Marking Scheme
CLASS: XII
Session: 2021-22
Mathematics (Code-041)
Term - 2

SECTION – A

1.	Find: $\int \frac{logx}{(1+logx)^2}dx$ Solution:$\int \frac{logx}{(1+logx)^2}dx = \int \frac{logx+1-1}{(1+logx)^2}dx = \int \frac{1}{1+logx}dx - \int \frac{1}{(1+logx)^2}dx$ $=\frac{1}{1+logx} \times x - \int \frac{-1}{(1+logx)^2} \times \frac{1}{x} \times xdx - \int \frac{1}{(1+logx)^2}dx = \frac{x}{1+logx} + c$ OR Find: $\int \frac{sin2x}{\sqrt{9-cos^4x}}dx$ Solution: Put $cos^2x = t \Rightarrow -2cosxsinxdx = dt \Rightarrow sin2xdx = -dt$ The given integral $= -\int \frac{dt}{\sqrt{3^2-t^2}} = -\sin^{-1}\frac{t}{3} + c = -\sin^{-1}\frac{cos^2x}{3} + c$	1/2 1+1/2 1 1
2.	Write the sum of the order and the degree of the following differential equation: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$ Solution: Order = 2 Degree = 1 Sum = 3	1 1/2 ½
3.	If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $\left\|\hat{a} + \hat{b}\right\| = 2cos\frac{\theta}{2}$, where θ is the angle between them. Solution: $(\hat{a} + \hat{b}).(\hat{a} + \hat{b}) = \|\hat{a}\|^2 + \left\|\hat{b}\right\|^2 + 2(\hat{a}.\hat{b})$ $\left\|\hat{a} + \hat{b}\right\|^2 = 1 + 1 + 2cos\theta$ $= 2(1 + cos\theta) = 4cos^2\frac{\theta}{2}$ $\therefore \left\|\hat{a} + \hat{b}\right\| = 2cos\frac{\theta}{2}$,	1 1/2 ½
4.	Find the direction cosines of the following line: $\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$ Solution: The given line is $\frac{x-3}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z}{4}$ Its direction ratios are <1, 1, 4> Its direction cosines are $\langle\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\rangle$	1 1/2 ½

5. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.

Solution: Let X be the random variable defined as the number of red balls.

Then X = 0, 1 — 1/2

P(X=0) = $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$ — 1/2

P(X=1) = $\frac{1}{4} \times \frac{3}{3} + \frac{3}{4} \times \frac{1}{3} = \frac{6}{12} = \frac{1}{2}$ — 1/2

Probability Distribution Table:

X	0	1
P(X)	$\frac{1}{2}$	$\frac{1}{2}$

½

6. Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?

Solution: The required probability = P((The first is a red jack card and The second is a jack card) or (The first is a red non-jack card and The second is a jack card)) — 1

$=\frac{2}{52} \times \frac{3}{51} + \frac{24}{52} \times \frac{4}{51} = \frac{1}{26}$ — 1

SECTION – B

7. Find: $\int \frac{x+1}{(x^2+1)x} dx$

Solution: Let $\frac{x+1}{(x^2+1)x} = \frac{Ax+B}{x^2+1} + \frac{C}{x} = \frac{(Ax+B)x+C(x^2+1)}{(x^2+1)x}$ — 1/2

$\Rightarrow x+1 = (Ax+B)x + C(x^2+1)$ (An identity)

Equating the coefficients, we get

B = 1, C = 1, A + C = 0

Hence, A = -1, B = 1, C = 1 — ½

The given integral = $\int \frac{-x+1}{x^2+1} dx + \int \frac{1}{x} dx$

$= \frac{-1}{2} \int \frac{2x-2}{x^2+1} dx + \int \frac{1}{x} dx$ — 1/2

$= \frac{-1}{2} \int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx + \int \frac{1}{x} dx$

$= \frac{-1}{2} \log(x^2+1) + \tan^{-1} x + log|x| + c$ — 1+1/2

8. Find the general solution of the following differential equation:

$x\frac{dy}{dx} = y - xsin(\frac{y}{x})$

Solution: We have the differential equation:

$\frac{dy}{dx} = \frac{y}{x} - sin(\frac{y}{x})$

The equation is a homogeneous differential equation.

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ — 1

The differential equation becomes

$v + x\frac{dv}{dx} = v - sinv$

$\Rightarrow \frac{dv}{sinv} = -\frac{dx}{x} \Rightarrow cosecvdv = -\frac{dx}{x}$ — ½

Integrating both sides, we get

	$log\|cosecv - cotv\| = -log\|x\| + logK, K > 0$ (Here, $logK$ is an arbitrary constant.) $\Rightarrow log\|(cosecv - cotv)x\| = logK$ $\Rightarrow \|(cosecv - cotv)x\| = K$ $\Rightarrow (cosecv - cotv)x = \pm K$ $\Rightarrow \left(cosec\frac{y}{x} - cot\frac{y}{x}\right)x = C$, which is the required general solution. OR Find the particular solution of the following differential equation, given that y = 0 when x = $\frac{\pi}{4}$: $\frac{dy}{dx} + ycotx = \frac{2}{1 + sinx}$ Solution: The differential equation is a linear differential equation I F = $e^{\int cotxdx} = e^{logsinx} = sinx$ The general solution is given by $ysinx = \int 2\frac{sinx}{1 + sinx}dx$ $\Rightarrow ysinx = 2\int \frac{sinx + 1 - 1}{1 + sinx}dx = 2\int [1 - \frac{1}{1 + sinx}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{1 + \cos\left(\frac{\pi}{2} - x\right)}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{2cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{2}sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)]dx$ $\Rightarrow ysinx = 2[x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)] + c$ Given that y = 0, when x = $\frac{\pi}{4}$, Hence, 0 = 2$[\frac{\pi}{4} + tan\frac{\pi}{8}] + c$ $\Rightarrow c = -\frac{\pi}{2} - 2tan\frac{\pi}{8}$ Hence, the particular solution is $y = cosecx[2\left\{x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\} - \left(\frac{\pi}{2} + 2tan\frac{\pi}{8}\right)]$	1 ½ 1 ½ 1 ½
9.	If $\vec{a} \neq \vec{0}, \vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$. Solution: We have $\vec{a}.\left(\vec{b} - \vec{c}\right) = 0$ $\Rightarrow \left(\vec{b} - \vec{c}\right) = \vec{0}$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$ $\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$ Also, $\vec{a} \times \left(\vec{b} - \vec{c}\right) = \vec{0}$ $\Rightarrow \left(\vec{b} - \vec{c}\right) = \vec{0}$ or $\vec{a} \parallel \left(\vec{b} - \vec{c}\right)$ $\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \parallel \left(\vec{b} - \vec{c}\right)$ $\vec{a}\ can\ not\ be\ both\ perpendicular\ to\ \left(\vec{b} - \vec{c}\right)\ and\ parallel\ to\ \left(\vec{b} - \vec{c}\right)$ Hence, $\vec{b} = \vec{c}$.	1 1 1
10.	Find the shortest distance between the following lines: $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + s\left(2\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ $\vec{r} = \left(\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) + t\left(4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$	

Solution: Here, the lines are parallel. The shortest distance = $\frac{\left|(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\right|}{|\vec{b}|}$

$$= \frac{\left|(3\hat{k})\times(2\hat{\imath}+\hat{\jmath}+\hat{k})\right|}{\sqrt{4+1+1}}$$ 1+1/2

$$\left(3\hat{k}\right)\times\left(2\hat{\imath}+\hat{\jmath}+\hat{k}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{\imath}+6\hat{\jmath}$$ 1

Hence, the required shortest distance = $\frac{3\sqrt{5}}{\sqrt{6}}$ units ½

OR

Find the vector and the cartesian equations of the plane containing the point $\hat{\imath}+2\hat{\jmath}-\hat{k}$ and parallel to the lines $\vec{r} = \left(\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)+s\left(2\hat{\imath}-3\hat{\jmath}+2\hat{k}\right)$ and $\vec{r} = \left(3\hat{\imath}+\hat{\jmath}-2\hat{k}\right)+t\left(\hat{\imath}-3\hat{\jmath}+\hat{k}\right)$

Solution: Since, the plane is parallel to the given lines, the cross product of the vectors $2\hat{\imath}-3\hat{\jmath}+2\hat{k}$ and $\hat{\imath}-3\hat{\jmath}+\hat{k}$ will be a normal to the plane

$$\left(2\hat{\imath}-3\hat{\jmath}+2\hat{k}\right)\times\left(\hat{\imath}-3\hat{\jmath}+\hat{k}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & 2 \\ 1 & -3 & 1 \end{vmatrix} = 3\hat{\imath}-3\hat{k}$$ 1

The vector equation of the plane is $\vec{r}.\left(3\hat{\imath}-3\hat{k}\right) = (\hat{\imath}+2\hat{\jmath}-\hat{k}).(3\hat{\imath}-3\widehat{k})$ 1

or, $\vec{r}.\left(\hat{\imath}-\hat{k}\right) = 2$

and the cartesian equation of the plane is x – z – 2 = 0 1

SECTION – C

11. Evaluate: $\int_{-1}^{2}|x^3-3x^2+2x|dx$

Solution: The given definite integral = $\int_{-1}^{2}|x(x-1)(x-2)|dx$

$$= \int_{-1}^{0}|x(x-1)(x-2)|dx + \int_{0}^{1}|x(x-1)(x-2)|dx + \int_{1}^{2}|x(x-1)(x-2)|dx$$ 1+1/2

$$= -\int_{-1}^{0}(x^3-3x^2+2x)dx + \int_{0}^{1}(x^3-3x^2+2x)dx - \int_{1}^{2}(x^3-3x^2+2x)dx$$ 1/2

$$= -[\frac{x^4}{4}-x^3+x^2]_{-1}^{0} + [\frac{x^4}{4}-x^3+x^2]_{0}^{1} - [\frac{x^4}{4}-x^3+x^2]_{1}^{2}$$

$$= \frac{9}{4}+\frac{1}{4}+\frac{1}{4} = \frac{11}{4}$$ 2

12. Using integration, find the area of the region in the first quadrant enclosed by the line x + y = 2, the parabola y ^{2} = x and the x-axis.

Solution: Solving x + y = 2 and y ^{2} = x simultaneously, we get the points of intersection as (1, 1) and (4, -2). 1

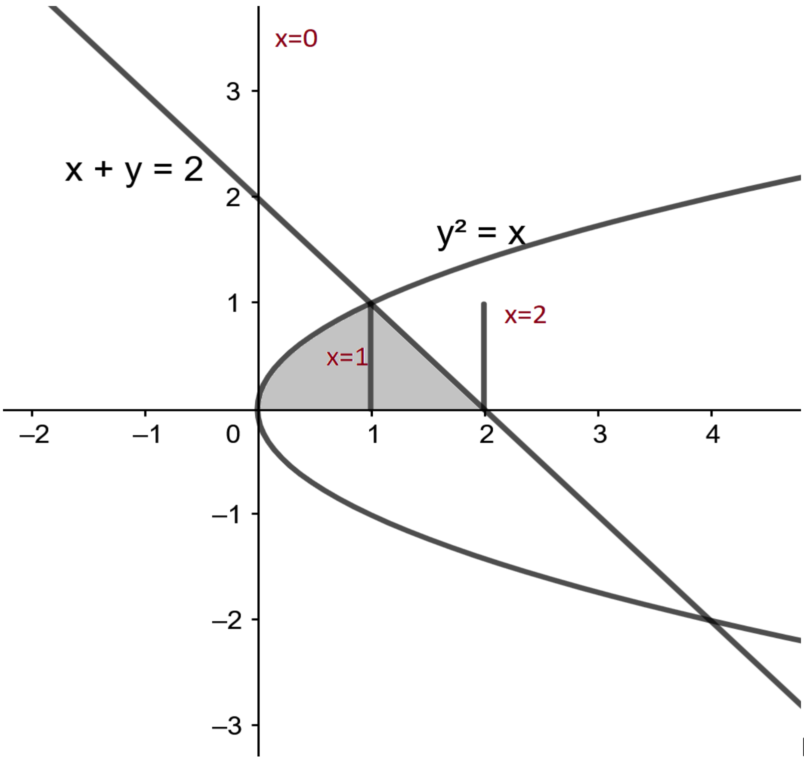

Fig 1 1

The required area = the shaded area = $\int_0^1 \sqrt{x}\, dx + \int_1^2 (2-x)dx$ 1

$= \frac{2}{3}[x^{\frac{3}{2}}]_0^1 + [2x - \frac{x^2}{2}]_1^2$

$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ square units 1

OR

Using integration, find the area of the region: $\{(x, y): 0 \leq y \leq \sqrt{3}x, x^2 + y^2 \leq 4\}$

Solution: Solving $y = \sqrt{3}x \; and \; x^2 + y^2 = 4$, we get the points of intersection as (1, $\sqrt{3}$) and (-1, $-\sqrt{3}$) 1

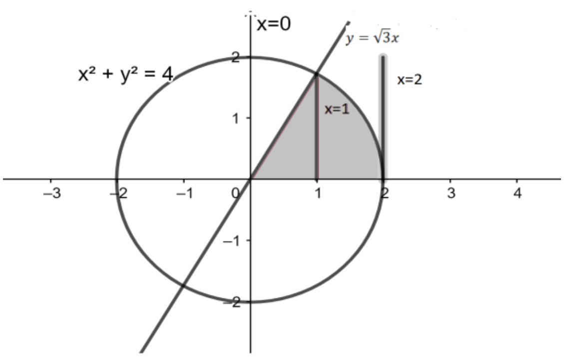

Fig 2 1

	The required area = the shaded area = $\int_0^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{4-x^2}dx$ $= \frac{\sqrt{3}}{2}[x^2]_0^1 + \frac{1}{2}[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}]_1^2$ $= \frac{\sqrt{3}}{2} + \frac{1}{2}[2\pi - \sqrt{3} - 2\frac{\pi}{3}]$ $= \frac{2\pi}{3}$ square units	1 1
13.	Find the foot of the perpendicular from the point (1, 2, 0) upon the plane x – 3y + 2z = 9. Hence, find the distance of the point (1, 2, 0) from the given plane. Solution: The equation of the line perpendicular to the plane and passing through the point (1, 2, 0) is $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z}{2}$ The coordinates of the foot of the perpendicular are $(\mu+1, -3\mu+2, 2\mu)$ for some μ These coordinates will satisfy the equation of the plane. Hence, we have $\mu + 1 - 3(-3\mu+2) + 2(2\mu) = 9$ $\Rightarrow \mu = 1$ The foot of the perpendicular is (2, -1, 2). Hence, the required distance = $\sqrt{(1-2)^2 + (2+1)^2 + (0-2)^2} = \sqrt{14}\,units$	1 ½ 1 ½ 1

14.	**CASE-BASED/DATA-BASED**  Fig 3 An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone. **Based on the given information, answer the following questions.**	
	(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?	
	(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?	
	Solution: Let E_1 = The policy holder is accident prone. E_2 = The policy holder is not accident prone. E = The new policy holder has an accident within a year of purchasing a policy. (i) $P(E)= P(E_1)\times P(E/E_1) + P(E_2)\times P(E/E_2)$ $=\frac{20}{100}\times\frac{6}{10}+\frac{80}{100}\times\frac{2}{10}=\frac{7}{25}$ (ii) By Bayes' Theorem, $P(E_1/E) = \frac{P(E_1)\times P(E/E_1)}{P(E)}$ $=\frac{\frac{20}{100}\times\frac{6}{10}}{\frac{280}{1000}}=\frac{3}{7}$	1 1 1 1
