = 1;$$

$$z_1 \overline{z_1} = 1;$$

$$z_1 = \frac{1}{\overline{z_1}}$$

$$|z_2| = 2 \Rightarrow z_2 = \frac{4}{\overline{z_2}}$$

$$|z_3| = 3 \Rightarrow z_3 = \frac{9}{\overline{z_3}}$$

$$|z_1 + z_2 + z_3| = \left|\frac{1}{\overline{z_1}} + \frac{4}{\overline{z_2}} + \frac{9}{\overline{z_3}}\right|$$

$$= \left|\frac{\overline{z_2 z_3} + 4\overline{z_1 z_3} + 9\overline{z_1 z_2}}{\overline{z_1 z_2 z_3}}\right|$$

$$= \left|\frac{\overline{z_2 z_3 + 4z_1 z_2 + 9z_1 z_2}}{\overline{z_1 z_2 z_3}}\right|$$

$$= \frac{|9z_1 z_2 + 4z_1 z_2 + z_2 z_3|}{|z_1||z_2||z_3|}$$

$$= \frac{|9z_1 z_2 + 4z_1 z_2 + z_2 z_3|}{1 \times 2 \times 3}$$

$$\therefore |9z_1 z_2 + 4z_1 z_2 + z_2 z_3| = 6$$

5. Er:2.6(2) If z = x+ iy is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$.

Solu:

Given: $z = x + iy$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$$

$$= \frac{(2x+1)+i2y}{(1-y)+ix}$$

$$\boxed{\text{Im}(Z) = \frac{bc-ad}{c^2+d^2}}$$

Here $a = 2x+1$, $b = 2y$, $c = (1-y)$, $d = x$

$$Im\left(\frac{2z+1}{iz+1}\right) = 0$$

$$\Rightarrow \frac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2} = 0$$

$$\Rightarrow y - y^2 - x^2 - x = 0$$

$$\Rightarrow x^2 + y^2 + x - y = 0$$

6. Example 2.18 Given the complex number z =3+i2, represent the complex numbers z, iz and z +iz in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.

Solu:

Given : $z = 3 + 2i = A(3,2)$

$iz = i(3 + 2i) = -2 + 3i = B(-2,3)$

$z + iz = 3 + 2i - 2 + 3i = \quad + 5i = C(1,5)$

The length of the sides of the triangle are

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(3 + 2)^2 + (2 - 3)^2}$$
$$= \sqrt{25 + 1}$$
$$= \sqrt{26}$$

$$BC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-2 - 1)^2 + (3 - 5)^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{13}$$

$$CA = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(1 - 3)^2 + (5 - 2)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

$AB^2 = 26$, $BC^2 = 13$, $CA^2 = 13$

Since (i) $BC = CA = \sqrt{13}$

(ii) $BC^2 + CA^2 = 13 + 13 = 26 = AB^2$,

So the given vertices form an isosceles right triangle.

3. Theory of Equations

1. Er:3.1(4) Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

Solu:

Given the product of two roots is 1

So let, $\alpha\beta = 1$

$$\Sigma_3 = \alpha\beta\gamma = \frac{-(-6)}{3} = 2$$
$$\alpha\beta\gamma = 2$$
$$\therefore 1 \times \gamma = 2$$
$$\gamma = 2$$

2	3	−16	23	−6
	0	6	−20	6
	3	−10	3	0

$3x^2 - 10x + 3 = 0$ is another factor.

$(x - \frac{1}{3})(x - 3) = 0$

$x = \frac{1}{3}$ or $x = 3$

The roots are $\frac{1}{3}, 3, 2$

2. Er:3.1(6) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.

Solu:

Given $x^3 - 9x^2 + 14x + 24 = 0$

−1	1	−9	14	24
	0	−1	−10	−24
	1	10	24	0

$x^2 + 10x - 24 = 0$ is another factor.

$(x - 4)(x - 6) = 0$

$x = 4$ or $x = 6$

Two of its roots are in the ratio 3 : 2 is 6, 4

Hence the roots are 6, 4 and −1

3. Er:3.1(10) If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

Solu:

Let α be the common root of the given equations. Then,

$$\alpha^2 + p\alpha + q = 0 \quad .....(1)$$

$$\alpha^2 + p'\alpha + q' = 0 \quad .....(2)$$

	α^2	α	1
p	q	1	p
p'	q'	1	p'

$$\frac{\alpha^2}{pq'-p'q} = \frac{\alpha}{q-q'} = \frac{1}{p'-p}$$

$$\frac{\alpha^2}{pq'-p'q} = \frac{\alpha}{q-q'} \quad \text{and} \quad \frac{\alpha}{q-q'} = \frac{1}{p'-p}$$

$$\frac{\alpha^2}{\alpha} = \frac{pq'-p'q}{q-q'}$$

$$\alpha = \frac{pq'-p'q}{q-q'} \qquad \alpha = \frac{q-q'}{p'-p}$$

Hence, $\alpha = \frac{pq'-p'q}{q-q'}$ or $\alpha = \frac{q-q'}{p'-p}$ is proved.

4. **Example 3.15** If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

Solu:

Given roots are $2 + i, 3 - \sqrt{2}$

Other roots will be $2 - i, 3 + \sqrt{2}, \alpha, \beta$

$$\sum_1 = \frac{\text{-co-effi of} x^5}{\text{co-effi of } x^6}$$

$$4 + 6 + \alpha + \beta = 13$$

$$\alpha + \beta = 3 .... ❶$$

$$\sum_6 = \frac{\text{constant}}{\text{co-effi of} x^6}$$

$$(2+i)(2-i)(3-\sqrt{5})(3+\sqrt{2})\alpha\beta = 140$$

$$(4+1)(9-2)\alpha\beta = -140$$

$$(5)(7)\alpha\beta = -140$$

$$\alpha\beta = -4$$

$$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$= \sqrt{9} + 16$$

$$\alpha - \beta = 5 .... ❷$$

$$❶ + ❷ \Rightarrow 2\alpha = 8$$

$$\alpha = 4$$

$$❶ - ❷ \Rightarrow 2\beta = -2$$

$$\beta = -1$$

The roots are

$$2 + i, 2 - i, 3 - \sqrt{2}, 3 + \sqrt{2}, -1, 4.$$

5. Er:3.3(4) Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

Solu:

Given $2x^3 - 6x^2 + 3x + k = 0$

Let α, β, γ be the roots.

one of its roots is twice the sum of the other two roots.

$$\alpha = 2(\beta+\gamma) \Rightarrow \beta + \gamma = \frac{\alpha}{2}$$

$$\therefore \Sigma_1 = (\alpha + \beta + \gamma) = -\frac{b}{a}$$

$$\alpha + \beta + \gamma = -\frac{(-6)}{2} = \frac{6}{2} = 3$$

$$\alpha + \frac{\alpha}{2} = 3 \Rightarrow \frac{2\alpha + \alpha}{2} = 3$$

$$3\alpha = 6$$

$$\alpha = 2$$

2	2	−6	3	k
	0	4	−4	−2
	2	−2	−1	0

$$\therefore k - 2 = 0 \Rightarrow k = 2$$

$2x^2 - 2x - 1 = 0$ is another factor.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{12}}{4}$$

$$= \frac{2 \pm \sqrt{4 \times 3}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{2(1 \pm \sqrt{3})}{4} = \frac{(1 \pm \sqrt{3})}{2}$$

$k = 2$, and the roots are $2, \frac{(1 \pm \sqrt{3})}{2}$.

6. Er:3.3(5) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

Solu:

Given roots are $1 + 2i$ and $\sqrt{3}$

Other roots will be

$$1 + 2i, 1 - 2i, \sqrt{3}, -\sqrt{3}, \alpha, \beta$$

$$\sum_1 = \frac{\text{-co-effi of} x^5}{\text{co-effi of } x^6}$$

$$1 + 2i + 1 - 2i + \sqrt{3} - \sqrt{3} + \alpha + \beta = 3$$

$$2 + \alpha + \beta = 3$$

$$\alpha + \beta = 1 \text{-----------} ❶$$

$$\sum_6 = \frac{\text{constant}}{\text{co-effi of} x^6}$$

$$(1 + 2i)(1 - 2i)(\sqrt{3})(-\sqrt{3})\ \alpha\beta = 135$$

$$(1 + 4)(-3)\alpha\beta = 135$$

$$-15\alpha\beta = 135$$

$$\alpha\beta = -\frac{135}{15}$$

$$\alpha\beta = -9$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= 1 - 4(-9)$$

$$= 1 + 36$$

$$\alpha - \beta = \sqrt{37} \text{-----------} ❷$$

$$❶ + ❷ \Rightarrow 2\alpha = 1 + \sqrt{37}$$

$$\alpha = \frac{1 + \sqrt{37}}{2}$$

$$❶ + ❷ \Rightarrow 2\beta = 1 - \sqrt{37}$$

$$\beta = \frac{1 - \sqrt{37}}{2}$$

The roots are

$$1 + 2i, -2i, \sqrt{3}, -\sqrt{3}, \frac{1 + \sqrt{37}}{2}, \frac{1 - \sqrt{37}}{2}$$

7. **Er:3.5(5)** Solve the equations $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Solu:

$$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$$

	6	-35	62	-35	6
2	0	12	-46	32	-6
	6	-23	16	-3	0
3	0	18	-15	3	
	6	-5	1	0	

$$6x^2 - 5x + 1 = 0$$

$$(2x - 1)(3x - 1) = 0$$

$$x = \frac{1}{2}, \frac{1}{3}$$

$$x = 2, \frac{1}{2}, 3, \frac{1}{3}$$

8. Er:3.5(7) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

Solu:

Given $\frac{1}{3}$ is a solution

$\therefore$ 3 is another root.

$\frac{1}{3}$	6	−5	−38	−5	6
	0	2	−1	−13	1
3	6	−3	−39	−18	0
	0	18	45	18	
	6	15	6	0	

$6x^2 + 15x + 6 = 0$ is another factor.

$$2x^2 + 5x + 2 = 0$$

$$(x + \tfrac{1}{2})(x + 2) = 0$$

$$x = -\frac{1}{2} \text{ and } x = -2$$

$$\text{Hence, } x = \frac{1}{3}, 3, -\frac{1}{2}, -2$$

4.Inverse Trigonometric Functions

1.**Er:4.1(6)**Find the domain of the following

$f(x) = \sin^{-1}\left[\frac{x^2+1}{2x}\right]$

Solu:

Domain of sin^{-1} is $[-1,1]$

So, $-1 \le \frac{x^2+1}{2x} \le 1$

$\Rightarrow -2x \le x^2 + 1 \le \;\; x$

$\Rightarrow -2x \le x^2 + 1$	$\Rightarrow x^2 + 1 \le \;\; x$
$\Rightarrow 0 \le x^2 + 1 + \;\; x$	$\Rightarrow x^2 + 1 - \;\; x \le 0$
$\Rightarrow 0 \le (x+1)^2$	$\Rightarrow (x-1)^2 \le 0$
$\Rightarrow (x+1)^2 \ge 0$	$\Rightarrow x - 1 \le 0$
$\Rightarrow x + 1 \ge 0$	$\Rightarrow x \le 1 \to (2)$
$\Rightarrow x \ge -1 \to (1)$	

From (1) & (2) $\Rightarrow \;\; -1 \le x \le 1$

$\therefore$ *Domain of* $f(x) = sin^{-1}\left(\frac{x^2+1}{2x}\right)$ *is* $[-1,1]$.

2. Example 4.7 Find the domain of

$\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

Solu:

The domain of $cos \;\; ^1x$ is $[-1,1]$. So

$$-1 \le \frac{2+\sin x}{3} \le 1$$

$\Rightarrow -3 \le 2 + \sin x \le 3$

$\Rightarrow -5 \le sinx \le 1$

$\Rightarrow -1 \le sinx \le 1$

$\Rightarrow -sin \;\; ^1(1) \le x \le sin \;\; ^1(1)$

$\Rightarrow -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\therefore$ The domain of $cos \;\; ^1\left(\frac{2+\sin x}{3}\right)$

is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

3. Example 4.4 Find the domain of

$\sin^{-1}(2 - 3x^2)$

Solu:

Domain of sin^{-1} *is* $[-1,1]$.

$$-1 \le 2 - 3x^2 \le 1.$$

$-1 \le \;\;\; - 3x^2$	$- 3x^2 \le 1$
$\Rightarrow -3 \le -3x^2$	$\Rightarrow -3x^2 \le -1$
$\Rightarrow 3x^2 \le 3$	$\Rightarrow 3x^2 \ge 1$
$\Rightarrow x^2 \le 1 \to (1)$	$\Rightarrow x^2 \ge \frac{1}{3} \to (2)$

From (1) & (2),

$$\frac{1}{3} \le x^2 \le 1$$

$$\frac{1}{\sqrt{3}} \le |x| \le 1$$

$$x \in \left[-1, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right].$$

4. Er:4.2(5) Find the domain of

$f(x) = \sin^{-1}\left[\frac{|x|-2}{3}\right] + \cos^{-1}\left[\frac{1-|x|}{4}\right]$

Solu:

The domain of $sin^{-1}x$ is $[-1,1]$

the domain of $cos^{-1}x$ is $[-1,1]$.

So, $-1 \le \frac{|x|-2}{3} \le 1 \;\; ; \; -1 \le \frac{1-|x|}{4} \le 1$

$-1 \le \frac{\lvert x\rvert-2}{3} \le 1$	$-1 \le \frac{1-\lvert x\rvert}{4} \le 1$
$\Rightarrow -3 \le \lvert x\rvert - 2 \le 3$	$\Rightarrow -4 \le 1 - \lvert x\rvert \le 4$
$\Rightarrow -1 \le \lvert x\rvert \le \;\; 5$	$\Rightarrow -5 \le -\lvert x\rvert \le 3$
$\Rightarrow \lvert x\rvert \le 5$	$\Rightarrow -3 \le \lvert x\rvert \le 5$
	$\Rightarrow \lvert x\rvert \le 5$

$.x \in [-5,5]$

5. Er:4.4(2) Find the value of

$\cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

Solu:

$cot \;\; ^1(1) + sin \;\; ^1\left(-\frac{\sqrt{3}}{2}\right) - sec \;\; ^1(-\sqrt{2})$

$= \tan(1) + sin \;\; ^1\left(-\sin\left(\frac{\sqrt{3}}{2}\right)\right) - \cos\left(-\frac{1}{\sqrt{2}}\right)$

$= \tan\left(\frac{\pi}{4}\right) + sin \;\; ^1\left(\sin\left(-\frac{\pi}{3}\right)\right) - \cos\left(\pi - \frac{\pi}{4}\right)$

$= \frac{\pi}{4} - \frac{\pi}{3} - \frac{3\pi}{4}$

$= -\frac{2\pi}{4} - \frac{\pi}{3}$

$= -\frac{\pi}{2} - \frac{\pi}{3}$

$= -\frac{5\pi}{6}$

6. Example 4.10 Find the value of

$$\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right).$$

Solu:

$$tan\ ^{1}(-1) = tan\ ^{1}\left(-tan\frac{\pi}{4}\right)$$

$$= tan\ ^{1}\left(tan\left(-\frac{\pi}{4}\right)\right)$$

$$= -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$cos\ ^{1}\left(\frac{1}{2}\right) = cos\ ^{1}\left(cos\left(\frac{\pi}{3}\right)\right)$$

$$= \frac{\pi}{3} \in [0, \pi]$$

$$sin\ ^{1}\left(-\frac{1}{2}\right) = sin\ ^{1}\left(-sin\frac{\pi}{6}\right)$$

$$= sin\ ^{1}\left(sin\left(-\frac{\pi}{6}\right)\right)$$

$$= -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$tan\ ^{1}(-1) + cos\ ^{1}\left(\frac{1}{2}\right) + sin\ ^{1}\left(-\frac{1}{2}\right)$$

$$= -\frac{\pi}{4} + \frac{\pi}{3} - \frac{\pi}{6}$$

$$= -\frac{\pi}{12}$$