BELIEVERS TUITION CENTER
AMMAPET, SALEM – 3
LESSON 1 ELECTRO STATICS REDUCED SYLLABUS STUDY MATERIAL

2 MARKS:

1. What is meant by quantisation of charges?
   The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.
   $q = ne$
   Here n is any integer (0, ±1, ±2, ±3, ±4). This is called quantisation of electric charge.

2. Write down Coulomb's law in vector form and mention what each term represents.
   The force on a charge q_1 exerted by a point charge q_1 is given by
   $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon 0}\frac{q_1q_2}{r^2}\hat{r}_{21}$$ BELIEVERS EDUCATION
   Here $\hat{r}_{21}$ is the unit vector from charge q_1 to q_1.
   But $\hat{r}_{21} = -\hat{r}_{12}$,
   $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\left(-\hat{r}_{12}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\left(\hat{r}_{12}\right) \quad \text{or} \quad \vec{F}_{12} = -\vec{F}_{21}$$
   Therefore, the electrostatic force obeys Newton's third law.

3. What are the differences between Coulomb force and gravitational force?
   - The gravitational force between two masses is always attractive but Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.
   - The value of the gravitational constant G = 6.626 x 10^{-11} N m^2 kg^{-2}. The value of the constant k in Coulomb law is k = 9 x 10^9 N m^2 C^2.
   - The gravitational force between two masses is independent of the medium. The electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.
   - The gravitational force between two point masses is the same whether two masses are at rest or in motion. If the charges are in motion, yet another force (Lorentz force) comes into play in addition to coloumb force.

4. Write a short note on superposition principle.
   According to this superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.
   $$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{1n}$$

5. Define 'electric field'.
   The electric field at the point P at a distance r from the point charge q is the force experienced
   by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The electric field is a vector quantity and its SI unit is NC^{-1}.

6. Define 'electric dipole'. Give the expression for the magnitiude of its electric dipole moment and the direction.
   Two equal and opposite charges separated by a small distance constitute an electric dipole.

   The dipole moment is a vector measure whose direction runs from negative to a positive charge. The formula for electric dipole moment for a pair of equal & opposite charges is p = qd, the magnitude of the charges multiplied by the distance between the two.

7. Write the general definition of electric dipole moment for a collection of point charge.
   The electric dipole moment vector lies along the line joining two charges and is directed from -q to + q. The SI unit of dipole moment is coulomb meter (Cm).
   $$\vec{P} = qa\hat{i} - qa(-\hat{i}) = 2qa\hat{i}$$

8. Define 'electrostatic potential".
   The electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external
   electric field $\vec{E}$.

9. Define 'electrostatic potential energy'.
   The potential energy of a system of point charges may be defined as the amount of work done in assembling the charges at their locations by bringing them in from infinity.

10. Define 'electric flux'.
    The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. Its unit is $N\ m^2\ C^{-1}$. Electric flux is a scalar quantity.
11. What is meant by electrostatic energy density?
    The energy stored per unit volume of space is defined as energy density $u_E = \frac{U}{Volume}$ BELIEVERS EDUCATION
    From equation $u_E = \frac{1}{2}\frac{(\varepsilon_0 A)}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 (Ad) E^2$ or $u_E = \frac{1}{2}\varepsilon_0 E^2$

12. Define 'capacitance'. Give its unit.
    The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.

$C = q / V$ or $Q \propto V$.

The SI unit of capacitance is coulomb per volt or farad (F).

13. What is corona discharge?

The electric field near the edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

5 MARKS:

1. Discuss the basic properties of electric charges.

Basic properties of charges

(i) Electric charge:

Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric charge is another intrinsic and fundamental property of particles. The SI unit of charge is coulomb.

(ii) Conservation of charges:

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged).

From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way. The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical process, the net change in charge will always be zero.

(iii) Quantisation of charges:

The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

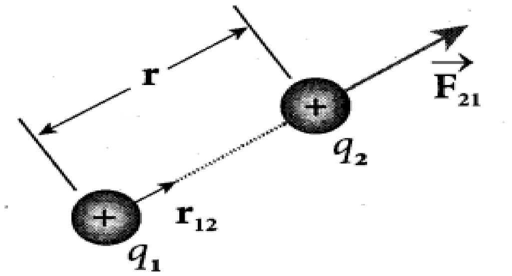
$q = ne$

Here n is any integer (0, ±1, ±2, ±3, ± ….). This is called quantisation of electric charge. Robert Millikan in his famous experiment found that the value of e = 1.6 x 10^{-19}C. The charge of an electron is -1.6 x 10^{-19} C and the charge of the proton is +1.6 x 10^{-19}C. When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of 10^{10}. So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

2. Explain in detail Coulomb's law and its various aspects.
Consider two point charges q_1 and q_2 at rest in vacuum, and separated by a distance of r. According to Coulomb, the force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{21} = K\frac{q_1 q_2}{r_2}\,\hat{r}_{12},$$

where [/latex] $\hat{r}_{12}$ is the unit vector directed from charge q_1 to charge q_2 and k is the proportionality constant.



Coulomb force between two point charges

Important aspects of Coulomb's law:
(i) Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.
(ii) The force on the charge $q_2$exerted by the charge q_1 always lies along the line joining the two charges. $\hat{r}_{21}$ is the unit vector pointing from charge q_1 to q_2 Likewise, the force on the charge q_1 exerted by q_2 is along – (i.e., in the direction opposite to $\hat{r}_{21}$).
(iii) In SI units, $k = 1 / 4\pi\varepsilon_0$ and its value is 9×10^9 Nm^2C^{-2}. Here e0 is the permittivity of free space or vacuum and the value of
$\varepsilon_0 = 1 / 4\pi\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$
(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$$[F] = \frac{9\times10^9\times1\times1}{1^2} = 9 \times 10^9 N.$$

This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of pC (micro coulomb) or nC (nano coulomb).

(v) In SI units, Coulomb's law in vacuum takes the form $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_2} \hat{r}_{12}$. sin Since ε > ε_0, the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as ε = $\frac{\varepsilon}{\varepsilon_0}$ .For vacuum or air, ε_r = 1 and for all other media ε_r > 1

(vi) Coulomb's law has same structure as Newton's law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses.

(vii) The force on a charge q_1 exerted by a point charge q_2 is given by $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_2} \hat{r}_{12}$ Here $\hat{r}_{21}$ is sthe unit vector from charge q_2 to q_1.

But $\hat{r}_{21} = -\hat{r}_{12}$, $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}\left(-\hat{r}_{12}\right)$

$= -\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}\left(\hat{r}_{12}\right)$ (or) $\vec{F}_{12} = -\vec{F}_{21}$ BELIEVERS EDUCATION

Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

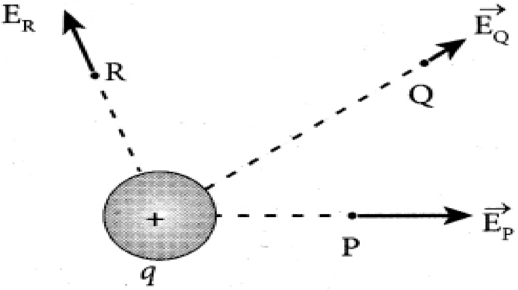
3. Define 'electric field' and discuss its various aspects.
The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

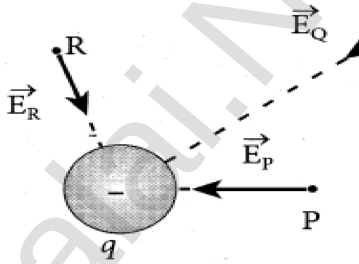
Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is Newton per Coulomb (NC^{-1}).
Important aspects of Electric field:
(i) If the charge q is positive then the electric field points away from the source charge and if q is negative, the electric field points towards the source charge q.

(ii) If the electric field at a point P is $\vec{E}$, then the force experienced by the test charge qo placed at the point P is A $\vec{F} = q_0 \vec{E}$. This is Coulomb's law in terms of electric field. This is shown in the below figure.

(iii) The equation implies that the electric field is independent of the test charge q0 and it depends only on the source charge q.
(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures (a) and (b). From equation, we can infer that as distance increases, the electric field decreases in magnitude. Note that in Figures (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the point Q and R because the point P is closer to the source charge.



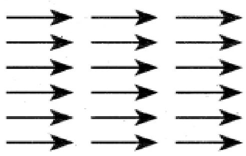
(a) Electric field due to positive charge

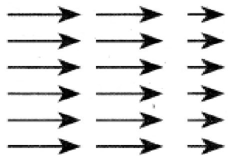


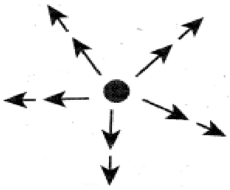
(b) Electric field due to negative charge

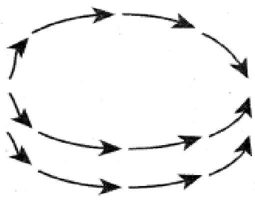
(v) In the definition of electric field, it is assumed that the test charge q_0 is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

(vi) The expression is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used. However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge.
(vii) There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non-uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases.


Uniform Electric field


Non-uniform electric field


Non-uniform electric field


Non-uniform electric field

4. Derive an expression for the torque experienced by a dipole due to a uniform electric field.

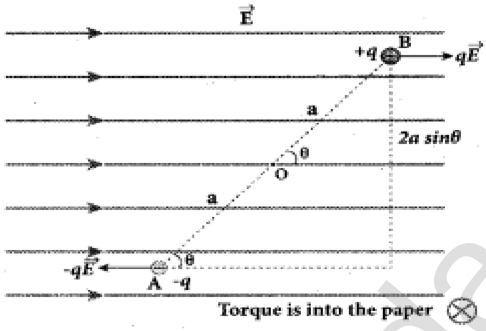
Torque experienced by an electric dipole in the uniform electric field:

Consider an electric dipole of dip ole moment $\vec{P}$ placed in a uniform electric field E whose field lines are equally spaced and point in the same direction. The charge +q will experience a force q$\vec{E}$ in the direction of the field and charge -q will experience a force -q$\vec{E}$ in a direction opposite to the field.

Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque. This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction). The total torque on the dipole about the point O

$$\vec{\tau} = \overrightarrow{OA} \times (-q\vec{E}) + \overrightarrow{OB} \times q\vec{E}$$

Using right-hand corkscrew rule, it is found that total torque is perpendicular to the plane of the paper and is directed into it.



Torque on dipole

The magnitude of the total torque

$$\vec{\tau} = |\overrightarrow{OA}|(-q\vec{E}) \sin\theta + |\overrightarrow{OB}|\,|q\vec{E}|\sin\theta$$

where θ is the angle made by $\vec{P}$ with $\vec{E}$. Since p = 2aq, the torque is written in terms of the vector product as

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The magnitude of this torque is $\tau = pE \sin\theta$ and is maximum Torque on dipole when θ =90°.

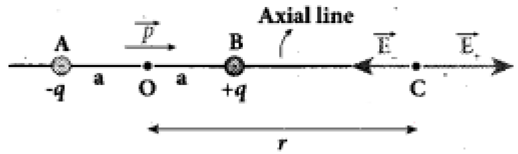
This torque tends to rotate the dipole and align it with the electric field $\vec{E}$.

Once $\vec{E}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero.

5. Calculate the electric field due to a dipole on its axial line and equatorial plane.

Case (I) :

Electric field due to an electric dipole at points on the axial line. Consider an electric dipole placed on the x-ax is as shown in figure. A point C is located at a distance of r from the midpoint O of the dipole along the axial line. Axial line



Electric field of the dipole along the axial line

The electric field at a point C due to +q is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ along BC}$$

Since the electric dipole moment vector $\vec{P}$ is from -q to +q and is directed along BC, the above equation is rewritten as

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} \qquad ...(1)$$

where $\hat{p}$ is the electric dipole moment unit vector from -q to +q. The electric field at a point C due to -q is

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p} \qquad ...(2)$$

Since +q is located closer to the point C than -q, . $\vec{E}_+$ us stronger than $\vec{E}_-$.

Therefore, the length of the E + vector is drawn large than that of $\vec{E}_-$ vector.
The total electric field at point C is calculated using the superposition principle of the electric field.

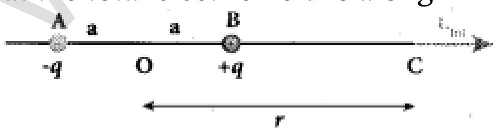
$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_- \qquad ...(3)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p}$$

$$\Rightarrow \vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p} \qquad ...(4)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p} \qquad ...(5)$$

Note that the total electric field is along $\vec{E}_+$, since +q is closer to C than -q.



Total electric field of the dipole on the axial line

The direction of $\vec{E}_{tot}$ is shown in Figure
If the point C is very far away from the dipole then (r >> a). Under this limit the term$(r^2 - a^2)^2 \approx r^4$ Substituting this into equation, we get

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p} \quad (r >> a) \quad ...(6)$$

since $2aq\ \hat{p} = \vec{p}$

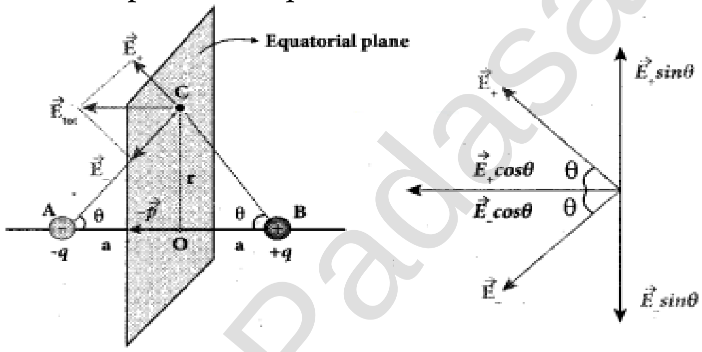
$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} \quad (r >> a)$$

If the point C is chosen on the left side of the dipole, the total electric field is still in the

Case (II) :

Electric field due to an electric dipole at a point on the equatorial plane Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in Figure. Since the point C is equi-distant from +q and -q, the magnitude of the electric fields of +q and -q are the same. The direction of E+ is along BC and the direction of E is along CA. E+ and E_ are resolved into two components; one component parallel to the dipole axis and the other perpendicular to it.

The perpendicular components $|\vec{E}_+|\sin\theta$ and $||\vec{E}_-||\sin\theta$ are oppositely directed and cancel each other. The magnitude of the total electric field at point C is the sum of the paralle component of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$.



Electric field due to a dipole at a point on the equatorial plane

$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\,\hat{p} - |\vec{E}_-|\cos\theta\,\hat{p} \quad ...(1)$$

The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the same and are given by

$$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \quad ...(2)$$

By substituting equation (1) into equation (2), we get

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{2q\cos\theta}{(r^2+a^2)}\hat{p} = -\frac{1}{4\pi\varepsilon_0}\frac{2qa}{(r^2+a^2)^{\frac{3}{2}}}\hat{p}$$

$$\text{since } \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}}$$

since $\vec{p} = 2qa\hat{p} \quad ...(3)$

At very large distances (r >> a), the equation becomes

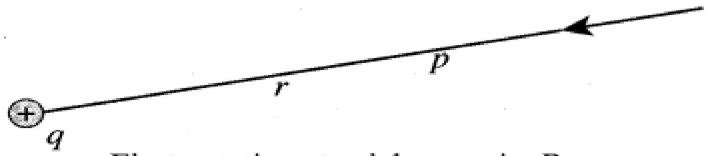
$$\vec{E}_{tot} \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3} \ (r >>) \ ...... (4)$$

6. Derive an expression for electrostatic potential due to a point charge.

Electric potential due to a point charge:

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q.

The electric potential at the point P is



Electrostatic potential at a point P

$$V = \int_{\infty}^{r} \left(-\vec{E}\right)\cdot d\vec{r} = -\int_{\infty}^{r} \vec{E}.\, d\vec{r} \qquad ...(1)$$

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad V = \frac{-1}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{q}{r^2}\hat{r}.d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr\hat{r}$ and using $\hat{r}\ .\ \hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{q}{r^2}\hat{r}.\, dr\hat{r} = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{q}{r^2}dr$$

After the integration,

$$V = -\frac{1}{4\pi\varepsilon_0}q\left\{-\frac{1}{r}\right\}_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \ ...... (2)$$

Important points (If asked in exam)

(i) If the source charge q is positive, V > 0. If q is negative, then V is negative and equal to

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

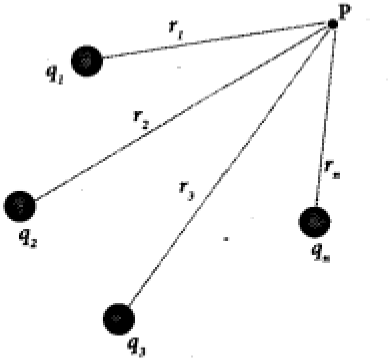
(ii) The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.

(iii) From expression (2), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity ($r = \infty$) electrostatic potential is zero (V = 0).

(iv) The electric potential at a point P due to a collection of charges $q_1, q_2, q_3 \ldots q_n$ is equal to sum of the electric potentials due to individual charges.

$$V_{tot} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + ... + \frac{kq_n}{r_n} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

Where r_1, r_2,r_3,.....r_n are the distances of q_1,q_2,q_3… q_n respectively from P

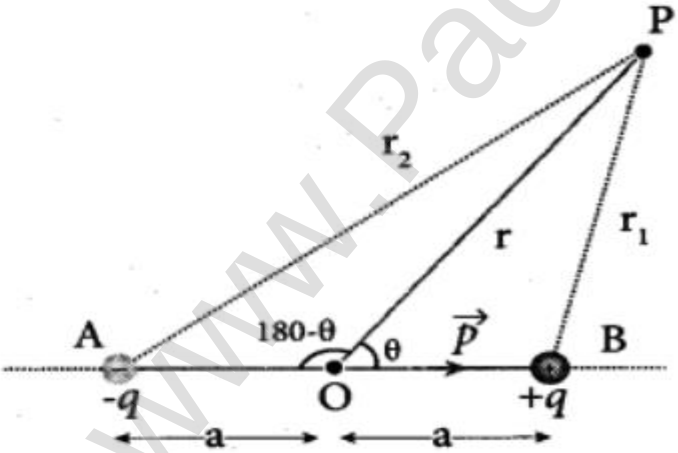


Electrostatic potential due to collection of charges

7. Derive an expression for electrostatic potential due to an electric dipole.
Consider two equal and opposite charges separated by a small distance 2a. The point P is located at a distance r from the midpoint of the dipole. Let 0 be the angle between the line OP and dipole axis AB.
Let r_1 be the distance of point P from +q and r_1 be the distance of point P from -q.
Potential at P due to charge +q = $(1 / 4\pi\varepsilon_0)(q / r_1)$
Potential at P due to charge -q = $(1 / 4\pi\varepsilon_0)(q / r_2)$
Total Potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \text{.........} (1)$$

Suppose if the point P is far away from the dipole, such that r >> a, then equation can be expressed in terms of r. By the cosine law for triangle BOP,



Potential due to electric dipole

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta = r^2$$
$$\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$$

Since the point P is very far from dipole, then r >> a. As a result the term a^2 / r^2 is very small and can be neglected. Therefore

$$r_1^2 = r^2\left(1 - 2a\frac{\cos\theta}{r}\right) \text{ (or) } r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}} \text{ (or)}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

since a / r << 1, we can use binominal theorem and retain the terms up to first order

1 / r = (1 + a / r cosθ) .... (2)

Similarly applying the cosine law for triangle AOP,

$r_2^2 = r^2 + a^2 - 2ra\cos(180 - \theta)$

Since cos (180 – θ) = cos θ we get

$r_2^2 = r^2 + a^2 + 2ra\cos\theta$

Neglecting the term $\frac{a^2}{r^2}$ (because r >> a)

$r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$ (or) $r_2 = r$

$\left(1 + \frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$

Using Binomial theorem,

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)$$

we get Substituting equations (3) and (2) in equation (1)

$$V = \frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{r}\left(1 + a\frac{\cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right)\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^2}\cos\theta$$

But the electric dipole moment p = 2qa and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write p cos θ = P⃗. r^ where r^ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \quad (r >> a) \text{ ..... (4)}$$

Equation (4) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (4) is valid for any distance.

Special cases:

Case (I): If the point P lies on the axial line of the dipole on the side of +q, then θ = 0. Then the electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

Case (II): If the point P lies on the axial line of the dipole on the side of -q, then θ = 180° , then

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

Case (III): If the point P lies on the equatorial line of the dipole, then θ = 90°. Hence, V = 0.

8. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.

Electrostatic potential energy for collection of point charges:

The electric potential at a point at a distance r from point charge ql is given by

$V = (1 / 4\pi\varepsilon_0) (q_1 / r)$ …… (1)

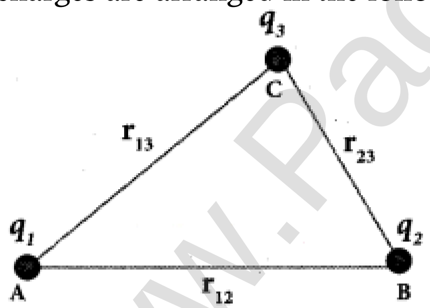
This potential V is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance r from qp the work done is the product of q_2 and the electric potential at that point. Thus we have $W = q_2V$ …… (2)

This work done is stored as the electrostatic potential energy U of a system of charges q_1 and q_2 separated by a distance r. Thus we have

$U = q_2 V = (1 / 4\pi\varepsilon_0) (q_1q_2 / r)$ …… (3)

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression (3) is derived by assuming that q_1 is fixed and q_2 is brought from infinity. The equation (3) holds true when q_2 is fixed and q_1 is brought from infinity or both $q_2$and q_2 are simultaneously brought from infinity to a distance r between them.

Three charges are arranged in the following configuration as shown in Figure.



Electrostatic potential
energy for Collection of point charges

To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration.

(i) Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1

(ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1 So the work done on the charge q_1 is $W = q_2V_{1B}$. Here V_{1B} is the electrostatic potential due to the charge q_1 at point B.

$U = (1 / 4\pi\varepsilon_0) (q_1q_2 / r_{12})$ ….. (4)

Note that the expression is same when q_2 is brought first and then q_1 later.

(iii) Similarly to bring the charge q3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is = q_3 ($V_{1C} + V_{2C}$). Here V_{1C} is the electrostatic potential due to charge q_1 at point C and V_{2C} is the electrostatic potential due to charge q_2 at point C. The electrostatic potential is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \quad ...(5)$$

(iv) Adding equations (4) and (5), the total electrostatic potential energy for the system of three charges q_1,q_2 and q_3 is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \quad ...(6)$$

Note that this stored potential energy U is equal to the total external work done to assemble the three charges at the given locations. The expression (6) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

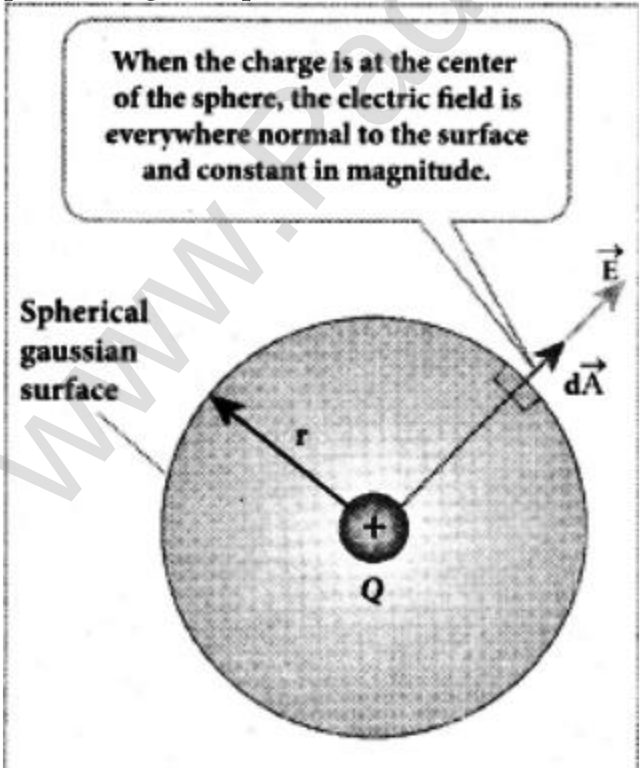
9. Obtain Gauss law from Coulomb's law.

Gauss law:

Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ through the closed surface is

$$\Phi_E = \oint \vec{E}\,.d\,\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in figure. We can calculate the total electric flux through the closed surface of the sphere using the equation.



Total electric flux of point charge

$$\Phi_E = \oint \vec{E}\,.d\,\vec{A} = \oint EdA\cos\theta \;......\;(1)$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element d is along the electric field and θ = 0°

$$\Phi_E = \oint EdA \text{ since } \cos 0° = 1 \;.....\;(2)$$

E is uniform on the surface of the sphere,

$$\Phi_E = \oint EdA \;.....\;(3)$$

$$\oint dA = 4\pi^2 \text{ and } E = \frac{1}{4\pi\varepsilon_0}$$

Substituting for Q in equation 3, we get

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2} \times 4\pi^2 = 4\pi\,\frac{1}{4\pi\varepsilon_0} = \frac{q}{\varepsilon_0} \;.......\;(4)$$

The equation (4) is called as Gauss's law. The remarkable point about this result is that the equation (4) is equally true for any arbitrary shaped surface which encloses the charge Q.

10. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.

Consider an electric dipole of dipole moment $\vec{P}$ = q(2a⃗) placed at an angle 0 in the direction of uniform electric field $\vec{E}$.

The dipole experiences torque

τ = P x E = pE sinθ

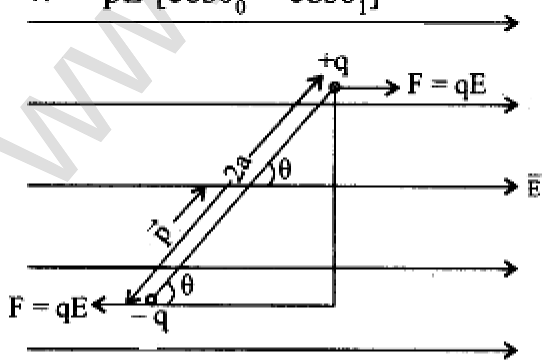
This torque tends to rotate the dipole in the direction of the electric field.

Suppose an external torque τ_{ext} is applied just to neutralise the torque (τ) and to rotate the dipole from θ_0 to θ_1 without angular acceleration then, the amount of work done by the external torque.

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}\, d\theta = \int_{\theta_0}^{\theta_1} pE\sin\theta \cdot d\theta \;\because\; [\tau_{ext} = \tau]$$

$$W = pE\left[-\cos\theta\right]_{\theta_0}^{\theta_1}$$

$$W = pE\,[\cos\theta_0 - \cos\theta_1]$$

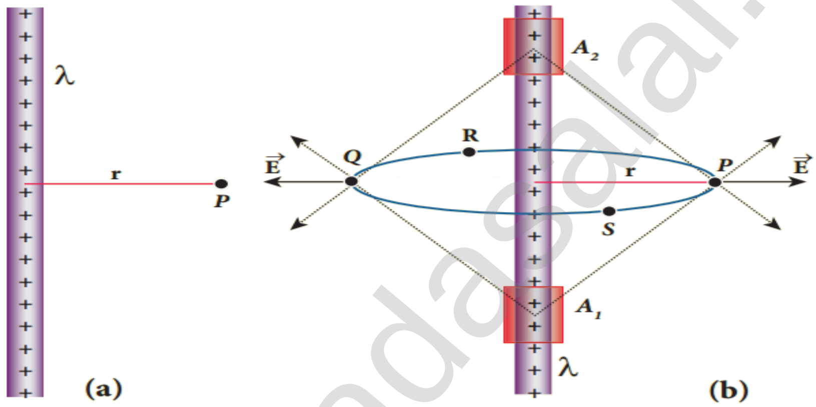


The work done in rotating the dipole from a position of zero potential energy ($\theta_0 = \pi / 2$) to any given position in the field (θ_1= θ) then,

$$W = pE\left[\cos\frac{\pi}{2} - \cos\theta\right]$$
$$= -pE\cos\theta$$

This quantity of work done is stored as potential energy U.
Thus, potential energy of an electric dipole in a uniform Electric field is
$U = -pE\cos\theta$

11. Obtain the expression for electric field due to an infinitely long charged wire.
Consider an infinitely long straight wire having uniform linear charge density λ. Let P be a point located at a perpendicular distance r from the wire. The electric field at the point P can be found using Gauss law. We choose two small charge elements A_1 and A_1 on the wire which are at equal distances from the point P. The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r. From this property, we can infer that the charged wire possesses a cylindrical symmetry.



Electric field due to infinite long charged wire

Let us choose a cylindrical Gaussian surface of radius r and length L. The total electric flux in this closed surface is

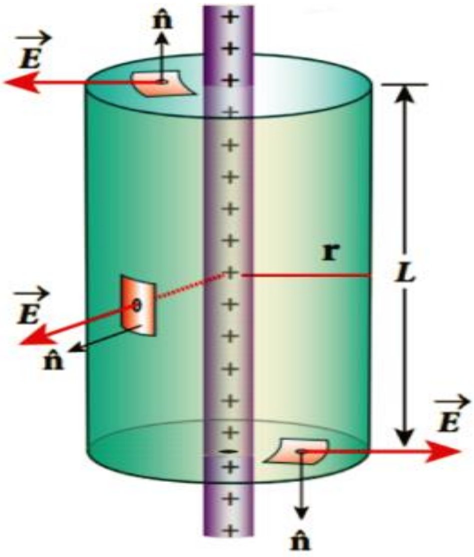
$$\Phi_E = \oint \vec{E}.d\vec{A} \quad ...(1)$$

$$\Phi_E = \int_{\text{Curved surface}} \vec{E}.d\vec{A} + \int_{\text{Top surface}} \vec{E}.d\vec{A} + \int_{\text{Bottom surface}} \vec{E}.d\vec{A} \quad ...(2)$$

It is seen that for the curved surface, is parallel to $\vec{A}$ and $\vec{E}$.d = EdA. For the top and $\vec{E}$ bottom surface, is perpendicular to and $\vec{A}$

$\vec{E}.d\vec{A} = 0$

Substituting these values in the equation (2) and applying Gauss law



Cylindrical Gaussian surface

$$\Phi_E = \int_{\text{Curved surface}} E\,dA = \frac{Q_{encl}}{\varepsilon_0} \quad ...(3)$$

Since the magnitude of the electric field for the entire curved surface is constant, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \lambda L$.

$$E \int_{\text{Curved surface}} dA = \frac{\lambda L}{\varepsilon_0} \quad ...(4)$$

Here,

$$\Phi_E = \int_{\text{Curved surface}} dA$$

dA = total area of the curved surface = $2\pi rL$. Substituting this in equation (4), We get

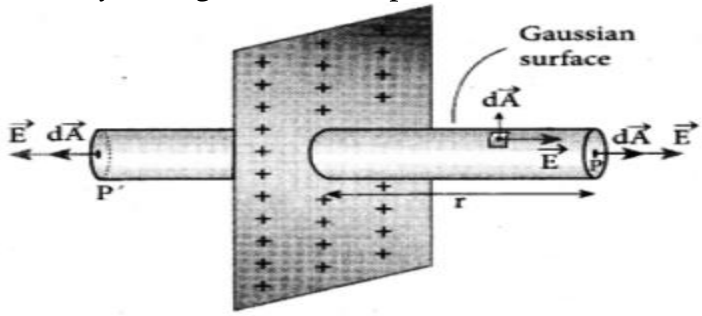
$$E.2\pi rL = \frac{\lambda L}{\varepsilon_0} \text{ (or) } E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} \quad ...(5)$$

$$\text{In vector form, } \vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r} \quad ...(6)$$

The electric field due to the infinite charged wire depends on $1/r$ rather than $1/r^2$ for a point charge.

Equation (6) indicates that the electric field is always along the perpendicular direction to wire. In fact, if $\lambda > 0$ then E points perpendicular outward from the wire and if $\lambda < 0$, then E points perpendicular inward.

12. Obtain the expression for electric field due to an charged infinite plane sheet.
Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet. Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points. A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.



Electric field due to charged infinite planar sheet

Applying Gauss law for this cylindrical surface,

$$\Phi_E = \oint \vec{E}.d\vec{A}$$

$$\Phi_E = \int_{\substack{\text{Curved}\\\text{surface}}} \vec{E}\cdot d\vec{A} + \int_P \vec{E}.d\vec{A} + \int_{P'} \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P'. Then,

$$\Phi_E = \int_P EdA + \int_{P'} EdA = \frac{Q_{encl}}{\varepsilon_0} \quad ...(2)$$

Since the magnitude of the electric field at these two equal surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E\int_P dA = \frac{\sigma A}{\varepsilon_0}$$

The total area of surface either at P or P'

$$\int_P dA = A$$

Hence $2EA = \frac{\sigma A}{\varepsilon_0}$ or $E = \frac{\sigma}{2\varepsilon_0}$ ...... (3)

In vector from, $E = \frac{\sigma}{2\varepsilon_0}\hat{n}$ ..... (4)

Hence n is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance r.
The electric field will be the same at any point farther away from the charged plane. Equation (4) implies that if σ > 0 the electric field at any point P is outward perpendicular n to the plane and if σ < 0 the electric field points inward perpendicularly (n) to the plane. For a finite charged plane sheet, equation (4) is

approximately true only in the middle region of the plane and at points far away from both ends.

13. Obtain the expression for electric field due to an uniformly charged spherical shell.

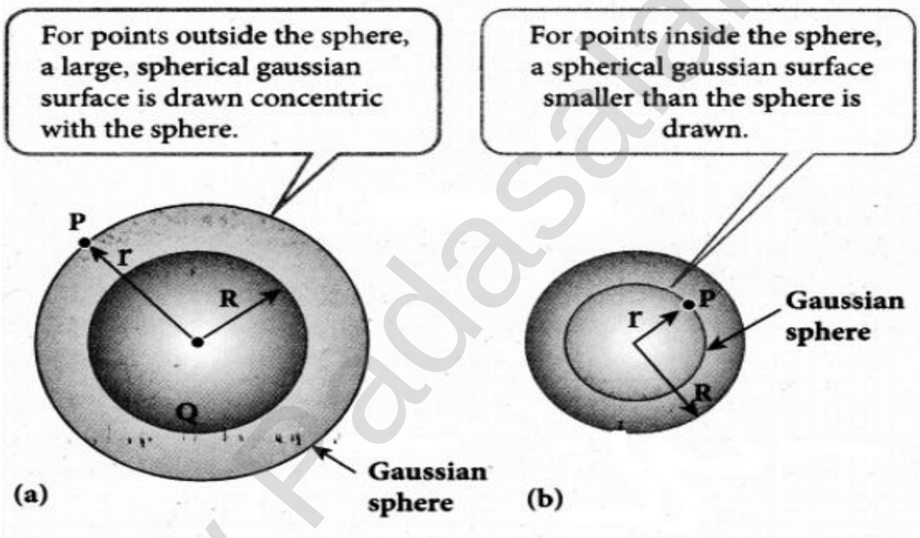
spherical shell:

Consider a uniformly charged spherical shell of radius R and total charge Q. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell (r > R):

Let us choose a point P outside the shell at a distance r from the center as shown in figure (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0. So we choose a spherical Gaussian surface of radius r and the total charge enclosed by this Gaussian surface is Q. Applying Gauss law,

$$\oint \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0} \quad ...(1)$$



The electric field due to a charged spherical shell

The electric field E and dA point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E\oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = \frac{Q}{\varepsilon_0}$$

$$\text{But } \oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = \text{total area of Gaussian surface} = 4\pi r^2$$

Substituting this value in equation (2)

$$E.4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E.4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \text{(or)} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

In vector form $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad ...(3)$$

The electric field is radially outward if $Q > 0$ and radially inward if $Q < 0$. From equation (3), we infer that the electric field at a point outside the shell will be same as if the entire charge Q is concentrated at the center of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell (r = R): The electrical field at points on the spherical shell (r = R) is given by

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{r} \quad .... (4)$$

Case (c) At a point inside the spherical shell (r < R): Consider a point P inside the shell at a distance r from the center. A Gaussian sphere of radius r is constructed as shown in the figure (b). Applying Gauss law.

$$\oint_{\substack{\text{Gussian} \\ \text{furface}}} \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E.4\pi r^2 = \frac{Q}{\varepsilon_0} \quad ...(5)$$

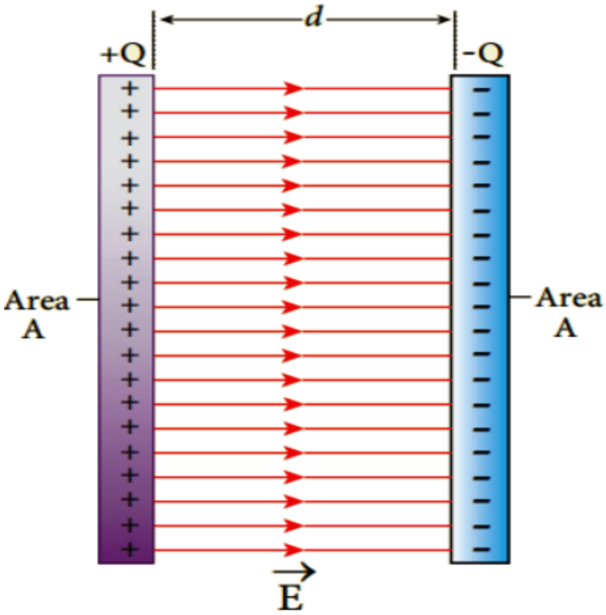
Since Gaussian surface encloses no charge, So Q = 0. The equation (5) becomes

$E = 0 \quad (r < R)$ .... (6)

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.

14. Obtain the expression for capacitance for a parallel plate capacitor.

Capacitance of a parallel plate capacitor: Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d. The electric field between two infinite parallel plates is uniform and is given by $E = \sigma / \varepsilon_0$ where σ is the surface charge density on the plates $\sigma = Q / A$.If the separation distance d is very much smaller than the size of the plate (d2 << A), then the above result is used even for finite-sized parallel plate capacitor.



Capacitance of a parallel plate capacitor The electric field between the plates is

$E = Q / A\varepsilon_0$ ....(1)

Since the electric field is uniform, the electric potential between the plates having separation d is given by

$V = Ed = Qd / A\varepsilon_0$ ....(2)

Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d} \quad \text{.....} \ (3)$$

From equation (3), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

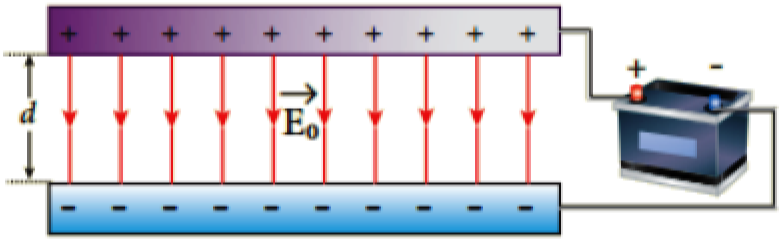
(ii) If the distance d between the two plates is reduced, the potential difference between the plates (V = Ed) decreases with E constant.

15. Explain in detail the effect of a dielectric placed in a parallel plate capacitor.

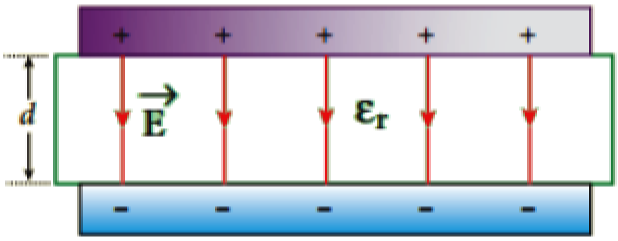
Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0. The capacitance of the capacitor without the dielectric is

$C_0 = Q_0 / V_0$ .... (1)

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by



(a)



(b)

(a) Capacitor is charged with a battery
(b) Dielectric is inserted after the battery is disconnected

$E = E_0 / \varepsilon_r$ .... (2)

Here E_0 is the electric field inside the capacitors when there is no dielectric and ε is the relative permeability of the dielectric or simply known as the dielectric constant. Since εr > 1, the electric field $E < E_0$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q will remain constant once the battery is disconnected. Hence the new potential difference is

$V = Ed = E_0 / \varepsilon r = V_0 / \varepsilon r$ . ... (3)

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases. Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \text{ ...... (4)}$$

Since $\varepsilon_r > 1$, we have $C > C_0$. Thus insertion of the dielectric constant ε_r increases the capacitance. Using equation,

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d} \text{ ...... (5)}$$

where $\varepsilon = \varepsilon_r \varepsilon_0$ is the permittivity of the dielectric medium. The energy stored in the capacitor before the insertion of a dielectric is given by

$U_0 = 1 / 2 (Q_0^2 / C_0$ .... (6)

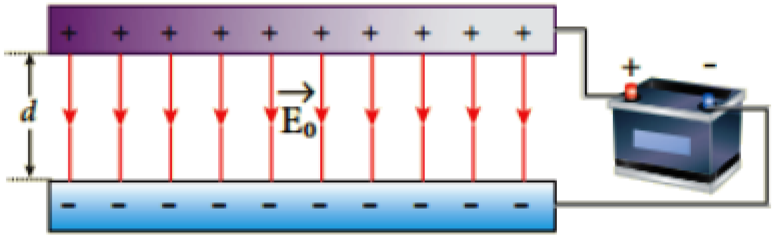
After the dielectric is inserted, the charge Q0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2}\frac{Q_0^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r}$$

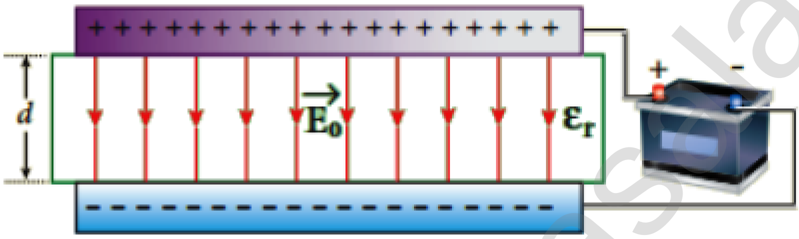
Since $\varepsilon_r > 1$ we get $U < U_0$. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor:

Let us now consider what happens when the battery of voltage V remains connected to the capacitor when the dielectric is inserted into the capacitor. The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.



(a)



(b)

(a) Capacitor is charged through a battery
(b) Dielectric is inserted when the battery is connected.

$Q = \varepsilon_r / Q_0$ .... (1)

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \text{ ...... (2)}$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

$$\text{Now, } C_0 = \frac{\varepsilon_0 A}{d} \text{ and, } C = \frac{\varepsilon A}{d} \text{ ...... (3)}$$

$$U_0 = \frac{1}{2} C_0 V_0^2 \text{ ..... (4)}$$

Note that here we have not used the expression

$$U_0 = \frac{1}{2} V_0^2 C_0$$

because here, both charge and capacitance are changed, whereas in equation 4, V remains constant. After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2} CV_0^2 = \frac{1}{2} \varepsilon_r CV_0^2 = \varepsilon_r U_0$$

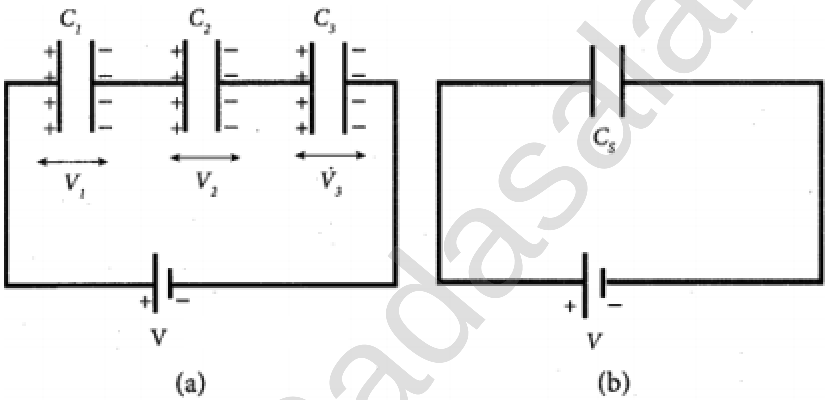
Since $e_r > 1$ we have $U > U_0$

It may be noted here that since voltage between the capacitor V is constant, the electric field between the plates also remains constant.

16. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.

(i) Capacitor in series:

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the figure (a). As soon as the battery is connected to the capacitors in series, the electrons of charge -Q are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount -Q from left plate of C_3 to the right plate of C_2 due to electrostatic induction. Similarly, the left plate of C_2 pushes the charges of Q to the right plate of which induces the positive charge +Q on the left plate of C_1 At the same time, electrons of charge -Q are transferred from left plate of C_1 to positive terminal of the battery.



(a) Capacitors connected in series (b) Equivalence capacitors C_s

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively. The total voltage across each capacitor must be equal to the voltage of the a battery. $V = V_1 + V_2 + V_3$ ….. (1)

Since Q = CV, we have $V = Q/C_1 + Q/C_2 + Q/C_3$

$$Q = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad ..... (2)$$

If three capacitors in series are considered to form an equivalent single capacitor C_s shown in figure (b), then we have $V = Q/C_s$

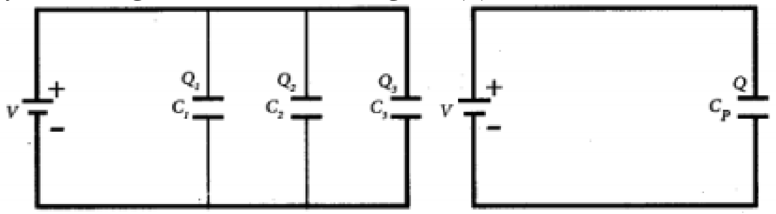
Substituting this expression into equation (2) we get

$$V = \frac{Q}{C_s} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad ..... (3)$$

Thus, the inverse of the equivalent capacitance C_s of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance C_s is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel:
Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in figure (a).



(a) (b)

(a) capacitors in parallel

(b) equivalent capacitance with the same total charge

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2 and Q_3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3 \ldots.. (1)$$

Now, since Q = CV, we have

$$Q = C_1V + C_2 V + C_3 V \ldots.. (2)$$

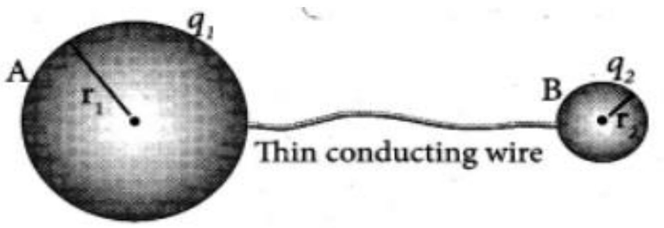
If these three capacitors are considered to form a single capacitance C which stores the total charge Q as shown in the figure (b), then we can write Q = CPV. Substituting this in equation (2), we get

$$C_p V = C_1 V + C_2 V + C_3 V$$

$$C_p = C_1 + C_2 + C_3$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitance. The equivalent capacitance C_p in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

17. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.
Distribution of charges in a conductor: Consider two conducting spheres A and B of radii r1 and r2 respectively connected to each other by a thin conducting wire as shown in the figure. The distance between the spheres is much greater than the radii of either spheres.



Two conductors are
connected through conducting wire

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q be the charge residing on the surface of sphere A and q is the charge residing on the surface of sphere B such that Q = q + q The charges are distributed only on the surface and there is no net charge inside the conductor. The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2} \quad .... (1)$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2} \quad .... (2)$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B \text{ or } \frac{q_1}{r_1} = \frac{q_2}{r_2} \quad ..... (3)$$

Let us take the charge density on the surface of sphere A is σ and charge density on the surface of sphere B is σ . This implies that

$q_1 = 4\pi r^2{}_1\, \sigma_1$ and $q_2 = 4\pi r_2{}^2\, \sigma_2$. Substituting these values into equation (3), we get

$\sigma_1\, r_1 = \sigma_2\, r_2$ ..... (4)

from which we conclude that σr = constant .... (5)

Thus the surface charge density o is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

Lightning arrester or lightning conductor: This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge. The device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle.

The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces a positive charge on the spike. Since the induced charge density on thin sharp spke is large, it results in a corona discharge.

This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod and is safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it divers the lightning to the ground safety.

18. Explain in detail the construction and working of a Van de Graaff generator.

Principle:

Electrostatic induction and action at points.

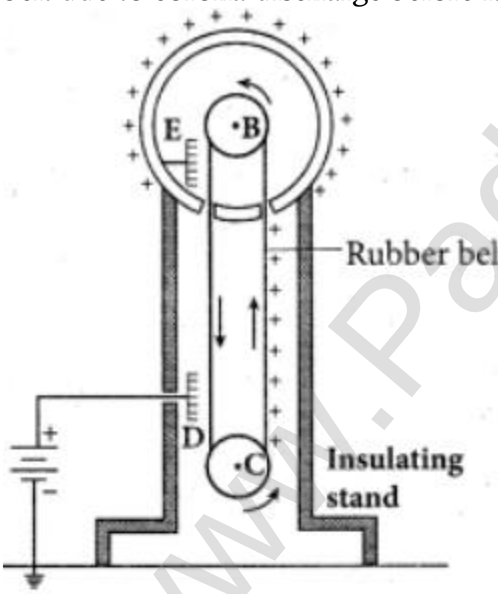
Construction:

A large hollow spherical conductor is fixed on the insulating stand. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor.

Two comb shaped metallic conductors E and D are fixed near the pulleys. The comb D is maintained at a positive potential of 104 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

Working:

Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.



Van de Graaff generator

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 107 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

Uses:

The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

19. Obtain the expression for energy stored in the parallel plate capacitor.

Energy stored in the capacitor:

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor. To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$dW = VdQ$ ….. (1)

Where $V = Q / C$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}\, dQ = \frac{Q^2}{2C} \quad \ldots(2)$$

This work done is stored as electrostatic potential energy (U_E) in the capacitor.

$U_E = Q^2 / 2C = 1/2\ CV^2$ ($\because Q = CV$) ….. (3)

where Q = CV is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor. But where is this energy stored in the capacitor? To understand this question, the equation (3) is rewritten as follows using the results

$C = \varepsilon_0 A / d$ and $V = Ed$

$U_E = 1/2\ (\varepsilon_0 A / d)\ (Ed)^2 = 1/2\ \varepsilon_0 (Ad)^2$ …… (4)

where Ad = volume of the space between the capacitor plates. The energy stored per unit volume of space is defined as energy density. Frome equation (4) we get

$u_E = 1/2\ (\varepsilon_0 E^2)$

From equation (5), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

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