SAMPLE QUESTION PAPER

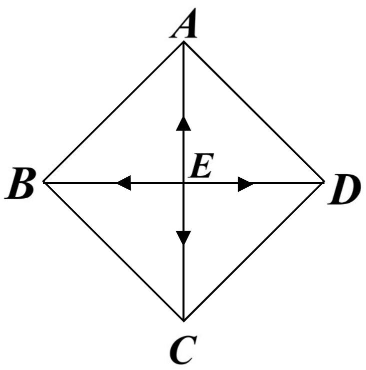
MARKING SCHEME

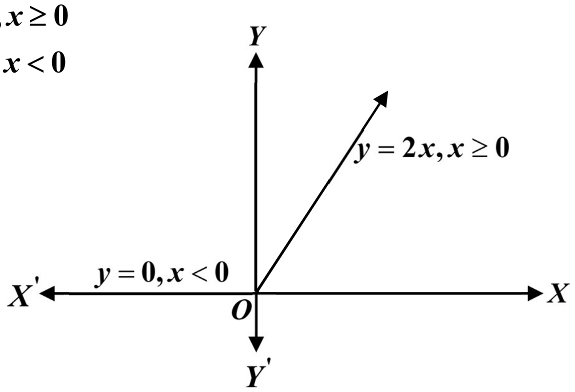
CLASS XII

MATHEMATICS (CODE-041)

SECTION: A (Solution of MCQs of 1 Mark each)

Q no.	ANS	HINTS/SOLUTION
1	(d)	$A=\begin{bmatrix}0&1\\1&0\end{bmatrix}, A^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}.$
2	(d)	$(A+B)^{-1}=B^{-1}+A^{-1}.$
3	(b)	**Area** $=\left\lvert\frac{1}{2}\begin{vmatrix}-3&0&1\\3&0&1\\0&k&1\end{vmatrix}\right\rvert$, given that the area $=9\,sq\,unit.$ $\Rightarrow \pm 9=\frac{1}{2}\begin{vmatrix}-3&0&1\\3&0&1\\0&k&1\end{vmatrix}$; expanding along C_2, we get $\Rightarrow k=\pm 3.$
4	(a)	Since, f is continuous at $x=0$, therefore, $L.H.L=R.H.L=f(0)=a\ finite\ quantity.$ $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$ $\Rightarrow \lim_{x\to 0^-}\frac{-kx}{x}=\lim_{x\to 0^+}3=3\Rightarrow k=-3.$
5	(d)	Vectors $2\hat{i}+3\hat{j}-6\hat{k}\ \&\ 6\hat{i}+9\hat{j}-18\hat{k}$ are parallel and the fixed point $\hat{i}+\hat{j}-\hat{k}$ on the line $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$ does not satisfy the other line $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(6\hat{i}+9\hat{j}-18\hat{k}\right)$; where $\lambda\ \&\ \mu$ are scalars.
6	(c)	The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$ *is* **2**
7	(b)	$Z\ =\ px+qy\ ---(i)$ At $(3,0)$, $Z=3p---(ii)$ and at $(1,1)$, $Z=p+q----(iii)$ From $(ii)\ \&\ (iii)$, $3p=p+q\Rightarrow 2p=q.$

8	(a)	Given, $ABCD$ is a rhombus whose diagonals bisect each other. $\left\|\overrightarrow{EA}\right\| = \left\|\overrightarrow{EC}\right\|$ and $\left\|\overrightarrow{EB}\right\| = \left\|\overrightarrow{ED}\right\|$ but since they are opposite to each other so they are of opposite signs $\Rightarrow \overrightarrow{EA} = -\overrightarrow{EC}$ and $\overrightarrow{EB} = -\overrightarrow{ED}$.  $\Rightarrow \overrightarrow{EA} + \overrightarrow{EC} = \vec{O}.....(i)$ and $\overrightarrow{EB} + \overrightarrow{ED} = \vec{O}....(ii)$ Adding (i) and (ii), we get $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \vec{O}$.
9	(b)	$f(x) = e^{\cos^2 x} \sin^3(2n+1)x$ $f(-x) = e^{\cos^2(-x)} \sin^3(2n+1)(-x)$ $f(-x) = -e^{\cos^2 x} \sin^3(2n+1)x$ $\because f(-x) = -f(x)$ $So, \int_{-\pi}^{\pi} e^{\cos^2 x} \sin^3(2n+1)x \, dx = 0$
10	(b)	Matrix A is a skew symmetric matrix of odd order. $\therefore \|A\| = 0$.
11	(c)	**We observe,** $(0,0)$ does not satisfy the inequality $x - y \geq 1$ **So, the half plane represented by the above inequality will not contain origin therefore, it will not contain the shaded feasible region.**
12	(b)	Vector component of $\vec{a}$ along $\vec{b} = \left(\frac{\vec{a}.\vec{b}}{\|\vec{b}\|^2}\right)\vec{b} = \frac{18}{25}\left(3\hat{j} + 4\hat{k}\right)$.
13	(d)	$\|adj(2A)\| = \|(2A)\|^2 = \left(2^3\|A\|\right)^2 = 2^6\|A\|^2 = 2^6 \times (-2)^2 = 2^8$.
14	(d)	**Method 1:** **Let** A, B, C be the respective events of solving the problem. Then, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$. Here, A, B, C are independent events. Problem is solved if at least one of them solves the problem. Required probability is $= P(A \cup B \cup C) = 1 - P(\overline{A})P(\overline{B})P(\overline{C})$

		$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=1-\frac{1}{4}=\frac{3}{4}.$ **Method 2:** The problem will be solved if one or more of them can solve the problem. The probability is $P\left(A\overline{B}\overline{C}\right)+P\left(\overline{A}B\overline{C}\right)+P\left(\overline{A}\overline{B}C\right)+P\left(AB\overline{C}\right)+P\left(A\overline{B}C\right)+P\left(\overline{A}BC\right)+P\left(ABC\right)$ $=\frac{1}{2}.\frac{2}{3}.\frac{3}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}=\frac{3}{4}.$ **Method 3:** Let us think quantitively. Let us assume that there are 100 questions given to A. A solves $\frac{1}{2}\times 100=50$ questions then remaining 50 questions is given to B and B solves $50\times\frac{1}{3}=16.67$ questions . Remaining $50\times\frac{2}{3}$ questions is given to C and C solves $50\times\frac{2}{3}\times\frac{1}{4}=8.33$ questions. Therefore, number of questions solved is $50+16.67+8.33=75$. So, required probability is $\frac{75}{100}=\frac{3}{4}.$
15	(c)	**Method 1:** $ydx-xdy=0\Rightarrow\frac{ydx-xdy}{y^2}=0\Rightarrow d\left(\frac{x}{y}\right)=0\Rightarrow x=\frac{1}{c}y\Rightarrow y=cx.$ **Method 2:** $ydx-xdy=0\Rightarrow ydx=xdy\Rightarrow\frac{dy}{y}=\frac{dx}{x}$**; on integrating** $\int\frac{dy}{y}=\int\frac{dx}{x}$ $\log_e\lvert y\rvert=\log_e\lvert x\rvert+\log_e\lvert c\rvert$ **since** $x,y,c>0$**, we write** $\log_e y=\log_e x+\log_e c\ \Rightarrow y=cx.$
16	(d)	Dot product of two mutually perpendicular vectors is zero. $\Rightarrow 2\times 3+(-1)\lambda+2\times 1=0\Rightarrow\lambda=8.$
17	(c)	**Method 1:** $f(x)=x+\lvert x\rvert=\begin{cases}2x,x\geq 0\\ 0\,,x<0\end{cases}$  There is a sharp corner at $x=0$, so $f(x)$ is not differentiable at $x=0$. **Method 2:**

		$Lf'(0)=0$ & $Rf'(0)=2$; so, the function is not differentiable at $x=0$ For $x\geq 0, f(x)=2x$ (linear function) & when $x<0, f(x)=0$ (constant function) Hence $f(x)$ is differentiable when $x\in(-\infty,0)\cup(0,\infty)$.
18	(d)	**We know,** $l^2+m^2+n^2=1\Rightarrow\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2=1\Rightarrow 3\left(\frac{1}{c}\right)^2=1\Rightarrow c=\pm\sqrt{3}.$
19	(a)	$\frac{d}{dx}(f(x))=(x-1)^3(x-3)^2$ Assertion : $f(x)$ has a minimum at $x=1$ is true as $\frac{d}{dx}(f(x))<0, \forall x\in(1-h,1)$ and $\frac{d}{dx}(f(x))>0, \forall x\in(1,1+h)$; where, $'h'$ is an infinitesimally small positive quantity , which is in accordance with the Reason statement.
20	(d)	**Assertion is false.** As element 4 has no image under f, so relation f is not a function. **Reason is true.** The given function $f:\{1,2,3\}\to\{x,y,z,p\}$ is one – one, as for each $a\in\{1,2,3\}$, there is different image in $\{x,y,z,p\}$ under f.

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

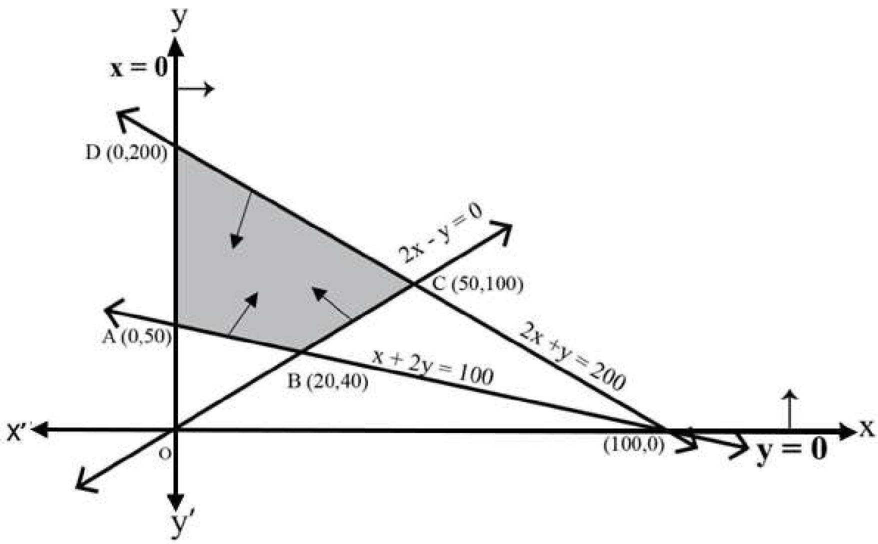
21	$\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)=\sin^{-1}\cos\left(6\pi+\frac{3\pi}{5}\right)=\sin^{-1}\cos\left(\frac{3\pi}{5}\right)=\sin^{-1}\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)$ $=\frac{\pi}{2}-\frac{3\pi}{5}=-\frac{\pi}{10}.$	1 1
21 OR	$-1\leq(x^2-4)\leq 1\Rightarrow 3\leq x^2\leq 5\Rightarrow\sqrt{3}\leq\lvert x\rvert\leq\sqrt{5}$ $\Rightarrow x\in\left[-\sqrt{5},-\sqrt{3}\right]\cup\left[\sqrt{3},\sqrt{5}\right]$. So, required domain is $\left[-\sqrt{5},-\sqrt{3}\right]\cup\left[\sqrt{3},\sqrt{5}\right]$.	1 1
22	$f(x)=xe^x\Rightarrow f'(x)=e^x(x+1)$ When $x\in[-1,\infty),(x+1)\geq 0$ & $e^x>0\Rightarrow f'(x)\geq 0$ $\therefore$ $f(x)$ increases in this interval. ***or, we can write*** $f(x)=xe^x\Rightarrow f'(x)=e^x(x+1)$ For $f(x)$ to be increasing, we have $f'(x)=e^x(x+1)\geq 0\Rightarrow x\geq -1$ as $e^x>0,\forall x\in\mathbb{R}$ Hence, the required interval where $f(x)$ increases is $[-1,\infty)$.	1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
23	**Method 1 :** $f(x)=\frac{1}{4x^2+2x+1}$,	

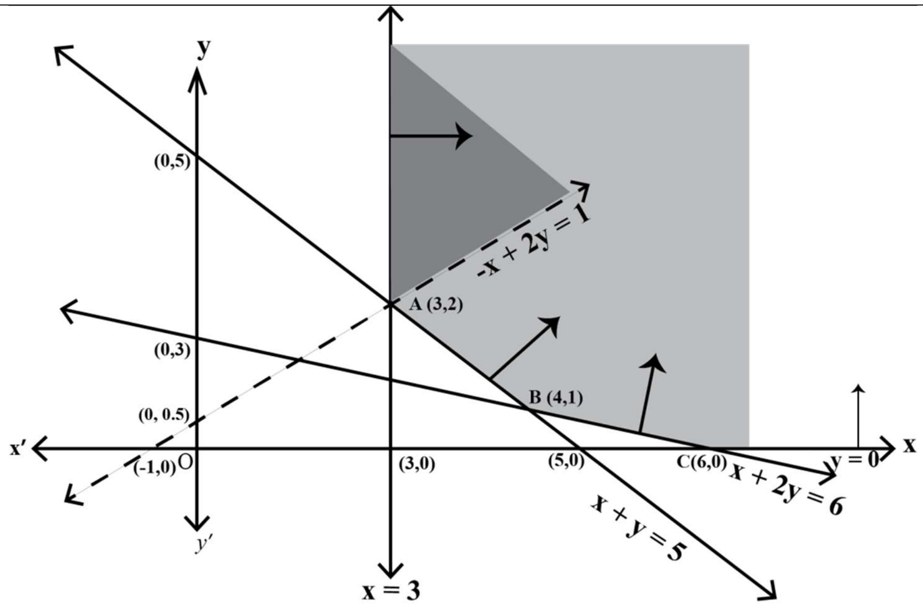
	Let $g(x)=4x^2+2x+1=4\left(x^2+2x\frac{1}{4}+\frac{1}{16}\right)+\frac{3}{4}=4\left(x+\frac{1}{4}\right)^2+\frac{3}{4}\geq\frac{3}{4}$	$1\frac{1}{2}$
	$\therefore$ maximum value of $f(x)=\frac{4}{3}$.	$\frac{1}{2}$
	Method 2 : $f(x)=\frac{1}{4x^2+2x+1}$, let $g(x)=4x^2+2x+1$	
	$\Rightarrow \frac{d}{dx}(g(x))=g'(x)=8x+2$ **and** $g'(x)=0$ **at** $x=-\frac{1}{4}$ **also** $\frac{d^2}{dx^2}(g(x))=g''(x)=8>0$	1
	$\Rightarrow g(x)$ is minimum when $x=-\frac{1}{4}$ so , $f(x)$ is maximum at $x=-\frac{1}{4}$	$\frac{1}{2}$
	$\therefore$ maximum value of $f(x)=f\left(-\frac{1}{4}\right)=\frac{1}{4\left(-\frac{1}{4}\right)^2+2\left(-\frac{1}{4}\right)+1}=\frac{4}{3}$.	$\frac{1}{2}$
	Method 3 : $f(x)=\frac{1}{4x^2+2x+1}$	
	On differentiating w.r.t x ,we get $f'(x)=\frac{-(8x+2)}{\left(4x^2+2x+1\right)^2}$ ....(i)	$\frac{1}{2}$
	For maxima or minima , we put $f'(x)=0\Rightarrow 8x+2=0\Rightarrow x=-\frac{1}{4}$.	$\frac{1}{2}$
	Again, differentiating equation (i) w.r.t. x ,we get	
	$f''(x)=-\left\{\frac{\left(4x^2+2x+1\right)^2(8)-(8x+2)2\times\left(4x^2+2x+1\right)(8x+2)}{\left(4x^2+2x+1\right)^4}\right\}$	$\frac{1}{2}$
	At $x=-\frac{1}{4}$, $f''\left(-\frac{1}{4}\right)<0$	
	$f(x)$ is maximum at $x=-\frac{1}{4}$.	
	$\therefore$ maximum value of $f(x)$ is $f\left(-\frac{1}{4}\right)=\frac{1}{4\left(-\frac{1}{4}\right)^2+2\left(-\frac{1}{4}\right)+1}=\frac{4}{3}$.	$\frac{1}{2}$
	Method 4: $f(x)=\frac{1}{4x^2+2x+1}$	
	On differentiating w.r.t x ,we get $f'(x)=\frac{-(8x+2)}{\left(4x^2+2x+1\right)^2}$ ....(i)	$\frac{1}{2}$
	For maxima or minima , we put $f'(x)=0\Rightarrow 8x+2=0\Rightarrow x=-\frac{1}{4}$.	$\frac{1}{2}$
	When $x\in\left(-h-\frac{1}{4},-\frac{1}{4}\right)$, where $'h'$ is infinitesimally small positive quantity.	
	$4x<-1\Rightarrow 8x<-2\Rightarrow 8x+2<0\Rightarrow -(8x+2)>0$ and $\left(4x^2+2x+1\right)^2>0\Rightarrow f'(x)>0$	

	and when $x \in \left(-\frac{1}{4}, -\frac{1}{4}+h\right), 4x > -1 \Rightarrow 8x > -2 \Rightarrow 8x+2 > 0 \Rightarrow -(8x+2) < 0$ and $\left(4x^2+2x+1\right)^2 > 0 \Rightarrow f'(x) < 0$. This shows that $x = -\frac{1}{4}$ is the point of local maxima. $\therefore$ maximum value of $f(x)$ is $f\left(-\frac{1}{4}\right) = \frac{1}{4\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right)+1} = \frac{4}{3}$.	$\frac{1}{2}$ $\frac{1}{2}$
23 OR	For maxima and minima, $P'(x) = 0 \Rightarrow 42 - 2x = 0$ $\Rightarrow x = 21$ and $P''(x) = -2 < 0$ So, $P(x)$ is maximum at $x = 21$. The maximum value of $P(x) = 72 + (42 \times 21) - (21)^2 = 513$ i.e., the maximum profit is **₹ 513.**	$\frac{1}{2}$ $\frac{1}{2}$ 1
24	Let $f(x) = \log\left(\frac{2-x}{2+x}\right)$ We have, $f(-x) = \log\left(\frac{2+x}{2-x}\right) = -\log\left(\frac{2-x}{2+x}\right) = -f(x)$ **So, $f(x)$ is an odd function.** $\therefore \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx = 0.$	1 1
25	$f(x) = x^3 + x, \quad \textbf{for all } x \in \mathbb{R}.$ $\frac{d}{dx}\left(f(x)\right) = f'(x) = 3x^2 + 1;$ **for all** $x \in \mathbb{R}, \; x^2 \geq 0 \Rightarrow f'(x) > 0$ Hence, no critical point exists.	$1\frac{1}{2}$ $\frac{1}{2}$
	Section –C **[This section comprises of solution short answer type questions (SA) of 3 marks each]**	
26	**We have,** $\frac{2x^2+3}{x^2\left(x^2+9\right)}$. Now , let $x^2 = t$ So, $\frac{2t+3}{t(t+9)} = \frac{A}{t} + \frac{B}{t+9}$, we get $A = \frac{1}{3}$ & $B = \frac{5}{3}$ $\int \frac{2x^2+3}{x^2\left(x^2+9\right)} dx = \frac{1}{3}\int \frac{dx}{x^2} + \frac{5}{3}\int \frac{dx}{x^2+9}$ $= -\frac{1}{3x} + \frac{5}{9}\tan^{-1}\left(\frac{x}{3}\right) + c,$ *where 'c' is an arbitrary constant of integration.*	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
27	We have, (i) $\sum P(X_i) = 1 \Rightarrow k + 2k + 3k = 1 \Rightarrow k = \frac{1}{6}.$	1 1

	(ii) $P(X<2)=P(X=0)+P(X=1)=k+2k=3k=3\times\frac{1}{6}=\frac{1}{2}.$ (iii) $P(X>2)=0.$	1
28	Let $x^{\frac{3}{2}}=t \Rightarrow dt=\frac{3}{2}x^{\frac{1}{2}}dx$ $\int\sqrt{\frac{x}{1-x^3}}\,dx=\frac{2}{3}\int\frac{dt}{\sqrt{1-t^2}}$ $=\frac{2}{3}\sin^{-1}(t)+c$ $=\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)+c$, *where 'c' is an arbitrary constant of integration.*	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
28 OR	Let $I=\int_0^{\frac{\pi}{4}}\log_e(1+\tan x)dx$ ------(i) $=\int_0^{\frac{\pi}{4}}\log_e\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$, Using, $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e\left(1+\frac{1-\tan x}{1+\tan x}\right)dx=\int_0^{\frac{\pi}{4}}\log_e\left(\frac{2}{1+\tan x}\right)dx=\int_0^{\frac{\pi}{4}}\log_e 2\,dx-I$ (Using ------(i) $I=\frac{\pi}{4}\log_e 2 \Rightarrow I=\frac{\pi}{8}\log_e 2.$	1 1 1
29	**Method 1:** $ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy \Rightarrow e^{\frac{x}{y}}(ydx-xdy)=y^2dy \Rightarrow e^{\frac{x}{y}}\left(\frac{ydx-xdy}{y^2}\right)=dy$ $\Rightarrow e^{\frac{x}{y}}d\left(\frac{x}{y}\right)=dy$ $\Rightarrow \int e^{\frac{x}{y}}d\left(\frac{x}{y}\right)=\int dy \Rightarrow e^{\frac{x}{y}}=y+c,$ *where 'c' is an arbitrary constant of integration.* **Method 2:** We have , $\frac{dx}{dy}=\frac{xe^{\frac{x}{y}}+y^2}{y.e^{\frac{x}{y}}}$ $\Rightarrow \frac{dx}{dy}=\frac{x}{y}+\frac{y}{e^{\frac{x}{y}}}$ ……………. (i) Let $x=vy \Rightarrow \frac{dx}{dy}=v+y.\frac{dv}{dy}$;	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$

	So equation (i) becomes $v + y\dfrac{dv}{dy} = v + \dfrac{y}{e^v}$ $\Rightarrow y\dfrac{dv}{dy} = \dfrac{y}{e^v}$ $\Rightarrow e^v\,dv = dy$ On integrating we get, $\int e^v\,dv = \int dy \Rightarrow e^v = y + c \Rightarrow e^{x/y} = y + c$ *where* $'c'$ *is an arbitrary constant of integration.*	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
29 OR	The given Differential equation is $\left(\cos^2 x\right)\dfrac{dy}{dx} + y = \tan x$ Dividing both the sides by $\cos^2 x$, we get $\dfrac{dy}{dx} + \dfrac{y}{\cos^2 x} = \dfrac{\tan x}{\cos^2 x}$ $\dfrac{dy}{dx} + y\left(\sec^2 x\right) = \tan x\left(\sec^2 x\right)........(i)$ Comparing with $\dfrac{dy}{dx} + Py = Q$ $P = \sec^2 x$, $Q = \tan x\,.\sec^2 x$ The Integrating factor is, $IF = e^{\int P\,dx} = e^{\int \sec^2 x\,dx} = e^{\tan x}$ On multiplying the equation (i) by $e^{\tan x}$, we get $\dfrac{d}{dx}\left(y.e^{\tan x}\right) = e^{\tan x}\tan x\left(\sec^2 x\right) \Rightarrow d\left(y.e^{\tan x}\right) = e^{\tan x}\tan x\left(\sec^2 x\right)dx$ On integrating we get , $y.e^{\tan x} = \int t.e^t\,dt + c_1;$ where, $t = \tan x$ so that $dt = \sec^2 x\,dx$ $= te^t - e^t + c = (\tan x)e^{\tan x} - e^{\tan x} + c$ $\therefore y = \tan x - 1 + c.\left(e^{-\tan x}\right),$ *where* $'c_1'\&'c'$ *are arbitrary constants of integration.*	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
30	The feasible region determined by the constraints, $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$, is given below.	

30 OR	 $A\ (0,\ 50)$, $B\ (20,\ 40)$, $C\ (50,\ 100)$ and $D\ (0,\ 200)$ are the corner points of the feasible region. The values of Z at these corner points are given below. <table><tr><td>Corner point</td><td>Corresponding value of $Z = x + 2y$</td><td></td></tr><tr><td>$A\ (0,\ 50)$</td><td>100</td><td>Minimum</td></tr><tr><td>$B\ (20,\ 40)$</td><td>100</td><td>Minimum</td></tr><tr><td>$C\ (50,\ 100)$</td><td>250</td><td></td></tr><tr><td>$D\ (0,\ 200)$</td><td>400</td><td></td></tr></table> The minimum value of Z is **100** at all the points on the line segment joining the points $(0,50)$ and $(20,40)$. The feasible region determined by the constraints, $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$ is given below.	$1\frac{1}{2}$ 1 $\frac{1}{2}$ $1\frac{1}{2}$



Here, it can be seen that the feasible region is unbounded. **1**

The values of Z at corner points $A\ (3,\ 2)$, $B\ (4,\ 1)$ and $C\ (6,\ 0)$ are given below. $\frac{1}{2}$

Corner point	Corresponding value of $Z=-x+2y$
$A\ (3,\ 2)$	**1 (** may or may not be the maximum value)
$B\ (4,\ 1)$	**-2**
$C\ (6,\ 0)$	**-6**

Since the feasible region is unbounded, $Z =\ 1$ may or may not be the maximum value.

Now, we draw the graph of the inequality, $-x\ +\ 2y\ >\ 1$, and we check whether the resulting open half-plane has any point/s, in common with the feasible region or not.

Here, the resulting open half plane has points in common with the feasible region.

Hence, $Z =\ 1$ is not the maximum value. We conclude, Z has no maximum value.

31

$$\frac{y}{x}=\log_e\left(\frac{x}{a+bx}\right)=\log_e x-\log_e(a+bx)$$ $\frac{1}{2}$

On differentiating with respect to x, we get

$$\Rightarrow \frac{x\frac{dy}{dx}-y}{x^2}=\frac{1}{x}-\frac{1}{a+bx}\frac{d}{dx}(a+bx)=\frac{1}{x}-\frac{b}{a+bx}$$ **1**

$$\Rightarrow x\frac{dy}{dx}-y=x^2\left(\frac{1}{x}-\frac{b}{a+bx}\right)=\frac{ax}{a+bx}$$ $\frac{1}{2}$

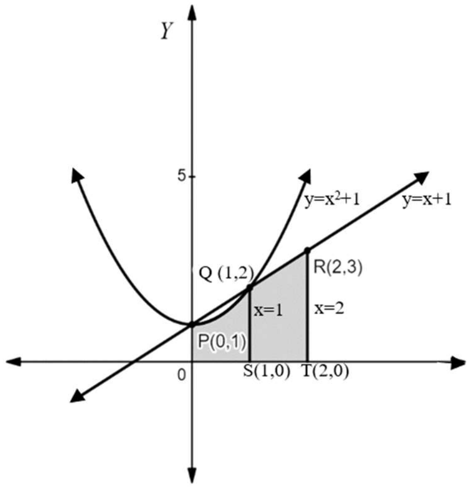
On differentiating again with respect to x, we get

$$\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)a-ax(b)}{(a+bx)^2}$$ $\frac{1}{2}$

	$\Rightarrow x\dfrac{d^2y}{dx^2}=\left(\dfrac{a}{a+bx}\right)^2$.	$\frac{1}{2}$

<u>Section –D</u>

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32	 	**1**
	To find the point of intersections of the curve $y=x^2+1$ and the line $y=x+1$,	
	we write $x^2+1=x+1\Rightarrow x(x-1)=0\Rightarrow x=0,1.$	
	So, the point of intersections $P(0,1)$ and $Q(1,2)$.	**1**
	Area of the shaded region $OPQRTSO$ = (Area of the region $OSQPO$ + Area of the region $STRQS$)	
	$=\int_0^1(x^2+1)dx+\int_1^2(x+1)dx$	**1**
	$=\left[\dfrac{x^3}{3}+x\right]_0^1+\left[\dfrac{x^2}{2}+x\right]_1^2$	$\frac{1}{2}$
	$=\left[\left(\dfrac{1}{3}+1\right)-0\right]+\left[(2+2)-\left(\dfrac{1}{2}+1\right)\right]$	$\frac{1}{2}$
	$=\dfrac{23}{6}$ **Hence the required area is** $\dfrac{23}{6}$ ***sq units.***	**1**
33	Let (a,b) be an arbitrary element of $\mathbb{N}\times\mathbb{N}$. Then, $(a,b)\in\mathbb{N}\times\mathbb{N}$ and $a,b\in\mathbb{N}$	
	We have, $ab=ba$; (As $a,b\in\mathbb{N}$ and multiplication is commutative on $\mathbb{N}$)	
	$\Rightarrow (a,b)R(a,b)$, according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$	
	Thus $(a,b)R(a,b),\ \forall(a,b)\in\mathbb{N}\times\mathbb{N}$.	
	So, R is reflexive relation on $\mathbb{N}\times\mathbb{N}$.	**1**
	Let $(a,b),(c,d)$ be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that $(a,b)R(c,d)$.	

	Then, $(a,b)R(c,d) \Rightarrow ad = bc \Rightarrow bc = ad$; (changing LHS and RHS) $\Rightarrow cb = da$; (As $a,b,c,d \in \mathbb{N}$ and multiplication is commutative on $\mathbb{N}$) $\Rightarrow (c,d)R(a,b)$; according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$ Thus $(a,b)R(c,d) \Rightarrow (c,d)R(a,b)$ So, R is symmetric relation on $\mathbb{N}\times\mathbb{N}$. Let $(a,b),(c,d),(e,f)$ be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that $(a,b)R(c,d)\ and\ (c,d)R(e,f)$. Then $\left.\begin{matrix}(a,b)R(c,d) \Rightarrow ad = bc \\ (c,d)R(e,f) \Rightarrow cf = de\end{matrix}\right\} \Rightarrow (ad)(cf) = (bc)(de) \Rightarrow af = be$ $\Rightarrow (a,b)R(e,f)$; (according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$) Thus $(a,b)R(c,d)$ **and** $(c,d)R(e,f) \Rightarrow (a,b)R(e,f)$ So, R is transitive relation on $\mathbb{N}\times\mathbb{N}$. As the relation R is reflexive, symmetric and transitive so, it is equivalence relation on $\mathbb{N}\times\mathbb{N}$. $[(2,6)] = \{(x,y) \in \mathbb{N}\times\mathbb{N} : (x,y)R(2,6)\}$ $= \{(x,y) \in \mathbb{N}\times\mathbb{N} : 3x = y\}$ $= \{(x,3x) : x \in \mathbb{N}\} = \{(1,3),(2,6),(3,9),.........\}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1
33 OR	We have, $f(x) = \begin{cases} \dfrac{x}{1+x}, if\ x \geq 0 \\ \dfrac{x}{1-x}, if\ x < 0 \end{cases}$ Now, we consider the following cases **Case 1:** when $x \geq 0$, we have $f(x) = \dfrac{x}{1+x}$ Injectivity: let $x, y \in \mathbb{R}^+ \cup \{0\}$ such that $f(x) = f(y)$, then $\Rightarrow \dfrac{x}{1+x} = \dfrac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$ So, f is injective function. Surjectivity : when $x \geq 0$, we have $f(x) = \dfrac{x}{1+x} \geq 0$ **and** $f(x) = 1 - \dfrac{1}{1+x} < 1, \text{as } x \geq 0$ Let $y \in [0,1)$, thus for each $y \in [0,1)$ there exists $x = \dfrac{y}{1-y} \geq 0$ such that $f(x) = \dfrac{\frac{y}{1-y}}{1+\frac{y}{1-y}} = y$.	1 1

	So, f is onto function on $[0,\infty)$ to $[0,1)$. **Case 2:** when $x<0$, we have $f(x)=\frac{x}{1-x}$ Injectivity: Let $x,y\in\mathbb{R}^-$ i.e., $x,y<0$, such that $f(x)=f(y)$, then $\Rightarrow \frac{x}{1-x}=\frac{y}{1-y}\Rightarrow x-xy=y-xy\Rightarrow x=y$ So, f is injective function. Surjectivity : $x<0$, we have $f(x)=\frac{x}{1-x}<0$ also, $f(x)=\frac{x}{1-x}=-1+\frac{1}{1-x}>-1$ $-1<f(x)<0$. Let $y\in(-1,0)$ be an arbitrary real number and there exists $x=\frac{y}{1+y}<0$ such that, $f(x)=f\left(\frac{y}{1+y}\right)=\frac{\frac{y}{1+y}}{1-\frac{y}{1+y}}=y.$ So, for $y\in(-1,0)$, there exists $x=\frac{y}{1+y}<0$ such that $f(x)=y$. Hence, f is onto function on $(-\infty,0)$ to $(-1,0)$. **Case 3:** (Injectivity): Let $x>0$ & $y<0$ such that $f(x)=f(y)\Rightarrow\frac{x}{1+x}=\frac{y}{1-y}$ $\Rightarrow x-xy=y+xy\Rightarrow x-y=2xy$, here $LHS>0$ but $RHS<0$, which is inadmissible. Hence, $f(x)\neq f(y)$ when $x\neq y.$ Hence f is one-one and onto function.	1 1 1
34	The given system of equations can be written in the form $AX=B$, Where, $A=\begin{bmatrix}2&3&10\\4&-6&5\\6&9&-20\end{bmatrix}, X=\begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}$ and $B=\begin{bmatrix}4\\1\\2\end{bmatrix}$ Now, $\|A\|=\begin{vmatrix}2&3&10\\4&-6&5\\6&9&-20\end{vmatrix}=2(120-45)-3(-80-30)+10(36+36)$ $=2(75)-3(-110)+10(72)=150+330+720=1200\neq0 \quad \therefore A^{-1}$ exists. $\therefore adj\,A=\begin{bmatrix}75&110&72\\150&-100&0\\75&30&-24\end{bmatrix}^T=\begin{bmatrix}75&150&75\\110&-100&30\\72&0&-24\end{bmatrix}$	$\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$

Hence, $A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ $\frac{1}{2}$

As, $AX = B \Rightarrow X = A^{-1}B \Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ $\frac{1}{2}$

$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$ $\frac{1}{2}$

Thus, $\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$ Hence, $x = 2, y = 3, z = 5$. **1**

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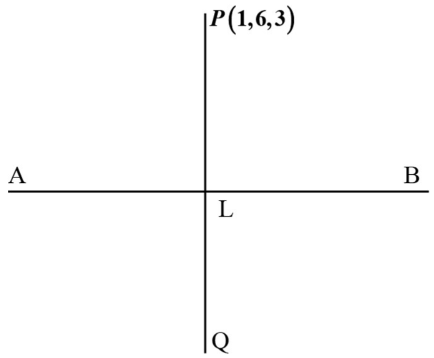
35 Let $P(1,6,3)$ be the given point, and let $'L'$ be the foot of the perpendicular from $'P'$ to the given line AB (as shown in the figure below). The coordinates of a general point on the given line are given by



$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda; \lambda$ is a scalar, i.e., $x = \lambda, y = 2\lambda + 1$ and $z = 3\lambda + 2$

Let the coordinates of L be $(\lambda, 2\lambda+1, 3\lambda+2)$.

So, direction ratios of PL are $\lambda - 1, 2\lambda + 1 - 6$ and $3\lambda + 2 - 3$, *i.e.* $\lambda - 1$, $2\lambda - 5$ and $3\lambda - 1$. $\frac{1}{2}$

Direction ratios of the given line are $1, 2$ and 3, which is perpendicular to PL. $\frac{1}{2}$

Therefore, $(\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0 \Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$

So, coordinates of L are $(1,3,5)$. **1**

Let $Q(x_1, y_1, z_1)$ be the image of $P(1,6,3)$ in the given line. Then, L is the mid-point of PQ. **1**

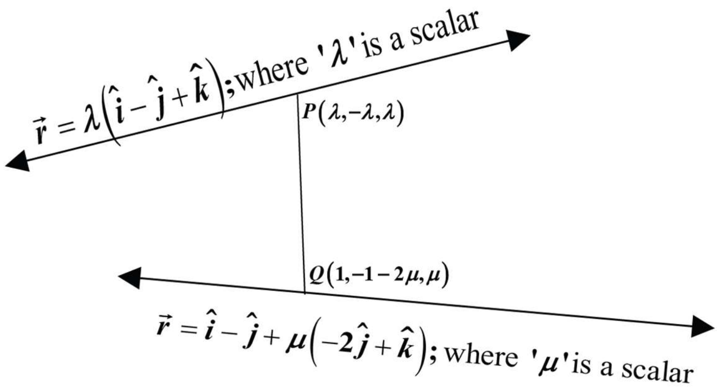
Therefore, $\frac{(x_1+1)}{2} = 1, \frac{(y_1+6)}{2} = 3$ and $\frac{(z_1+3)}{2} = 5 \Rightarrow x_1 = 1, y_1 = 0$ and $z_1 = 7$

Hence, the image of $P(1,6,3)$ in the given line is $(1,0,7)$. **1**

Now, the distance of the point $(1,0,7)$ from the $y-$ axis is $\sqrt{1^2+7^2} = \sqrt{50}$ *units*. **1**

35 OR

Method 1:



Given that equation of lines are

$\vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$..............$(i)$ and $\vec{r} = \hat{i} - \hat{j} + \mu(-2\hat{j} + \hat{k})$..............$(ii)$

The given lines are non-parallel lines as vectors $\hat{i} - \hat{j} + \hat{k}$ and $-2\hat{j} + \hat{k}$ are not parallel. There is a unique line segment PQ (P lying on line (i) and Q on the other line (ii)), which is at right angles to both the lines. PQ is the shortest distance between the lines. Hence, the shortest possible distance between the aeroplanes $= PQ$.

Let the position vector of the point P lying on the line $\vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$ where 'λ' is a scalar, is $\lambda(\hat{i} - \hat{j} + \hat{k})$, for some λ and the position vector of the point Q lying on the line $\vec{r} = \hat{i} - \hat{j} + \mu(-2\hat{j} + \hat{k})$; where '$\mu$' is a scalar, is $\hat{i} + (-1-2\mu)\hat{j} + (\mu)\hat{k}$, for some μ. **$\frac{1}{2}$**

Now, the vector $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (1-\lambda)\hat{i} + (-1-2\mu+\lambda)\hat{j} + (\mu-\lambda)\hat{k}$; (where '$O$' is the origin), is perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $\hat{i} - \hat{j} + \hat{k}$ and $-2\hat{j} + \hat{k}$. **$\frac{1}{2}$**

$\Rightarrow (1-\lambda).1 + (-1-2\mu+\lambda).(-1) + (\mu-\lambda).1 = 0$ &

$\Rightarrow (1-\lambda).0 + (-1-2\mu+\lambda).(-2) + (\mu-\lambda).1 = 0$ **$\frac{1}{2}$**

$\Rightarrow 2 + 3\mu - 3\lambda = 0$ & $2 + 5\mu - 3\lambda = 0$ **$\frac{1}{2}$**

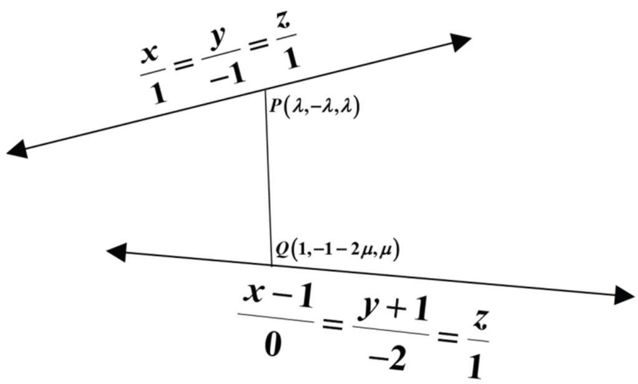
On solving the above equations , we get $\lambda=\frac{2}{3}$ and $\mu=0$

So, the position vector of the points, at which they should be so that the distance between them is the shortest, are $\frac{2}{3}\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\hat{i}-\hat{j}$. **1** **1**

$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=\frac{1}{3}\hat{i}-\frac{1}{3}\hat{j}-\frac{2}{3}\hat{k}$ and $\left|\overrightarrow{PQ}\right|=\sqrt{\left(\frac{1}{3}\right)^2+\left(-\frac{1}{3}\right)^2+\left(-\frac{2}{3}\right)^2}=\sqrt{\frac{2}{3}}$ **1**

The shortest distance $=\sqrt{\frac{2}{3}}$ units.

Method 2:



The equation of two given straight lines in the Cartesian form are $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$........$(i)$ and $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}$.........$(ii)$

The lines are not parallel as direction ratios are not proportional. Let P be a point on straight line (i) and Q be a point on straight line (ii) such that line PQ is perpendicular to both of the lines. $\frac{1}{2}$

Let the coordinates of P be $(\lambda,-\lambda,\lambda)$ and that of Q be $(1,-2\mu-1,\mu)$; where 'λ' and 'μ' are scalars. $\frac{1}{2}$

Then the direction ratios of the line PQ are $(\lambda-1,-\lambda+2\mu+1,\lambda-\mu)$

Since PQ is perpendicular to straight line (i), we have,

$(\lambda-1).1+(-\lambda+2\mu+1).(-1)+(\lambda-\mu).1=0$ $\frac{1}{2}$

$\Rightarrow 3\lambda-3\mu=2$......$(iii)$

Since , PQ is perpendicular to straight line (ii), we have $\frac{1}{2}$

$0.(\lambda-1)+(-\lambda+2\mu+1).(-2)+(\lambda-\mu).1=0\Rightarrow 3\lambda-5\mu=2$........$(iv)$ **1**

Solving (iii) and (iv), we get $\mu=0,\lambda=\frac{2}{3}$ **1**

Therfore, the ***Coordinates*** of P are $\left(\frac{2}{3},-\frac{2}{3},\frac{2}{3}\right)$ and that of Q are $(1,-1,0)$ **1**

	So, the required shortest distance is $\sqrt{\left(1-\frac{2}{3}\right)^2+\left(-1+\frac{2}{3}\right)^2+\left(0-\frac{2}{3}\right)^2}=\sqrt{\frac{2}{3}}$ *units.*	

Section –E

[This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36	Let E_1, E_2, E_3 be the events that Jayant, Sonia and Oliver processed the form, which are clearly pairwise mutually exclusive and exhaustive set of events. Then $P(E_1)=\frac{50}{100}=\frac{5}{10}$, $P(E_2)=\frac{20}{100}=\frac{1}{5}$ and $P(E_3)=\frac{30}{100}=\frac{3}{10}$. Also, let E be the event of committing an error. We have, $P(E\mid E_1)=0.06$, $P(E\mid E_2)=0.04$, $P(E\mid E_3)=0.03$. (i) The probability that Sonia processed the form and committed an error is given by $P(E\cap E_2)=P(E_2).P(E\mid E_2)=\frac{1}{5}\times 0.04=0.008.$	1
	(ii) The total probability of committing an error in processing the form is given by $P(E)=P(E_1).P(E\mid E_1)+P(E_2).P(E\mid E_2)+P(E_3).P(E\mid E_3)$ $P(E)=\frac{50}{100}\times 0.06+\frac{20}{100}\times 0.04+\frac{30}{100}\times 0.03=0.047.$	1
	(iii) The probability that the form is processed by Jayant given that form has an error is given by $P(E_1\mid E)=\frac{P(E\mid E_1)\times P(E_1)}{P(E\mid E_1).P(E_1)+P(E\mid E_2).P(E_2)+P(E\mid E_3).P(E_3)}$ $=\frac{0.06\times\frac{50}{100}}{0.06\times\frac{50}{100}+0.04\times\frac{20}{100}+0.03\times\frac{30}{100}}=\frac{30}{47}$	1
	Therefore, the required probability that the form is **not** processed by Jayant given that form has an error $=P(\overline{E_1}\mid E)=1-P(E_1\mid E)=1-\frac{30}{47}=\frac{17}{47}$.	1
	(iii) **OR** $\sum_{i=1}^{3}P(E_i\mid E)=P(E_1\mid E)+P(E_2\mid E)+P(E_3\mid E)=1$	1
	Since, sum of the posterior probabilities is 1.	1

	(We have , $\sum_{i=1}^{3} P(E_i\|E) = P(E_1 \mid E) + P(E_2 \mid E) + P(E_3 \mid E)$ $= \dfrac{P(E \cap E_1) + P(E \cap E_2) + P(E \cap E_3)}{P(E)}$ $= \dfrac{P((E \cap E_1) \cup (E \cap E_2) \cup (E \cap E_3))}{P(E)}$ as $E_i \,\&\, E_j ; i \neq j$, are mutually exclusive events $= \dfrac{P(E \cap (E_1 \cup E_2 \cup E_3))}{P(E)} = \dfrac{P(E \cap S)}{P(E)} = \dfrac{P(E)}{P(E)} = 1;$ $'S'$ being the sample space)	
37	**We have ,** $\left\|\vec{F_1}\right\| = \sqrt{6^2 + 0^2} = 6kN, \left\|\vec{F_2}\right\| = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}\,kN, \left\|\vec{F_3}\right\| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}\,kN.$ (i) Magnitude of force of Team $A = 6kN.$ **(ii) Since** $\vec{a} + \vec{c} = 3(\hat{i} - \hat{j})$ *and* $\vec{b} = -4\ (\hat{i} - \hat{j})$ *So*, $\vec{b}$ **and** $\vec{a} + \vec{c}$ **are unlike vectors having same intial point** **and** $\left\|\vec{b}\right\| = 4\sqrt{2}$ **&** $\left\|\vec{a} + \vec{c}\right\| = 3\sqrt{2}$ *Thus* $\left\|\vec{F_2}\right\| > \left\|\vec{F_1} + \vec{F_3}\right\|$ **also** $\vec{F_2}$ **and** $\vec{F_1} + \vec{F_3}$ **are unlike** **Hence B will win the game** (iii) $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 6\hat{i} + 0\hat{j} - 4\hat{i} + 4\hat{j} - 3\hat{i} - 3\hat{j} = -\hat{i} + \hat{j}$ $\therefore \left\|\vec{F}\right\| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}\,kN.$ **OR** $\vec{F} = -\hat{i} + \hat{j}$ $\therefore \theta = \pi - \tan^{-1}\left(\dfrac{1}{1}\right) = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$; **where** $'\theta'$ is the angle made by the resultant force with the $+ve$ direction of the $x-$ axis.	1 1 1 1 1 1
38	$y = 4x - \dfrac{1}{2}x^2$ (i) The rate of growth of the plant with respect to the number of days exposed to sunlight is given by $\dfrac{dy}{dx} = 4 - x.$ (ii) Let rate of growth be represented by the function $g(x) = \dfrac{dy}{dx}$.	2

	Now, $g'(x)=\frac{d}{dx}\left(\frac{dy}{dx}\right)=-1<0$ $\Rightarrow g(x)$ decreases. So the rate of growth of the plant decreases for the first three days. Height of the plant after 2 days is $y=4\times 2-\frac{1}{2}(2)^2=6cm.$	 1 1