Sri Raghavendra Tuition Center

COMPLEX UNIT 2

12th Standard

Maths

Date : 13-May-24

Reg.No. : | | | | | | |

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APPLICATION NAME: ARCHANGEL

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ONLINE / OFFLINE CLASSES AVAILABLE

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Exam Time : 01:30:00 Hrs

Total Marks : 50

I. ANSWER ALL QUESTION 10 x 1 = 10

1) If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

2) If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

3) The solution of the equation $|z|$ - z = 1 + 2i is

(a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$

4) The principal argument of $\frac{3}{-1+i}$ is

(a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

5) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

6) The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

7) The area of the triangle formed by the complex numbers z, iz and z+iz in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

8) If z is a complex number such that $z \in \mathbb{C}\backslash\mathbb{R}$ and $z + \frac{1}{z}\epsilon R$, then $|z|$ is

(a) 0 (b) 1 (c) 2 (d) 3

9) If (1+i)(1+2i)(1+3i)...(1+ni) = x + iy, then $2 \cdot 5 \cdot 10... \left(1 + n^2\right)$ is

(a) 1 (b) i (c) x^2+y^2 (d) $1+n^2$

10) If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}$ = 3k, then k is equal to

(a) 1 (b) -1 (c) $\sqrt{3i}$ (d) $-\sqrt{3i}$

II.ANSWER ANY 5 QUESTION 5 x 2 = 10

11) Evaluate the following if z = 5−2i and w = −1+3i

z − iw

12) Find the following $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$

13) Find the modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$

14) Find the following $\left|\frac{2+i}{-1+2i}\right|$

III. ANSWER ANY 3 QUESTION 5 x 3 = 15

15) The complex numbers u, v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If v = 3−4i and w = 4+3i, find u in rectangular form.

16) If z_1 = 3, z_2 = -7i, and z_3 = 5 + 4i, show that $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$

17) If $\omega \neq 1$ is a cube root of unity, then the show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$

18) For any two complex number z_1 and z_2 such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that $\frac{z_1+z_2}{1+z_1 z_2}$ is real number.

19) If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3)\ldots(x_n + iy_n) = a + ib$, show that
$\sum_{r=1}^{n} tan^{-1}\left(\frac{y_r}{x_r}\right) = tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k\epsilon Z$

20) Show that the following equations represent a circle, and, find its centre and radius
$|3z-6+12i| = 8$

IV. ANSWER ANY 3 QUESTION 3 x 5 = 15

21) If z = x + iy is a complex number such that Im $\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

22) If $\frac{1+z}{1-z} = cos2\theta + isin2\theta$, show that z = i tan$\theta$

23) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

24) If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

25) Find the cube roots of unity.

All The Best

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