Unit 1.Electrostatics

II Short Answer Question and answer

1. What is meant by quantisation of charges?

The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

$$q = ne$$

Here n is any integer (0, ±1, ±2, ±3, ±4……..).This is called quantisation of electric charge.

2. Write down Coulomb's law in vector form and mention what each term represents.

The force on the point charge q_2 exerted by another point charge q_1 is given by

$$\vec{F}_{21} = K\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector directed from charge q_1 to charge q_2 and k is the proportionality constant.

$$\text{But } \hat{r}_{12} = -\hat{r}_{21}$$

$$\vec{F}_{12} = -K\frac{q_1 q_2}{r^2}\hat{r}_{21} \text{ or } \vec{F}_{21} = -\vec{F}_{12}$$

Therefore, the electrostatic force obeys Newton's third law.

3. What are the differences between Coulomb force and gravitational force?

Coulomb force	Gravitational force
The electrostatic force is directly proportional to the product of the magnitude of two point charges	gravitational force is directly proportional to the product of two masses
Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges	The gravitational force between two masses is always attractive
The value of the constant k in Coulomb law is $k = 9 \times 10^9$ N m^2 C^{-2}	The value of the gravitational constant $G = 6.626 \times 10^{-11}$ N m^2 kg^{-2}
Since k is much more greater than G,	
The electrostatic force is always greater in magnitude than gravitational force for smaller size objects	
The gravitational force between two masses is independent of the medium. For example, if 1 kg of two masses are kept in air or inside water, the gravitational force between two masses remains the same	But the electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.
The gravitational force between two point masses is the same whether two masses are at rest or in motion.	If the charges are in motion, yet another force (Lorentz force) comes into play in addition to coulomb force.

T.Vediyappan.,M.Sc.,B.Ed.,M.Phil., Department Of Physics

4. Write a short note on superposition principle

According to this superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

Consider a system of n charges, namely q_1, q_2, q_3 …. q_n. The force on q_1 exerted by the charge q_2

$$\vec{F}_{12} = K\,\frac{q_1 q_2}{r_{21}^2}\,\hat{r}_{21}$$

Here $\hat{r}_{12}$ is the unit vector from q_2 to q_1 along the line joining the two charges and r_{21} is the distance between the charges q_1 and q_2. The electrostatic force between two charges is not affected by the presence of other charges in the neighbourhood.

The force on q_1 exerted by the charge q_3 is

$$\vec{F}_{13} = K\,\frac{q_1 q_2}{r_{31}^2}\,\hat{r}_{31}$$

By continuing this, the total force acting on the charge q_1 due to all other charges is given by

$$\vec{F}_1^{\,tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots\ldots\ldots + \vec{F}_{1n}$$

$$\vec{F}_1^{\,tot} = K\,\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + K\,\frac{q_1 q_2}{r_{31}^2}\,\hat{r}_{31} + K\,\frac{q_1 q_2}{r_{41}^2}\,\hat{r}_{41} + \ldots\ldots + K\,\frac{q_1 q_2}{r_{n1}^2}\,\hat{r}_{n1}$$

$$= K\left\{\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_2}{r_{31}^2}\,\hat{r}_{31} + \frac{q_1 q_2}{r_{41}^2}\,\hat{r}_{41} + \ldots\ldots + \frac{q_1 q_2}{r_{n1}^2}\,\hat{r}_{n1}\right\}$$

5. Define "electric field"

The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{Kq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is Newton per Coulomb (NC^{-1})

6. What is meant by "Electric field lines"?

Electric field vectors are visualized by the concept of electric field lines. They form a set of continuous lines which are the visual representation of the electric field in some region of space.

7. The electric field lines never intersect. Justify.

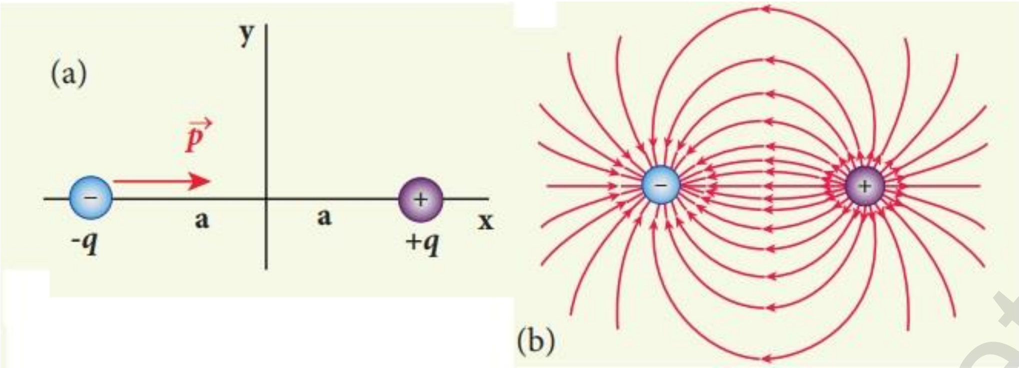
As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

8. Define 'Electric dipole

Two equal and opposite charges separated by a small distance constitute an electric dipole. $\vec{P} = 2qa\hat{\imath}$

9. What is the general definition of electric dipole moment?

The electric dipole moment is defined as $\vec{P} = q\hat{r}_+ - q\hat{r}_-$. Here $\hat{r}_+$ is the position vector of +q from the origin and $\hat{r}_-$ is the position vector of –q from the origin.



$$\vec{P} = qa\hat{\imath} - qa(-\hat{\imath}) = 2qa\hat{\imath}$$

The electric dipole moment vector lies along the line joining two charges and is directed from –q to +q. The SI unit of dipole moment is coulomb meter (Cm).

10. Define electric potential.

The electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$.

11. What is an equipotential surface?

An equipotential surface is a surface on which all the points are at the same potential.

12. What are the properties of an equipotential surface?

(i) The work done to move a charge q between any two points A and B, W = q (V_B – V_A). If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$.

(ii) The electric field is normal to an equipotential surface. If it is not normal, then there is a component of the field parallel to the surface. Then work must be done to move a charge between two points on the same surface. This is a contradiction. Therefore, the electric field must always be normal to equipotential surface.

13. Give the relation between electric field and electric potential.

Consider a positive charge q kept fixed at the origin. To move a unit positive charge by a small distance dx in the electric field E, the work done is given by dW=-Edx. The minus sign implies that work is done against the electric field. This work done is equal to electric potential difference. Therefore,

$$dW = dV.$$

$$\text{(or) } dV = -E\,dx$$

$$\text{Hence } E = -\frac{dV}{dx}$$

The electric field is the negative gradient of the electric potential. In general,

$$\vec{E} = (\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k})$$

14. Define electrostatic potential energy?

This potential V is the work done to bring a unit positive charge from infinity to the point. W = qV. This work done is stored as the electrostatic potential energy U.

15. Define electric flux?

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter Φ_E and its unit is N m^2 C^{-1}. Electric flux is a scalar quantity and it can be positive or negative. $\Phi E = \vec{E}.\vec{A} = E A \cos\theta$

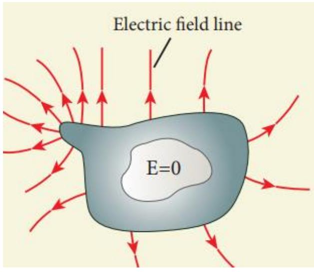
16. What is meant by electrostatic energy density?

This work done is stored in the form of electrostatic energy per unit volume is called energy density.

17. Write a short note on electrostatic shielding.

Using Gauss law, we proved that the electric field inside the charged spherical shell is zero, Further, we showed that the electric field inside both hollow and solid conductors is zero.

It is a very interesting property which has an important consequence. Consider a cavity inside the conductor as shown in Figure 1.48 (a).



Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero.

A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity. This is called electrostatic shielding.

18. What is polarization?

Polarisation $\vec{P}$ is defined as the total dipole moment per unit volume of the dielectric. $\vec{P} = \chi_e \vec{E}_{ext}$.

19. What is dielectric strength?

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. **The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength. For example, the dielectric strength of air is 3×10^6 V m^{-1}.**

20. Define capacitance? Give units.

The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors. $C = \frac{Q}{V}$ The SI unit of capacitance is coulomb per volt or farad (F) in honor of Michael Faraday.

T.Vediyappan.,M.Sc.,B.Ed.,M.Phil., Department Of Physics

21. What is corona discharge?

The electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

III Long Answer question and answer

1. Discuss the basic properties of electric charges.

(i) Electric charge

Electric charge Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric charge is another intrinsic and fundamental property of particles. The SI unit of charge is coulomb

(ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other.

Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other.

(For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk.

As a result, the glass rod is positively charged and silk cloth becomes negatively charged).

From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other.

This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical process, the net change in charge will always be zero.

(iii) Quantisation of charges

What is the smallest amount of charge that can be found in nature? Experiments show that the charge on an electron is -e and the charge on the proton is +e. Here, e denotes the fundamental unit of charge.

The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

$$q = ne$$

Here n is any integer (0, ± 1, ± 2, ± 3, ± 4………). This is called quantisation of electric charge.

Robert Millikan in his famous experiment found that the value of $e = 1.6 \times 10^{-19}$C. The charge of an electron is -1.6×10^{-19} C and the charge of the proton is $+1.6 \times 10^{-19}$C.

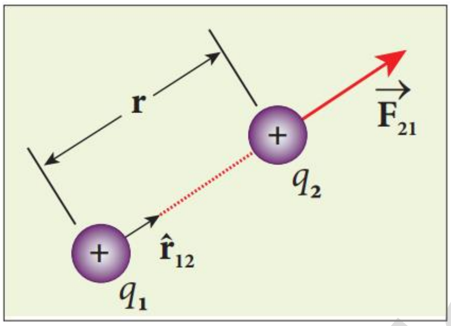
When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of 10^{10}. So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

2. Explain in detail Coulomb's law and its various aspects.

Consider two point charges q_1 and q_2 at rest in vacuum, and separated by a distance of r. According to Coulomb, the force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{21} = K \frac{q_1 q_2}{r_{21}^2} \hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector directed from charge q_1 to charge q_2 and k is the proportionality constant.



Important aspects of Coulomb's law

(i) Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

(ii) The force on the charge q_2 exerted by the charge q_1 always lies along the line joining the two charges. $\hat{r}_{12}$ is the unit vector pointing from charge q_1 to q_2. It is shown in the above Figure. Likewise, the force on the charge q_1 exerted by q_2 is along $-\hat{r}_{12}$ (i.e., in the direction opposite to $\hat{r}_{12}$).

(iii) In SI units, $K = \frac{1}{4\pi\varepsilon_0}$ and its value is 9×10^9 N m^2 C^{-2}. Here ε_0 is the permittivity of free space or vacuum and the value of $\varepsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12}$ C^2 m^2 N^{-1}.

(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$$|F| = \frac{9 \times 10^9 \times 1 \times 1}{1^2} = 9 \times 10^9 \text{ N}$$

This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of µC (micro coulomb) or nC (nano coulomb).

(v) In SI units, Coulomb's law in vacuum takes the form $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{12}$. In a medium of permittivity ε, the force between two point charges is given by $\vec{F}_{21} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{12}$.

Since $\varepsilon > \varepsilon_0$, the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$. For vacuum or air, $\varepsilon_r = 1$ and for all other media $\varepsilon_r > 1$.

T.Vediyappan.,M.Sc.,B.Ed.,M.Phil., Department Of Physics

(vi) Coulomb's law has same structure as Newton's law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses. But there are some important differences between these two laws.

(vii) The force on a charge q1 exerted by a point charge q2 is given by $\vec{F}_{12}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{21}$

Here $\hat{r}_{12}$ is the unit vector from charge q_2 to q_1. But $\hat{r}_{21} = -\hat{r}_{12}$,

$\vec{F}_{12}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}(-\hat{r}_{12}) = -\vec{F}_{12}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}(\hat{r}_{12})$, $\vec{F}_{12} = -\vec{F}_{21}$ Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However, we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

3. Define 'Electric field' and discuss its various aspects.

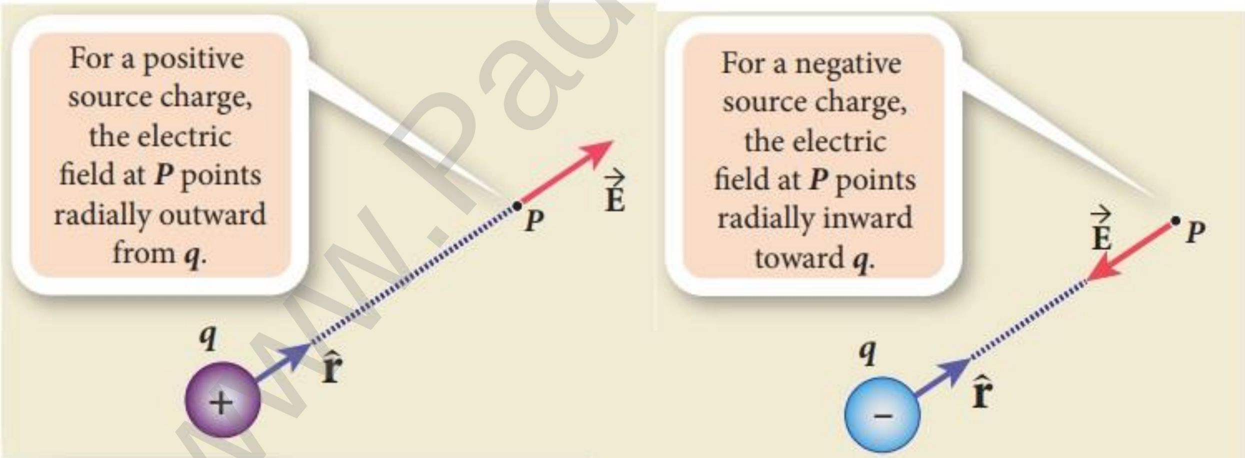
The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{Kq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\;\ldots\ldots\ldots\ldots(1)$$

Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is Newton per Coulomb (NC^{-1}).

Important aspects of Electric field:

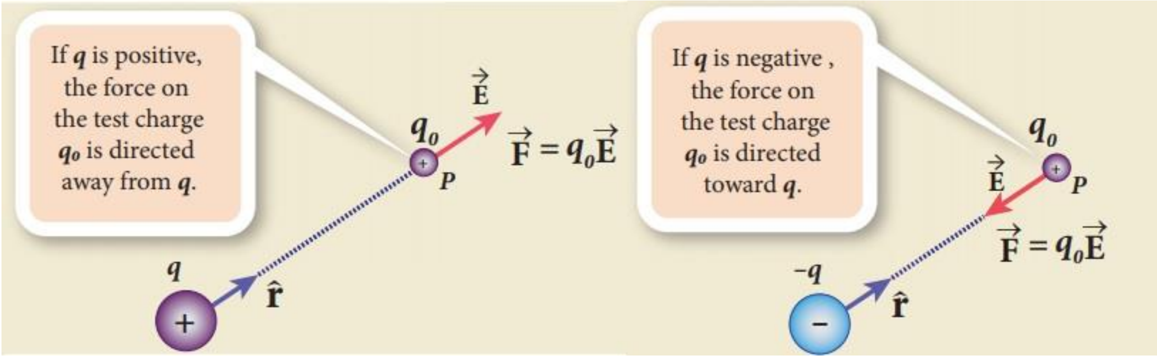
(i) If the charge q is positive then the electric field points away from the source charge and if q is negative, the electric field points towards the source charge q. This is shown in the Figure.



(ii) If the electric field at a point P is $\vec{E}$,then the force experienced by the test charge qo placed at the point P is $\vec{F} = q\vec{E}$

This is Coulomb's law in terms of electric field. This is shown in Figure below

T.Vediyappan.,M.Sc.,B.Ed.,M.Phil., Department Of Physics



(iii) The equation (1) implies that the electric field is independent of the test charge qo and it depends only on the source charge q.

(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures 1.6(a) and (b). From equation (1), we can infer that as distance increases, the electric field decreases in magnitude. Note that in Figures 1.6 (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the points Q and R because the point P is closer to the source charge.

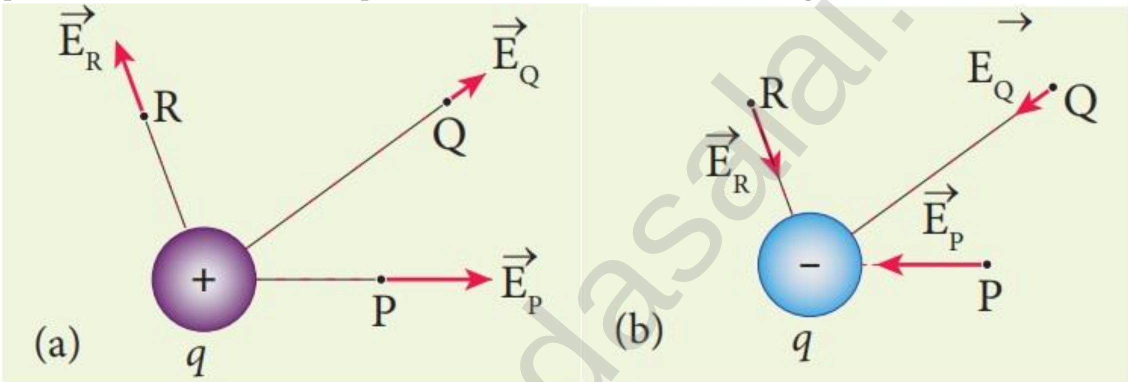


Figure 1.6 (a) Electric field due to positive charge
(b) Electric field due to negative charge

(v) In the definition of electric field, it is assumed that the test charge q_0 is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

(vi) The expression (1) is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used These will be explained later in the same section. However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge. Note that we similarly treat the Earth as a point mass when we calculate the gravitational field of the Sun on the Earth (refer unit 6, volume 2, XI physics).

(vii)There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non-uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases. These are shown in Figure 1.7.

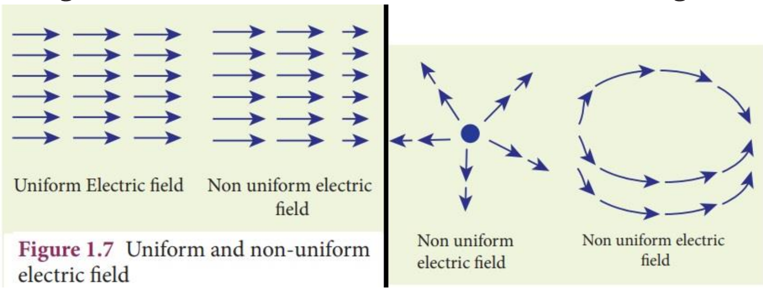


Figure 1.7 Uniform and non-uniform electric field

4. How do we determine the electric field due to a continuous charge distribution? Explain

The electric charge is quantized microscopically. The expressions (1.2), (1.3), (1.4) are applicable to only point charges. While dealing with the electric field due to a charged sphere or a charged wire etc., it is very difficult to look at individual charges in these charged bodies. Therefore, it is assumed that charge is distributed continuously on the charged bodies and the discrete nature of charges is not considered here. The electric field due to such continuous charge distributions is found by invoking the method of calculus.

Consider the following charged object of irregular shape as shown in Figure 1.9. The entire charged object is divided into a large number of charge elements Δq_1, Δq_2,Δq_3 Δq_4 ...... Δq_n and each charge element Δq is taken as a point charge.

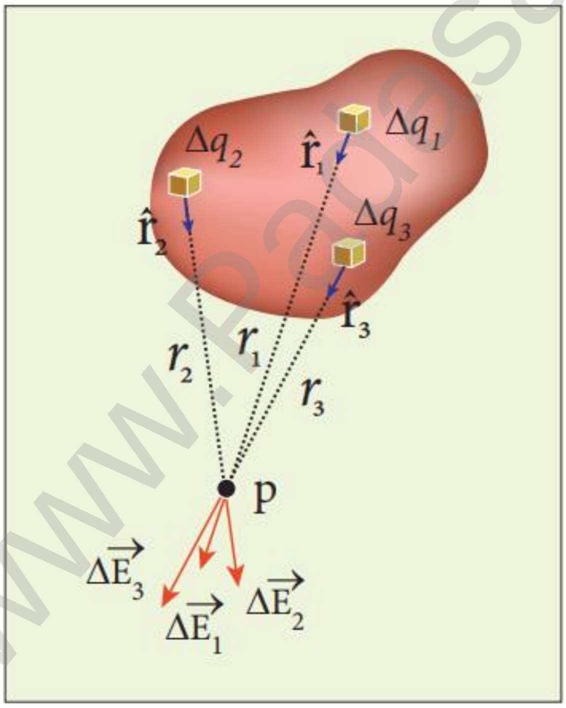


Figure 1.9 Continuous charge distributions

The electric field at a point P due to a charged object is approximately given by the sum of the fields at P due to all such charge elements.

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + \dots\dots\dots + \frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right) \approx \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP} \qquad (1.9)$$

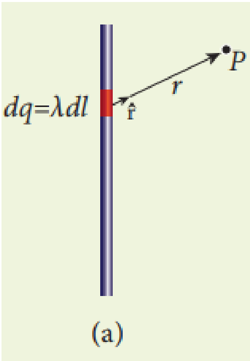
Here Δq_i is the i[th] charge element, r_{iP} is the distance of the point P from the i[th] charge element and $\hat{r}_{iP}$ is the unit vector from i[th] charge element to the point P.

However the equation (1.9) is only an approximation. To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (1.9) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r} \qquad (1.10)$$

Here r is the distance of the point P from the infinitesimal charge Δq and $\hat{r}$is the unit vector from Δq to point P. Even though the electric field for a continuous charge distribution is difficult to evaluate, the force experienced by some test charge q in this electric field is still given by $\vec{F} = q\vec{E}$.

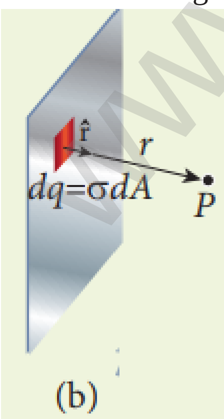
(a) If the charge Q is uniformly distributed along the wire of length L, then linear charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. Its unit is coulomb per meter (Cm^{-1}). The charge present in the infinitesimal length dl is dq = λdl. This is shown in Figure 1.10 (a).



(a)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl}{r^2}\hat{r} = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{r^2}\hat{r}$$

(b) If the charge Q is uniformly distributed on a surface of area A, then surface charge density (charge per unit area) is $\sigma = \frac{Q}{A}$. Its unit is coulomb per square meter ($C\ m^{-2}$). The charge present in the infinitesimal area dA is dq = σdA. This is shown in the figure 1.10 (b). The electric field due to a total charge Q is given by

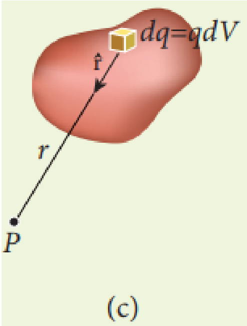


(b)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma da}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\sigma\int \frac{da}{r^2}\hat{r}$$

This is shown in Figure 1.10(b).

(c) If the charge Q is uniformly distributed in a volume V, then volume charge density (charge per unit volume) is given by $\rho = \frac{Q}{V}$. Its unit is coulomb per cubic meter (C m^{-3}). The charge present in the infinitesimal volume element dV is dq = ρdV. This is shown in Figure 1.10(c). The electric field due to a volume of total charge Q is given by

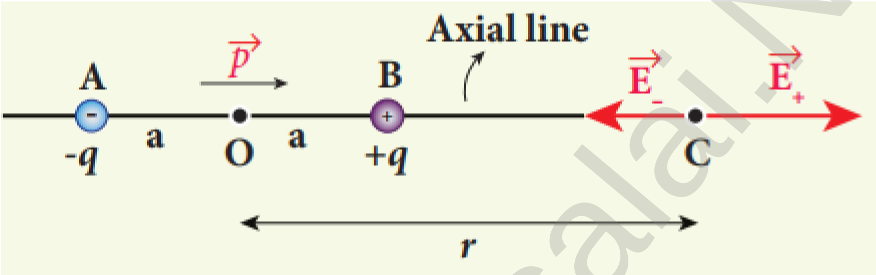


(c)

$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\rho dV}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_\circ}\rho\int \frac{dV}{r^2}\hat{r}.$$

5. Calculate the electric field due to a dipole on its axial line and equatorial plane.

Consider an electric dipole placed on the x-axis as shown in Figure. A point C is located at a distance of r from the midpoint O of the dipole along the axial line.



The electric field at a point C due to +q is $\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$ along BC

Since the electric dipole moment vector $\bar{p}$ is from −q to +q and is directed along BC, the above equation is rewritten as

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} \qquad (1.13)$$

where $\hat{p}$ is the electric dipole moment unit vector from −q to +q.

The electric field at a point C due to −q is

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p} \qquad (1.14)$$

Since +q is located closer to the point C than −q, $\vec{E}_+$ is stronger than $\vec{E}_-$. Therefore, the length of the $\vec{E}_+$ vector is drawn larger than that of $\vec{E}_-$ vector.

The total electric field at point C is calculated using the superposition principle of the electric field.

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p} \quad (1.15)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p} \quad (1.16)$$

Note that the total electric field is along $\vec{E}_+$, since $+q$ is closer to C than –q. The direction of $\vec{E}_{tot}$ is shown in Figure 1.18.

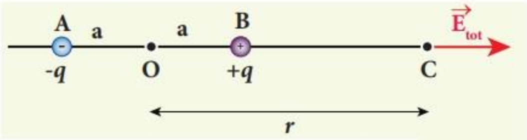


Figure 1.18 Total electric field of the dipole on the axial line

6. Derive an expression for the torque experienced by a dipole due to a uniform electric field.

Consider an electric dipole of dipole moment $\vec{P}$ placed in a uniform electric field $\vec{E}$ whose field lines are equally spaced and point in the same direction. The charge +q will experience a force q$\vec{E}$ in the direction of the field and charge –q will experience a force –q$\vec{E}$ in a direction opposite to the field. Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque as shown in Figure 1.20. This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction).

The total torque on the dipole about the point O

$$\vec{\tau} = \overline{OA} \times (-q\vec{E}) + \overline{OB} \times q\vec{E} \qquad (1.22)$$

Using right-hand corkscrew rule (Refer XI, volume 1, unit 2), it is found that total torque is perpendicular to the plane of the paper and is directed into it.

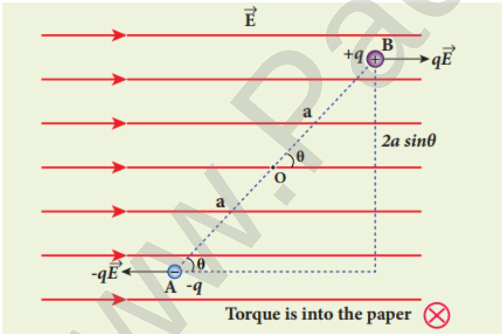


Figure 1.20 Torque on dipole

The magnitude of the total torque $\tau = |\overline{OA}||(-q\vec{E})|\sin\theta + |\overline{OB}||q\vec{E}|\sin\theta$

$$\tau = qE \cdot 2a\sin\theta \qquad (1.23)$$

where θ is the angle made by $\vec{p}$ with $\vec{E}$. Since p = 2aq, the torque is written in terms of the vector product as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad (1.24)$$

The magnitude of this torque is $\tau = pE\sin\theta$ and is maximum when $\theta = 90^\circ$.

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The magnitude of this torque is $\tau = pE \sin\theta$ and is maximum when $\theta = 90^o$. This torque tends to rotate the dipole and align it with the electric field $\vec{E}$. Once $\vec{P}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero. If the electric field is not uniform, then the force experienced by +q is different from that experienced by –q. In addition to the torque, there will be net force acting on the dipole. This is shown in Figure

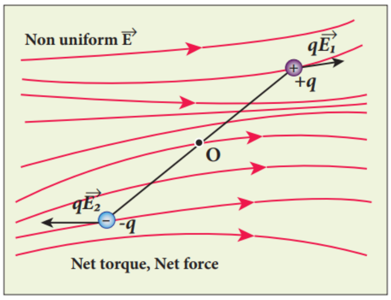

Figure 1.21 The dipole in a non-uniform electric field

7. **Derive an expression for electrostatic potential due to a point charge.**

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q. This is shown in Figure 1.23.

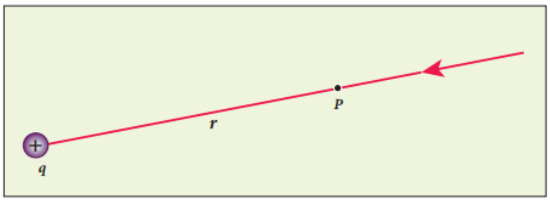

Figure 1.23 Electrostatic potential at a point P

The electric potential at the point P is $V = \int_{\infty}^{r} (-\vec{E}) \cdot d\vec{r} = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$ (1.32)

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = \frac{-1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} \hat{r}.d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr\hat{r}$ and using $\hat{r}.\hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\varepsilon_\circ} \int_{\infty}^{r} \frac{q}{r^2} \hat{r} \cdot dr\hat{r} = -\frac{1}{4\pi\varepsilon_\circ} \int_{\infty}^{r} \frac{q}{r^2} dr$$

After the integration,

$$V = -\frac{1}{4\pi\varepsilon_\circ} q \left\{ -\frac{1}{r} \right\}_{\infty}^{r} = \frac{1}{4\pi\varepsilon_\circ} \frac{q}{r}$$

Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \qquad (1.33)$$

Important points:

(i) If the source charge q is positive, V > 0. If q is negative, then V is negative and equal to $V = -\frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

(ii) The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.

(iii) From expression (1.33), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity ($r = \infty$) electrostatic potential is zero (V = 0). In the case of gravitational force, mass moves from a point of higher gravitational potential to a point of lower gravitational potential. Similarly, a positive charge moves from a point of higher electrostatic potential to lower electrostatic potential. However a negative charge moves from lower electrostatic potential to higher electrostatic potential. This comparison is shown in Figure 1.24.

(iv) The electric potential at a point P due to a collection of charges $q_1, q_2, q_3 \ldots q_n$ is equal to sum of the electric potentials due to individual charges.

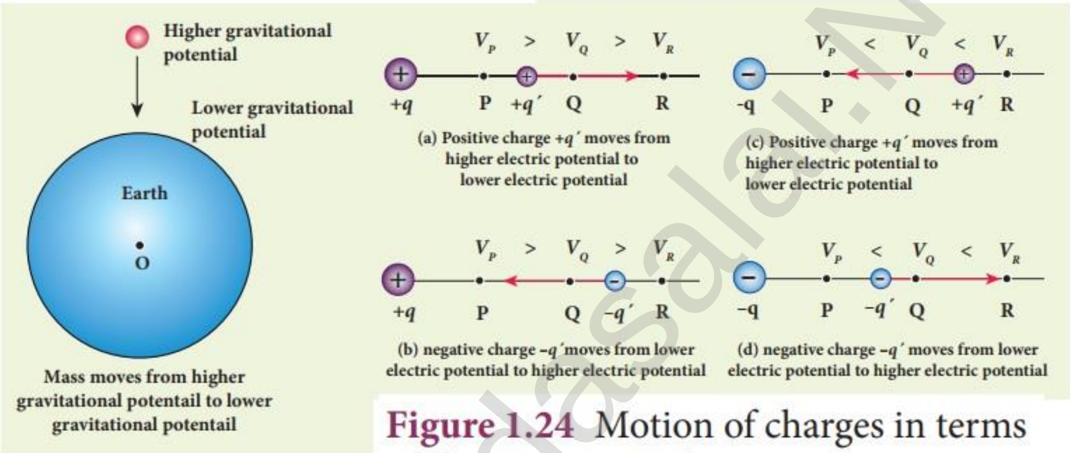

Figure 1.24 Motion of charges in terms of electric potential

8. Derive an expression for electrostatic potential due to an electric dipole.

Consider two equal and opposite charges separated by a small distance 2a as shown in Figure 1.26. The point P is located at a distance r from the midpoint of the dipole. Let θ be the angle between the line OP and dipole axis AB.

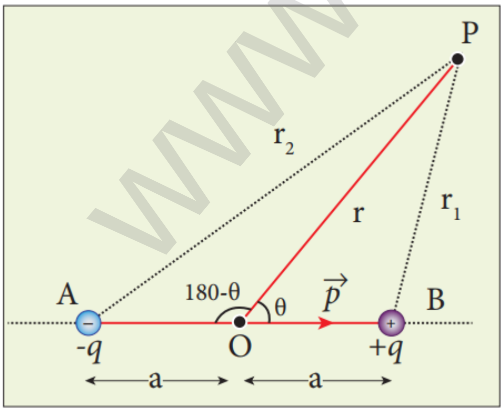

Figure 1.26 Potential due to electric dipole

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Let r_1 be the distance of point P from +q and r_2 be the distance of point P from –q.

Potential at P due to charge +q $= \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_1}$

Potential at P due to charge –q $= -\frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_2}$

Total potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_\circ}q\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad (1.35)$$

Suppose if the point P is far away from the dipole, such that r>>a, then equation (1.35) can be expressed in terms of r.

By the cosine law for triangle BOP, $r_1^2 = r^2 + a^2 - 2ra\cos\theta$

$$r_1^2 = r^2\left(1+\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right)$$

Since the point P is very far from dipole, then r>>a. As a result the term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore

$$r_1^2 = r^2\left(1-2a\frac{\cos\theta}{r}\right) \quad \text{(or)} \quad r_1 = r\left(1-\frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$ Since $\frac{a}{r} << 1$, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1+\frac{a}{r}\cos\theta\right) \qquad (1.36)$$

Similarly applying the cosine law for triangle AOP,

$$r_2^2 = r^2 + a^2 - 2ra\cos(180-\theta)$$

since $\cos(180-\theta) = -\cos\theta$ we get

$$r_2^2 = r^2 + a^2 + 2ra\cos\theta$$ Neglecting the term $\frac{a^2}{r^2}$ (because r>>a)

$$r_2^2 = r^2\left(1+\frac{2a\cos\theta}{r}\right), \qquad r_2 = r\left(1+\frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

Using Binomial theorem, we get

$$\frac{1}{r_2} = \frac{1}{r}\left(1-a\frac{\cos\theta}{r}\right) \qquad (1.37)$$

Substituting equation (1.37) and (1.36) in equation (1.35),

$$V = \frac{1}{4\pi\varepsilon_\circ} q\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{q}{4\pi\varepsilon_\circ}\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{2aq}{r^2}\cos\theta$$

But the electric dipole moment $p = 2qa$ and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write p $\cos\theta = \vec{p}\cdot\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad (r>>a) \quad (1.38)$$

Equation (1.38) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (1.38) is valid for any distance.

Special cases

Case (i) If the point P lies on the axial line of the dipole on the side of +q, then θ = 0. Then the electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1.39)$$

Case (ii) If the point P lies on the axial line of the dipole on the side of –q, then θ = 180o, then $V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ ……………………………….(1.40)

Case (iii) If the point P lies on the equatorial line of the dipole, then θ = 90o. Hence, V = 0

9. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.

The electric potential at a point at a distance r from point charge q_1 is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$

This potential V is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance r from q_1, the work done is the product of q_2 and the electric potential at that point. Thus we have $W = q_2V$

This work done is stored as the electrostatic potential energy U of a system of charges q1 and q2 separated by a distance r. Thus we have

$$U = q_2V = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.45)$$

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression (1.45) is derived by assuming that q_1 is fixed and q_2 is brought from infinity. The equation (1.45) holds true when q_2 is fixed and q1 is brought from infinity or both q_1 and q_2 are simultaneously brought from infinity to a distance r between them. Three charges are arranged in the following configuration as shown in Figure 1.30.

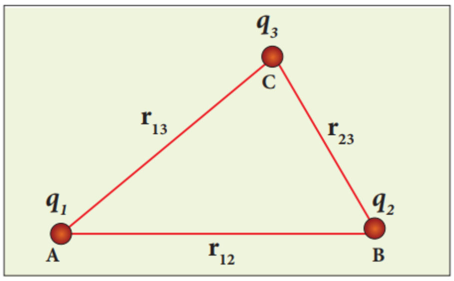

Figure 1.30 Electrostatic potential energy for Collection of point charges

To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown in Figure 1.30.

(i) Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1.

(ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1. So the work done on the charge q_2 is $W = q_2V_{1B}$. Here V_{1B} is the electrostatic potential due to the charge q_1 at point B.

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$$

Note that the expression is same when q_2 is brought first and then q_1 later.

(iii) Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is $= q_3 (V_{1C} + V_{2C})$. Here V_{1C} is the electrostatic potential due to charge q_1 at point C and V_{2C} is the electrostatic potential due to charge q_2 at point C.

The electrostatic potential is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.47)$$

(iv) Adding equations (1.46) and (1.47), the total electrostatic potential energy for the system of three charges q_1, q_2 and q_3 is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.48)$$

Note that this stored potential energy U is equal to the total external work done to assemble the three charges at the given locations. The expression (1.48) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

10. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.

Consider a dipole placed in the uniform electric field $\vec{E}$ as shown in the Figure 1.31. A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle θ' to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

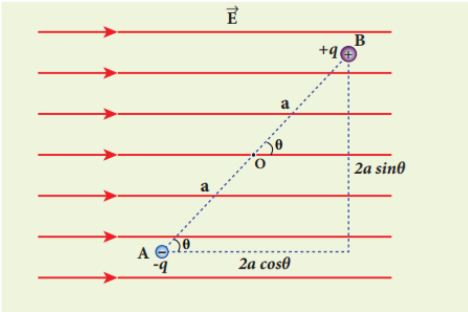

Figure 1.31 The dipole in a uniform electric field

The work done by the external torque to rotate the dipole from angle θ' to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext}\, d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.49)$$

Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have

$$|\vec{\tau}_{ext}| = |\vec{\tau}_E| = |\vec{p} \times \vec{E}| \qquad (1.50)$$

Substituting equation (1.50) in equation (1.49), we get

$$W = \int_{\theta'}^{\theta} pE \sin\theta\, d\theta$$

$$W = pE(\cos\theta' - \cos\theta)$$

This work done is equal to the potential energy difference between the angular positions θ and θ′.

$$U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$$

If the initial angle is $\theta' = 90^\circ$ and is taken as reference point, then $U(\theta') = pE\cos 90^\circ = 0$.

The potential energy stored in the system of dipole kept in the uniform electric field is given by U = - E p cosθ = $-\vec{P}\,\vec{E}$ (1.51) In addition to p and E, the potential energy also depends on the orientation θ of the electric dipole with respect to the external electric field. The potential energy is maximum when the dipole is aligned anti-parallel (θ = π) to the external electric field and minimum when the dipole is aligned parallel (θ = 0) to the external electric field.

11. Obtain Gauss law from Coulomb's law.

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure 1.36. We can calculate the total electric flux through the closed surface of the sphere using the equation (1.58).

$$\phi_E = \oint \vec{E}.d\vec{A} = \oint EdA\cos\theta$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and $\theta = 0°$.

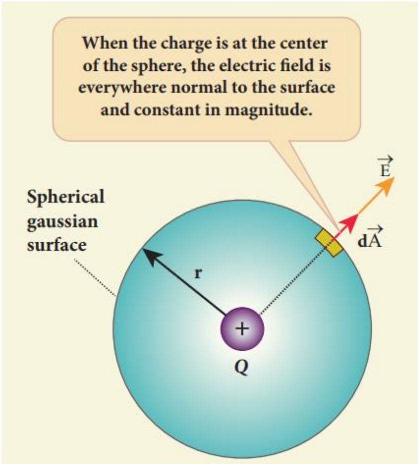

Figure 1.36 Total electric flux of point charge

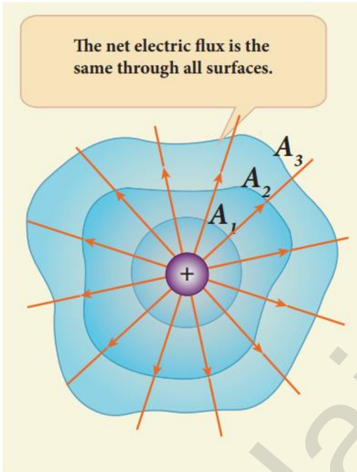

Figure 1.37 Gauss law for arbitrarily shaped surface

$$\phi_E = \oint E\,dA \quad \text{since } \cos 0^0 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.59)$$

E is uniform on the surface of the sphere,

$$\phi_E = E\oint dA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.60)$$

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ in equation 1.60, we get

$$\Phi_E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi\frac{1}{4\pi\varepsilon_o}Q$$

$$\Phi_E = \frac{Q}{\varepsilon_o} \qquad (1.61)$$

The equation (1.61) is called as Gauss's law. The remarkable point about this result is that the equation (1.61) is equally true for any arbitrary shaped surface which encloses the charge Q and as shown in the Figure 1.37. It is seen that the total electric flux is the same for closed surfaces A_1, A_2 and A_3 as shown in the Figure 1.37.

Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Here Qencl denotes the charges inside the closed surface.

Discussion of Gauss law a

(i) The total electric flux through the closed surface depends only on the charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

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(ii) The total electric flux is independent of the location of the charges inside the closed surface.

(iii) To arrive at equation (1.62), we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge; therefore, spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation (1.62), the electric field $\vec{E}$ is due to charges present inside and outside the Gaussian surface but the charge Q_{encl} denotes the charges which lie only inside the Gaussian surface.

(v) The Gaussian surface cannot pass through any discrete charge but it can pass through continuous charge distributions. It is because, very close to the discrete charges, the electric field is not well defined.

(vi) Gauss law is another form of Coulomb's law and it is also applicable to the charges in motion. Because of this reason, Gauss law is treated as much more general law than Coulomb's law.

12. Obtain the expression for electric field due to an infinitely long charged wire.

Consider an infinitely long straight wire having uniform linear charge density λ. Let P be a point located at a perpendicular distance r from the wire (Figure 1.38(a)).

The electric field at the point P can be found using Gauss law. We choose two small charge elements A_1 and A_2 on the wire which are at equal distances from the point P.

The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r.

This is shown in the Figure 1.38(b). From this property, we can infer that the charged wire possesses a cylindrical symmetry.

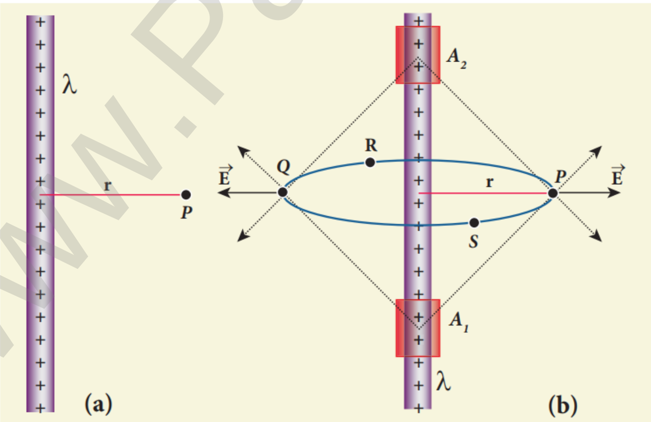

Figure 1.38 Electric field due to infinite long charged wire

Let us choose a cylindrical Gaussian surface of radius r and length L as shown in the Figure 1.39. The total electric flux in this closed surface is calculated as follows.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$
$$= \int_{\substack{\text{Curved}\\ \text{surface}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{top}\\ \text{surface}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{bottom}\\ \text{surface}}} \vec{E}\cdot d\vec{A} \quad (1.63)$$

It is seen from Figure (1.39) that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E}\ d\vec{A} = E\ dA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$ and $\vec{E}\ d\vec{A} = 0$.

Substituting these values in the equation (1.63) and applying Gauss law to the cylindrical surface, we have

$$\Phi_E = \int_{\substack{\text{Curved}\\ \text{surface}}} E\,dA = \frac{Q_{encl}}{\varepsilon_\circ} \qquad (1.64)$$

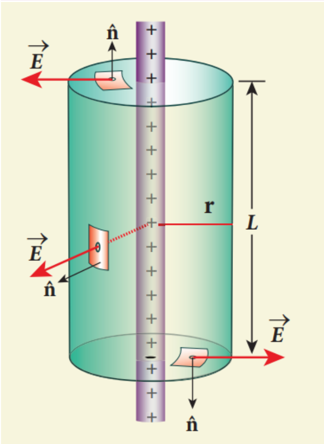

Figure 1.39 Cylindrical Gaussian surface

Since the magnitude of the electric field for the entire curved surface is constant, E is taken out of the integration and Q_{encl} is given by $QL_{encl} = \lambda$.

$$E \int_{\substack{\text{Curved}\\ \text{surface}}} dA = \frac{\lambda L}{\varepsilon_\circ} \qquad (1.65)$$

Here $\Phi_E = \int_{\substack{\text{Curved}\\ \text{surface}}} dA =$ tc

surface = 2πrL. Substituting this in equation (1.65), we get

$$E\cdot 2\pi r L = \frac{\lambda L}{\varepsilon_\circ} \qquad E = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r} \qquad (1.66)$$

In vector form $\vec{E} = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r}\hat{r}$ (1.67)

The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ for a point charge.

Equation (1.67) indicates that the electric field is always along the perpendicular direction ($\hat{r}$) to wire. In fact, if $\lambda > 0$ then $\vec{E}$ points perpendicular outward ($\hat{r}$) from the wire and if $\lambda < 0$, then $\vec{E}$ points perpendicular inward ($-\hat{r}$). The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

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13. Obtain the expression for electric field due to a charged infinite plane sheet.

Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet as shown in the Figure 1.40.

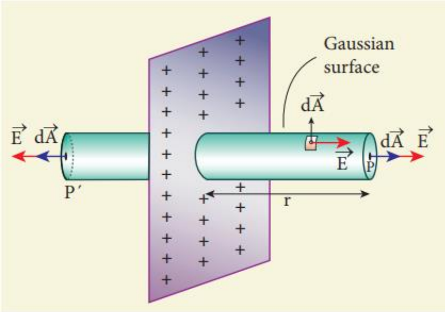

Figure 1.40 Electric field due to charged infinite planar sheet

Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points. A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface. Applying Gauss law for this cylindrical surface,

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_{\substack{Curved\\surface}} \vec{E}\cdot d\vec{A} + \int_P \vec{E}\cdot d\vec{A} + \int_{P'} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad (1.68)$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P′ (Figure 1.40). Then,

$$\Phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0} \quad (1.69)$$

Since the magnitude of the electric field at these two equal surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get,

$$2E\int_P dA = \frac{\sigma A}{\varepsilon_0}$$

The total area of surface either at P or P′

$$\int_P dA = A$$

$$\text{Hence } 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0} \quad (1.70)$$

$$\text{In vector form, } \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \quad (1.71)$$

Here $\hat{n}$ is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance r.

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The electric field will be the same at any point farther away from the charged plane. Equation (1.71) implies that if $\sigma > 0$ the electric field at any point P is outward perpendicular $\hat{n}$ to the plane and if $\sigma < 0$ the electric field points inward perpendicularly ($-\hat{n}$) to the plane. For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

14. **Obtain the expression for electric field due to an uniformly charged spherical shell.**

Consider a uniformly charged spherical shell of radius R and total charge Q as shown in Figure 1.42. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell (r > R)

Let us choose a point P outside the shell at a distance r from the center as shown in Figure 1.42 (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0. So we choose a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q. Applying Gauss law

$$\oint_{\substack{\text{Gaussian}\\\text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_\circ} \qquad (1.73)$$

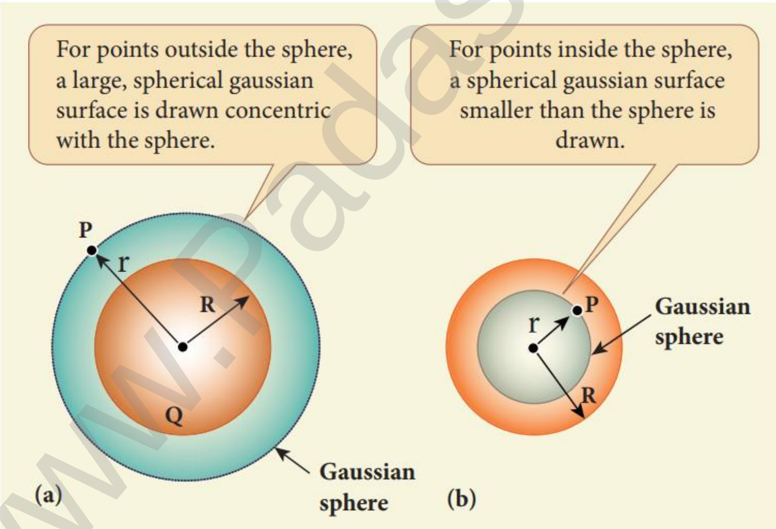

Figure 1.42 The electric field due to a charged spherical shell

The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E \oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = \frac{Q}{\varepsilon_\circ} \qquad (1.74)$$

But $\oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA =$ total area of Gaussian surface $= 4\pi r^2$. Substituting this value in equation (1.74)

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_\circ}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_\circ} \text{ (or) } E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2}$$

In vector form $$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2}\hat{r} \qquad (1.75)$$

The electric field is radially outward if Q>0 and radially inward if Q< 0. From equation (1.75), we infer that the electric field at a point outside the shell will be same as if the entire charge Q is concentrated at the centre of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell (r = R).

The electrical field at points on the spherical shell (r = R) is given by

$$\vec{E} = \frac{Q}{4\pi\varepsilon_\circ R^2}\hat{r} \qquad (1.76)$$

Case (c) At a point inside the spherical shell (r < R)

Consider a point P inside the shell at a distance r from the centre. A Gaussian sphere of radius r is constructed as shown in the Figure 1.42 (b). Applying Gauss law

$$\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_\circ}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_\circ} \qquad (1.77)$$

Since Gaussian surface encloses no charge, So Q = 0. The equation (1.77) becomes

$$E = 0 \qquad (r < R) \qquad (1.78)$$

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell. A graph is plotted between the electric field and radial distance. This is shown in Figure 1.43.

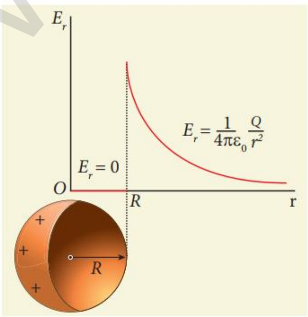

Figure 1.43 Electric field versus distance for a spherical shell of radius R

15. Discuss the various properties of conductors in electrostatic equilibrium.

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown in Figure 1.44.

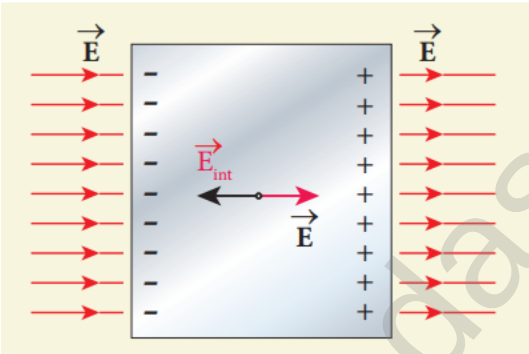

Figure 1.44 Electric field of conductors

Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown in Figure 1.44. Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of 10^{-16}s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.45. A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside the conductor, the net electric flux is also zero over this Gaussian surface. From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced inside the conductor, it immediately reaches the surface of the conductor.

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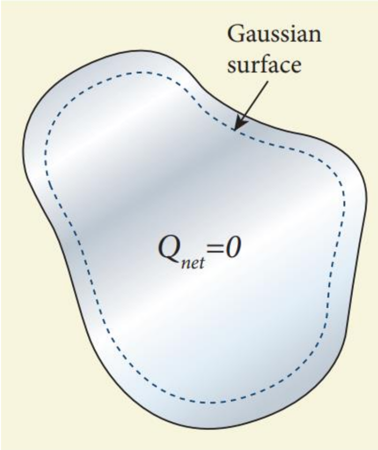

Figure 1.45 No net charge inside the conductor

(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density at that point.

If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure 1.46(a)). This means that the conductor is not in equilibrium. Therefore, at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure 1.46 (b).

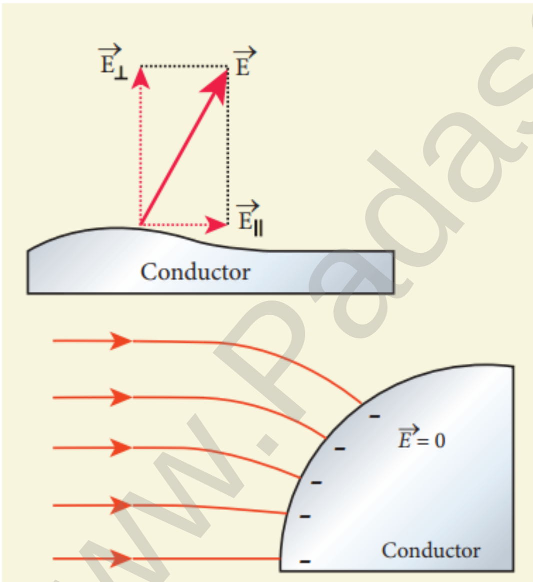

Figure 1.46 (a) Electric field is along the surface (b)Electric field is perpendicular to the surface of the conductor

We now prove that the electric field has magnitude $\frac{\sigma}{\varepsilon_0}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the Figure 1.47. One half of this cylinder is embedded inside the conductor.

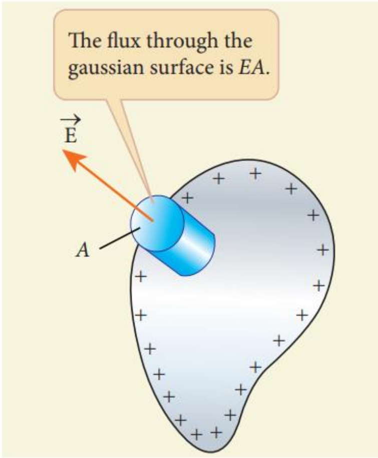

Figure 1.47 The electric field on the surface of the conductor

Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux. Therefore, the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is σA. By applying Gaus's law,

$$EA = \frac{\sigma A}{\varepsilon_\circ}$$

In vector form, $$\vec{E} = \frac{\sigma}{\varepsilon_\circ}\hat{n} \quad (1.79)$$

Here $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface. Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

16. Explain the process of electrostatic induction.

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of charging without actual contact is called electrostatic induction.

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure 1.49(a). The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side. Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

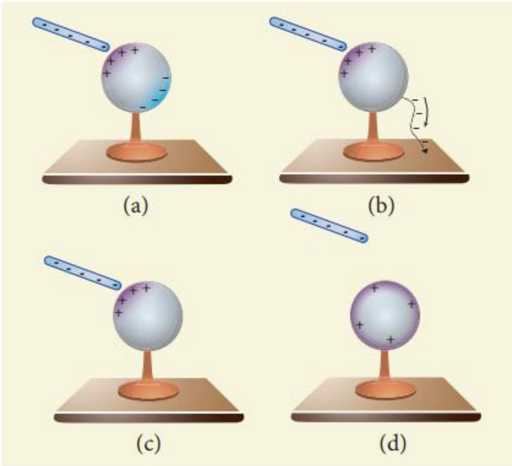

Figure 1.49 Various steps in electrostatic induction

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure 1.49(b)).

(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure 1.49(c))

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure 1.49 (d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section 1.9

17. Explain dielectrics in detail and how an electric field is induced inside a dielectric.

When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field. Therefore, the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the Figure 1.52(b).

The uniform electric field between the plates acts as an external electric field $\vec{E}_{ext}$ which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface.

But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the Figure 1.52(b).

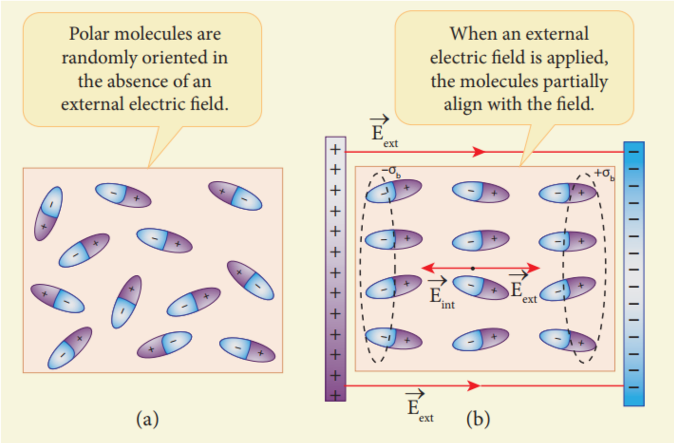

Figure 1.52 Induced electric field lines inside the dielectric

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown in Figure 1.53.

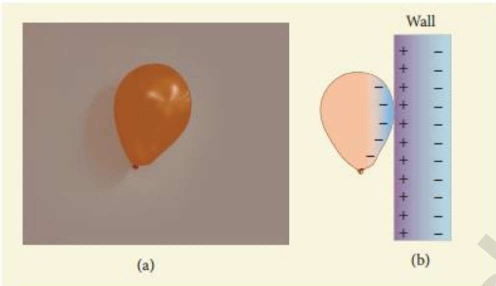

Figure 1.53 (a) Balloon sticks to the wall (b) Polarisation of wall due to the electric field created by the balloon

18. Obtain the expression for capacitance for a parallel plate capacitor.

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown in Figure 1.56.

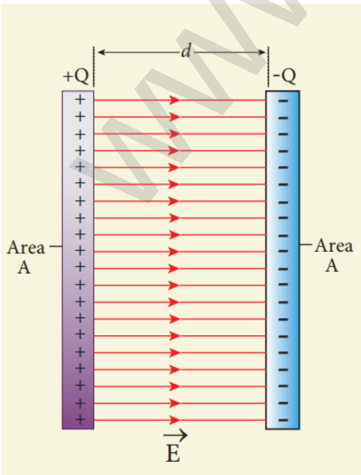

Figure 1.56 Capacitance of a parallel plate capacitor

The electric field between two infinite parallel plates is uniform and is given by E = $\frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density on the plates $\sigma = \frac{Q}{A}$. If the separation distance d is very much smaller than the size of the plate (d2 << A), then the above result is used even for finite–sized parallel plate capacitor. The electric field between the plates is

$$E = \frac{\sigma}{A\varepsilon_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.82)$$

Since the electric field is uniform, the electric potential between the plates having separation d is given by

$$V = Ed = \frac{Qd}{A\varepsilon_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.83)$$

Therefore, the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} = \frac{A\varepsilon_0}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.84)$$

From equation (1.84), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance d between the two plates is reduced, the potential difference between the plates ($V = Ed$) decreases with E constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery's terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

19. Obtain the expression for energy stored in the parallel plate capacitor.

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery.

This work done is stored as electrostatic potential energy in the capacitor. To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = VdQ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.85)$$

where $V = \frac{Q}{C}$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}\, dQ = \frac{Q^2}{2C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.86)$$

This work done is stored as electrostatic potential energy (UE) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (\because Q = CV) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.87)$$

where Q = CV is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor. But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the results

$C = \frac{\varepsilon_0 A}{d}$ and V = Ed

$U_E = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0(Ad)E^2$ ....(1.88)

where Ad = volume of the space between the capacitor plates. The energy stored per unit volume of space is defined as energy density $u_E = \frac{U}{volume}$ From equation (1.88), we get

$u_E = \frac{1}{2}\varepsilon_0 E^2$.....................................(1.89)

From equation (1.89), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

20. Explain in detail the effect of a dielectric placed in a parallel plate capacitor.

In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery

(i) when the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0} \qquad (1.90)$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in Figure 1.58.

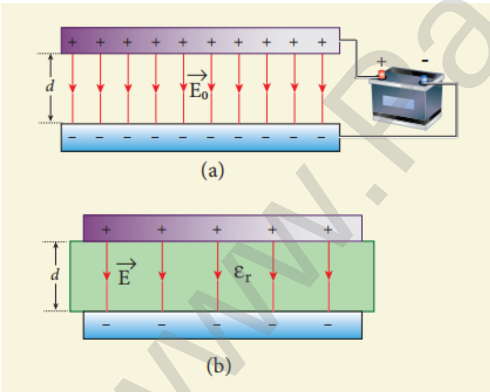

Figure 1.58 (a) Capacitor is charged with a battery (b) Dielectric is inserted after the battery is disconnected

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by

$$E = \frac{E_0}{\varepsilon_r} \qquad (1.91)$$

Here E o is the electric field inside the capacitors when there is no dielectric and ε_r is the relative permeability of the dielectric or simply known as the dielectric constant.

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Since $\varepsilon_r > 1$, the electric field $E < E_o$. As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected. Hence the new potential difference is

$$V = Ed = \frac{E_0}{\varepsilon_r}d = \frac{V_0}{\varepsilon_r} \qquad (1.92)$$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases. Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \qquad (1.93)$$

Since $\varepsilon_r > 1$, we have $C > C_o$. Thus insertion of the dielectric constant ε_r increases the capacitance. Using equation (1.84),

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d} \qquad (1.94)$$

where $\varepsilon = \varepsilon_r \varepsilon_o$ is the permittivity of the dielectric medium. The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0} \qquad (1.95)$$

After the dielectric is inserted, the charge Q0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r} \qquad (1.96)$$

Since $\varepsilon_r > 1$ we get $U < U_o$. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage V_0 remains connected to the capacitor when the dielectric is inserted into the capacitor. This is shown in Figure1.59. The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.

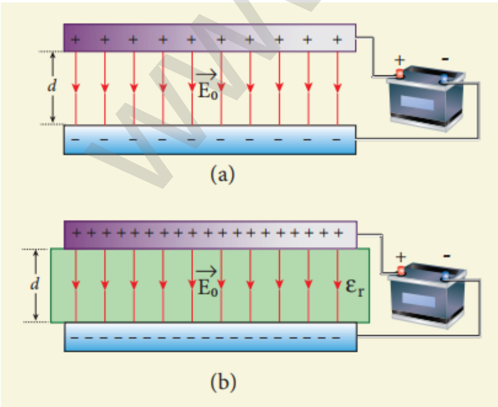

Figure 1.59 (a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

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$Q = \varepsilon_r Q_0$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \qquad (1.98)$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

$$\text{Now,} \quad C_0 = \frac{\varepsilon_0 A}{d}$$

$$\text{and} \quad C = \frac{\varepsilon A}{d} \qquad (1.99)$$

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2 \qquad (1.100)$$

Note that here we have not used the expression $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ because here, both charge and capacitance are changed, whereas in equation (1.100), V_o remains constant.

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} \varepsilon_r C_0 V_0^2 = \varepsilon_r U_0 \qquad (1.101)$$

Since $\varepsilon_r > 1$ we have $U > U_o$.

It may be noted here that since voltage between the capacitor V0 is constant, the electric field between the plates also remains constant.

The energy density is given by

$$u = \frac{1}{2} \varepsilon E_0^2 \qquad (1.102)$$

where ε is the permittivity of the given dielectric material. The results of the above discussions are summarised in the following Table 1.2

Table 1.2

S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

21. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.

(i) Capacitor in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the Figure 1.60 (a). As soon as the battery is connected to the capacitors in series, the electrons of charge –Q are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount -Q from left plate of C_3 to the right plate of C2 due to electrostatic induction.

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Similarly, the left plate of C_2 pushes the charges of –Q to the right plate of C1 which induces the positive charge +Q on the left plate of C_1. At the same time, electrons of charge –Q are transferred from left plate of C_1 to positive terminal of the battery.

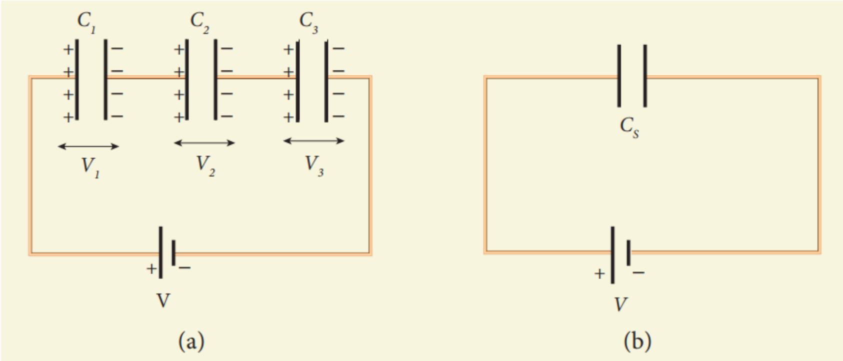

Figure 1.60 (a) Capacitors connected in series (b) Equivalence capacitors C_S

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively. The total voltage across each capacitor must be equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3 \tag{1.103}$$

Since, $Q = CV$, we have $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \tag{1.104}$$

If three capacitors in series are considered to form an equivalent single capacitor C_s shown in Figure 1.60(b), then we have $V = \frac{Q}{C_S}$. Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{1.105}$$

Thus, the inverse of the equivalent capacitance CS of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance CS is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance C1, C2 and C3 connected in parallel with a battery of voltage V as shown in Figure 1.61 (a).

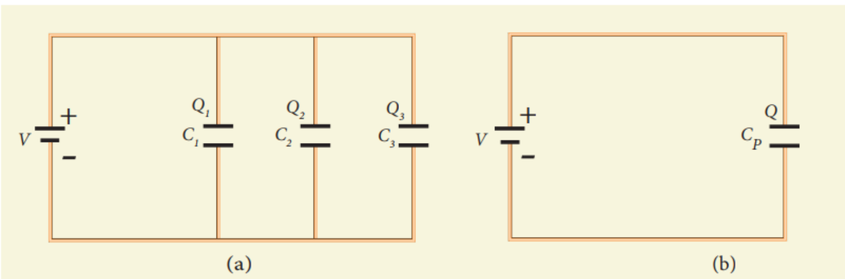

Figure 1.61 (a) capacitors in parallel (b) equivalent capacitance with the same total charge

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2, and Q_3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3 \qquad (1.106)$$

Now, since Q=CV, we have

$$Q = C_1V + C_2V + C_3V \qquad (1.107)$$

If these three capacitors are considered to form a single capacitance C_P which stores the total charge Q as shown in the Figure 1.61(b), then we can write $Q = C_PV$. Substituting this in equation (1.107), we get

$$C_PV = C_1V + C_2V + C_3V$$

$$C_P = C_1 + C_2 + C_3 \qquad (1.108)$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances. The equivalent capacitance CP in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

22. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.

Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in the Figure 1.62. The distance between the spheres is much greater than the radii of either spheres.

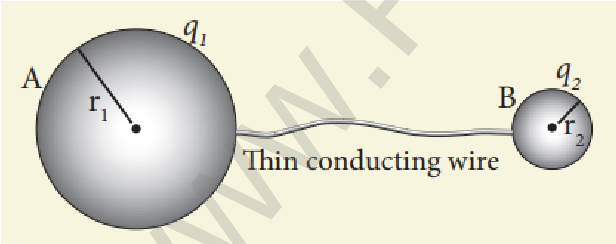

Figure 1.62 Two conductors are connected through conducting wire

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that $Q = q_1 + q_2$. The charges are distributed only on the surface and there is no net charge inside the conductor. The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1} \quad (1.110)$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_2} \quad (1.111)$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B$$

$$\text{or} \quad \frac{q_1}{r_1} = \frac{q_2}{r_2} \quad (1.112)$$

Let us take the charge density on the surface of sphere A is σ_1 and charge density on the surface of sphere B is σ_2. This implies that $q_1 = 4\pi r_1^2\sigma_1$ and $q_2 = 4\pi r_2^2\sigma_2$. Substituting these values into equation (1.112), we get

$$\sigma_1 r_1 = \sigma_2 r_2 \quad (1.113)$$

from which we conclude that

$$\sigma r = \text{constant} \quad (1.114)$$

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

23. Explain in detail the construction and working of a Van de Graaff generator.

In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V). This Van de Graff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.65. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys. The comb D is maintained at a positive potential of 10^4 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere. Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

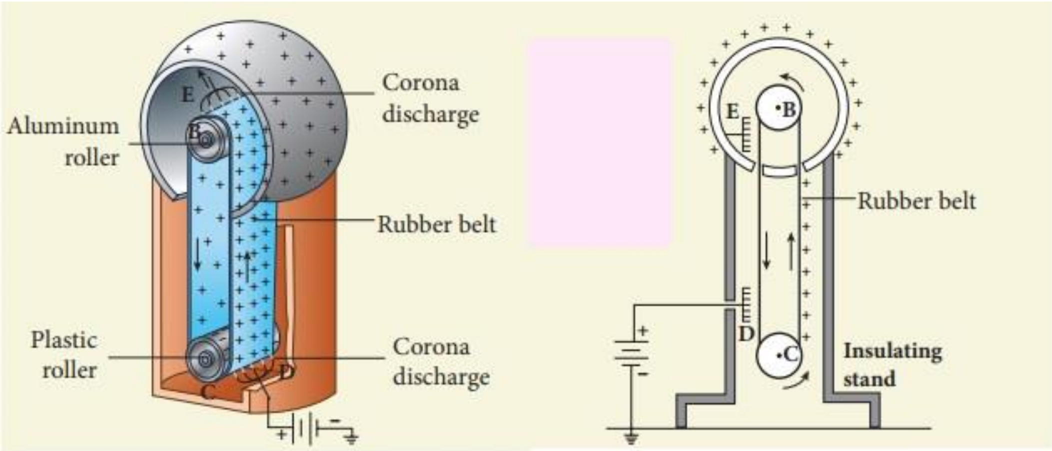

Figure 1.65 Van de Graaff generator

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure. The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

Thank You

T.Vediyappan.,M.Sc.,B.Ed.,M.Phil., Department Of Physics