TRB –UG – MATHEMATICS

Exponential series and Matrix

SELLAVEL K

Topics :

- **Exponential series**
- **Type of matrix**
- **Eigen values**
- **Characteristic equation**

Unit : 1 Algebra

Series

T.R.B. 2006 – 2007

1. The value of $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\cdots$

2) the value of $\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\cdots$

3) In $n=\frac{1}{e}-\frac{1}{2e^2}+\frac{1}{3e^3}-\cdots$ then

T.R.B – 2005 – 2006

1. The co.eff. of x^n in $\frac{1}{1-3x}$ is

2. $\frac{1}{n+1}+\frac{1}{2(n+1)^2}+\frac{1}{3(n+1)^3}+\cdots\infty$ is

3) $\frac{x}{1!}+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots\infty$ equal to.

4. The validity of the expression $(1-2x)^{-1}$ when expanded is.

T.R.B – 2001

1. The co-eff of x^n in $(1-x)^{-1}$ is.

2) $1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots\infty$ is equal to.

3) $\log 2 =$

TRB – 2002

1. The co.eff of x^7 in $\frac{1}{1+2x}$ is

2. $1-\log 2+\frac{1}{2!}(\log 2)^2-\frac{1}{3!}(\log 2)^3+\cdots\infty$ is.

3) $1+\frac{1}{3!}+\frac{1}{5!}+\cdots=?$

4) If $|x|<1$ which of the following true

TRB – 2003 – 2004

1. The coeff of x^n of e^{a+bx} is.

2. $\frac{1+\frac{1}{2!}+\frac{1}{4!}+\cdots}{1+\frac{1}{3!}+\frac{1}{5!}+\cdots}$

3. $\frac{2n}{(n^2+1)}+\frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3+\frac{1}{5}\left(\frac{2n}{n^2+1}\right)^5+\cdots=?$

4. If $|x|<1$ which of the following is true
$\log(1+x)$, $\log(1-x)$, $\log\left(\frac{1+x}{1-x}\right)$, $\log\left(\frac{1+x}{1-x}\right)$

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T.R.B 2004-2005

1. $\frac{x^2+1}{(x-1)^4} = ?$

2) $1 + 2(\frac{1}{3}) + 3(\frac{1}{3})^2 + 4(\frac{1}{3})^3 + \ldots \infty$,

3) $\log 3 + \frac{(\log 3)^2}{2!} + \frac{(\log 3)^3}{3!} + \ldots$

4) The expansion $(3x-4)^{-7}$ is valid if.

TRB - 2007 - 2008

1. $1 + n\left(\frac{2a}{1+a}\right) + \frac{n(n+1)}{1\cdot 2}\left(\frac{2a}{1+a}\right)^2 + \ldots = ?$

2) $\log 2 - \frac{(\log 2)^2}{2!} + \frac{(\log 2)^3}{3!} + \ldots \infty =$

3) If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ then $x = ?$

TRB - old

1. $f(x) = \frac{1+x}{(1-x)^2}$, the coeff of x^n.

2) $\frac{2n}{n^2+1} + \frac{1}{3}\cdot\left(\frac{2n}{n^2+1}\right)^3 + \frac{1}{5}\left(\frac{2n}{n^2+1}\right)^5 + \ldots$

3). $\frac{1}{2}\log a + \frac{1}{2}\log c + \frac{1}{2ac+1} + \frac{1}{3}\frac{1}{(2ac+1)^3} + \ldots \infty$

4) $e = ?$

5) $(1+3x)^{5/2}$ is valid for ?

Exponential Series

Results:

$$e^{x} = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots = \sum_{r=0}^{\infty}\frac{x^r}{r!}$$

$$e^{-x} = 1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots = \sum_{r=0}^{\infty}\frac{(-1)^r x^r}{r!}$$

Problems:

1. Find the co.eff of x^n in $1+\frac{(a+bx)}{1!}+\frac{(a+bx)^2}{2!}\cdots$

Soln

$$LHS = \sum_{r=0}^{\infty}\frac{(a+bx)}{r!}$$

$$= e^{a+bx}$$

$$= e^{a}\cdot e^{bx}$$

$$= e^{a}\sum_{r=0}^{\infty}\frac{(bx)^r}{r!}$$

$\therefore$ Co.eff of $x^n = e^{a}\cdot\frac{b^n}{n!}$

2) Find the co.eff of x^n in $(3+2x)e^{3x}$.

$$(3+2x)e^{3x} = (3+2x)\sum_{r=0}^{\infty}\frac{(3x)^r}{r!}$$

$\therefore$ Co.eff $x^n = \frac{3\cdot 3^n}{n!}+\frac{2(3^{n-1})}{(n-1)!}$

3) Find the co.eff of x^n in $(1-x)e^{1+x}$.

$$(1-x)e^{1+x} = (1-x)\cdot e\cdot e^{x}$$

$$= (1-x)e\sum_{r=0}^{\infty}\frac{x^r}{r!}$$

$$= \frac{e}{n!}-\frac{e}{(n-1)!} = \frac{e}{n!}(1-n)$$

4) Find the co.eff of x^n in $\frac{2+3x}{e^x}$

$$(2+3x)e^{-x} = (2+3x)\sum_{r=0}^{\infty}\frac{(-1)^r(x)^r}{r!}$$

Co.eff of $x^n = \frac{2(-1)^n}{n!}+\frac{3(-1)^{n-1}}{(n-1)!}$

$$= \frac{2(-1)^n}{n!}(2-3n)$$

5) Find the co.eff of x^n in $\left(\frac{a+bx}{e^x}\right)^2$

$$\left(\frac{a+bx}{e^x}\right)^2 = (a^2+b^2x^2+2abx)\sum_{r=0}^{\infty}\frac{(2x)^r(-1)^r}{r!}$$

$$\therefore \text{coeff} \cdot x^n = \frac{2^n a^2(-1)^n}{n!} + \frac{b^2(-1)^{n-2}2^{n-2}}{(n-2)!} + \frac{2ab\,(-1)^{n-1}}{(n-1)!}\cdot 2^{n-1}$$

$$= \frac{2^{n-2}}{n!}\{4a^2+b^2n(n-1)+4abn\}$$

6) Find the co.eff of x^n in $\sum_{n=1}^{\infty}\frac{(1+2x)^n}{n!}$

Soln

$$\sum_{n=1}^{\infty}\frac{(1+2x)^n}{n!} = e^{1+2x}-1$$

$$= e\cdot e^{2x}-1$$

$$= e\sum_{r=0}^{\infty}(-1)^r\frac{(2x)^r}{r!}-1$$

$$\therefore \text{coeff of } x^n = \frac{e\cdot 2^n}{n!}$$

7) Find the co.eff of x^n in $(1-x)e^{1-x}$

$$e(1-x)e^{-x} = e(1-x)\sum_{r=0}^{\infty}\frac{(-1)^r(x)^r}{r!}$$

$$\text{co.eff}, x^n = \frac{e(-1)^n}{n!} - \frac{(-1)^{n-1}e}{(n-1)!}$$

$$= \frac{e(-1)^n}{n!}\{1+n\}$$

8) Find the co.eff of x^n in $\frac{2+3x+x^2}{e^{2x}}$

$$(2+3x+x^2)e^{-2x} = (2+3x+x^2)\sum_{r=0}^{\infty}\frac{(-1)^r(2x)^r}{r!}$$

$$\text{co.eff of } x^n = \frac{2(-1)^n 2^n}{n!} + \frac{3(-1)^{n-1}2^{n-1}}{(n-1)!} + \frac{(-1)^{n-2}(2)^{n-2}}{(n-2)!}$$

9) Find the co.eff of x^n in $\frac{e^{5x}+e^x}{e^{3x}}$

$$e^{2x}+e^{-2x} = \sum_{r=0}^{\infty}\frac{(2x)^r}{r!} + \sum_{r=0}^{\infty}\frac{(-1)^r(2x)^r}{r!}$$

$$\text{co.eff of } x^n = \frac{2^n}{n!} + \frac{(-1)^n 2^n}{n!}$$

$$= \frac{2^n}{n!}\{1+(-1)^n\}$$

10) Find the co.eff of x^n in $\sum_{r=0}^{\infty} \frac{(2x+3x)^r}{r!}$

$$\sum_{r=0}^{\infty} \frac{(2+3x)^r}{r!} = e^{2+3x}$$

$$= e^2 \cdot e^{3x}$$

$$= e^2 \sum_{r=0}^{\infty} \frac{(3x)^r}{r!}$$

Co.eff of $x^n = \frac{e^2 \cdot 3^n}{n!}$

11. Find the co.eff of x^n in $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\dots\right)^2$,

$$LHS = \left(\frac{e^x+e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x}+e^{-2x}+2}{4}$$

$$= \frac{1}{4}\left[\sum_{r=0}^{\infty}\frac{(2x)^r}{r!} + \sum_{r=0}^{\infty}\frac{(-1)^r(2x)^r}{r!}\right\}$$

$$\therefore \text{co.eff } x^n = \frac{1}{4}\left\{\frac{2^r}{r!}+\frac{(-1)^r(2)^r}{r!}\right\}$$

$$= \frac{1}{4}\left(\frac{2^n}{n!}\right\}\left\{1+(-1)^n\right\}.$$

If n is even then co.eff $x^n = \frac{2^{n-1}}{n!}$

If n is odd then co.ff of $x^n = 0$.

12) $\log 2 - \frac{(\log 2)^2}{2!} + \frac{(\log 2)^3}{3!} + \dots \infty = ?$

$$LHS = -\left\{-\log 2 + \frac{(\log 2)^2}{2!} - \dots\right\}$$

$$= -\left\{e^{-\log 2} - 1\right\}$$

$$= 1 - e^{\log 1/2}$$

$$= 1 - 1/2 = 1/2$$

13) $1 + \frac{(\log n)^2}{2!} + \frac{(\log n)^4}{4!} + \dots = ?$

$$LHS = \frac{e^{\log n} + e^{-\log n}}{2}$$

$$= \frac{1}{2}\left(n + 1/n\right\}$$

14) $2\left\{1+\frac{(\log n)^2}{2!}+\frac{(\log n)^4}{4!}\dots\right\} = (n+1/n)$

15) $\log 3 - \frac{1}{2!}(\log 3)^2 + \frac{(\log 3)^3}{3!} + \dots = ?$

$$LHS = -\left\{-\log 3 + \frac{(\log 3)^2}{2!} - \dots\right.$$

$$= -\left\{e^{-\log 3} - 1\right\}$$

$$= 1 - e^{-\log 3}$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

16) $\log 3 + \frac{(\log 3)^2}{2!} + \frac{(\log 3)^3}{3!} + \dots = ?$

$$= e^{\log 3} - 1$$

$$= 3 - 1$$

$$= 2$$

17) $\frac{2^2}{1!} + \frac{2^4}{3!} + \frac{2^6}{5!} + \dots = ?$

$$= 2\left(\frac{2}{1!} + \frac{2^3}{3!} + \frac{2^5}{5!} + \dots\right)$$

$$= 2\left(\frac{e^2 - e^{-2}}{2}\right)$$

$$= e^2 - \frac{1}{e^2}$$

$$= \frac{e^4 - 1}{e^2}$$

18) $\dfrac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots} = ?$

$$= \frac{\frac{e^1 + e^{-1}}{2} - 1}{\frac{e^1 - e^{-1}}{2}}$$

$$= \frac{e + e^{-1} - 2}{e - e^{-1}}$$

$$= \frac{(e - e^{-1})^2}{(e - e^{-1})}$$

$$= \frac{e + e^{-1}}{e^2 - 1}$$

$$= \frac{e - 1}{e + 1}$$

19) $\dfrac{1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots}{1+\frac{1}{3!}+\frac{1}{5!}+\cdots} = ?$

$$= \frac{\frac{e+e^{-1}}{2}}{\frac{e-e^{-1}}{2}} = \frac{e+1/e}{e-1/e} = \frac{e^2+1}{e^2-1}.$$

20) $1+\dfrac{x\log a}{1!}+\dfrac{(x\log a)^2}{2!}+\dfrac{(x\log a)^3}{3!}+\cdots = ?$

$$= e^{x\log a} = e^{\log a^x} = a^x$$

21) $1+\dfrac{2^2}{2!}+\dfrac{2^4}{4!}+\dfrac{2^6}{6!}+\cdots = ?$

$$= \frac{e^2+e^{-2}}{2} = \frac{e^2+\frac{1}{e^2}}{2} = \frac{e^4+1}{2e^2}$$

22) $\left(1+\frac{1}{1!}+\frac{1}{2!}+\cdots\right)\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots\right) = ?$

$$= e^x \cdot e^{-x}$$

$$= e\cdot e^{-1} = e^0 = 1.$$

23) $\dfrac{2^2}{1!}+\dfrac{2^4}{2!}+\dfrac{2^6}{3!}+\cdots = ?$

$$= 2\left(\frac{e^2-e^{-2}}{2}\right)$$

$$= e^2 - 1/e^2 = \frac{e^4-1}{e^2}$$

24) $\dfrac{1}{1.3}+\dfrac{1}{1.2.3.5}+\dfrac{1}{1.2.3.4.5.7}+\cdots\infty = ?$

$$= \frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\cdots$$

$$= \frac{2^{\cancel{2}}}{3\cancel{!}}+\frac{2^{\cancel{2}}}{5\cancel{!}}+$$

$$= \sum_{r=0}^{\infty}\frac{2n}{(2n+1)!}$$

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25) $1+\left(x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots\right)^2=?$

$=1+\left(\frac{e^x-e^{-x}}{2}\right)^2$

$=1+\frac{e^{2x}+e^{-2x}-2}{4}$

$=\frac{e^{2x}+e^{-2x}+2}{4}=\left(\frac{e^x+e^{-x}}{2}\right)^2$

26) $\left(1+\frac{1}{2!}+\frac{1}{4!}+\cdots\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+\cdots\right)^2=?$

$=\left(\frac{e+e^{-1}}{2}\right)^2-\left(\frac{e-e^{-1}}{2}\right)^2.$

$=\frac{1}{4}\{(e+e^{-1})^2-(e-e^{-1})^2\}$

$=\frac{1}{4}\{e^2+e^{-2}-2-\{e^2+e^{-2}-2\}$

$=1.$

27) $1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\cdots=?$

$T_n=\frac{2^n-1}{n!}$

$S=\Sigma T_n=e^2-e=e(e-1)$

28) $1+\frac{1+3}{2!}+\frac{1+3+3^2}{3!}+\cdots$

$T_n=\frac{3^n-1}{n!}$

$S=\Sigma T_n=e^3-e=e(e^2-1).$

29) $2\left\{\log 2+\frac{(\log 2)^3}{3!}+\frac{(\log 2)^5}{5!}+\cdots\right\}=?$

$=2\left\{\frac{e^{\log 2}-e^{-\log 2}}{2}\right\}$

$=2-\frac{1}{2}=\frac{3}{2}$

30) $\log_3 e-\log_9 e+\log_{27} e-\cdots=?$

$=\frac{1}{\log_e 3}-\frac{1}{\log_e 3^2}+\frac{1}{\log_e 3^3}+\cdots$

$=\left(\frac{1}{\log 3}\right)-\frac{1}{2\log 3}+\frac{1}{3\log 3}+\cdot$

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$=\frac{1}{\log_e 3}\left\{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right\}=\frac{\log 2}{\log 3}$

logarithmic series

Matrix

TRB - 2005 - 2006

1

1. The no. of divisor of 480 is.
2. The no. of integers less than 1125 and prime to it is.
3. The matrix $\begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}$ is.
4. The eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ are
5.

TRB - 2001

1. Which of the following is not composite no.
2. Which of the following is ~~not~~ a Hermition matrix.
3. If A is a matrix of order 4x5, its rank
4. Characterstic roots of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
5.

TRB - 2002

1. Which of the following is not true.
   (i) A is singular ⇒ $|A| = 0$.
   (ii)
2) The rank of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ is
3) The eigen values of $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 5 & 0 \\ 7 & 1 & 3 \end{bmatrix}$ is
4. A is Hermition iA is.
5. The inverse of the orthogonal matrix A is.

TRB - 2003 - 2004

1. The no. of integers less than n and prime to it, when n = 729 is
2) If A & B are skew symmetric then A+B is.

3) The rank of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 1 & 2 \end{pmatrix}$ is

4) The eigen values of $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$ is

TRB – 2004 – 2005

1. If the eigen values of $A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$ are 2, 2, 2 then the eigen values of A^2 are.

2) The matrix $A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & -2 \\ -3 & 2 & 0 \end{pmatrix}$ is.

3) The highest power of 7 which divides 900! exactly is.

4) The no. of divisors of 360 is.

5) $A^T = (\overline{A^T})$ then A is.

6) The sum of eigen values of $\begin{pmatrix} 1 & 2 & 5 \\ 2 & 2 & 4 \\ 1 & 2 & 7 \end{pmatrix}$ is.

7) If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ then A^n equal to.

TRB – 2007 – 2008

1. The rank of $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ is

2. The matrix $\begin{pmatrix} 4 & 2+3i & -i \\ 2-3i & -1 & 1-i \\ i & 1+i & 0 \end{pmatrix}$ is.

3) The sum of the eigenvalue of $\begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$ is.

4. The sum of all divisors of 360 is

5. The highest power of 3 that divides 1000! is.

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TRB – 2006-2007

1. The no. of integers less than 600 & prime to it is.

2) The matrix $\begin{bmatrix} i & 1+i \\ -1+i & i \end{bmatrix}$ is a.

3) Which of the following is an orthogonal.

4. The rank of $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ is.

5) The sum of the eigen values of $\begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$ is

6) If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ then

logarithmic Series

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \infty$$

$$\log(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots\right)$$

$$\log\left(\frac{1+x}{1-x}\right) = 2\left\{x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \infty\right\}$$

Theory of numbers

~~II-N~~

Matrix

Symmetric matrix: $A^T = A$.

1) $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ 2) $\begin{bmatrix} 0 & -1 & 3 \\ -1 & -2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$ 3) $\begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$

4) $\begin{bmatrix} x & -a & b \\ -a & y & c \\ b & c & 0 \end{bmatrix}$ 5) $\begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{bmatrix}$ 6) $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Skew symmetric matrix: $A^T = -A$.

1) $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 4 \\ -2 & -4 & 0 \end{bmatrix}$ 2) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$ 3) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

4) $\begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$ 5) $\begin{bmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{bmatrix}$ 6) $\begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}$

Hermitian matrix: $(\bar{A})^T = A$

1) $\begin{pmatrix} a & \bar{z}_1 & \bar{z}_2 \\ z_1 & b & \bar{z}_3 \\ z_2 & z_3 & c \end{pmatrix}$ 2) $\begin{bmatrix} a & z_1 & \bar{z}_2 \\ \bar{z}_1 & b & \bar{z}_3 \\ z_2 & z_3 & c \end{bmatrix}$

3) $\begin{bmatrix} 2 & 1-i & -1+2i \\ 1+i & -3 & 3-i \\ -1-2i & 3+i & 4 \end{bmatrix}$ 4) $\begin{bmatrix} 0 & i & 2 \\ -i & 2 & -2+i \\ 2 & -2-i & 1 \end{bmatrix}$

5) $\begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}$ Hermitian But not Symmetric matrix.

6) Symmetric $\Rightarrow$ Hermitian.

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7) $\begin{bmatrix} 3 & 4+5i \\ 4-5i & 6 \end{bmatrix}$ 8) $\begin{bmatrix} 4 & 6+4i & 3-7i \\ 6-4i & 7 & 2+5i \\ 3+7i & 2-5i & 0 \end{bmatrix}$

9) $\begin{bmatrix} 1 & 1+3i & -i \\ 1-3i & 3 & -2-3i \\ i & -2+3i & 6 \end{bmatrix}$ 10) $\begin{bmatrix} 3 & 1+2i \\ 1-2i & 2 \end{bmatrix}$ 11) $\begin{bmatrix} 4 & 1-i \\ 1+i & 2 \end{bmatrix}$

Skew Hermitian: $(\bar{A})^T = -A$.

1) $\begin{bmatrix} 0 & 3-4i \\ -3-4i & 0 \end{bmatrix}$ 2) $\begin{bmatrix} -2i & 4-7i \\ -4-7i & 0 \end{bmatrix}$

3) $\begin{bmatrix} 0 & -1+i \\ 1+i & 0 \end{bmatrix}$ 4) $\begin{bmatrix} i & 3+i & -2-i \\ -3+i & 0 & 3-4i \\ 2-i & -3-4i & -2 \end{bmatrix}$

5) $\begin{bmatrix} 3 & 2-3i & 3+5i \\ 2+3i & 5 & i \\ 3-5i & -i & 7 \end{bmatrix}$ is Hermitian.

6) $\begin{bmatrix} i & 1+i & 2 \\ -1-i & 3i & i \\ -2 & i & 0 \end{bmatrix}$ 7) $\begin{bmatrix} i & 1+i & 2-i \\ -(1-i) & 2i & 3i \\ -(2+i) & 3i & 4i \end{bmatrix}$

8) $\begin{bmatrix} i & 2 & 1+i \\ -2 & 0 & -1-2i \\ -1+i & 1-2i & -2i \end{bmatrix}$

Orthogonal matrix: $AA^T = A^TA = I$ $\therefore$ $\boxed{A^{-1} = A^T}$

$|A| = \pm 1$

1) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ 4) $\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

5) $\frac{1}{7}\begin{bmatrix} 6 & -3 & 2 \\ -3 & -2 & 6 \\ 2 & 6 & 3 \end{bmatrix}$ 6) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

7) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ 8) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Unitary matrix: $(\bar{A})^T A = I$ $\therefore$ $A^{-1} = (\bar{A})^T$ $|A| = \pm 1$

1) $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ 2) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

3) $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ 4) $\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -1+i \\ 1+i & 0 \end{bmatrix}$

5) $\frac{1}{5}\begin{bmatrix} -1+2i & 2-4i \\ -4-2i & -2-i \end{bmatrix}$ 6) $\begin{bmatrix} \frac{1+i}{\sqrt{7}} & \frac{2+i}{\sqrt{7}} \\ \frac{2-i}{\sqrt{7}} & \frac{-1+i}{\sqrt{7}} \end{bmatrix}$

7) $\begin{bmatrix} a+ib & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary $\Leftrightarrow$ $a^2+b^2+c^2+d^2=1$

Rank of matrix:

	Rank
1) $\begin{bmatrix} 1 & 3 & 4 & 5 & 10 \\ 3 & 2 & 5 & 4 & 8 \end{bmatrix}$	2
2) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$	1
3) $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 3 & 6 & 9 & -3 \\ 2 & 4 & 6 & -2 \end{bmatrix}$	1
4) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$	2
5) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$	2
6) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$	2
7) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$	3
8) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 2 & 4 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$	3.
9) $\begin{bmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & 4 & -1 \\ 2 & 0 & -4 & -2 \end{bmatrix}$	2
10) $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{bmatrix}$	3.

Sellavel K Tiruvannamalai

Eigen values and characteristic equations

ch. equation : $\lambda^3 - \lambda^2(a_{11}+a_{22}+a_{33}) + \lambda(A_{11}+A_{22}+A_{33}) - |A| = 0$

1) $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ $\quad \lambda^2 - 6\lambda + 5 = 0 \quad \lambda = 1, 5$

2) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ $\quad \lambda^3 - 18\lambda^2 + 45\lambda = 0 \quad \lambda = 0, 3, 15$

3) $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ $\quad \lambda^3 - 13\lambda + 12 = 0, \quad \lambda = 1, 3, -4$

4) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ $\quad \lambda^3 - 11\lambda^2 + 34\lambda - 24, \quad \lambda = 1, 4, 6$

5) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ $\quad \lambda^3 - 4\lambda^2 - 25\lambda + 28 = 0 \quad \lambda = 1, -4, 7$

6) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ $\quad \lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0, \quad \lambda = 1, 2, 4$

7) $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ $\quad \lambda^3 - 4\lambda^2 + \lambda + 6 = 0, \quad \lambda = -1, 2, 3$

8) $\begin{bmatrix} 5 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ $\quad \lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0, \quad \lambda = 2, 3, 6$

9) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ $\quad \lambda^3 - 7\lambda^2 + 36 = 0 \quad \lambda = -2, 3, 6$

10) $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ $\quad \lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0 \quad \lambda = 2, 3, 6$

11) $\begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$ $\quad \lambda^3 - 9\lambda^2 - 9\lambda + 81 = 0, \quad \lambda = 3, -3, 9$

12) $\begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{bmatrix}$ $\quad \lambda^3 - 18\lambda^2 + 81\lambda = 0 \quad \lambda = 0, 9, 9$

13) $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ $\quad \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0 \quad \lambda = 5, 1, 1$

14) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ $\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$, $\lambda = 5, 1, 1$

15) $\begin{bmatrix} 5 & -2 & 2 \\ -2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}$ $\lambda^3 - 9\lambda^2 + 15\lambda - 7 = 0$ $\lambda = 1, 1, 7$

16) $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ $\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$ $\lambda = 2, 2, 3$

17) $\begin{bmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{bmatrix}$ $\lambda^3 - \lambda^2 - 2\lambda = 0$ $\lambda = 0, -1, 2$

Assignments

1) The matrix $\begin{pmatrix} 0 & 6 \\ -6 & 0 \end{pmatrix}$ is

2) The matrix $\begin{bmatrix} 6 & 3+i \\ -3+i & i \end{bmatrix}$ is

3) The matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is

4) All the diagonal elements of a skew symmetric matrix are.

5) If A is square matrix then $A - A^T$ is.

6) A is Hermitian then A^2 is.

7) $\rho(AB)$ is

8) Rank of $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ is

9) Rank of $\begin{bmatrix} 2 & 1 & -1 \\ 4 & 2 & \lambda \end{bmatrix}$ is 2 then λ is

10) Eigen values of $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ are.

11) Char. roots of $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ are.

12) " " $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ are.

13) Characteristic roots of a hermitial matrix are all real.

13) ch. roots of real symmetric matrix are.

14) The characteristic vector of $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ corresponding to the characteristic root 1 is.

Sellavel K Tiruvannamalai

16) Char. roots of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 6 \\ 0 & 2 & 7 \end{bmatrix}$ are.

17) Char. roots of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ are.

18) If $\begin{bmatrix} 0 & -2 & x \\ 2 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$ is skew Hermitian then x is

19) If $\begin{bmatrix} 1 & -1+2i & 3+4i \\ -1-2i & -2 & 3 \\ x & 3 & 2 \end{bmatrix}$ is hermitian then x is

20) If $\begin{bmatrix} 0 & -1+2i & 3-4i \\ x & 1 & -2i \\ -3-4i & -2i & 3i \end{bmatrix}$ is Skew Hermitian then x is

21) If $\begin{pmatrix} 0 & m & 0 \\ 4 & 0 & 6 \\ 0 & 0 & 1 \end{pmatrix}$ is orthogonal then x is.

22) If A is Hermitian then $|A|$ is

23) The inverse of the orthogonal matrix A is

24) ch. roots of $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$

25) Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 1 & 2 \end{bmatrix}$

26) If A is symmetric and skew symmetric then $A = 0$

27) The char. roots of $\begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$ are.

28) The char. equ. of $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ is

29) Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \end{bmatrix}$

30) " " $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$

31) " " $\begin{pmatrix} 3 & 2 & -1 \\ 4 & -1 & -2 \\ -6 & 7 & 8 \end{pmatrix}$

32) " " $\begin{bmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 2 & 0 & 4 & -2 \end{bmatrix}$

33) The matrix $\begin{bmatrix} 3 & 5 & 6 \\ 5 & 0 & 8 \\ 6 & 8 & 1 \end{bmatrix}$ is both

34) Char. roots of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ are

35) ch. roots of $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are

36) " " " $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ are.

37) ch. equ. of $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ is

38) ch. equ. of $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ is.

39) All the elements in leading diagonal of skew symmetric matrix is

40) In a skew Hermitian matrix the leading diagonal matrix are

41) If A is any sqr. matrix then AA^T is.

42) If A and B are Hermitian then AB−BA is.

43) The matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is.

44) The rank of $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is

45) The characteristic equ. of $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ is.

46) The char. roots are $\begin{bmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{bmatrix}$ are

47) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ then A satisfies.

Theory of equations

Assignments:

1. A quadratic equation whose roots ~~are~~ is $3+2i$

2. cubic equation with $1, i$ are roots.

3. i is a root of $x^3+x^2+x+1=0$ then other roots are.

4. If x^3+ax^2+x-6 is divided by $x-1$ the remainder is zero then $a=$

5. If x^3+5x^2-2 is divided by $x-1$ then the remainder is

6. If $2+i\sqrt{3}$ is a root of $x^2+px+q=0$ then $p+q=$

7. The lowest degree of an equ. having $\sqrt{2}+\sqrt{3}+i$ as one roots is.

8) cubic equ. with roots $1, 3-\sqrt{-1}$ is

9) one root of $x^4+4x^3+5x^2+2x-2=0$ is $-1+i$ the other roots are

10) one root of $x^4-5x^3+4x^2+8x-8=0$ is $1-\sqrt{5}$ the other roots are.

11. $\sqrt{2}+\sqrt{3}$ is a root of $3x^5-4x^4-42x^3+56x^2+27x-36=0$ its rational roots.

12) The product of the roots of $x^5-x^3-4x^2-7x+10=0$

13) If 3 is a root of $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ then $\frac{1}{3}$ is also a root

14) Every polynomial of odd degree has atleast one real root.

15) The product of the roots of the equation $x^5 - x^3 - 4x^2 - 7x + 10 = 0$. is.

16) If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP then the roots are.

Ans: 1, 4, 7

17)

Sellavel K Tiruvannamalai