Sri Raghavendra Tuition Center

Slip test 14 - Unit 2.1 to 2.4 , 12.2

12th Standard

Date : 18-May-24

Reg.No. :

Maths

TEACHER NAME: P. DEEPAK M.Sc., M.A.,B.Ed.,DCA.,TET-1.,TET-2.,
APPLICATION NAME: ARCHANGEL
PHONE NUMBER: 9944249262
ONLINE / OFFLINE CLASSES AVAILABLE
TRICHY(DT), THOTTIYAM(TK), 621207

Time : 01:30:00 Hrs

Total Marks : 50

I. Answer all questions 10 x 1 = 10

1) (b) Multiplication

2) (b) (SxS) $\longrightarrow$ S

3) (d) $\sqrt{5}$ is an irrational number

4) (b) Z

5) (a) ¬r→(¬p∧¬q)

6) (b) 8

7) (b)

(a)	(b)	(c)	(d)
F	T	T	T

8) (c) 3

9) (d) If p and q are any two statements then p↔q is a tautology.

10) (c) logically equivalent to p ∧ q

II. Answer any 5 question 5 x 2 = 10

11) Given z_1= 2 - i and z_2= -4+3i

z_1z_2 = (2-i)(-4+3i)

= -8 + 6i + 4i - $3i^2$

= -8 +10i - 3(-1)

= -8 +10i + 3 = -5 +10i

Inverse of z_1z_1 is $\frac{1}{z_1z_2}$

= $\frac{1}{-5+10i} \times \frac{-5-10i}{-5-10i}$

= $\frac{-5-10i}{(-5)^2-(10i)^2}$

= $\frac{-5-10i}{25-100i^2}$

= $\frac{-5-10i}{25+100}$ [∵ i^2 = -1]

$= \frac{5\!\!\!/(-1-2i)}{5\!\!\!/(25)} = \frac{-1-2i}{25}$

∴ Inverse of z_1z_2 is $\frac{1}{25}$ (-1-2i)

Inverse of $\frac{z_1}{z_2}$ is $\frac{1}{\frac{z_1}{z_2}} = \frac{z_2}{z_2}$

∴ Inverse of $\frac{z_1}{z_2} = \frac{z_2}{z_1} = \frac{-4+3i}{2-i} \times \frac{2+i}{2+i}$

= $\frac{-4+3i}{2-i} \times \frac{2+i}{2+i}$

= $\frac{-8-4i+6i+3i^2}{2^2-(i^2)}$

= $\frac{-8+2i-3}{4+1} = \frac{-11+2i}{5}$

$= \frac{1}{5}$(-11 + 2i)

$\therefore$ Inverse of $\frac{z_1}{z_2}$ is $\frac{-11+2i}{5}$ or $\frac{1}{5}$(-11 + 2i)

12) We have, $z^2 = \bar{z}$

$\Rightarrow |z|^2 = |z|$

$|z|\left(|z| - 1\right) = 0$

$\Rightarrow |z| = 0, or\, |z| = 1$

$|z| = 0 \Rightarrow z = 0$ is a solution $|z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$

Given $z^2 = \bar{z} \Rightarrow z^2 = \frac{1}{z} \Rightarrow z^3 = 1$

It has 3 non-zero solutions. Hence including zero solution, there are four solutions.

13) let z = |4+3i|

= $\sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25}$

$\sqrt{a+ib} = \pm\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}$

[Here |z| = 5, a = 4, b = 3]

$\sqrt{4+3i} = \pm\sqrt{\frac{5+4}{2}} + i\frac{3}{|3|}\sqrt{\frac{5-4}{2}}$

= $\pm\sqrt{\frac{9}{2}} + i\frac{3}{3}\sqrt{\frac{1}{2}}$

= $\pm\frac{3}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

Aliter :

Square root of 4 + 3i

Formula method

$\sqrt{a+ib} = \pm\left[\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$

Now, $|4+3i| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore \sqrt{4+3i} = \pm\left[\sqrt{\frac{5+4}{2}} + i\sqrt{\frac{5-4}{2}}\right]$

$= \pm\left[\frac{3}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$

14) Then A∨ B = $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0\vee 1 & 1\vee 1 \\ 1\vee 0 & 1\vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0\wedge 1 & 1\wedge 1 \\ 1\wedge 0 & 1\wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

15) p ∨ ¬ t (p ∨ ¬s) contains 3 variables p, s, and t. Hence the corresponding truth table will contain 2^3 = 8 rows

16) ((p ∧ q) ∨ (¬r ∨¬s)) ∧ (¬ t ∧ v)) contains 6 variables p, q, r, s, t, and v. Hence the corresponding truth table will contain 2^6 = 64 rows.

17) Truth Table for ~p ∧ ~q

p	**q**	**~p**	**~q**	**~p ∧ ~q**
T	T	F	F	F
T	F	F	T	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

18) Truth Table for ~(p ∧ ~q)

p	**q**	**~q**	**p ∧ ~q**	**~(p ∧ ~q)**

T	T	F	F	T
T	F	T	T	F
F	T	T	F	T
F	F	T	F	T

19) Truth Table for (p V q) ∧ ~q

p	**q**	**p V q**	**~q**	**(p V q) ∧ ~q**
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

III. Answer any 5 questions 5 x 3 = 15

20) $|z-2-i|=3$

⇒ |z-(2+i)| = 3

It is of the form |z - z_0| = r and so it represents a circle.

Centre is (2, 1) and radius = 3 units.

Aliter:

Let z = x +iy

|z-2-i| = 3

|x + iy-2-i| = 3

$|(x-2)+i(y-1)|=3$

$\sqrt{(x-2)^2+(y-1)^2}=3$

Squaring on both sides

$(x-2)^2+(y-1)^2=9$

$x^2-4x+4+y^2-2y+1-9=0$

$x^2+y^2-4x-2y-4=0$

Comparing with General form of equation of circle

$ax^2+by^2+2gx+2fy+c=0$

we get a = 1, b = 1, g = -2, f = -1, c = -4

Centre (- g, - f) = (2, 1)

radius = $\sqrt{g^2+f^2-c}=\sqrt{4+1+4}=\sqrt{9}$

= 3 units

21) Given 2cos α = x+$\frac{1}{x}$

⇒ 2cos α = $\frac{x^2+1}{x}$

⇒ x^2+1 = 2xcos α

⇒ x^2-2x cos α+1 = 0

⇒ $\frac{2cos\alpha\pm\sqrt{(-2cos\alpha)^2-4(1)(1)}}{2}$

= $\frac{2cos\alpha\pm\sqrt{4cos^2\alpha-4}}{2}$ $\left[\because \frac{b\pm\sqrt{b^2-4ac}}{2a}\right]$

= $\frac{2cos\alpha\pm\sqrt{-sin^2\alpha}}{2}$

= $\frac{2cos\alpha\pm isin\alpha}{2}$ [∵ sin^2α+cos^2α = 1]

⇒ x^2 = cos α ± sin α

Also, 2cos β = y+$\frac{1}{y}$

⇒ 2cos β = $\frac{y^2+1}{y}$

⇒ y^2-2y cos β+1 = 0

⇒ $\frac{2cos\beta\pm\sqrt{(-2cos^2\beta^2-4(1)(1)}}{2}$

= $\frac{2cos\beta\pm\sqrt{4cos^2\beta-4}}{2}=\frac{2cos\beta\pm2isin\beta}{2}$

$\Rightarrow y = \cos\beta \pm i \sin\beta$

$$x^m y^n + \frac{1}{x^m y^n} = 2cos(m\alpha + n\beta)$$

$x^m y^n = (\cos \alpha + i \sin m\alpha)(\cos n\beta + i \sin n\beta)$

$\cos(m\alpha+n\beta)+i \sin(m\alpha+n\beta)$

$\frac{1}{x^m y^n} = \cos(m\alpha+n\beta)-i \sin(m\alpha+n\beta)$

$$\therefore x^m y^n + \frac{1}{x^m y^n} = \cos(m\alpha+n\beta)+i\sin(\not{m\alpha+n\beta}) + \cos(m\alpha+n\beta)-i\sin(\not{m\alpha+n\beta})$$

$= 2\cos(m\alpha+n\beta)$

22)

p	q	¬ q	$r : (p \vee q)$	s:$(p \vee \neg q)$	r ∧ s
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

Also the above result can be proved without using truth tables. This proof will be provided after studying the logical equivalence

23)

p	q	q → p	~p	~q	~q → ~p
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

The entries in the columns corresponding q → p and ~p → ~q are identical and hence they are equivalent.

q → p ≡ ~p → ~q

Hence proved

24)

p	q	p → q	q⟶ p	p ↔ q	(p → q) ∧ (q⟶ p)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

The entries in the columns corresponding to p ↔ q and (p ⟶ q) ∧ (q ⟶ p) are identical and hence they are equivalent

25)

p	q	p↔q	~(p↔q)	~q	p↔~q
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

The entries in column (4) and (6) are identical ~(p↔q) ≡ p↔~q

Any 3 Question 3 x 5 = 15

26) Area of the triangle formed by the vertices z, iz and Z+ iz is 50 sq. units

Let z = x + iy

Then $iz = i(x + iy) = ix + i^2y = -y + ix$

z + iz = x + iy-y + ix

= (x - y) + i(x + y)

If A denotes the area of the triangle formed by z, iz and z + iz, then

$$A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix}$$

$R_2 \longrightarrow R_2$-R_1-R_3, we get

$$A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{vmatrix}$$

Expanding along R_2 we get

$$A = \frac{1}{2}\left[+1\begin{vmatrix} x & y \\ -y & x \end{vmatrix}\right] = \frac{1}{2}(x^2+y^2)$$

Given A = 50 sq units

$\therefore 50 = \frac{1}{2}(x^2+y^2) \Rightarrow 100 = x^2+ y^2$

Then $\sqrt{x^2+y^2} = \sqrt{100} = 10$

$\therefore |z| = 10$ $[\because |z| = \sqrt{x^2+y^2}]$

Aliter :

Given area of triangle = 50 sq. unit

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ -x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix} = 50$$

$$\xrightarrow{R_2\to R_2-R_3} \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{vmatrix} = 50$$

$$\frac{1}{2}\left[\begin{vmatrix} x & y \\ -y & x \end{vmatrix}\right] = 50$$

$$\frac{1}{2}\left[x^2+y^2\right] = 50$$

$$x^2+y^2 = 100$$

$$|z|^2 = 100$$

$|z| = 10$

27) Prove that p⟶(q⟶r) = (p ∧ q)⟶r without using truth table. From example we know that p⟶ q = ~p V q

Consider LHS = p⟶(q⟶r)

= p ⟶ (~q V r) [Implication Law]

= ~p V (~q V r) [Implication Law]

= (~p V ~q) V r [associative property]

= ~(p ∧ q) V r [using Demorgan's law]

= (p ∧ q) ⟶ r [Implication Law]

Hence proved.

28) 29)

p	q	r	~ q	~q V r	p→(¬qVr)	~p	~pV(~qVr)
T	T	T	F	T	T	F	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the table, it is clear that the column of p→(¬q V ~r) and ~pV(~q V r) are identical

∴ p→(¬q V ~r) ≡ ~pV(~q V r)

Hence proved.

30) To prove $\neg(p \to q) \equiv p \wedge \neg q$

Truth table for $\neg(p \to q)$

p	**q**	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Truth table for $p \wedge \neg q$

p	**q**	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	T

The entries in the column $\neg(p \to q)$ and $p \wedge \neg q$ are identical and they are equivalent.