APPLICATIONS OF MATRICES AND DETERMINANTS

Modulus :

1.) $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ $|A| = ?$

$|A| = \begin{vmatrix} 1 & 3 \\ 4 & 5 \end{vmatrix}$

$|A| = 5 - 12$

$|A| = -7 \neq 0$

2.) $A = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix}$ $|A| = ?$

$|A| = \begin{vmatrix} 1 & 2 \\ -3 & -4 \end{vmatrix}$

$|A| = -4 - (-6)$

$|A| = -4 + 6$

$|A| = 2 \neq 0$

3.)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$|A| = ad - cb \neq 0$$

(or)

$$= ad - bc \neq 0$$

4.)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -2 & 4 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -2 & 4 & -5 \end{vmatrix}$$

$$|A| = 2(-10-12) - 3(-5+6) + 4(4+4)$$

$$= 2(-22) - 3(1) + 4(8)$$

$$= -44 - 3 + 32$$

$$= -47 + 32$$

$$|A| = -15 \neq 0$$

5.)

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{vmatrix}$$

$$|A| = 2(9-2) + 1(-15+3) + 3(-10+9)$$

$$|A| = 2(7) + 1(-12) + 3(-1)$$

$$= 14 - 12 - 3$$

$$= 14 - 15$$

$$\boxed{|A| = -1 \neq 0}$$

6.)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix}$$

$$|A| = 2(1-2) - 1(1-2) + 1(2-2)$$

$$= 2(-1) - 1(-1) + 1(0)$$

$$= -2 + 1$$

$$\boxed{|A| = -1 \neq 0}$$

Adj A = ?

1.)

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \quad adj\ A = ?$$

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$$

2.)

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

3.)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Exercise 1.1

1. Find the adjoint of the following

i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

$$adj\ A = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$$

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ii) $\begin{bmatrix} \overset{+}{2} & \overset{-}{3} & \overset{+}{1} \\ \overset{-}{3} & \overset{+}{4} & \overset{-}{1} \\ \overset{+}{3} & \overset{-}{7} & \overset{+}{2} \end{bmatrix}$

Dr. G. THIRUMOORTHI, M.Sc., B.Ed., Ph.D.,
Guest Lecture
PG and Research Department of Physics
Government Arts College (Autonomous)
SALEM - 636 007.
8610560810

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{11} = +2 \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix}$$

$$= 2(8-7) \quad = 2(1) \quad = 2$$

$$a_{12} = -3 \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= -3(6-3)$$

$$= -3(3) = -9$$

$A_{11} = +(8-7)$

$= 1$

$A_{12} = -(6-3)$

$= -(3) = -3$

$A_{13} = +(21-12)$

$= (9)$

$A_{21} = -(6-7)$

$= -(-1)$

$A_{22} = +(4-3)$

$= 1$

$A_{23} = -(14-9)$

$= -(5) = -5$

$A_{31} = +(3-4)$

$= -1$

$A_{32} = -(2-3) = -(-1) = 1$

$A_{33} = +(8-9)$

$= -1$

$$A_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{ij} = \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T$$

T = Transpose

$$\text{adj } A = \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

(ii)

Creative Sums :

$$\frac{1}{3} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

$a_{11} = +(2+4) = 6$

$a_{12} = -(-2-2) = -(-4) = 4$ $\quad -(2-2) = 0$

$a_{13} = +(2-1) = 1$ $\quad -2-1 = -3$

$a_{21} = -(4+2) = -6$

$a_{22} = +(4-1) = 3$

$a_{23} = -(-4-2) = -(-6) = 6$

$a_{31} = +(4-1) = 3$

$a_{32} = -(4-1) = -(3) = -3$

$a_{33} = +(2-2) = 0$

$$A_{ij} = \begin{bmatrix} 6 & 0 & -3 \\ -6 & 3 & 6 \\ 3 & -3 & 0 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T$$

$$= \begin{bmatrix} 6 & 0 & -3 \\ -6 & 3 & 6 \\ 3 & -3 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -6 & 3 \\ 0 & 3 & -3 \\ -3 & 6 & 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 6 & -6 & 3 \\ 0 & 3 & -3 \\ -3 & 6 & 0 \end{bmatrix}$$

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iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

$a_{11} = +(2+4) = 6$

$a_{12} = -(-4-2) = -(-6) = 6$

$a_{13} = +(4-1) = 3$

$a_{21} = -(4+2) = -6$

$a_{22} = +(4-1) = 3$

$a_{23} = -(-4-2) = 6$

$a_{31} = +(4-1) = 3$

$a_{32} = -(4+2) = -6$

$a_{33} = +(2+4) = 6$

$$A_{ij} = \begin{bmatrix} 6 & 6 & 3 \\ -6 & 3 & 6 \\ 3 & -6 & 6 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T$$

$$= \begin{bmatrix} 6 & 6 & 3 \\ -6 & 3 & 6 \\ 3 & -6 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6/3 & -6/3 & 3/3 \\ 6/3 & 3/3 & -6/3 \\ 3/3 & 6/3 & 6/3 \end{bmatrix}$$

$$\text{adj} = \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

Dr. G. THIRUMOORTHI, M.Sc.,B.Ed.,Ph.D.,
Guest Lecture
PG and Research Department of Physics
Government Arts College (Autonomous)
SALEM - 636 007.
8610560810

FORMULA:

λA^{n-1} (adj A)

$\lambda A = \lambda^{n-1}$ (adj A)

$$= \left(\frac{1}{3}\right)^{n-1} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$= \left(\frac{1}{3}\right)^{3-1} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

n = 3

$$= \left(\frac{1}{3}\right)^{2} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$= \frac{(1)^2}{(3)^2} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

Taking Common on (3)

$$= \frac{1}{9} \times 3 \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \frac{3}{9} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

Home Work Sum :

Modulus

1.) $A = \begin{bmatrix} 3 & -2 & -1 \\ 5 & 4 & 3 \\ -1 & 2 & -3 \end{bmatrix}$

2.) adj A = ?

$A = \begin{bmatrix} 2 & 3 & 5 \\ -1 & -2 & 4 \\ -1 & 2 & 3 \end{bmatrix}$

3.) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ adj A = ?

4.) adj A = ?

$A = \begin{bmatrix} 3 & -1 & -2 \\ 1 & 2 & 3 \\ -3 & 2 & 4 \end{bmatrix}$

5.) Adj A = ?

$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

09.04.24
Tuesday

1. Modulus

$$A = \begin{bmatrix} 3 & -2 & -1 \\ 5 & 4 & 3 \\ -1 & 2 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 & -1 \\ 5 & 4 & 3 \\ -1 & 2 & 3 \end{vmatrix}$$

$$|A| = 3(-12-6) + 2(-15+3) - 1(10+4)$$

$$= 3(-18) + 2(-12) - 1(14)$$

$$= -54 - 24 - 14$$

$$|A| = -92 \neq 0$$

2. adj A = ?

$$|A| = \begin{bmatrix} 2 & 3 & 5 \\ -1 & -2 & 4 \\ -1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{11} = +(-6-8) = +(-14) = -14$

$a_{12} = -(-3+4) = -(1) = -1$

$a_{13} = +(-2-2) = +(-4) = -4$

$a_{21} = -(9-10) = -(-1) = 1$

$a_{22} = +(6+5) = +(11) = 11$

$a_{23} = -(4+3) = -(7) = -7$

$a_{31} = +(12+10) = +(22) = 22$

$a_{32} = -(8+5) = -(13) = -13$

$a_{33} = +(-4+3) = +(-1) = -1$

$$A_{ij} = \begin{bmatrix} -14 & -1 & -4 \\ 1 & 11 & -7 \\ 22 & -13 & -1 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T$$

$$= \begin{bmatrix} -14 & -1 & -4 \\ 1 & 11 & -7 \\ 22 & -13 & -1 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} -14 & 1 & 22 \\ -1 & 11 & -13 \\ -4 & -7 & -1 \end{bmatrix}$$

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3. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ adj $A = ?$

$$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

---

4. adj $A = ?$

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 1 & 2 & 3 \\ -3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{11} = +(8-6) = +(2) = 2$

$a_{12} = -(4+9) = -(13) = -13$

$a_{13} = +(2+6) = +(8) = 8$

$a_{21} = -(-4+4) = -(0) = 0$

$a_{22} = +(12-6) = +(6) = 6$

$a_{23} = -(6-3) = -(3) = -3$

$a_{31} = +(-3+4) = +(1) = 1$

$a_{32} = -(9+2) = -(11) = -11$

$a_{33} = +(6+1) = +(7) = 7$

$$A_{ij} = \begin{bmatrix} 2 & -13 & 8 \\ 0 & 6 & -9 \\ 1 & -11 & 7 \end{bmatrix}$$

$$\text{adj}_A = [A_{ij}]^T$$

$$\text{adj}_A = \begin{bmatrix} 2 & 0 & 1 \\ -13 & 6 & -11 \\ 8 & -3 & 7 \end{bmatrix}$$

5.

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

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Inverse Matrix :

$$A^{-1} = \frac{1}{|A|} (adj A)$$

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Exercise : 1.1

2. Find the inverse (if it exists) of the following

i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

Dr. G. THIRUMOORTHI, M.Sc.,B.Ed.,Ph.D.,
Guest Lecture
PG and Research Department of Physics
Government Arts College (Autonomous)
SALEM - 636 007.
8610560810

$$|A| = \begin{vmatrix} -2 & 4 \\ 1 & -3 \end{vmatrix}$$

$$= 6 - 4 = 2 \neq 0$$

$$|A| = 2$$

$$\text{adj } A = \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$$

ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ →

Example : 1.2

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = ?$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$|A| = ad - bc \neq 0$

$$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$A^{-1} = \frac{1}{ab-dc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{vmatrix}$$

$= 5(25-1) - 1(5-1) + 1(1-5)$

$= 5(24) - 1(4) + 1(-4)$

$= 120 - 4 - 4$

$= 112 \neq 0$

adj A

$a_{11} = +(25-1) = 24$

$a_{12} = -(5-1) = -4$

$a_{13} = +(1-5) = -4$

$a_{21} = -(5-1) = -4$

$a_{22} = +(25-1) = 24$

$a_{23} = -(5-1) = -4$

$a_{31} = +(1-5) = -4$

$a_{32} = -(5-1) = -4$

$a_{33} = +(25-1) = 24$

$$A_{ij} = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$$

$$\text{adj} A = [A_{ij}]^T$$

$$\text{adj} A = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj} A)$$

$$A^{-1} = \frac{1}{112}\begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{112} \times 4 \begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{28}\begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}$$

iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{vmatrix}$

$= 2(8-7) - 3(6-3) + 1(21-12)$

$= 2(1) - 3(3) + 1(9)$

$= 2 - 9 + 9$

$= 2 \neq 0$

adj A

$a_{11} = +(8-7) = +1$

$a_{12} = -(6-3) = -3$

$a_{13} = +(21-12) = 9$

$a_{21} = -(6-7) = 1$

$a_{22} = +(4-3) = 1$

$a_{23} = -(14-9) = -5$

$a_{31} = +(3-4) = -1$

$a_{32} = -(2-3) = 1$

$a_{33} = +(8-9) = -1$

$$A_{ij} = \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T$$

$$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{1}{|A|}(\text{adj } A)}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

Example : 1.11.

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

$A A^T$ = orthogonal

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \text{—(1)}$$

$$A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{—(2)}$$

$$AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$AA^T = A^TA = I$$

A orthogonal

Example : 1.9

verify $(AB)^{-1} = B^{-1}A^{-1}$

$$A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|}(\text{adj } AB)$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$B^{-1} = \frac{1}{|B|}(\text{adj } B)$$

$$AB = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -0-0 & -0+3 \\ -2+0 & -3-4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -7 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 0 & 3 \\ -2 & -7 \end{vmatrix}$$

$$= -0+6$$

$$= 6 \neq 0$$

$$\text{adj } AB = \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$$

$$AB^{-1} = \frac{1}{AB} (\text{adj } AB)$$

$$AB^{-1} = \frac{1}{6} \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \rightarrow (1)$$

$$|A| = \begin{vmatrix} 0 & -3 \\ 1 & 4 \end{vmatrix}$$

$$= 0+3$$

$$= 3 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\operatorname{adj} A)$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} \rightarrow (2)$$

$$|B| = \begin{vmatrix} -2 & -3 \\ 0 & -1 \end{vmatrix}$$

$$= 2 + 0$$

$$= 2 \neq 0$$

$$\operatorname{adj} B = \begin{bmatrix} -1 & 3 \\ -0 & -2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} (\operatorname{adj} B)$$

$$B^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 3 \\ -0 & -2 \end{bmatrix} \rightarrow (3)$$

$$B^{-1} A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$$

$$B^{-1} A^{-1} = \frac{1}{6} \begin{bmatrix} -4 - 3 & -3 + 0 \\ 0 + 2 & 0 - 0 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \rightarrow (4)$$

(i) = (ii)

$(AB^{-1}) = B^{-1}A^{-1}$.

Hence Prove that it.

Exercise 1.1

7. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$

Verify that $(AB)^{-1} = B^{-1}A^{-1}$

$AB^{-1} = \frac{1}{|AB|}(\text{adj } AB)$

$A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$B^{-1} = \frac{1}{|B|}(\text{adj } B)$

$AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} -3+10 & -9+4 \\ -7+25 & -21+10 \end{bmatrix}$

$AB = \begin{bmatrix} 7 & -5 \\ 18 & -11 \end{bmatrix}$

$$|AB| = \begin{vmatrix} 7 & -5 \\ 15 & -11 \end{vmatrix}$$

$$= -77 + 90$$

$$= 13 \neq 0$$

$$\text{adj } AB = \begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} (\text{adj } AB)$$

$$(AB)^{-1} = \frac{1}{13} \begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix} \rightarrow (1)$$

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix}$$

$$= 15 - 14$$

$$= 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$(A)^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$(A)^{-1} = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$(A)^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \rightarrow (2)$$

$$B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$$

$$|B| = \begin{vmatrix} -1 & -3 \\ 5 & 2 \end{vmatrix}$$

$$= -2 + 15$$

$$= 13 \neq 0$$

$$\text{adj } B = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$$

$$(B)^{-1} = \frac{1}{|B|} (\text{adj } B)$$

$$(B)^{-1} = \frac{1}{13} \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix} \rightarrow (3)$$

$$(B)^{-1}(A)^{-1} = \frac{1}{13} \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} 10-21 & -4+9 \\ -25+7 & 10-3 \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix} \rightarrow (4)$$

$$(1) = (4)$$

$(AB^{-1}) = B^{-1}A^{-1}$

Hence Proved.

Example 1.6

If $adj\,A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Find A^{-1}

$$|adj\,A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= -1(1-4) - 2(1-4) + 2(2-2)$$

$$= -1(-3) - 2(-3) + 2(0)$$

$$= 3 + 6$$

$$|adj\,A| = 9 \neq 0$$

$$A^{-1} = \pm \frac{1}{\sqrt{|adj\,A|}} (adj\,A)$$

$$A^{-1} = \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

If $(adj\,A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}

$$A^{-1} = \pm \frac{1}{\sqrt{|adj\,A|}}(adj\,A)$$

$$|adj\,A| = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix}$$

$$= 0(12+6) + 2(36-18) - 0(0+6)$$

$$= 2(+18)$$

$$= +36 \neq 0$$

$$A^{-1} = \pm \frac{1}{\sqrt{36}} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

Dr. G. THIRUMOORTHI, M.Sc., B.Ed., Ph.D.,
Guest Lecture
PG and Research Department of Physics
Government Arts College (Autonomous)
SALEM - 636 007.
86 10 56 0810