12th STD

Reg. No.

PUBLIC EXAMINATION - 2022

Part - III

Mathematics (with answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

PART - I

Note : (i) **All** questions are **compulsory.**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer. **20 × 1 = 20**

1. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is :

(a) 3 (b) 1 (c) 4 (d) 2

2. Which one of the following is not true in the case of discrete random variable X?

(a) $\lim_{x \to \infty} F(x) = F(\infty) = 1$

(b) $0 \le F(x) \le 1$ for all $x \in \mathbb{R}$

(c) $F(x)$ is real valued decreasing function.

(d) $\lim_{x \to -\infty} F(x) = F(-\infty) = 0$

3. If $f(x) = \frac{x}{x+1}$ then its differential is :

(a) $\frac{1}{x+1}dx$ (b) $\frac{-1}{(x+1)^2}dx$

(c) $\frac{-1}{x+1}dx$ (d) $\frac{1}{(x+1)^2}dx$

4. The value of $\int_0^1 x(1-x)^{99}\, dx$ is :

(a) $\frac{1}{10010}$ (b) $\frac{1}{11000}$

(c) $\frac{1}{10001}$ (d) $\frac{1}{10100}$

5. The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

6. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is :

(a) 19 (b) 17 (c) 21 (d) 14

7. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is :

(a) $\frac{q}{r}$ (b) $-\frac{q}{r}$ (c) $-\frac{q}{P}$ (d) $-\frac{p}{r}$

8. If $(1 + i)(1 + 2i)(1+ 3i) \ldots (1 + ni) = x + iy$ then the value $2.5.10 \ldots (1 + n^2)$ is :

(a) $x^2 + y^2$ (b) 1

(c) $1 + n^2$ (d) i

9. The minimum value of the function $|3-x| + 9$ is :

(a) 6 (b) 0 (c) 9 (d) 3

10. The value of $\sum_{n=1}^{12} i^n$ is :

(a) 0 (b) 1 (c) –1 (d) i

11. If the vectors $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, then the value of m is :

(a) 2 (b) 3 (c) –2 (d) –3

12. The general equation of a circle with centre (–3, –4) and radius 3 units is :

(a) $x^2 + y^2 - 6x + 8y - 16 = 0$

(b) $x^2 + y^2 - 6x - 8y + 16 = 0$

(c) $x^2 + y^2 + 6x - 8y + 16 = 0$

(d) $x^2 + y^2 + 6x + 8y + 16 = 0$

13. The solution of $\frac{dy}{dx} + p(x)y = 0$ is :

(a) $x = ce^{-\int pdy}$ (b) $y = ce^{\int pdx}$

(c) $x = ce^{\int pdy}$ (d) $y = ce^{-\int pdx}$

14. The value of $\int_0^\infty e^{-3x} x^2 dx$ is :

(a) $\frac{4}{27}$ (b) $\frac{7}{27}$ (c) $\frac{2}{27}$ (d) $\frac{5}{27}$

15. The point of inflection of the curve $y = (x-1)^3$ is :

(a) (1,0) (b) (0,0) (c) (1,1) (d) (0,1)

16. The angle between the lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

17. Which one of the following is a binary operation on N?

(a) Multiplication (b) Division
(c) Subtraction (d) All the above

18. Which one of the following is **incorrect**?

(a) If A is a square matrix of order n, and λ is a scalar, then Adj $(\lambda A) = \lambda^n$ (Adj A).
(b) Adjoint of a symmetric matrix is also a symmetric matrix.
(c) A(Adj A) = (Adj A) A = |A|I.
(d) Adjoint of a diagonal matrix is also a diagonal matrix.

19. If sin$x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is :

(a) $\tan x$ (b) $\log \sin x$
(c) $\cot x$ (d) $\cos x$

20. The length of the latus rectum of the parabola $x^2 = 24y$ is :

(a) 8 (b) 24 (c) 6 (d) 12

PART - II

Note : (i) Answer **any seven** questions.
(ii) Question number **30** is **compulsory.**

7 × 2 = 14

21. Prove the following properties :

$\text{Re}(z) = \frac{z+\bar{z}}{2}$ and $\text{Im}(z) = \frac{z-\bar{z}}{2i}$

22. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

23. Find the principal value of $\tan^{-1}(\sqrt{3})$

24. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.

25. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$.

26. Show that the differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants, is $\frac{d^2y}{dx^2} - y = 0$.

27. Solve : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

28. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find k.

29. X is the number of tails occurred when three fair coins are tossed simultaneously. Find the values of the random variable X and number of points in its reverse images.

30. Show that the distance from the origin to the plane $3x + 6y + 2z + 7 = 0$ is 1.

PART - III

Note : (i) Answer **any seven** questions.
(ii) Question number **40** is **compulsory.**

7 × 3 = 21

31. Show that the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ is 3.

32. Solve the following system of linear equations, using matrix inversion method : $5x + 2y = 3$, $3x + 2y = 5$.

33. Which one of the points $10 - 8i$, $11 + 6i$ is closest to $1 + i$.

34. Solve the equation $2x^3 - 9x^2 + 10x = 3$, if 1 is a root, find the other roots.

35. Find the magnitude and the direction cosines of the torque about the point (2, 0, –1) of a force, $2\hat{i} + \hat{j} - \hat{k}$ whose line of action passes through the origin.

36. Evaluate : $\lim\limits_{x\to\infty}\dfrac{2x^2-3}{x^2-5x+3}$

37. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1. mm, how much is cross-sectional area increased approximately.

38. Show that $\displaystyle\int_0^{\frac{\pi}{3}}\frac{\sec x\tan x}{1+\sec^2 x}dx=\tan^{-1}(2)-\frac{\pi}{4}$.

39. Let * be defined on R by $(a * b) = a + b + ab - 7$. Is * binary on R? If so, find $3*\left(\dfrac{-7}{15}\right)$.

40. Prove that the general equation of the circle whose diameter is the line segment joining the points (–4, –2) and (–1, –1), is $x^2 + y^2 + 5x + 3y + 6 = 0$.

PART - IV

Note : Answer **all** the following questions. **7 × 5 = 35**

41. (a) Cramer's rule is not applicable to solve the system $3x + y + z = 2$, $x - 3y + 2z = 1$, $7x - y + 4z = 5$. Why?

OR

(b) Prove that the local minimum values for the function $f(x) = 4x^6 - 6x^4$ attain at –1 and 1.

42. (a) Show that the locus of $z = x + iy$ if $|z + i| = |z - 1|$, is $x + y = 0$.

OR

(b) Show that $\displaystyle\int_0^a\frac{f(x)}{f(x)+f(a-x)}dx=\frac{a}{2}$.

43. (a) Show that the equation of the parabola with focus $\left(-\sqrt{2},0\right)$ and directrix $x=\sqrt{2}$ is $y^2 = -4\sqrt{2}\,x$.

OR

(b) Find the value of

$\cot^{-1}(1)+\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$.

44. (a) The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Show that the distance from the Sun to the other focus is 575×10^5 km.

OR

(b) Prove by vector method

sin (A + B) = sin A. cos B + cos A. sin B.

45. (a) Find the vector equation (any form) or Cartesian equation of a plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

OR

(b) Show that the angle between the curves $y = x^2$ and $x = y^2$ at (1, 1) is $\tan^{-1}\left(\dfrac{3}{4}\right)$.

46. (a) The distribution function of a continuous random variable X is :

$$F(x)=\begin{cases}0, & x<1\\ \dfrac{x-1}{4}, & 1\le x\le 5\\ 1, & x>5\end{cases}$$

Find (i) $P(X < 3)$ (ii) $P(2 < X < 4)$ (iii) $P(3 \le X)$

OR

(b) Show that the area of the region bounded by $3x - 2y + 6 = 0$, $x = -3$, $x = 1$ and x - axis, is $\dfrac{15}{2}$.

47. (a) Show that the solution of the differential equation $(1 + x^2)\dfrac{dy}{dx} = 1 + y^2$ is $\tan^{-1}y=\tan^{-1}x+C$ (or) $\tan^{-1}x=\tan^{-1}y+C$.

OR

(b) Prove $p\to(q\to r)\equiv(p\wedge q)\to r$ using truth table.

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Answers

PART - I

1. (b) 1
2. (c) F(x) is real valued decreasing function
3. (d) $\frac{1}{(x+1)^2}\ dx$
4. (d) $\frac{1}{10100}$
5. (d) $\frac{\pi}{6}$
6. (a) 19
7. (b) $-\frac{q}{r}$
8. (a) $x^2 + y^2$
9. (c) 9
10. (a) 0
11. (d) –3
12. (d) $x^2 + y^2 + 6x + 8y + 16 = 0$
13. (d) $y = ce^{-\int pdx}$
14. (c) $\frac{2}{27}$
15. (a) (1,0)
16. (a) $\frac{\pi}{2}$
17. (a) Multiplication
18. (a) If A is a square matrix of order n, and λ is a scalar, then Adj (λ A) = λ^n (Adj A).
19. (c) cot x
20. (b) 24

PART - II

21. Let $z = x + iy$ where x is the Re (z) and y is the Im (z).

$$\text{Then } \bar{z} = x - iy$$
$$z + \bar{z} = x + iy + x - iy = 2x$$
$$\therefore \frac{z+\bar{z}}{2} = x$$
$$\Rightarrow \frac{z+\bar{z}}{2} = \text{Re}\,(z)$$

$$\text{Also } z - \bar{z} = x + iy - (x - iy)$$
$$= x + iy - x + iy = 2iy$$
$$\therefore \frac{z - \bar{z}}{2i} = y$$
$$\Rightarrow \frac{z - \bar{z}}{2i} = \text{Im}(z)$$

22. Since $2 - \sqrt{3}$ is a root and the coefficients are rational numbers, $2 + \sqrt{3}$ is also a root. A required polynomial equation is given by x^2 − (Sum of the roots) x + Product of the roots = 0 and hence $x^2 - 4x + 1 = 0$ is a required equation.

23. Let $\tan^{-1}(\sqrt{3}) = y$. Then,
$$\tan y = \sqrt{3}. \text{ Thus, } y = \frac{\pi}{3}$$
Since $\frac{\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Thus the principal value of $\tan^{-1}(\sqrt{3})$ is $\frac{\pi}{3}$.

24. The slope of the line $y = x$ is 1. The tangent to the given curve will be parallel to the line, if the slope of the tangent to the curve at a point is also 1. Hence,
$$\frac{dy}{dx} = 3x^2 - 6x + 1 = 1$$
$$\text{which gives } 3x^2 - 6x = 0$$
Hence, $x = 0$ and $x = 2$.

Therefore, at (0, –2) and (2, –4) the tangent is parallel to the line $y = x$.

25. Taking differentials,
$$df = (2x + 3)\,dx$$
$$\text{When } x = 2,\ dx = 0.1$$
$$df = [2(2) + 3]\,(0.1) = 7(0.1)=0.7$$

26. Given : $y = Ae^x + Be^{-x}$ ... (1)

Differentiate (1) with respect to x, we get,
$$\frac{dy}{dx} = Ae^x - Be^{-x} \quad ...(2)$$
Differentiate (2) with respect to x, we get,
$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x}$$
$$\Rightarrow \frac{d^2y}{dx^2} = y$$
$\Rightarrow \frac{d^2y}{dx^2} - y = 0$ is the required differential equation.

Hence proved.

27. Separating the variables we get,

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}y = \sin^{-1}x + C \text{ [on integrating both sides we get]}$$

28. Since the given function is a probability mass function, the total probability is one. That is $\sum f(x) = 1$.

From the given data $k + 2k + 6k + 5k + 6k + 10k = 30k = 1 \Rightarrow k = \frac{1}{30}$

29. When 3 fair coins are tossed, sample space

s = {HHH, HHT, THH, HTH, HTT, THT, TTH, TTT}

Let X denote the number of tails occured.

X (no tail) = {HHH} = 1

X (1 tail) = {HHT, THH, HTH} = 3

X (2 tails) = {HTT, THT, TTH} = 3

X (3 tails) = {TTT} = 1

∴ X takes the values 0, 1, 2, 3.

Values of random variable X	0	1	2	3	Total
Number of elements in inverse images	1	3	3	1	8

30. Distance from the origin to the plane

$$\delta = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|7|}{\sqrt{(3)^2+(6)^2+(2)^2}} = \frac{|7|}{\sqrt{9+36+4}}$$

$$= \frac{|7|}{\sqrt{49}} = \frac{7}{7} = 1$$

PART - III

31. Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow[R_4 \to R_4 - R_1]{\substack{R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1}} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 \div 4} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -1 & 1 \\ 0 & -3 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 - 3R_3} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{R_3 \to 7R_3 - R_2} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_4 \to 2R_4 + R_3} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

The last equivalent matrix is in row echelon form and it has three non-zero rows

$\therefore \rho(A) = 3$

32. The matrix form of the system is AX = B, where $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

We find $|A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 10 - 6 = 4 \neq 0$. So A^{-1} exists and $A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$.

Then, applying the formula $X = A^{-1}B$, we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6 & -10 \\ -9 & +25 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{-4}{4} \\ \frac{16}{4} \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

So the solution is $x = -1, y = 4$.

33. Let the points be A $(10 - 8i)$, B $(11 + 6i)$ and C $(1 + i)$

Distance between A and C is $|(10 - 8i) - (1 + i)|$

$= |10 - 8i - 1 - i| = |9 - 9i|$

$= \sqrt{9^2 + (-9)^2} = \sqrt{81 + 81} = \sqrt{2 \times 81} = \sqrt{162}$

Distance between B and C is $|(11 + 6i) - (1 + i)|$

$= |11 + 6i - 1 - i| = |10 + 5i|$

$= \sqrt{10^2 + 5^2} = \sqrt{100 + 25} = \sqrt{125}$

Since $\sqrt{125} < \sqrt{162}$, B is closest to C.

$\therefore$ $11 + 6i$ is closest to $1 + i$

34. Since the sum of the co-efficients is

$2 - 9 + 10 - 3 = 12 - 12 = 0$

$\Rightarrow \quad x = 1$ is a root of $f(x)$

$\therefore (x - 1)$ is a factor of $f(x)$

To find the other factor, let us divide $f(x)$ by $x - 1$

1	2	–9	10	–3
	0	2	–7	3
3	2	–7	3	0
	0	6	–3	
	2	–1	0	

[Using synthetic division]

$f(x) = (x-1)(x-3)(2x-1) = 0$

$\Rightarrow x-1=0, x-3=0$ or $2x-1=0$

$\Rightarrow x = 1, x = 3, x = \frac{1}{2}$

Hence the roots are $1, 3, \frac{1}{2}$.

35. Let A be the point (2, 0, – 1). Then the position vector of A is $\overrightarrow{OA} = 2\hat{i} - \hat{k}$ and therefore $\vec{r} = \overrightarrow{AO} = -2\hat{i} + \hat{k}$.

Then the given force is $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$ So, the torque is

$$\vec{t} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} - 2\hat{k}$$

The magnitude of the torque $= |-\hat{i} - 2\hat{k}| = \sqrt{5}$ and the direction cosines of the torque are $-\frac{1}{\sqrt{5}}, 0, -\frac{2}{\sqrt{5}}$.

36. $$\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3} = \lim_{\frac{1}{x}\to 0} \frac{2-\frac{3}{x^2}}{1-\frac{5}{x}+\frac{3}{x^2}}$$ [Dividing the numerator and denominator by x^2]

$$= \frac{2-0}{1-0+0} = \frac{2}{1} = 2$$

37. Given

$r = 2$ mm

$dr = (2.1 - 2) = 0.1$ mm

Area $= \pi r^2$

Approximate area $dA = 2\pi r\, dr = 2\pi(2)(0.1)$

$= 4\pi(0.1) = 0.4\pi$ mm^2

38. Let $I = \int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x} dx$

Put $\sec x = u$. Then, $\sec x \tan x\, dx = du$

When $x = 0$, $u = \sec 0 = 1$.

When $x = \frac{\pi}{3}$, $u = \sec\frac{\pi}{3} = 2$.

$$\therefore I = \int_1^2 \frac{du}{1+u^2} = [\tan^{-1} u]_1^2 = \tan^{-1}(2) - \tan^{-1}1 = \tan^{-1}(2) - \frac{\pi}{4}$$

39. Given $a * b = a + b + ab - 7$

Let $a, b \in \mathbb{R}$

$a * b = a + b + ab - 7 \in \mathbb{R}$,

$\therefore$ * is binary operation on $\mathbb{R}$.

$$3 * \left(\frac{-7}{15}\right) = 3 - \frac{7}{15} + 3\left(\frac{-7}{15}\right) - 7$$

[Here $a = 3$, $b = \frac{-7}{15}$]

$$= 3 - \frac{7}{15} - \frac{7}{5} - 7$$

$$= \frac{45 - 7 - 21 - 105}{15} = \frac{-88}{15}$$

$$\therefore 3 * \left(\frac{-7}{15}\right) = \frac{-88}{15}$$

40. Equation of the circle with end points of the diameter as (x_1, y_1) and (x_2, y_2) is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$
$$(x + 4)(x + 1) + (y + 2)(y + 1) = 0$$
$$x^2 + x + 4x + 4 + y^2 + y + 2y + 2 = 0$$
$$x^2 + y^2 + 5x + 3y + 6 = 0$$

Hence proved.

PART - IV

41. (a) When the coefficient matrix is a square matrix and non-singular, then the matrix can solved by Cramer's rule.

Coefficient matrix

$$\begin{vmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{vmatrix} = 3(-12 + 2) - 1(4 - 14) + 1(-1 + 21)$$
$$= 3(-10) - 1(-10) + 1(20)$$
$$= -30 + 10 + 20 = 0$$

$\therefore$ Coefficient matrix is singular

$\therefore$ Cramer's rule is not applicable.

OR

(b) Differentiating with respect to x, we get

$$f'(x) = 24x^5 - 24x^3$$
$$= 24x^3(x^2 - 1)$$
$$= 24x^3(x + 1)(x - 1)$$
$$f'(x) = 0 \Rightarrow x = -1, 0, 1.$$

Hence the critical numbers are $x = -1, 0, 1$.

Now, $f''(x) = 120x^4 - 72x^2 = 24x^2(5x^2 - 3)$

$\Rightarrow f''(-1) = 48, f''(0) = 0, f''(1) = 48.$

As $f''(-1)$ and $f''(1)$ are positive by the second derivative test, the function $f(x)$ has local minimum at -1 and 1.

42. (a)

$$\Rightarrow \quad |x + iy + i| = |x + iy - 1|$$

$$\Rightarrow \quad |x + i(y + 1)| = |(x - 1) + iy|$$

$$\Rightarrow \quad \sqrt{x^2 + (y+1)^2} = \sqrt{(x-1)^2 + y^2}$$

$$\Rightarrow \quad x^2 + (y + 1)^2 = (x - 1)^2 + y^2 \quad \text{[squaring both sides]}$$

$$\Rightarrow \quad \cancel{x^2} + \cancel{y^2} + 2y + \cancel{1} = \cancel{x^2} - 2x + \cancel{1} + \cancel{y^2}$$

$$\Rightarrow \quad 2y + 2x = 0$$

$$\Rightarrow \quad x + y = 0$$

Hence, the required Cartesian equation is $x + y = 0$

OR

(b)

$$\text{Let I} = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx \quad \text{... (1)}$$

Applying the formula $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ in equation (1), we get

$$\text{I} = \int_a^0 \frac{f(a-x)}{f(a-x) + f(a-(a-x))} dx$$

$$= \int_0^a \frac{f(a-x)}{f(x) + f(a-x)} dx$$

Adding equation (1) and (2), we get

$$2\text{I} = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx + \int_0^a \frac{f(a-x)}{f(x) + f(a-x)}$$

$$= \int_0^a \frac{f(x) + f(a-x)}{f(x) + f(a-x)} dx = \int_0^a dx = a$$

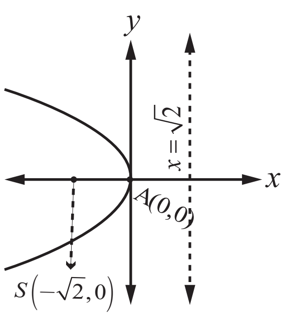
Hence , we get $\text{I} = \frac{a}{2}$

43. (a) Parabola is open left and axis of symmetry as x-axis and vertex (0,0).

Then the equation of the required parabola is

$$(y - 0)^2 = -4\sqrt{2}\,(x - 0).$$

$$y^2 = -4\sqrt{2}\,x$$

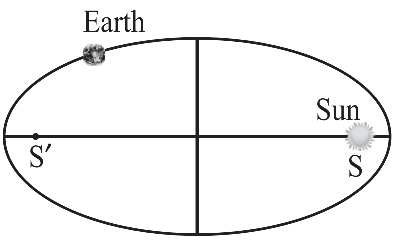


OR

(b)

$$\text{Let } \cot^{-1}(1) = x$$
$$\Rightarrow \cot x = 1$$
$$\Rightarrow \tan x = 1$$
$$\Rightarrow \tan x = \tan\frac{\pi}{4}$$
$$\Rightarrow x = \frac{\pi}{4} \qquad [\because \frac{\pi}{4} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]]$$
$$\text{Let } \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y$$
$$\Rightarrow \sin y = \frac{-\sqrt{3}}{2} = -\sin\frac{\pi}{3}$$
$$\Rightarrow \sin y = \sin\left(\frac{-\pi}{3}\right) \qquad [\because \sin(-\theta) = -\sin\theta]$$
$$\Rightarrow y = \frac{-\pi}{3}$$
$$\text{Let } \sec^{-1}(-\sqrt{2}) = z$$
$$\Rightarrow \sec z = -\sqrt{2} \Rightarrow \cos z = \frac{-1}{\sqrt{2}}$$
$$\Rightarrow \cos z = -\cos\frac{\pi}{4}$$
$$\Rightarrow \cos z = \cos\left(\pi - \frac{\pi}{4}\right)$$
$$\Rightarrow \cos z = \cos\left(\frac{3\pi}{4}\right) \Rightarrow z = \frac{3\pi}{4}$$
$$\therefore \cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2}) = \frac{\pi}{4} - \frac{\pi}{3} - \frac{3\pi}{4} = \frac{3\pi - 4\pi - 9\pi}{12} = \frac{-10\pi}{12} = \frac{-5\pi}{6}$$
$$\therefore \cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2}) = \frac{-5\pi}{6}$$

44. (a)



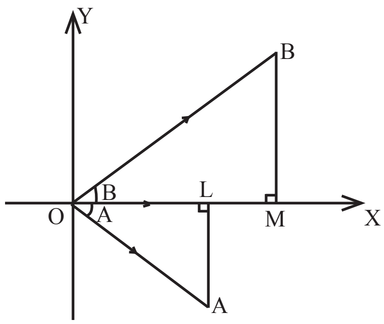
$$AS = 94.5 \times 10^6 \text{ km}, SA' = 152 \times 10^6 \text{ km}$$
$$a + c = 152 \times 10^6$$
$$a - c = 94.5 \times 10^6$$

Subtracting $2c = 57.5 \times 10^6 = 575 \times 10^5$ km

Distance of the Sun from the other focus is $SS' = 575 \times 10^5$ km.

OR

(b)



Let $\hat{a} = \overrightarrow{OA}$ and $\hat{b} = \overrightarrow{OB}$ be the unit vectors and which make angles α, β respectively with positive x-axis

Draw AL and BM $\perp^r$ to x-axis.

$$\text{Then } |\overrightarrow{OL}| = |\overrightarrow{OA}|\cos A$$

$$\Rightarrow \overrightarrow{OL} = |\overrightarrow{OA}|\hat{i} = \cos A\hat{i}$$

$$|\overrightarrow{LA}| = |\overrightarrow{OA}|\sin A$$

$$\Rightarrow \overrightarrow{LA} = |\overrightarrow{OA}|\hat{j} = \sin A\left(-\hat{j}\right) = -\sin A\hat{j}$$

[$\overrightarrow{LA}$ is in the opp direction of y axis]

$$\hat{a} = \overrightarrow{OA} = \overrightarrow{OL} + \overrightarrow{LA} = \cos A\hat{i} - \sin A\hat{j} \quad ...(1)$$

$$\text{Similarly } \hat{b} = \overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB} = \cos B\hat{i} + \sin B\hat{j} \quad ...(2)$$

$$\text{Now } \hat{a}\times\hat{b} = |\hat{a}|\,|\hat{b}|\sin(A+B)\,\hat{k} = \sin(A+B)\hat{k} \quad ...(3)$$

[$|\hat{a}| = |\hat{b}| = 1$]

$$\text{Also } \hat{a}\times\hat{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos A & -\sin A & 0 \\ \cos B & \sin B & 0 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}\,(\cos A \sin B + \sin A \cos B)$$

$$= (\sin A \cos B + \cos A \sin B)\,\hat{k} \quad ...(4)$$

Using (3) and (4) we get,

sin(A + B) = sin A cos B + cos A sin B

Hence proved.

45. (a) Given plane is passing through the points

$$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{b} = 9\hat{i} + 3\hat{j} + 6\hat{k}.$$

Equation of the given plane is $2x + 6y + 6z = 9$. It can be written as $\vec{r}\cdot(2\hat{i} + 6\hat{j} + 6\hat{k}) = 9$.

Since the given plane is perpendicular to $2\hat{i} + 6\hat{j} + 6\hat{k}$, the required plane is parallel to $\vec{c} = 2\hat{i} + 6\hat{j} + 6\hat{k}$. Hence, parametric form of vector equation of plane passing through two points and parallel to a vector is

$$\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t\vec{c},\ s, t \in \mathbb{R}$$

$$\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k} + s(7\hat{i} + \hat{j} + 5\hat{k}) + t(2\hat{i} + 6\hat{j} + 6\hat{k}),\ s, t \in \mathbb{R}$$

Cartesian equation of the plane is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(6-30)-(y-2)(42-10)+(z-1)(42-2) = 0$$

$$\Rightarrow (x-2)(-24)-(y-2)(32)+(z-1)(40) = 0$$

$$\Rightarrow -24x+48-32y+64+40z-40 = 0$$

$$\Rightarrow -24x-32y+40z+72 = 0$$

$\div -8$ we get, $3x+4y-5z-9=0$ is the Cartesian form.

$\therefore$ The parametric form of vector equation is $\vec{r}.\left(3\vec{i}+4\vec{j}-5\vec{k}\right)=9$.

OR

(b) Let us now find the slopes of the curves.

Let m_1 be the slope of the curve $y=x^2$,

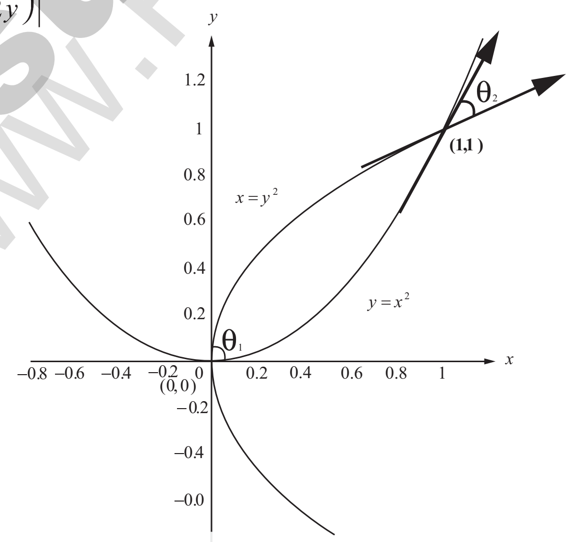
$$\text{then } m_1 = \frac{dy}{dx} = 2x$$

Let m_2 be the slope of the curve $x=y^2$

$$\text{then } m_2 = \frac{dy}{dx} = \frac{1}{2y}$$

Let θ_1 and θ_2 be the angles at (0,0) and (1,1) respectively.

At (0,0) we come across the indeterminate form of $0 \times \infty$ in the denominator of $\tan\theta_1 = \left|\dfrac{2x-\dfrac{1}{2y}}{1+2x\left(\dfrac{1}{2y}\right)}\right|$ and so we follow the limiting process.



$$\tan\theta_1 = \lim_{(x,y)\to(0,0)}\left|\frac{2x-\frac{1}{2y}}{1+2x\left(\frac{1}{2y}\right)}\right| = \lim_{(x,y)\to(0,0)}\left|\frac{4xy-1}{2(y+x)}\right| = \infty$$

Which gives $\theta_1 = \tan^{-1}(\infty) = \frac{\pi}{2}$

At (1,1), $m_1 = 2, m_2 = \frac{1}{2}$,

$$\tan\theta_1 = \left|\frac{2-\frac{1}{2}}{1+(2)\left(\frac{1}{2}\right)}\right| = \frac{3}{4}$$

Which gives $\theta_2 = \tan^{-1}\left(\frac{3}{4}\right)$ Hence proved.

46. (a) (i) $P(X<3) = P(X\le 3) = F(3) = \frac{3-1}{4} = \frac{1}{2}$ (since F (x) is continuous).

(ii) $P(2<X<4) = P(2\le X\le 4) = F(4) - F(2) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

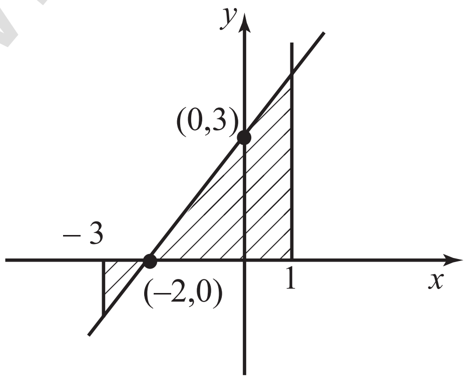
(iii) $P(3\le X) = P(X\ge 3) = 1 - P(X<3) = 1 - \frac{1}{2} = \frac{1}{2}$

OR

(b) Given equation of line is $3x - 2y + 6 = 0$

$2y = 3x + 6 \Rightarrow y = \frac{3x+6}{2}$

x	0	–2
y	3	0



$\therefore$ Area $= \int_{-3}^{-2} -y\,dx + \int_{-2}^{1} y\,dx$ [$\because$ the Area is below the x - axis]

$$= \frac{-1}{2}\int_{-3}^{-2}(3x+6)dx + \frac{1}{2}\int_{-2}^{1}(3x+6)dx$$

$$= \frac{-1}{2}\left[\frac{3x^2}{2}+6x\right]_{-3}^{-2} + \frac{1}{2}\left[\frac{3x^2}{2}+6x\right]_{-2}^{1}$$

$$= -\frac{1}{2}\left[\left(\frac{12}{2}-12\right)-\left(\frac{27}{2}-18\right)\right] + \frac{1}{2}\left[\left(\frac{3}{2}+6\right)-\left(\frac{12}{2}-12\right)\right]$$

$$= -\frac{1}{2}\left[(-6)-\left(\frac{27-36}{2}\right)\right] + \frac{1}{2}\left[\left(\frac{3+12}{2}\right)-(-6)\right]$$

$$= -\frac{1}{2}\left[-6+\frac{9}{2}\right] + \frac{1}{2}\left[\frac{15}{2}+6\right]$$

$$= -\frac{1}{2}\left[\frac{-3}{2}\right] + \frac{1}{2}\left[\frac{27}{2}\right] = \frac{3}{4}+\frac{27}{4} = \frac{30}{4} = \frac{15}{2}$$

$\therefore$ A = 7.5 sq.units

47. (a) Given that $(1+x^2)\frac{dy}{dx} = 1+y^2$. ... (1)

The given equation is written in the variables separable form

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2} \quad \text{... (2)}$$

Integrating both sides of (2), we get $\tan^{-1} y = \tan^{-1} x + C$. ... (3)

But $\tan^{-1} y - \tan^{-1} x = \tan^{-1}\left(\frac{y-x}{1+xy}\right)$ ... (4)

Using (4) in (3) leads to $\tan^{-1}\left(\frac{y-x}{1+xy}\right) = C$, which implies $\frac{y-x}{1+xy} = \tan C = a$ (say)

Thus, $y - x = a(1+xy)$ gives the required solution.

OR

(b)

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

$\therefore p \to (q \to r) \equiv (p \wedge q) \to r$

Hence proved.

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