Sri Raghavendra Tuition Center

unit - 2 Full Test - 3

12th Standard

Date : 17-May-24

Reg.No. :

Maths

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Time : 01:30:00 Hrs

Total Marks : 50

I. ANSWER ALL QUESTION 10 x 1 = 10

1) (b) $\frac{-1}{i+2}$

2) (a) $\frac{1}{2}$

3) (a) 1

4) (a) z

5) (b) 1

6) (a) −110°

7) (d) $-\sqrt{3i}$

8) (a) 1

9) (d) Vertices of a square

10) (b) 0

II. ANSWER ALL QUESTION 4 x 2 = 8

11) $\overline{3i} + \frac{1}{2-i}$

= - 3i + $\frac{1}{2-i} \times \frac{2+i}{2+i}$

[∴ Conjugate of 3i is -3i]

= - 3i + $\frac{2+i}{2^2-i^2} = -3i + \frac{2+i}{4+1}$

= - 3i + $\frac{2+i}{5}$

= $\frac{-15i+2+i}{5} = \frac{-14i+2}{5}$

$= \frac{2}{5} - \frac{14}{5}i.$

12) Since z is purely imaginary

z = -$\overline{z}$

∴ 2n$\left[cos\frac{n\pi}{6} + isin\frac{n\pi}{6}\right]$

= -2n$\left[cos\frac{n\pi}{6} + isin\frac{n\pi}{6}\right]$ [From (1) & (2)]

$\Rightarrow$ cos $\frac{n\pi}{6} + i$ sin $\frac{m\pi}{6} = -$ cos $\frac{n\pi}{6} + i$ sin $\frac{m\pi}{6}$

⇒ 2cos$\frac{n\pi}{6}$ = 0

⇒ $cosn\frac{\pi}{6} = 0 = cos\frac{\pi}{2}$

[∴ cos $\frac{\pi}{2}$ = 0]

$\Rightarrow \frac{n\pi}{6} = \frac{\pi}{2}$

⇒ n = $\frac{6}{2}$

⇒ n = 3

Hence z is purely imaginary

13) |z+i| = |z-1|

⇒ |x + iy +i| = |x + iy-1|

$\Rightarrow |x + i(y + 1)| = |(x - 1) + iy|$

$\Rightarrow \sqrt{x^2 + (y+1)^2} = \sqrt{(x-1)^2 + y^2}$

$\Rightarrow x^2 + (y + 1)^2 = (x - 1)^2 + y^2$

[squaring both sides]

$\Rightarrow \cancel{x^2} + \cancel{y^2} + 2y + \cancel{1} = \cancel{x^2} - 2x + \cancel{1} + \cancel{y^2}$

$\Rightarrow 2y + 2x = 0$

$\Rightarrow x + y = 0$

Hence, the Cartesian equation is $x + y = 0$

14) $\sum_{n=1}^{102} i^n = (i^1+i^2+i^3+i^4)+(i^5+i^6+i^7+i^8)+\ldots+(i^{97}+i^{98}+i^{99}+i^{100})+i^{101}+i^{102}$

$= (i^1+i^2+i^3+i^4)+(i^1+i^2+i^3+i^4)+\ldots+(i^1+i^2+i^3+i^4)+i^{-1}+i^{-2}$

$= \{i+(-1)+(-i)+1\}+\{i+(-1)+(-i)\}+\ldots\ldots+\{i+(-1)+(-i)+1\}+i+(-1)$

$= 0+0+\ldots0+i-1$

$= -1+i$

III. ANSWER ALL QUESTION 4 x 3 = 12

15) $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$

Given $z_1 = 3$, $z_2 = -7i$, $z_3 = 5+4i$

LHS $= z_1(z_2 + z_3)$

$= 3\,[-7i + 5 + 4i]$

$= 3[5-3i]$

$= 15-9i$

RHS $= z_1z_2 + z_1z_3$

$= 3(-7i) + 3(5 + 4i)$

$= -21i + 15 + 12i$

$= -9i + 15$

$= 15-9i$

LHS = RHS

$\therefore z_1(z_1 + z_3) = z_1z_2 + z_1z_3$

Hence proved

16) Given $v = 3-4i$, $w = 4+3i$ and $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$

$\therefore \frac{1}{u} = \frac{1}{3-4i} + \frac{1}{4+3i}$

$= \frac{3+4i}{(3-4i)(3+4i)} + \frac{4-3i}{(4+3i)(4-3i)}$

$= \frac{3+4i}{9-(4i)^2} + \frac{4-3i}{16-(3i)^2} = \frac{3+4i}{9+16} + \frac{4-3i}{16+9}$

$= \frac{3+4i}{25} + \frac{4-3i}{25} = \frac{3+4i+4-3i}{25}$

$\frac{1}{u} = \frac{7+i}{25}$

$\therefore u = \frac{25}{7+i} \times \frac{7-i}{7-i} = \frac{25(7-i)}{7^2-(i^2)}$

$= \frac{25(7-i)}{49+1} = \frac{25(7-i)}{50} = \frac{1}{2}(7-i)$

$\therefore u = \frac{1}{2}(7-i)$ or $\frac{7}{2} - \frac{i}{2}$

17) $|z^2-3| \le |z^2|+|-3|$ [Triangle law of inequality]

$\le |z|^2+3 \le 1+3$ [$\therefore |z| = 1$]

$|z^2-3| \le 4$ ....(1)

Also, $|z^2- 3| \ge ||z^2|-|-3||$

$\ge ||z|^2-3|$ [$\because |-3| = 3$]

$\ge |1^2-3|$ [$\because |z| = 1$]

$\ge |-2|$.

$|z^2-3| \ge 2$.............(2)

From (1) and (2) we get $2 \le |z^2-3| \le 4$

Hence proved.

18) $cis\frac{2\pi}{9} + cis\frac{4\pi}{9} + cis\frac{6\pi}{9} + cis\frac{8\pi}{9} + cis\left(\frac{10\pi}{9}\right) + cis\frac{12\pi}{9} + cis\frac{14\pi}{9} + cis\frac{16\pi}{9}$

$= cis\left(\frac{2\pi}{9} + \frac{4\pi}{9} + \frac{6\pi}{9} + \frac{8\pi}{9} + \frac{10\pi}{9} + \frac{12\pi}{9} + \frac{14\pi}{9} + \frac{16\pi}{9}\right)$

$= cis\frac{2\pi}{9}(1+2+3+\ldots+8) = cis\frac{\cancel{2}\pi}{\cancel{9}} \times \frac{8\times\cancel{9}}{\cancel{2}}$

$= \left[\because 1+2+3+\ldots.+n = \frac{n(n+1)}{2}\right]$

= c is 8π = [cos(8π)+i sin 8π]

= -1 + i(0) [∴ cos8π = -1 and sin 8π = 0 = -1

IV. ANSWER ANY 4 QUESTION 4 x 5 = 20

19) Given that $|z_1| = |z_2| = |z_3| = r \Rightarrow z_1\bar{z_1} = z_2\bar{z_2} = r^2$

$\Rightarrow z_1 = \frac{r^2}{\bar{z_1}}, z_2 = \frac{r^2}{\bar{z_2}}, z_3 = \frac{r^2}{\bar{z}_3}$

Therefore $z_1 + z_2 + z_3 = \frac{r^2}{\bar{z}_1} + \frac{r^2}{\bar{z_2}} + \frac{r^2}{\bar{z_3}}$

= $r^2 \left(\frac{\bar{z_2}\bar{z_3}+\bar{z_1}\bar{z_3}+\bar{z_1}\bar{z_2}}{\overline{z_1}\bar{z_2}\bar{z_3}}\right)$

$|z_1 + z_2 + z_3| = |r^2| \left|\frac{\overline{z_2 z_3+z_1 z_3+z_1 z_2}}{\overline{z_1 z_2 z_3}}\right| \left(\because \bar{z}_1 + \bar{z}_2 = \overline{z_1 + z_2}\right)$

= $r^2 \frac{|z_2 z_3+z_1 z_3+z_1 z_2|}{|z_1||z_2||z_3|} \quad (\because |z| = |\bar{z}| and\ |z_1 z_2 z_3| = |z_1| |z_2| |z_3|)$

= $|z_1 + z_2 + z_3| = r^2 \frac{|z_2 z_3+z_1 z_3+z_1 z_2|}{r^3} = \frac{|z_2 z_3+z_1 z_3+z_1 z_2|}{r}$

$\frac{|z_2 z_3+z_1 z_3+z_1 z_2|}{|z_1+z_2+z_3|}$ = r (given that $z_1 + z_2 + z_3 \neq 0$)

Thus, $\left|\frac{z_2 z_3+z_1 z_3+z_1 z_2}{z_1+z_2+z_3}\right|$ = r

20) Given z = x + iy

Im $\left(\frac{2z+1}{iz+1}\right)$ = 0

⇒ Im $\left(\frac{2(x+iy)+1}{i(x+iy)+1}\right)$ = 0

⇒ Im $\left(\frac{(2x+1)+2iy}{ix+i^2y+1}\right)$

⇒ Im $\left(\frac{(2x+1)+2iy}{ix-y+1}\right)$

$\left(\frac{(2x+1)+iy}{(1-y)+ix}\right)$

Multiply and divide by the conjugate of the denominator

We get Im $\left(\frac{(2x+1)+2iy}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}\right)$=0

⇒ Im $\left(\frac{(2x+1)+2iy\times(1-y)-ix}{(1-y)^2+x^2}\right)$

Choosing the imaginably part we get,

$\frac{(2x+1)(-x)+2y(1-y)}{(1-y)^2+x^2}$

⇒ (2x+1)-x+2y(1-y) = 0

⇒ $-2x^2-x+2y-2y^2 = 0$

⇒ $2x^2+2y^2+x-2y = 0$

Hence, locus of z is $2x^2+2y^2+x-2y = 0$

21) Given $(z-1)^3 + 8 = 0$

$(z-2)^3 = -8 = -1 \times 2^3$

⇒ $z-1 = (-1)^{1/3} \times 2$

⇒ z-1 = 2[cos π + i sin π]$^{1/3}$

z-1 = 2

[cos$\frac{1}{3}$(2kπ + π) + i sin$\frac{1}{3}$(2kπ + π)], k = 0, 1, 2

When k = 0

z-1 = $2\left[cos\frac{\pi}{3} + isin\frac{\pi}{3}\right]$

= $2\left[\frac{1}{2} + \frac{-\sqrt{3}}{2}\right]$ = 1+i$\sqrt{3}$c

= $-2\omega^2$ ....(1)

[∵ $\omega^2 = \left[\frac{-1i-\sqrt{3}}{2}\right]$ ⇒ $2\omega^2 = -1-i\sqrt{3}$

⇒ $2\omega^2 = -1 + i\sqrt{3}$

⇒ $z = 1 - 2\omega^2$....(2)

When k = 1, z -1 = 2 = $2\left[cos\frac{3\pi}{3} + isin\frac{3\pi}{3}\right]$

= 2[cos π + isin π] = -2

⇒ z = -2+1 = -1 ...(2)

z-1 = $2\left[cos\frac{5\pi}{3} + isin\frac{5\pi}{3}\right]$

= $2\left[cos\left(2\pi - \frac{\pi}{3}\right) + isin\left(2\pi - \frac{\pi}{3}\right)\right]$

= $2\left[cos\frac{\pi}{3} - isin\frac{\pi}{3}\right] = 2\left[\frac{1}{2} + \frac{-\sqrt{3}}{2}\right]$

= $1 - i\sqrt{3} = -2\omega$

[∵ ω = $\frac{-1+i\sqrt{3}}{2} \Rightarrow 2\omega = -1 + i\sqrt{3}$

⇒ -2ω = 1-i$\sqrt{3}$

⇒ z = 1-2ω^2 …… (3)

From (1), (2) and (3), the roots are -1, 1-2ω and 1-2ω^2.

22) $\frac{2\left(cos\frac{9\pi}{4}+isin\frac{9\pi}{4}\right)}{4\left(cos\left(\frac{-3\pi}{2}+\right)sin\left(\frac{-3\pi}{2}\right)\right)}$

= $\frac{1}{2}\left(cos\left(\frac{9\pi}{4} - \left(\frac{-3\pi}{2}\right)\right) + isin\left(\frac{9\pi}{4} - \left(\frac{-3\pi}{2}\right)\right)\right)$

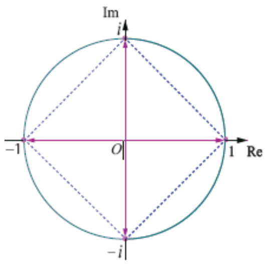
= $\frac{1}{2}\left(cos\left(\frac{9\pi}{4} + \frac{3\pi}{2}\right) + isin\left(\frac{9\pi}{4} + \frac{3\pi}{2}\right)\right)$

= $\frac{1}{2}\left(cos\left(\frac{15\pi}{4}\right) + isin\left(\frac{15\pi}{4}\right)\right) = \frac{1}{2}\left(cos\left(4\pi - \frac{\pi}{4}\right) + isin\left(4\pi - \frac{\pi}{4}\right)\right)$

= $\frac{1}{2}\left(cos\left(\frac{\pi}{4}\right) - isin\left(\frac{\pi}{4}\right)\right) = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$\frac{2\left(cos\frac{9\pi}{4}+isin\frac{9\pi}{4}\right)}{4\left(cos\left(\frac{-3\pi}{2}+\right)sin\left(\frac{-3\pi}{2}\right)\right)}$ = $\frac{1}{2\sqrt{2}} - i\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} + i\frac{\sqrt{2}}{4}$ Which is in rectangular form.

23)



Fourth roots of unity

We have to find $1^{\frac{1}{4}}$. Let z^4 = $1^{\frac{1}{4}}$. Then z^4 = 1.

In polar form, the equation z = 1 can be written as

$z^4 = cos\left(0 + 2k\pi\right) + isin\left(0 + 2k\pi\right) = e^{i2k\pi}$, k = 0, 1, 2,…

Therefore,$(z)^{\frac{1}{4}} = cos\left(\frac{2k\pi}{4}\right) + isin\left(\frac{2k\pi}{4}\right) = e^{i\frac{2k\pi}{4}}$, k=0,1,2,3.

Taking k = 0, 1, 2, 3, we get

k = 0, z = cos 0 + isin 0 = 1

k = 1, $z = cos\left(\frac{\pi}{2}\right) + isin\left(\frac{\pi}{2}\right) = i$

k = 2, $z = cos\pi + isin\pi = -1$

k = 3, $z = cos\frac{3\pi}{2} + isin\frac{3\pi}{2} = -cos\frac{\pi}{2} - isin\frac{\pi}{2} = -i$

Fourth roots of unity are 1, i, −1, −i ⇒ 1, ω, ω^2 and ω^3, where $\omega = e^{i\frac{2\pi}{4}} = i$

24) $(1+i)^{18}$

Let 1+ i = $r(cos\theta + isin\theta)$. Then , we get

$r = \sqrt{1^2 + 1^2} = \sqrt{2}; \alpha = tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$\theta = \alpha = \frac{\pi}{4}$ (∵ 1+i lies in the first Quadrant)

Therefore 1+ i = $\sqrt{2}\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$

Raising the power 18 on both sides

$(1+i)^{18} = \left[\sqrt{2}\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)\right]^{18} = \sqrt{12}^{18}\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$

By de Moivre's theorem

$(1+i)^{18} = 2^9\left(cos\frac{18\pi}{4} + isin\frac{18\pi}{4}\right)$

= $2^9\left(cos\left(4\pi + \frac{\pi}{2}\right) + isin\left(4\pi + \frac{\pi}{2}\right)\right) = 2^9\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)$

$(1+i)^{18} = 2^9\left(cos\frac{18\pi}{4} + isin\frac{18\pi}{4}\right)$

= $(1+i)^{18} = 2^9(i) = 512i$