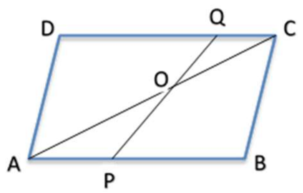
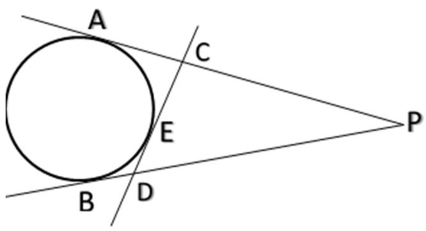
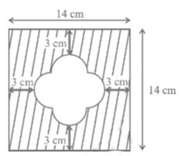
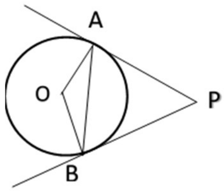
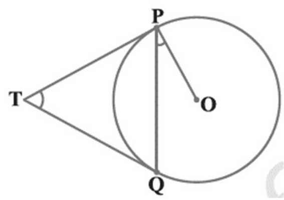
Marking Scheme
Class X Session 2023-24
MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	(b) xy^2	1
2.	(b) 1 zero and the zero is '3'	1
3.	(b) $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	1
4.	(c) 2 distinct real roots	1
5.	(c) 7	1
6.	(a) 1:2	1
7.	(d) infinitely many	1
8.	(b) $\frac{ac}{b+c}$	1
9.	(b) 100°	1
10.	(d) 11 cm	1
11.	(c) $\frac{\sqrt{b^2-a^2}}{b}$	1
12.	(d) cos A	1
13.	(a) 60°	1
14.	(a) 2 units	1
15.	(a) 10m	1
16.	(b) $\frac{4-\pi}{4}$	1
17.	(b) $\frac{22}{46}$	1
18.	(d) 150	1
19.	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20.	(c) Assertion (A) is true but reason (R) is false.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers *a* and *b* such that $\sqrt{2} = \frac{a}{b}$ where *a* and *b* are coprime.	½
	So, b $\sqrt{2}$ = a. Squaring both sides, we get $2b^2 = a^2$.	½
	Therefore, 2 divides a^2 and so 2 divides a. So, we can write a = 2c for some integer c. Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.	½
	This means that 2 divides b^2, and so 2 divides b Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.	½

22.	ABCD is a parallelogram. AB = DC = a Point P divides AB in the ratio 2:3 AP = $\frac{2}{5}$ a , BP = $\frac{3}{5}$ a point Q divides DC in the ratio 4:1. DQ = $\frac{4}{5}$ a , CQ = $\frac{1}{5}$ a Δ APO ~ Δ CQO [AA similarity] $\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$ $\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1} \Rightarrow OC = ½\ OA$  	½ ½ ½ ½
23.	PA = PB; CA = CE; DE = DB [Tangents to a circle] Perimeter of ΔPCD = PC + CD + PD = PC + CE + ED + PD = PC + CA + BD + PD = PA + PB Perimeter of ΔPCD = PA + PA = 2PA = 2(10) = 20 cm  	½ 1 ½
24.	$\because \tan(A+B) = \sqrt{3} \quad \therefore A+B = 60^0$ ...(1) $\because \tan(A-B) = \frac{1}{\sqrt{3}} \quad \therefore A-B = 30^0$ ...(2) Adding (1) & (2), we get 2A=$90^0 \Longrightarrow A = 45^0$ Also (1) –(2), we get $2B = 30^0 \Longrightarrow B = 45^0$	½ ½ ½ ½
	[or]	
	2 cosec²30 + x sin²60 – $\frac{3}{4}$ tan²30 = 10 $\Rightarrow \quad 2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$ $\Rightarrow \quad 2(4) + x\left(\frac{3}{4}\right) - \frac{3}{4}\left(\frac{1}{3}\right) = 10$ $\Rightarrow \quad 8 + x\left(\frac{3}{4}\right) - \frac{1}{4} = 10$ $\Rightarrow \quad 32 + x(3) - 1 = 40$ $\Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$	1 ½ ½
25.	Total area removed = $\frac{\angle A}{360}\pi r^2 + \frac{\angle B}{360}\pi r^2 + \frac{\angle C}{360}\pi r^2$ $= \frac{\angle A + \angle B + \angle C}{360}\pi r^2$ $= \frac{180}{360}\pi r^2$ $= \frac{180}{360} \times \frac{22}{7} \times (14)^2$ ½ = 308 cm²	½ ½ ½
	[or]	
	The side of a square = Diameter of the semi-circle = a Area of the unshaded region = Area of a square of side ‘a’ + 4(Area of a semi-circle of diameter ‘a’) The horizontal/vertical extent of the white region = 14-3-3 = 8 cm Radius of the semi-circle + side of a square + Radius of the semi-circle = 8 cm  	½ ½

	2 (radius of the semi-circle) + side of a square = 8 cm $2a = 8$ cm $\Rightarrow a = 4$ cm Area of the unshaded region = Area of a square of side 4 cm + 4 (Area of a semi-circle of diameter 4 cm) $= (4)^2 + 4 \times \frac{1}{2}\pi(2)^2 = (16 + 8\pi)$ cm^2	½ ½
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	Number of students in each group subject to the given condition = HCF (60,84,108) HCF (60,84,108) = 12 Number of groups in Music = $\frac{60}{12} = 5$ Number of groups in Dance = $\frac{84}{12} = 7$ Number of groups in Handicrafts = $\frac{108}{12} = 9$ Total number of rooms required = 21	½ ½ ½ ½ ½ ½
27.	$P(x) = 5x^2 + 5x + 1$ $\alpha+\beta = \frac{-b}{a} = \frac{-5}{5} = -1$ $\alpha\beta = \frac{c}{a} = \frac{1}{5}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= (-1)^2 - 2\left(\frac{1}{5}\right)$ $= 1 - \frac{2}{5} = \frac{3}{5}$ $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ $= \frac{(\alpha+\beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$	½ ½ ½ ½ ½ ½
28.	Let the ten's and the unit's digits in the first number be x and y, respectively. So, the original number = $10x + y$ When the digits are reversed, x becomes the unit's digit and y becomes the ten's Digit. So the obtain by reversing the digits= $10y + x$ According to the given condition. $(10x + y) + (10y + x) = 66$ i.e., $11(x + y) = 66$ i.e., $x + y = 6$ ---- (1) We are also given that the digits differ by 2, therefore, either $x - y = 2$ ---- (2) or $y - x = 2$ ---- (3) If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$. In this case, we get the number 42. If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$. In this case, we get the number 24. Thus, there are two such numbers 42 and 24.	½ ½ ½ ½ ½ ½
	[or]	
	Let $\frac{1}{\sqrt{x}}$ be 'm' and $\frac{1}{\sqrt{y}}$ be 'n', Then the given equations become $2m + 3n = 2$ $4m - 9n = -1$	½ ½

	$(2m + 3n = 2)$ X- 2 $\Rightarrow -4m - 6n = -4$ ...(1)	
	$4m - 9n = -1$ $\quad 4m - 9n = -1$ ...(2)	
	Adding (1) and (2)	
	We get $-15n = -5 \Rightarrow n = \frac{1}{3}$	½
	Substituting $n = \frac{1}{3}$ in $2m + 3n = 2$, we get	½
	$2m + 1 = 2$	
	$2m = 1$	
	$m = \frac{1}{2}$	1
	$m = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ and $n = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$	
29.		
	$\angle OAB = 30°$	
	$\angle OAP = 90°$ [Angle between the tangent and the radius at the point of contact]	
		
	$\angle PAB = 90° - 30° = 60°$	½
	AP = BP [Tangents to a circle from an external point]	
	$\angle PAB = \angle PBA$ [Angles opposite to equal sides of a triangle]	½
	In ΔABP, $\angle PAB + \angle PBA + \angle APB = 180°$ [Angle Sum Property]	
	$60° + 60° + \angle APB = 180°$	
	$\angle APB = 60°$	½
	$\therefore$ ΔABP is an equilateral triangle, where AP = BP = AB.	
	PA = 6 cm	½
	In Right ΔOAP, $\angle OPA = 30°$	
	$\tan 30° = \frac{OA}{PA}$	
	$\frac{1}{\sqrt{3}} = \frac{OA}{6}$	½
	$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3}cm$	½
	[or]	
	Let $\angle TPQ = \theta$	
	$\angle TPO = 90°$ [Angle between the tangent and the radius at the point of contact]	½
		
	$\angle OPQ = 90° - \theta$	
	TP = TQ [Tangents to a circle from an external point]	
	$\angle TPQ = \angle TQP = \theta$ [Angles opposite to equal sides of a triangle]	½
	In ΔPQT, $\angle PQT + \angle QPT + \angle PTQ = 180°$ [Angle Sum Property]	½
	$\theta + \theta + \angle PTQ = 180°$	½
	$\angle PTQ = 180° - 2\theta$	½
	$\angle PTQ = 2(90° - \theta)$	½
	$\angle PTQ = 2\angle OPQ$ [using (1)]	
30.	Given, $1 + \sin^2\theta = 3 \sin\theta \cos\theta$	
	Dividing both sides by $\cos^2\theta$,	
	$\frac{1}{\cos^2\theta} + \tan^2\theta = 3\tan\theta$	
	$\sec^2\theta + \tan^2\theta = 3\tan\theta$	½
	$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$	½
	$1 + 2\tan^2\theta = 3\tan\theta$	½
	$2\tan^2\theta - 3\tan\theta + 1 = 0$	½
	If $\tan\theta = x$, then the equation becomes $2x^2 - 3x + 1 = 0$	

	$\Rightarrow (x-1)(2x-1) = 0$ x = 1 or $\frac{1}{2}$ tan θ = 1 or $\frac{1}{2}$	1
31.	(table below) Mean = a + $\frac{\sum fd}{\sum f}$ = 149 + $\frac{-8}{40}$ = 149 – 2.025 = 146.975 Average length of the leaves = 146.975	2 ½ ½

Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd
118 – 126	3	117.5- 126.5	122	-27	-81
127 – 135	5	126.5– 135.5	131	-18	-90
136 – 144	9	135.5– 144.5	140	-9	-81
145 – 153	12	144.5 – 153.5	a = 149	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

SECTION D

Section D consists of 4 questions of 5 marks each

32. Let the speed of the stream be x km/h.

The speed of the boat upstream = (18 – x) km/h and
the speed of the boat downstream = (18 + x) km/h. 1

The time taken to go upstream = $\frac{distance}{speed} = \frac{24}{18-x}$ hours

the time taken to go downstream = $\frac{distance}{spe} = \frac{24}{18+x}$ hours 1

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ 1

24(18 + x) – 24(18 – x) = (18 – x) (18 + x)

$x^2 + 48x - 324 = 0$

x = 6 or – 54 1

Since x is the speed of the stream, it cannot be negative.
Therefore, x = 6 gives the speed of the stream = 6 km/h. 1

[or]

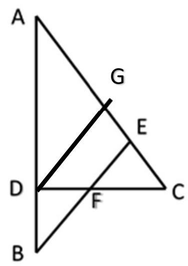
Let the time taken by the smaller pipe to fill the tank = x hr.
Time taken by the larger pipe = (x – 10) hr ½

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

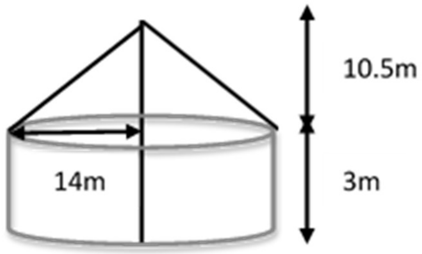
Part of the tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$ 1

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. ½

Part of the tank filled by both the pipes in 1 hour = $\frac{8}{75}$ ½

	Therefore, $\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$	½
	$8x^2 - 230x + 750 = 0$	
	$x = 25, \frac{30}{8}$	1
	Time taken by the smaller pipe cannot be 30/8 = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible.	½
	Therefore, the time taken individually by the smaller pipe is 25 hours and the larger pipe will be 25 – 10 =15 hours.	½
33.	(a) Statement – ½	
	Given and To Prove – ½	
	Figure and Construction ½	3
	Proof – 1 ½	
		
	[b] Draw DG || BE	
	In Δ ABE, $\frac{AB}{BD}=\frac{AE}{GE}$ [BPT]	½
	CF = FD [F is the midpoint of DC] ---(i)	½
	In Δ CDG, $\frac{DF}{CF}=\frac{GE}{CE}=1$ [Mid point theorem]	½
	GE = CE ---(ii)	
	∠CEF = ∠CFE [Given]	
	CF = CE [Sides opposite to equal angles] ---(iii)	½
	From (ii) & (iii) CF = GE ---(iv)	
	From (i) & (iv) GE = FD	
	$\therefore \frac{AB}{BD}=\frac{AE}{GE} \Rightarrow \frac{AB}{BD}=\frac{AE}{FD}$	
34.		
	Length of the pond, l= 50m, width of the pond, b = 44m	
	Water level is to rise by, h = 21 cm = $\frac{21}{100}$ m	
	Volume of water in the pond = lbh = 50 x 44 x $\frac{21}{100}$ m^3 =462 m^3	1
	Diameter of the pipe = 14 cm	
	Radius of the pipe, r = 7cm = $\frac{7}{100}$ m	
	Area of cross-section of pipe = πr^2	
	$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = \frac{154}{10000}$ m^2	1
		½
	Rate at which the water is flowing through the pipe, h = 15km/h = 15000 m/h	
	Volume of water flowing in 1 hour = Area of cross-section of pipe x height of water coming out of pipe	½
	$= \left(\frac{154}{10000} \times 15000\right) m^3$	1
	Time required to fill the pond $= \frac{Volume\ of\ the\ pond}{Volume\ of\ water\ flowing\ in\ 1\ hour}$	1
	$= \frac{462 \times 10000}{154 \times 15000} = 2$ hours	
	Speed of water if the rise in water level is to be attained in 1 hour = 30km/h	
	[or]	

Radius of the cylindrical tent (r) = 14 m



Total height of the tent = 13.5 m

Height of the cylinder = 3 m

Height of the Conical part = 10.5 m ½

Slant height of the cone $(l) = \sqrt{h^2 + r^2}$

$= \sqrt{(10.5)^2 + (14)^2}$

$= \sqrt{110.25 + 196}$ 1

$= \sqrt{306.25} = 17.5$ m

Curved surface area of cylindrical portion

= 2πrh

$= 2 \times \frac{22}{7} \times 14 \times 3$ 1

= 264 m^2

Curved surface area of conical portion

=πrl

$= \frac{22}{7} \times 14 \times 17.5$ 1

=770 m^2 ½

Total curved surface area = 264 m^2 + 770 m^2 = 1034 m^2

Provision for stitching and wastage = 26 m^2 ½

Area of canvas to be purchased = 1060 m^2

Cost of canvas = Rate × Surface area ½

= 500 x 1060 = ₹ 5,30,000/-

35.

Marks obtained	Number of students	Cumulative frequency	
20 – 30	p	p	
30 – 40	15	p + 15	
40 – 50	25	p + 40	1
50 – 60	20	p + 60	
60 – 70	q	p + q + 60	
70 – 80	8	p + q + 68	½
80 - 90	10	p + q + 78	½
	90		

p + q + 78 = 90

p + q = 12

Median $= (l) + \frac{\frac{n}{2} - c}{f} \cdot h$

$50 = 50 + \frac{45-(p+40)}{20} \cdot 10$ ½

$\frac{45-(p+40)}{20} \cdot 10 = 0$ ½

$45 - (p + 40) = 0$ ½

P = 5 ½

5 + q = 12

q = 7 1

Mode $= l + \frac{f1-f0}{2f1-f0-f2} \cdot h$

	$= 40 + \frac{25-15}{2(25)-15-20} \cdot 10$ $= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$	
	SECTION E	
36.	(i) Number of throws during camp. a = 40; d = 12 $t_{11} = a + 10d$ $= 40 + 10 \times 12$ $= 160\ throws$	1
	(ii) a = 7.56 m; d = 9cm = 0.09 m n = 6 weeks $t_n = a + (n-1)\ d$ = 7.56 + 6(0.09) = 7.56 + 0.54 Sanjitha's throw distance at the end of 6 weeks = 8.1 m (or) a = 7.56 m; d = 9cm = 0.09 m t_n =11.16 m $t_n = a + (n-1)\ d$ 11.16 = 7.56 + (n-1) (0.09) 3.6 = (n-1) (0.09) $n-1 = \frac{3.6}{0.09} = 40$ n = 41 Sanjitha's will be able to throw 11.16 m in 41 weeks.	½ ½ ½ ½ ½ ½ ½ ½
	(iii) a = 40; d = 12; n = 15 $S_n = \frac{n}{2}[2a + (n-1)\ d]$ $S_n = \frac{15}{2}[2(40) + (15-1)(12)]$ $= \frac{15}{2}[80 + 168]$ $= \frac{15}{2}[248]$ =1860 throws	½ ½
37.	(i) Let D be (a,b), then Mid point of AC = Midpoint of BD $\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$ 4 + a = 7 3+ b = 8 a = 3 b = 5 Central midfielder is at (3,5)	 ½ ½

	(ii) GH = $\sqrt{(-3-3)^2+(5-1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ GK = $\sqrt{(0+3)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$ HK = $\sqrt{(3-0)^2+(1-3)^2} = \sqrt{9+4} = \sqrt{13}$ GK +HK = GH ⇒G,H & K lie on a same straight line [or] CJ = $\sqrt{(0-5)^2+(1+3)^2} = \sqrt{25+16} = \sqrt{41}$ CI = $\sqrt{(0+4)^2+(1-6)^2} = \sqrt{16+25} = \sqrt{41}$ Full-back J(5,-3) and centre-back I(-4,6) are equidistant from forward C(0,1) Mid-point of IJ = $\left(\frac{5-4}{2}, \frac{-3+6}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$ C is NOT the mid-point of IJ	½ ½ ½ ½ ½ ½ ½ ½
	(iii) A,B and E lie on the same straight line and B is equidistant from A and E ⇒ B is the mid-point of AE $\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2,-3)$ $1+a=4$; a = 3. 4+b = -6; b = -10 E is (3,-10)	 ½ ½
38.	(i) tan 45° = $\frac{80}{CB}$ ⇒ CB = 80m	1
	(ii) tan 30° = $\frac{80}{CE}$ ⇒ $\frac{1}{\sqrt{3}} = \frac{80}{CE}$ ⇒ CE = $80\sqrt{3}$ Distance the bird flew = AD = BE = CE-CB = $80\sqrt{3} - 80 = 80(\sqrt{3}-1)$ m (or) tan 60° = $\frac{80}{CG}$ ⇒ $\sqrt{3} = \frac{80}{CG}$ ⇒ CG = $\frac{80}{\sqrt{3}}$ Distance the ball travelled after hitting the tree =FA=GB = CB -CG GB = 80 - $\frac{80}{\sqrt{3}} = 80\left(1-\frac{1}{\sqrt{3}}\right)$ m	½ ½ ½ ½ ½ ½ ½ ½
	(iii) Speed of the bird = $\frac{Distance}{Time\ taken} = \frac{20(\sqrt{3}+1)}{2}$ m/sec $= \frac{20(\sqrt{3}+1)}{2}$ x 60 m/min = $600(\sqrt{3}+1)$ m/min	½ ½