| Basic Mathematics (241) Marking Scheme 2023-24 |  |
| :---: | :---: |
| Section A |  |
| 1) (b) $x y^{2}$ | 1 |
| 2) (c) 20 | 1 |
| 3) (b) $1 / 2$ | 1 |
| 4) (d) No Solution | 1 |
| 5) (d) 0,8 | 1 |
| 6) (c) 5 Unit | 1 |
| 7) (a) $\triangle P Q R \sim \triangle C A B$ | 1 |
| 8) (d) RHS | 1 |
| 9) (b) $70^{\circ}$ | 1 |
| 10) (b) $3 / 4$ | 1 |
| 11) (b) $45^{\circ}$ | 1 |
| 12) (a) $\sin ^{2} A$ | 1 |
| 13) (c) $\pi: 2$ | 1 |
| 14) (a) 7 cm | 1 |
| 15) (d) $\frac{1}{6}$ | 1 |
| 16) (a) 15 | 1 |
| 17) (a) 3.5 cm | 1 |
| 18) (b) 12-18 | 1 |
| 19) (a) Both assertion and reason are true and reason is the correct explanation of assertion. | 1 |
| 20) (d) Assertion (A) is false but reason(R) is true. | 1 |

21) $3 x+2 y=8$
$6 x-4 y=9$
$a_{1}=3, \quad b_{1}=2, \quad c_{1}=8$
$a_{2}=6, \quad b_{2}=-4, \quad c_{2}=9$
$\frac{a_{1}}{a_{2}}=\frac{3}{6}=\frac{1}{2} \quad \frac{b_{1}}{b_{2}}=\frac{2}{-4}=\frac{-1}{2} \quad \frac{c_{1}}{c_{2}}=\frac{8}{9}$
$\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$
The given pair of linear equations are consistent.
22) Given:-AB II CD II EF

To prove: $-\frac{A E}{E D}=\frac{B F}{F C}$
Construction:- Join BD to
intersect EF at G.
Proof:- in $\triangle \mathrm{ABD}$


In $\triangle D B C$
GF IICD (EF IICD )
$\frac{B F}{F C}=\frac{B G}{G D} \quad($ by $B P T$ $\qquad$ (2)
from (1) \& (2)
$\frac{A E}{E D}=\frac{B F}{F C}$

## OR

Given $A D=6 \mathrm{~cm}, \mathrm{DB}=9 \mathrm{~cm}$
$A E=8 \mathrm{~cm}, E C=12 \mathrm{~cm}, \angle A D E=48$
To find:- $\angle A B C=$ ?
Proof:
In $\triangle A B C$
$\frac{A D}{D B}=\frac{6}{9}=\frac{2}{3}$

$\frac{A E}{E C}=\frac{8}{12}=\frac{2}{3}$
From (1) \& (2)
$\frac{A D}{D B}=\frac{A E}{E C}$
DE II BC (Converse of BPT)
$\angle A D E=\angle A B C$ (Corresponding angles)
$\Rightarrow \angle A B C=48^{\circ}$
23) In $\triangle \mathrm{OTA}, \angle \mathrm{OTA}=90^{\circ}$

By Pythagoras theorem
$O A^{2}=O T^{2}+A T^{2}$
$(5)^{2}=\mathrm{OT}^{2}+(4)^{2}$
$25-16=\mathrm{OT}^{2}$

$9=\mathrm{OT}^{2}$
$\mathrm{OT}=3 \mathrm{~cm}$
radius of circle $=3 \mathrm{~cm}$.
24) $\operatorname{Sin}^{2} 60^{\circ}+2 \tan 45^{\circ}-\cos ^{2} 30^{\circ}$
$=\left(\frac{\sqrt{3}}{2}\right)^{2}+2(1)-\left(\frac{\sqrt{3}}{2}\right)^{2}$
$=\frac{3}{4}+2-\frac{3}{4}$
$=2$
25) Area of the circle= sum of areas of 2 circles
$\pi R^{2}=\pi(40)^{2}+\pi(9)^{2}$
$\pi R^{2}=\pi \times\left(40^{2}+9^{2}\right)$
$R^{2}=1600+81$
$R^{2}=1681$
$R=41 \mathrm{~cm}$.
Diameter of given circle $=41 \times 2=82 \mathrm{~cm}$

## OR

radius of circle $=10 \mathrm{~cm}, \quad \theta=90^{\circ}$
Area of minor segment $=\frac{\theta}{360^{\circ}} \pi r^{2}-$ Area of $\Delta$
$=\frac{\theta}{360^{\circ}} \times \pi r^{2}-\frac{1}{2} \times b \times h$
$=\frac{90^{\circ}}{360^{\circ}} \times 3.14 \times 10 \times 10-\frac{1}{2} \times 10 \times 10$
$=\frac{314}{4}-50$
$=78.5-50=28.5 \mathrm{~cm}^{2}$

Area of minor segment $=28.5 \mathrm{~cm}^{2}$

## Section C

26) Let us assume that $\sqrt{3}$ be a rational number
$\sqrt{3}=\frac{a}{b} \quad$ where $a$ and $b$ are co-prime.
squaring both the sides
$(\sqrt{3})^{2}=\left(\frac{a}{b}\right)^{2}$
$3=\frac{a^{2}}{b^{2}} \Rightarrow a^{2}=3 b^{2}$
$a^{2}$ is divisible by 3 so $a$ is also divisible by 3 $\qquad$ (1)
let $a=3 c$ for any integer $c$.
$(3 c)^{2}=3 b^{2}$
$9 c^{2}=3 b^{2}$
$b^{2}=3 c^{2}$
since $b^{2}$ is divisible by 3 so, b is also divisible by 3 $\qquad$ (2)

From (1) \& (2) we can say that 3 in a factor of $a$ and $b$
which is contradicting the fact that a and b are co- prime.
Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.
Hence, $\sqrt{3}$ is an irrational number.
27) $P(S)=4 S^{2}-4 S+1$
$4 S^{2}-2 S-2 S+1=0$
$2 S(2 S-1)-1(2 S-1)=0$
$(2 S-1)(2 S-1)=0$
$S=1 / 2 \quad S=1 / 2$
$\mathrm{a}=4 \quad \mathrm{~b}=-4 \quad \mathrm{c}=1 \quad \alpha=1 / 2 \quad \beta=1 / 2$
$\alpha+\beta=\frac{-b}{a}, \quad \alpha \beta=\frac{c}{a}$
$\frac{1}{2}+\frac{1}{2}=\frac{-4}{4}, \quad\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$
$\frac{1+1}{2}=\frac{+4}{4}, \quad \frac{1}{4}=\frac{1}{4}$
$\frac{2}{2}=1$
$1=1$
28) Let cost of one bat be Rs $x$

Let cost of one ball be Rs $y$
ATQ
$4 x+1 y=2050$
$3 x+2 y=1600$
(2)
from (1) $4 x+1 y=2050$
$y=2050-4 x$

Substite value of $y$ in (2)
$3 x+2(2050-4 x)=1600$

$$
\begin{aligned}
3 \mathrm{x}+4100-8 \mathrm{x} & =1600 \\
-5 \mathrm{x} & =-2500 \\
x & =500
\end{aligned}
$$

Substiture value of $x$ in (1)

$$
\begin{aligned}
& 4 x+1 y=2050 \\
& 4(500)+y=2050 \\
& 2000+y=2050 \\
& y=50
\end{aligned}
$$

Hence
Cost of one bat $=$ Rs. 500
Cost of one ball $=$ Rs. 50

## OR

Let the fixed charge for first 3 days= Rs. $x$
And additional charge after 3 days= Rs. $y$ 1/2
ATQ
$x+4 y=27$
$x+2 y=21$
Subtract eq ${ }^{\text {n }}$ (2) from (1)
$2 y=6$
$y=3$
Substitute value of $y$ in (2)
$x+2(3)=21$
$x=21-6$
$x=15$
Fixed charge= Rs. 15
Additional charge per day = Rs. 3
29) Given circle touching sides of $A B C D$ at $P, Q, R$ and $S$

To prove- $A B+C D=A D+B C$
Proof-
AP=AS-------(1) tangents from an external point
PB=BQ-------(2) to a circle are equal in length
DR=DS-------(3)


CR=CQ-------(4)
Adding eq ${ }^{\mathrm{n}}$ (1),(2),(3) \& (4)
$A P+B P+D R+C R=A S+D S+B Q+C Q$
$A B+D C=A D+B C$
30) $(\operatorname{cosec} \theta-\cot \theta)^{2}=\frac{1-\cos \theta}{1+\cos \theta}$
$\mathrm{LHS}=(\operatorname{cosec} \theta-\cot \theta)^{2}$
$=\left(\frac{1}{\sin \theta}-\frac{\cos \theta}{\sin \theta}\right)^{2}$
$=\left(\frac{1-\cos \theta}{\sin \theta}\right)^{2}$
$=\frac{(1-\cos \theta)^{2}}{\sin ^{2} \theta}$
$=\frac{(1-\cos \theta)^{2}}{1-\cos ^{2} \theta}$
$=\frac{(1-\cos \theta)^{2}}{(1-\cos \theta)(1+\cos \theta)}$
$=\frac{1-\cos \theta}{1+\cos \theta}=\mathrm{RHS}$
LHS $=$ RHS, Hence Proved

## OR

$\sec A(1-\sin A)(\sec A+\tan A)=1$
LHS $=\frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$
$=\frac{(1-\sin \mathrm{A})}{\cos A} \frac{(1+\sin \mathrm{A})}{\cos A}$
$=\frac{(1-\sin \mathrm{A})(1+\sin \mathrm{A})}{\cos ^{2} A}$
$=\frac{1-\sin ^{2} \mathrm{~A}}{\cos ^{2} A} \quad\left(1-\sin ^{2} A=\cos ^{2} A\right)$
$=\frac{\cos ^{2} A}{\cos ^{2} A}$
$=1=$ RHS
LHS=RHS. Hence Proved
31) (i) Red balls=6, Black balls $=4$, White balls $=x$
$\mathrm{P}($ white ball $)=\frac{x}{10+x}=\frac{1}{3}$
$\Rightarrow 3 x=10+x \Rightarrow x=5$ white balls
(ii) Let y red balls be removed, black balls $=4$, white balls $=5$
$\mathrm{P}($ white balls $)=\frac{5}{(6-y)+4+5}=\frac{1}{2}$
$\Rightarrow \frac{5}{15-y}=\frac{1}{2} \Rightarrow 10=15-y \Rightarrow y=5$
So 5 balls should be removed.

## Section D

32) Let the speed of train be $x \mathrm{~km} / \mathrm{hr}$
distance $=360 \mathrm{~km}$
Speed $=\frac{\text { distance }}{\text { time }}$
Time $=\frac{360}{x}$
New speed $=(x+5) \mathrm{km} / \mathrm{hr}$
Time $=\frac{D}{5}$
$x+5=\frac{360}{\left(\frac{360}{x}-1\right)}$
$(x+5)\left(\frac{360}{x}-1\right)=360$
$(x+5)(360-x)=360 x$
$-x^{2}-5 x+1800=0$
$x^{2}+5 x-1800=0$
$x^{2}+45 x-40 x-1800=0$
$x(x+45)-40(x+45)=0$
$(x+45)(x-40)=0$
$x+45=0 \quad, \quad x-40=0$
$x=-45 \quad, \quad x=40$
Speed cannot be negative
Speed of train $=40 \mathrm{~km} / \mathrm{hr}$

## OR

Let the speed of the stream $=x \mathrm{~km} / \mathrm{hr}$
Speed of boat= $18 \mathrm{~km} / \mathrm{hr}$
Upstream speed= $(18-x) \mathrm{km} / \mathrm{hr}$
Downstream speed $=(18+x) \mathrm{km} / \mathrm{hr}$
Time taken (upstream) $=\frac{24}{(18-x)}$
Time taken (downstream) $=\frac{24}{(18+\mathrm{x})}$
ATQ

$$
\begin{equation*}
\frac{24}{(18-x)}=\frac{24}{(18+x)}+1 \tag{1}
\end{equation*}
$$

$$
\frac{24}{(18-x)}-\frac{24}{(18+x)}=1
$$

$24(18+x)-24(18-x)=(18-x)(18+x)$
$24(18+x-18+x)=(18)^{2}-x^{2}$
$24(2 x)=324-x^{2}$
$48 x-324+x^{2}=0$
$x^{2}+48 x-324=0$
$x^{2}-6 x+54 x-324=0$
$x(x-6)+54(x-6)=0$
$(x-6)(x+54)=0$
$x-6=0, \quad x+54=0$
$x=6 \quad, \quad x=-54$
Speed cannot be negative
Speed of stream $=6 \mathrm{~km} / \mathrm{hr}$
33) Given $\triangle A B C, D E| | B C$

To prove $\frac{A D}{D B}=\frac{A E}{E C}$
Construction: join BE and CD
Draw $D M \perp A C$ and $E N \perp A B$
Proof: Area of $\triangle A D E=\frac{1}{2} \times \mathrm{b} \times \mathrm{h}$
$=\frac{1}{2} \times A D \times E N-$
Area $(\triangle D B E)=\frac{1}{2} \times D B \times E N$ -
Divide eq ${ }^{n}(1)$ by (2)
$\frac{\operatorname{ar} \triangle A D E}{\operatorname{ar} \triangle D B E}=\frac{\frac{1}{2} \mathrm{X} A D \times E N}{\frac{1}{2} \mathrm{X} \mathrm{DB} \mathrm{X} E N}=\frac{A D}{D B}$

area $\triangle A D E=\frac{1}{2} \times \mathrm{AE} \times \mathrm{DM}$
area $\triangle D E C=\frac{1}{2} \times E C \times D M$
Divide eq ${ }^{\mathrm{n}}$ (4) by (5)
$\frac{\operatorname{ar} \triangle A D E}{\operatorname{ar} \triangle D E C}=\frac{\frac{1}{2} \mathrm{X} A E \times D M}{\frac{1}{2} \mathrm{X} E C \times D M}=\frac{A E}{E C}$
$\triangle B D E$ and $\triangle D E C$ are on the same base $D E$ and between same parallel lines $B C$ and $D E$
$\therefore \operatorname{area}(\triangle D B E)=\operatorname{ar}(D E C)$
hence
$\frac{\operatorname{ar}(\triangle A D E)}{\operatorname{ar}(\triangle D B E}=\frac{\operatorname{ar}(\triangle A D E)}{\operatorname{ar}(\triangle D E C)} \quad[$ LHS of (3) $=$ RHS of (6)]
$\frac{A D}{D B}=\frac{A E}{E C} \quad[\mathrm{RHS}$ of $(3)=\mathrm{RHS}$ of (6)
Since $\frac{P S}{S Q}=\frac{P T}{T R} \therefore S T \| Q R$ (by converse of $B P T$ )
$\angle \mathrm{PST}=\angle \mathrm{PQR} \quad$ (Corresponding angles)
But $\angle \mathrm{PST}=\angle \mathrm{PRQ}$ (given)
$\angle P Q R=\angle P R Q$
$P R=P Q$ ( sides opposite to equal angles are equal


Hence $\triangle P Q R$ is isosceles.
34) Diameter of cylinder and hemisphere $=5 \mathrm{~mm}$ radius, $(r)=\frac{5}{2}$

Total length $=14 \mathrm{~mm}$
Height of cylinder $=14-5=9 \mathrm{~mm}$
CSA of cylinder $=2 \pi$ rh
$=2 \times \frac{22}{7} \times \frac{5}{2} \times 9$
$=\frac{990}{7} \mathrm{~mm}^{2}$
CSA of hemispheres $=2 \pi r^{2}$
$=2 \times \frac{22}{7} \times\left(\frac{5}{2}\right)^{2}$
$=\frac{275}{7} \mathrm{~mm}^{2}$
CSA of 2 hemispheres $=2 \times \frac{275}{7}$
$=\frac{550}{7} \mathrm{~mm}^{2}$
Total area of capsule $=\frac{990}{7}+\frac{550}{7}$
$=\frac{1540}{7}$
$=220 \mathrm{~mm}^{2}$

Diameter of cylinder $=2.8 \mathrm{~cm}$
radius of cylinder $=\frac{2.8}{2}=1.4 \mathrm{~cm}$
radius of cylinder $=$ radius of hemisphere $=1.4 \mathrm{~cm}$
Height of cylinder $=5-2.8$
$=2.2 \mathrm{~cm}$
Volume of 1 Gulab jamun = vol. of cylinder $+2 x$ vol. of hemisphere
$=\pi r^{2} \mathrm{~h}+2 \times \frac{2}{3} \pi r^{3}$
$\frac{22}{7} \times(1.4)^{2} \times 2.2+2 \times \frac{2}{3} \times \frac{22}{7} \times(1.4)^{3}$
$=13.55+11.50$
$=25.05 \mathrm{~cm}^{3}$
volume of 45 Gulab jamun $=45 \times 25.05$
syrup in 45 Gulab jamun $=30 \% \times 45 \times 25.05$

$$
\begin{gathered}
=\frac{30}{100} \times 45 \times 25.05 \\
=338.175 \mathrm{~cm}^{3} \\
\approx 338 \mathrm{~cm}^{3}
\end{gathered}
$$

35) 

| Life time (in hours) | Number of lamps(f) | Mid $x$ | d | fd |
| :---: | :---: | :---: | :---: | :---: |
| $1500-2000$ | 14 | 1750 | -1500 | -21000 |
| $2000-2500$ | 56 | 2250 | -1000 | -56000 |
| $2500-3000$ | 60 | 2750 | -500 | -30000 |
| $3000-3500$ | 74 | 3750 | 500 | 37000 |
| $3500-4000$ | 62 | 4250 | 1000 | 62000 |
| $4000-4500$ | 48 | 4750 | 1500 | 72000 |
| $4500-5000$ | 400 |  |  | 64000 |

$$
\text { Mean }=\mathrm{a}+\frac{\Sigma f d}{\Sigma \mathrm{f}}
$$

$$
\begin{aligned}
\text { Mean } & =3250+\frac{64000}{400} \\
& =3250+160 \\
& =3410
\end{aligned}
$$

Average life of lamp is 3410 hr

## Section E

36) $\mathrm{a}_{6}=16000 \quad \mathrm{a}_{9}=22600$
$a+5 d=16000$
$a+8 d=22600$
substitute $a=1600-5 d$ from (1)
$16000-5 d+8 d=22600$
$3 \mathrm{~d}=22600-16000$
$3 \mathrm{~d}=6600$
$\mathrm{d}=\frac{6600}{3}=2200$
$a=16000-5(2200)$
$a=16000-11000$
$a=5000$
(i) $a_{n}=29200, a=5000, \quad d=2200$
$a_{n}=a+(n-1) d$
$29200=5000+(n-1) 2200$
$29200-5000=2200 \mathrm{n}-2200$
$24200+2200=2200 n$
$26400=2200 n$
$n=\frac{264}{22}$
$\mathrm{n}=12$
in $12^{\text {th }}$ year the production was Rs 29200
(ii) $n=8, \quad a=5000, \quad d=2200$
$\mathrm{a}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d} \quad 1 / 2$
$=5000+(8-1) 2200 \quad 1 / 2$
$=5000+7 \times 2200$
$=5000+15400$
= 20400
The production during $8^{\text {th }}$ year is $=20400$

## OR

$\mathrm{n}=3, \quad \mathrm{a}=5000, \quad \mathrm{~d}=2200$
$\mathrm{S}_{\mathrm{n}}=\frac{n}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$=\frac{3}{2}[2(5000)+(3-1) 2200]$
$\mathrm{S}_{3}=\frac{3}{2}(10000+2 \times 2200)$
$=\frac{3}{2}(10000+4400)$
$=3 \times 7200$
$=21600$
The production during first 3 year is 21600
(iii) $a_{4}=a+3 d$
$=5000+3(2200)$
$=5000+6600$
$=11600$
$a_{7}=a+6 d$
$=5000+6 \times 2200$
$=5000+13200$
= 18200
$\mathrm{a}_{7}-\mathrm{a}_{4}=18200-11600=6600$
37) coordinates of $A(2,3)$ Alia's house
coordinates of $B(2,1)$ Shagun's house
coordinates of $C(4,1) \quad$ Library
(i) $\mathrm{AB}=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$

$$
=\sqrt{(2-2)^{2}+(1-3)^{2}}
$$

$=\sqrt{\left(0^{2}+(-2)\right.}{ }^{2}$
$A B=\sqrt{0+4}=\sqrt{4}=2$ units
Alia's house from shagun's house is 2 units
(ii) $\mathrm{C}(4,1), \mathrm{B}(2,1)$
$\mathrm{CB}=\sqrt{(2-4)^{2}+(1-1)^{2}}$
$=\sqrt{(-2)^{2}+0^{2}}$
$=\sqrt{4+0}=\sqrt{4}=2$ unit
(iii) $0(0,0), B(2,1)$
$\mathrm{OB}=\sqrt{(2-0)^{2}+(1-0)^{2}}$
$=\sqrt{2^{2}+1^{2}}=\sqrt{4+1}=\sqrt{5}$ units
Distance between Alia's house and Shagun's house, $\mathrm{AB}=2$ units
Distance between Library and Shagun's house, $C B=2$ units
$O B$ is greater than $A B$ and $C B$,
For shagun, school [O] is farther than Alia's house [A] and Library [C]
$\mathrm{C}(4,1), \quad \mathrm{A}(2,3)$
$C A=\sqrt{(2-4)^{2}+(3-1)^{2}}$
$=\sqrt{(-2)^{2}+2^{2}}+=\sqrt{4+4} \quad=\sqrt{8}$
$=2 \sqrt{2}$ units $\quad A C^{2}=8$
Distance between Alia's house and Shagun's house, $\mathrm{AB}=2$ units
Distance between Library and Shagun's house, CB $=2$ units
$\mathrm{AB}^{2}+\mathrm{BC}^{2}=2^{2}+2^{2}=4+4=8=\mathrm{AC}^{2}$
Therefore $\mathrm{A}, \mathrm{B}$ and C form an isosceles right triangle.
(i) $X Y \| P Q$ and $A P$ is transversal.
$\angle A P D=\angle P A X$ (alternative interior angles)
$\angle A P D=45^{\circ}$
(ii) Since $X Y$ || $P Q$ and $A Q$ is a transversal so alternate interior angles are equal hence $\angle \mathrm{YAQ}=\angle \mathrm{AQD}=30^{\circ}$
(iii) $\operatorname{In} \triangle A D P, \theta=45^{\circ}$

$\tan \theta=\frac{P}{B}$
$\tan 45^{\circ}=\frac{100}{P D}$
$P D=100 \mathrm{~m}$

Boat $P$ is 100 m from the light house

## OR

In $\triangle A D Q, \theta=30^{\circ}$
$\tan \theta=\frac{P}{B}$
$\tan 30=\frac{100}{D Q}$
$\frac{1}{\sqrt{3}}=\frac{100}{D Q}$
$D Q=100 \sqrt{3} \mathrm{~m}$

