

Applied Mathematics (241)
Marking Scheme
Class XII (2023-24)

Section A
(1 Mark each)

Q-1 Option (c) Here $X < 0$ and $Y > 0$, hence $-70 \pmod{13}$ is $8 \because 8 > 0$. 1 Mark

Q-2 Option (b) $x \in (-\infty, -2)$ 1 Mark

$$\frac{x+1}{x+2} \geq 1 \Rightarrow \frac{x+1}{x+2} - 1 \geq 0$$

$$\Rightarrow \frac{x+1-x-2}{x+2} \geq 0$$

$$\Rightarrow \frac{-1}{x+2} \geq 0 \Rightarrow x+2 < 0 \left[\because \frac{a}{b} > 0 \text{ and } a < 0 \Rightarrow b < 0 \right]$$

$$\Rightarrow x < -2$$

Q-3 Option (b) \bar{x} is a statistic 1 Mark

Q-4 Option (a) 1 Mark

Q-5 Option (a)

Let man's rate upstream = x km/h

Let man's rate downstream = $2x$ km/h

Hence, Man's rate in still water = $\frac{1}{2}(x + 2x) = \frac{3x}{2}$ km/h

Therefore $\frac{3x}{2} = 6 \Rightarrow x = 4$ km/h

Man's rate downstream = 8 km/h

Hence rate of stream $\frac{1}{2}(8 - 4) = 2$ km/h 1 Mark

Q-6 Option (d)

| x_i | Sample Event | P(x_i) = p_i | $x_i p_i$ |
|-------|--------------|--------------------|---------------|
| 0 | TT | $\frac{1}{4}$ | 0 |
| 1 | HT, TH | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 2 | HH | $\frac{1}{4}$ | $\frac{1}{2}$ |

Mathematical Expectation $E(X) = \sum p_i x_i = 1$ 1 Mark

Q-7 Option (c) $3^1 \equiv 3 \pmod{7} \Rightarrow 3^2 \equiv 3 \times 3 = 2 \pmod{7}$

$$\Rightarrow 3^3 = 3 \times 2 = 6 = -1 \pmod{7}$$

$$\Rightarrow (3^3)^{16} = (-1)^{16} \pmod{7}$$

$$\Rightarrow (3^3)^{16} = 1 \pmod{7} \Rightarrow (3^3)^{16} \times 3^2 = 1 \times 3^2 \pmod{7}$$

$$\Rightarrow 3^{50} = 2 \pmod{7}$$

1 Mark

Q-8 Option (a) $i = \frac{r}{400}$.
 $P = \frac{R}{i} \Rightarrow 24000 = \frac{300 \times 400}{r} \Rightarrow r = \frac{120}{24} = 5\%$ 1 Mark

Q-9 Option (b) $\int \frac{\log x}{x} dx$
 Put $\log x = t$
 Differentiating $\frac{1}{x} dx = dt$
 Hence, $\int \frac{\log x}{x} dx = \int t dt = \frac{t^2}{2} + C = \frac{(\log x)^2}{2} + C$ 1 Mark

Q-10 Option (d) 1 Mark

Q-11 Option (b) 1 mark

$$D = \frac{C-S}{n} \Rightarrow D = \frac{30,000-4000}{4} = \frac{26000}{4} = 6500$$

Hence, the depreciation is ₹ 6500 1 Mark

Q-12. Option (c)

$$r_{eff} = \left[\left(1 + \frac{r}{m} \right)^m - 1 \right] \times 100$$

$$r_{eff} = [(1.03)^2 - 1] \times 100 = (1.0609 - 1) \times 100 = 6.09\%$$

Q-13 Option (b)

$$CAGR = \left[\left(\frac{EV}{SV} \right)^{\frac{1}{5}} - 1 \right] \times 100$$

$$= \left[\left(\frac{32000}{20000} \right)^{\frac{1}{5}} - 1 \right] \times 100$$

$$= \left[(1.6)^{\frac{1}{5}} - 1 \right] \times 100$$

$$= [1.098 - 1] \times 100 = 0.098 \times 100 = 9.8\%. \quad 1 \text{ Mark}$$

Q-14 Option (c) $x \frac{dy}{dx} + 2y = x^3$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{x^3}{x}$$

$$\frac{dy}{dx} + \frac{2y}{x} = x^2$$

$$I.F = e^{\int \frac{2}{x}} = e^{2 \ln x} = e^{\ln x^2} = x^2 \quad 1 \text{ Mark}$$

Q-15 Option (a) 1 Mark

Q-16 Option (a) 1 Mark

$$3P(X = 2) = 2P(X = 1)$$

$$\Rightarrow 3 \frac{m^2 e^{-m}}{2!} = 2 \frac{m e^{-m}}{1!} \Rightarrow m = \frac{4}{3}$$

Q-17 Option (c) 1 Mark

Q-18 Option (d)

$$Z = \frac{x-\mu}{\sigma} \Rightarrow 5 = \frac{x-12}{4} \Rightarrow x = 32. \quad 1 \text{ Mark}$$

Q-19 Option(c)

Assertion : $P(x) = 41 + 24x - 8x^2$

$$P'(x) = 24 - 16x$$

$$P'(x) = 0 \Rightarrow 24 - 16x = 0 \Rightarrow x = \frac{24}{16} = \frac{3}{2}$$

$$P''(x) = -16 < 0 \Rightarrow x = \frac{3}{2} \text{ is a point of maxima}$$

$$\text{Max Profit} = P = 41 + 24 \times \frac{3}{2} - 8 \times \frac{9}{4} = 41 + 36 - 18 = 59$$

Assertion is true but Reason is false , for Maximum

Profit at $x = a, P'(a) = 0$ and $P''(a) < 0$.

1 Mark

Q-20 Option (a) Both A and R are true and R is the correct explanation of A 1 Mark

Section B (2 Marks each)

Q-21 Let rate of interest be $r\%$ per annum, then $i = \frac{r}{200}$

Given $R = ₹1500$ and $P = \text{Rs } 20,000$

$$P = \frac{R}{i} \Rightarrow i = \frac{R}{P} = \frac{1500}{20000}$$

1 Mark

$$\Rightarrow \frac{r}{200} = \frac{1500}{20000} \Rightarrow r = 15\%$$

1 Mark

$$\text{Q-22 } A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

1 Mark

$$A^2 = pA \Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = p \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix} \Rightarrow p = 4$$

1 Mark

OR

$$\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} \quad \frac{1}{2} \text{ Mark}$$

Comparing $a = -2$; $b = -b \Rightarrow 2b = 0 \Rightarrow b = 0$ and $c = -3$ 1½ Mark

Hence $a+b+c = -2+0-3 = -5$. ½ Mark

Q-23 Let 'x' hectares and 'y' hectares of land be allocated to crop A and Crop B

$$\text{Max } Z = 8000x + 9500y. \quad \frac{1}{2} \text{ Mark}$$

Subject to $x + y \leq 10$; $2x + y \leq 50$; $x \geq 0$ and $y \geq 0$ 1½ mark

Q-24 Let speed of boat and stream be x km/h and y km/h respectively

$$(x + y) \times 5 = 25 \quad \frac{1}{2} \text{ Mark}$$

$$(x - y) \times 5 = 15 \quad \frac{1}{2} \text{ Mark}$$

$$\Rightarrow x + y = 5 \text{ and } x - y = 3$$

Solving, $x = 4$ and $y = 1$ ½ Mark

So, speed of stream is 1km/h ½ Mark

OR

When B runs 50 m A runs 40 m ½ Mark

When B runs 1 m , A runs $= \frac{40}{50} = \frac{4}{5}$ ½ Mark

When B runs 1000 m , A runs $= \frac{4}{5} \times 1000 = 800$ m ½ Mark

Hence B beats A by 200 m ½ Mark

Q-25 Define Null hypothesis H_0 alternate hypothesis H_1 as follows:

$$H_0: \mu = 0.50 \text{ mm}$$

$$H_1: \mu \neq 0.50 \text{ mm}$$

Thus a two-tailed test is applied under hypothesis H_0 , we have

$$t = \frac{\bar{X} - \mu}{s} \sqrt{n - 1} = \frac{0.53 - 0.50}{0.03} \times 3 = 3. \quad 1 \text{ Mark}$$

Since the calculated value of t i.e. $t_{\text{cal}} (=3) > t_{\text{tab}} (=2.262)$, the null hypothesis H_0 can be rejected. Hence, we conclude that machine is not working properly. 1 Mark

Section C
(3 Marks each)

Q-26 $\int \frac{x^3}{(x+2)} dx = \left(\int x^2 - 2x + 4 - \frac{8}{x+2} \right) dx$ 2 Mark
 $= \frac{x^3}{3} - x^2 + 4x - 8 \log|x+2| + C.$ 1 Mark
 (where C is an arbitrary constant of integration)

OR

$\int (x^2 + 1) \ln x dx$
 Integrating by parts
 $\ln x \left(\frac{x^3}{3} + x \right) - \int \frac{1}{x} \left(\frac{x^3}{3} + x \right) dx.$ 2 mark
 $\ln x \left(\frac{x^3}{3} + x \right) - \int \left(\frac{x^2}{3} + 1 \right) dx$
 $\ln x \left(\frac{x^3}{3} + x \right) - \left(\frac{x^3}{9} + x \right) + C.$ 1 Mark

Q-27

Toy A. Toy B

$\begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix}$ Here Row 1 and Row 2 indicate Shopkeeper 1 and Shopkeeper 2

Cost Matrix = $\begin{bmatrix} 50 \\ 75 \end{bmatrix}$ 1 Mark

Amount = $\begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix} \begin{bmatrix} 50 \\ 75 \end{bmatrix} = \begin{bmatrix} 350 + 750 \\ 400 + 450 \end{bmatrix} = \begin{bmatrix} 1100 \\ 850 \end{bmatrix}$
 Income of Shopkeeper P is ₹1100 and shopkeeper Q is ₹850 2 Marks

Q-28 $f(x) = 2x^3 - 9x^2 + 12x - 5$
 $f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$
 $f'(x) = 6(x - 1)(x - 2)$ 1 Mark
 $f'(x) = 0 \Rightarrow x = 1 \text{ and } x = 2 \text{ are the critical points.}$ ½ Mark

The intervals are $(-\infty, 1); (1,2); (2, \infty)$ ½ Mark

Increasing in $(-\infty, 1] \cup [2, \infty)$, Decreasing in $[1,2]$ 1 Mark

Q-29 Under pure competition

$p_d = p_s$
 $\Rightarrow 16 - x^2 = 2x^2 + 4$
 $\Rightarrow 3x^2 = 12 \Rightarrow x = 2, -2$; since x can't be -ve, so $x=2$ 1 Mark
 When $x_0 = 2$; $p_0 = 12$ ½ Mark
 Hence, Consumer's surplus = $\int_0^2 p_d dx - p_0 x_0$ ½ Mark

$$= \int_0^2 (16 - x^2) dx - 12 \times 2$$

$$= 16/3 \text{ units}$$

1 Mark

OR

$$p_d = p_s$$

$$\Rightarrow 56 - x^2 = 8 + \frac{x^2}{3}$$

$$\Rightarrow \frac{4}{3}x^2 = 48 \Rightarrow x^2 = 36 \Rightarrow x = 6, -6; \text{ since } x \text{ can't be } -ve, \text{ so } x=6$$

1 Mark

$$\text{When } x_0 = 6; p_0 = 20$$

½ Mark

$$\text{Hence, Producer's surplus} = p_0 x_0 - \int_0^6 p_s dx$$

½ Mark

$$= 6 \times 20 - \int_0^6 \left(8 + \frac{x^2}{3}\right) dx$$

$$= 120 - [48 + 24]$$

$$= 48 \text{ units}$$

1 Mark

Q-30 Here P = 5,00,000 ; I = 2,00,000; EMI = 12,500

$$EMI = \frac{P+I}{n}$$

1½ Mark

$$12,500 = \frac{5,00,000 + 2,00,000}{n} \Rightarrow n = \frac{7,00,000}{12,500} = 56 \text{ months.}$$

1½ Mark

Q-31 Let Rs. R be set aside biannually for 10 years in order to have ₹ 500,000 after 10 years

$$\text{Here } S = 500,000. ; n = 10 \times 2 = 20$$

$$i = \frac{5}{2 \times 100} = 0.025$$

½ Mark

$$R = \frac{iS}{(1+i)^n - 1} = \frac{0.025 \times 500,000}{(1.025)^{20} - 1} = \frac{12,500}{1.6386 - 1} = 19,574.07.$$

2½ Mark

Section D (5 Marks each)

Q-32 Here m = 0.4

$$P(X = r) = \frac{e^{-m} \cdot m^r}{r!} = \frac{e^{-0.4 \times (0.4)^r}}{r!}$$

$$= \frac{0.6703 \times (0.4)^r}{r!}$$

1 Mark

$$\text{In 1000 pages error} = 1000 \times \frac{0.6703 \times (0.4)^r}{r!}$$

½ Mark

$$\text{For zero error } P(X = 0) = 1000 \times \frac{e^{-m} \cdot m^0}{0!} = \frac{e^{-0.4 \times (0.4)^0}}{0!}$$

$$= 1000 \times 0.6703 = 670.3$$

1½ Mark

$$\text{For one error } P(X = 1) = 1000 \times \frac{e^{-m} \cdot m^1}{1!} = \frac{e^{-0.4 \times (0.4)^1}}{1!}$$

$$= 670.3 \times 0.4 = 268.12$$

2 Mark

OR

$$\text{Here } p = \frac{1}{2} \text{ and } q = \frac{1}{2}$$

$$P(X=r) = C(n, r) p^r q^{n-r}$$

$$1 - P(r = 0) > \frac{90}{100}$$

1 Mark

$$1- C(n,0)\left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > \frac{9}{10}$$

$$\Rightarrow \frac{n!}{0!(n-0)!} \left(\frac{1}{2}\right)^n < \frac{1}{10} \quad 2 \text{ Mark}$$

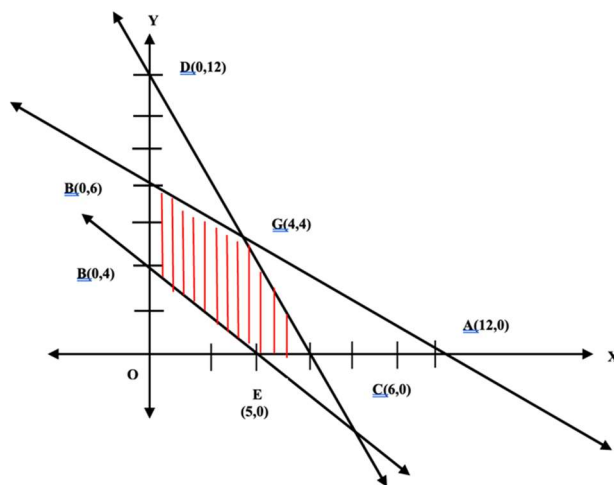
$$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{10} \Rightarrow 2^n > 10 \Rightarrow n \text{ is } 4 \text{ or more times} \quad 2 \text{ Mark}$$

Q-33 Let 'x' and 'y' be the number of units of items M and N respectively.

We have : $x \geq 0, y \geq 0$

$$x + 2y \leq 12; 2x + y \leq 12; x + \frac{5}{4}y \geq 5. \quad 1\frac{1}{2} \text{ Mark}$$

$$\text{Max } Z = 600x + 400y \quad 1 \text{ Mark}$$



Graph $1\frac{1}{2}$ Mark

| Corner Point | $Z = 600x + 400y$ |
|------------------|-----------------------|
| E : (5,0) | 3000 |
| C : (6,0) | 3600 |
| G : (4,4) | 4000 (Maximum) |
| B : (0,6) | 2400 |
| F : (0,4) | 1600 |

Hence maximum profit is ₹4000 when 4 units of each item M and N are produced. 1 Mark

Q-34. Let 'x' units of product be produced and sold. As selling price of one unit is Rs 8 total revenue on 'x' units = Rs 8x

(i) Cost Function $C(x) = \text{Fixed Cost} + 25\% \text{ of } 8x$

$$= 24000 + \frac{25}{100} \times 8x$$

$$= 24000 + 2x.$$

$1\frac{1}{2}$ Mark

(ii) Revenue Function = $8x$

1 Mark

(iii) Breakeven Point $8x = 24000 + 2x$

$$x = 4000$$

$1\frac{1}{2}$ Mark

(iv) Profit function = $R(x) - C(x) = 6x - 24000$

1 Mark

OR

Let x and y be the dimension of the printed pages then $x \cdot y = 180$.

$$A = \text{Area of the page} = (x+4)(y+5)$$

$$= xy + 5x + 4y + 20$$

$$= 180 + 5x + 4 \times \left(\frac{180}{x}\right) + 20$$

$$= 200 + 5x + \frac{720}{x}$$

2½ Mark

For most economical dimension $\frac{dA}{dx} = 0 \Rightarrow 5 - \frac{720}{x^2} = 0.$
 $\Rightarrow x^2 = 144 \Rightarrow x = 12$

Now $\frac{d^2A}{dx^2} = \frac{1440}{x^3}$
 $\left(\frac{d^2A}{dx^2}\right)_{x=12} = \frac{1440}{12^3} > 0. \therefore A \text{ is minimum}$

Hence, the most economical dimensions are 16cm and 20 cm

2½ Mark

Q-35 $x + y + z = 12$
 $2x + 3y + 3z = 33$
 $x - 2y + z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

1 Mark

$|A| = 3 \neq 0$

½ Mark

$$\text{adj}A = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

2½ Mark

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 108 - 99 + 0 \\ 12 + 0 - 0 \\ -84 + 99 + 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Hence $x = 3, y = 4, z = 5$

1 Mark

Section E (Case Studies Based Questions)

Q- 36 Case Study – I

(i)

A + B fill the tank in 6 hrs

B + C fill the tank in 10 hrs

A + C fill the tank in $\frac{15}{2}$ hrs

$$2(A + B + C) = \frac{6 \times 10 \times \frac{15}{2}}{6 \times 10 + 6 \times \frac{15}{2} + 10 \times \frac{15}{2}} = \frac{450}{60 + 45 + 75} = \frac{450}{180} = \frac{5}{2} \text{ hrs}$$

Hence A, B and C together will fill the tank in 5 Hrs

2 Mark

(ii) A will in $[(A+B+C) - (B+C)] = \frac{10 \times 5}{10 - 5} = 10 \text{ hrs}$

1 Mark

(iii) B will fill in $\frac{\frac{15}{2} \times 5}{\frac{15}{2} - 5} = 15 \text{ hrs}$

1 Mark

OR

C will fill in $\frac{5 \times 6}{6 - 5} = 30 \text{ hrs}$

Q-37 Case Study – II

| | | | | | | |
|------------|-----|---|----|----|---|---|
| x_i | 0 | 1 | 2 | 3 | 4 | 5 |
| $P(X=x_i)$ | 0.2 | k | 2k | 2k | 0 | 0 |

(i) Since $\sum P = 1 \Rightarrow 0.2 + k + 2k + 2k = 1 \Rightarrow 0.2 + 5k = 1 \Rightarrow 5k = 0.8$

$\Rightarrow k = \frac{4}{25}$

1½ Mark

(ii) $P(X=2) = 2k = \frac{8}{25}$

1 Mark

(iii) $P(X \geq 2) = 4k = \frac{16}{25}$

1½ Mark

OR

$P(X \leq 2) = 0.2 + 3k = \frac{17}{25}$

Q-38 Case Study – III

| Year | Y | X=Year - 2003 | X ² | XY |
|------|------|---------------|----------------|------|
| 2001 | 160 | -2 | 4 | -320 |
| 2002 | 185 | -1 | 1 | -185 |
| 2003 | 220 | 0 | 0 | 0 |
| 2004 | 300 | 1 | 1 | 300 |
| 2005 | 510 | 2 | 4 | 1020 |
| | 1375 | | 10 | 815 |

2 Marks for table

$a = \frac{\sum Y}{n} = \frac{1375}{5} = 275$

½ Mark

$b = \frac{\sum XY}{\sum X^2} = \frac{815}{10} = 81.5$

½ Mark

$Y_c = a + bX$

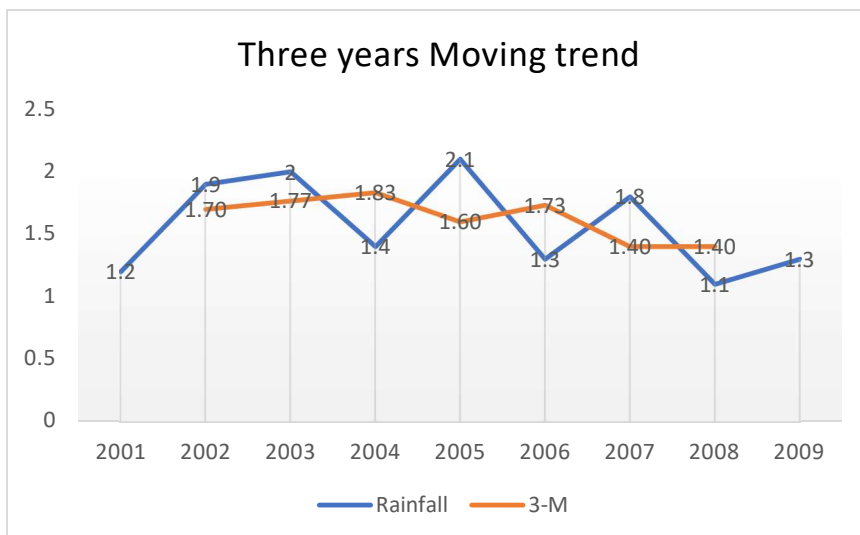
$Y_c = 275 + 81.5 X$

The estimated value for 2008 will be $275 + 81.5 \times 5 = 275 + 407.5 = 682.5$ 1 Mark

OR

| Year | Rainfall(in cm) | 3 years moving total | 3 years moving average |
|------|-----------------|----------------------|------------------------|
| 2001 | 1.2 | | |
| 2002 | 1.9 | 5.1 | 1.70 |
| 2003 | 2 | 5.3 | 1.77 |
| 2004 | 1.4 | 5.5 | 1.83 |
| 2005 | 2.1 | 4.8 | 1.60 |
| 2006 | 1.3 | 5.2 | 1.73 |
| 2007 | 1.8 | 4.2 | 1.40 |
| 2008 | 1.1 | 4.2 | 1.40 |
| 2009 | 1.3 | | |

1½ Marks for table



2½ Marks for graph