No. of Printed pages: **12**

Register number

PART – III

MATHEMATICS

MODEL QUESTION PAPER – 2021

Time allowed : 3.00Hours] [Maximum Marks : 90

Instructions:

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the hall supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) All the questions are **compulsory.** **20x1=20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$

(b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

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(c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$

(d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

2. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then the value of $A^2 - 3I$ is:

(a) $(A^{-1})^T$ (b) $2A^T$ (c) $2A^{-1}$ (d) $3A^{-1}$

3. If z_1, z_2 and z_3 are the complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$, then the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$ is:

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) z

4. If z is the complex number such that $|z + 2|=|z - 2|$, then the locus of z is:

(a) Real axis (b) Imaginary axis (c) ellipse (d) Circle

5. A zero of $x^3 + 64$ is:

(a) 0 (b) 4 (c) 4 i (d) -4

6. The domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ is:

(a) [1, 2] (b) [-1, 1] (c) [0, 1] (d) [-1, 0]

7. The point of intersection of the tangents 5 and 7 on parabola $y^2 = 4x$ is:

(a) (-35, 12) (b) (35, 12) (c) (-12, 35) (d) (12, 35)

8. Consider an ellipse whose centre is of the origin and its major axis is along x – axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is:

(a) 8 (b) 32 (c) 80 (d) 40

9. The acute angle between the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) + t(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$ and the straight line passing through the points (5, 1, 4) and (9, 2, 12) is:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $cos^{-1}\left(\frac{2}{3}\right)$ (d) $cos^{-1}\left(\frac{3}{2}\right)$

10. If the length of the perpendicular from the origin to the plane 2x + 3y +λz = 1, λ > 0 is $\frac{1}{5}$ then the value of λ is:

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

11. In which of the following interval the function $f(x) = x^2 - 2x - 3$ is strictly increasing?

(a) [2,∞) (b) (2, ∞) (c) [2, 7) (d) [2, 7]

12. A stone is thrown up vertically. The height it reaches at time t seconds is given by x= 80t – $16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

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13. A circular template has a radius 10cm. the measurement of the radius has an approximate error of 0.02 cm. then the percentage error in calculating area of this template is:

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

14. $\int_0^{\frac{\pi}{4}} \frac{dx}{4sin^2x+4cos^2x}$ =

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{9}$ (d) $\frac{\pi}{16}$

15. The area of the region enclosed by the circle $x^2 + y^2 = 1$ is:

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) $\frac{\pi}{3}$

16. The order of the differential equation of all circles with the centre at (h, k) and radius 'a' is:

(a) 2 (b) 3 (c) 4 (d) 1

17. If p and q are the order and degree of the differential equation $y\frac{dy}{dx} + x^3\left(\frac{d^2x}{dx^2}\right) + xy = \cos x$, when

(a) P < q (b) p = q (c) p > q (d) none

18. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$ is:

(a) 20 (b) 10 (c) $\sqrt{10}$ (d) $\frac{2}{5}$

19. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denotes this sum. Then the number of elements in the inverse image of 7 is:

(a) 1 (b) 2 (c) 3 (d) 4

20. Which one of the following is a binary operation on N?

(a) Subtraction (b) Multiplication (c) Division (d) None

PART – II

Note: (i) Answer **any seven** questions. **7x2=14**

(ii) Question number **30 is compulsory.**

21. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form. Also find the cofactor of M_{21}.

22. Simplify the following:

(a) $i^{-1924} + i^{2018}$ (b) $\sum_{n=1}^{12} i^{n+50}$

23. For what value x does $\sin x = sin^{-1}x$?

24. The line $3x + 4y - 12 = 0$ meets the coordinate axes at A and B. find the equation of the circle drawn on AB as diameter.

25. Prove that $\left[\vec{a} - \vec{b}, \vec{b} - \vec{c},\ \vec{c} - \vec{a}\right] = 0$

26. Evaluate: $\lim_{x \to 0} \frac{sin2x}{\sqrt{5x}}$

27. Let $f, g: (a, b) \to \mathbb{R}$ be differentiable functions. Show that $d(fg) = fdg + gdf$.

28. Solve: $(1 + x^2)\frac{dy}{dx} = 1 + y^2$.

29. A random variable X has the following probability mass function:

x	1	2	3	4	5
f(x)	K^2	$2K^2$	$3K^2$	2K	3K

Find (i) the value of K (ii) P(3 < X)

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30. Let a function $f(x)$ is defined by $f(x) = e^{-|x|}$ and if $f(-x) = f(x)$ then prove that $\int_{-log2}^{log2} f(x)dx - 1 = 0.$

PART – III

Note: (i) Answer **any seven** questions. **7x3=21**

(ii) Question number **40 is compulsory.**

31. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

32. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0.$

33. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

34. Find the equations of the tangent and normal to the curve given by x = 7 cos t and y = 2 sin t, t ∈ ℝ at any point on the curve.

35. A right circular cylinder has radius r=10cm and height h=20cm. suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also calculate the relative error and percentage error.

36. Evaluate the following:

(i) $\int_0^{\frac{\pi}{2}} \begin{vmatrix} cos^4x & 7 \\ sin^5x & 3 \end{vmatrix} dx.$

(ii) $\int_0^1 x^5(1 - x^2)^5 \, dx.$

37. Find the differential equation of the family of circles passing through the points (a, 0) and (-a, 0).

38. The probability density function of X is $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & otherwise \end{cases}$

Find (i) P(0.2 ≤ X < 0.6) (ii) P(1.2 ≤ X < 1.8) (iii) P(0.5 ≤ X < 1.5)

39. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.

40. Find the shortest distance between two given straight lines $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + t(-2\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$.

PART – IV

Note: (i) Answer **all** the questions. **7x5=35**

41. An amount of Rs.65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is Rs.4,800. The income from the third bond is Rs.600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method).

(or)

If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

42. Solve the following equation:
$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

(or)

Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at (1, -3).

43. A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in the contact with x-axis is an ellipse. Find the eccentricity.

(or)

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Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

44. Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, -2, 3) and parallel to the straight line passing through the points (2, 1, -3) and (-1, 5, -8).

(or)

Find the area of the region bounded by the line $7x - 5y = 35$, x – axis and the lines x = -2 and x = 3.

45. Find the local extrema of the function $f(x) = 4x^6 - 6x^4$.

(or)

Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x} dx$.

46. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

(or)

Solve the following linear differential equation:

$$\left(y - e^{sin^{-1}x}\right)\frac{dx}{dy} + \sqrt{1 - x^2} = 0$$

47. The rate of increase in the number of bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 100hours?

(or)

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

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