UG - TRB MATHEMATICS

ANALYTICAL GEOMETRY UG TRB

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2023

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Analytical Geometry

Sphere:

Find the equation of Sphere:

Model I: (radius and Centre are given)

The equation of Sphere with centre (c_1, c_2, c_3) and radius 'a' is

$(x-c_1)^2 + (y-c_2)^2 + (z-c_3)^2 = a^2$

Problems:

1. Centre $(2,-3,4)$ radius 5

Soln

$x^2+y^2+z^2-4x+6y-8z+d=0.$

$x^2+y^2+z^2-4x+6y-8z+4=0.$

$d = u^2+v^2+w^2-a^2$
$= 4+9+16-25$

2) Centre $(1,2,3)$ radius 4.

$x^2+y^2+z^2-2x-4y-6z+d=0$

$x^2+y^2+z^2-2x-4y-6z-2=0.$

$d = u^2+v^2+w^2-a^2$
$= 1+4+9-16$
$= -2$

3) $(1,-2,3)$ radius 5 units.

$x^2+y^2+z^2-2x+4y-6z+d=0$

$x^2+y^2+z^2-2x+4y-6z-11=0$

$d = 1+4+9-25$
$= -11.$

4) Centre $(2,-3,4)$ radius 5 units.

$x^2+y^2+z^2-4x+6y-8z+d=0$

$x^2+y^2+z^2-4x+6y-8z+4=0.$

$d = 4+9+16-25$

5) $(1,2,-2)$ and radius 3 units.

$x^2+y^2+z^2-2x-4y+4z+d=0$

$x^2+y^2+z^2-2x-4y+4z=0.$

$d = 1+4+4-9$
$d=0$

6) Find the equation of sphere whose centre is $(1,-2,3)$ and it is passing through $(2,1,2)$

$x^2+y^2+z^2-2x+4y-6z+d=0 \longrightarrow (1)$

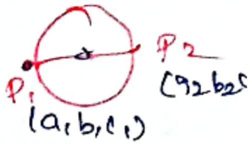
it is passing through $(2,1,2)$

$4+1+4-4+4-12+d=0$

$d=3$

Model 2: (End pts of diameters are given)

Formula:



If (a_1,b_1,c_1) & (a_2,b_2,c_2) are the end pts of the diameter of the sphere then its equation is

$$(x-a_1)(x-a_2)+(y-b_1)(y-b_2)+(z-c_1)(z-c_2)=0.$$

problems:

1. Find the equation of sphere ~~whose centre~~ is which as the pts ~~(2,7,-4)~~ (2,7,-4) & (4,5,-1) as the extremities.

Soln

$$x^2+y^2+z^2-6x-12y+5z+d=0 \rightarrow (1)$$

$(4,5,-1)$ $16+25+1-24-60-5+d=0$

$$-47+d=0$$
$$d=47.$$

2) End pts are $(1,-2,3)$ $(3,-1,-2)$

$$x^2+y^2+z^2-4x+y-z+d=0. \longrightarrow (1)$$

$(1,-2,3)$ $1+4+9-4-2-3+d=0$

$$d=-5$$

3) End pts are $(2,-3,4)$ & $(-1,5,7)$

$$x^2+y^2+z^2-x-2y-11z+d=0$$

$(2,-3,4)$ $4+9+16-2+6-44+d=0$

$$-11+d=0 \Rightarrow d=11.$$

4) End pts are $(1,-3,4)$ & $(3,-1,3)$

$$x^2+y^2+z^2-4x+4y-7z+d=0$$

$(1,-3,4)$ $1+9+16-4-12-28+d=0.$

$$-18+d=0$$
$$d=18.$$

5) Find pts are $(2,7,-4)$ & $(4,5,-1)$

$$x^2+y^2+z^2-6x-12y+5z+d=0$$

$(4,5,-1)$ $16+25+1-24-60-5+d=0$

$$-47+d=0$$

$$d=47.$$

-65
18
47

Find the centre & radius of the sphere

The centre and radius of the sphere

$x^2+y^2+z^2+2ux+2uy+2wz+d=0$ are

$$\text{centre} = (-u,-v,-w)$$

$$\text{radius} = \sqrt{u^2+v^2+w^2-d}.$$

1. $x^2+y^2+z^2+2x-4y-6z+10=0.$

$$\text{Centre} = (-1,2,3)$$

$$\text{radius} = \sqrt{1+4+9-10} = 2;$$

2. $x^2+y^2+z^2-4x+6y-2z-11=0.$

$$\text{Centre} = (2,-3,1)$$

$$\text{radius} = \sqrt{4+9+1+11} = \sqrt{25}, 5$$

3. $x^2+y^2+z^2+2x-4y+8z-2=0.$

$$\text{Centre} = (-1,2,-4)$$

$$\text{radius} = \sqrt{1+4+16+2} = \sqrt{23}.$$

4. $x^2+y^2+z^2+4x-2y+6z+10=0.$

$$\text{Centre} = (-2,1,-3)$$

$$\text{radius} = \sqrt{4+1+9-10} = 2.$$

5) $2x^2+2y^2+2z^2-2x+4y-6z-1=0$.

$x^2+y^2+z^2-x+2y-3z-\frac{1}{2}=0$.

centre $=\left(\frac{1}{2},-1,\frac{3}{2}\right)$

radius $=\sqrt{\frac{1}{4}+1+\frac{9}{4}+\frac{1}{2}}=\sqrt{\frac{1+4+9+2}{4}}=2$.

6) $16x^2+16y^2+16z^2-16x-8y-16z-35=0$.

$x^2+y^2+z^2-x-\frac{1}{2}y-z-\frac{35}{16}=0$

centre $=\left(\frac{1}{2},\frac{1}{4},\frac{1}{2}\right)$

radius $=\sqrt{\frac{1}{4}+\frac{1}{16}+\frac{1}{4}+\frac{35}{16}}=\sqrt{\frac{4+1+4+35}{16}}$

$=\sqrt{45/16}=\frac{\sqrt{45}}{4}=\frac{3\sqrt{5}}{4}$.

7) $2(x^2+y^2+z^2)+6x-6y+8z+9=0$

$x^2+y^2+z^2+3x-3y+4z+9/2=0$

centre $=\left(-3/2,3/2,-2\right)$

radius $=\sqrt{\frac{9}{4}+\frac{9}{4}+4-\frac{9}{2}}=\sqrt{4}=2$.

8) $3(x^2+y^2+z^2)+2x+2y+2z-10=0$

$x^2+y^2+z^2+\frac{2}{3}x+\frac{2}{3}y+\frac{2z}{3}-\frac{10}{3}=0$

centre $\left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3}\right)$

radius $=\sqrt{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{10}{3}}$

$=\sqrt{\frac{3}{9}+\frac{10}{3}}$

$=\sqrt{\frac{1}{3}+\frac{10}{3}}=\sqrt{\frac{11}{3}}$.

9) $16x^2+16y^2+16z^2-16x-8y-16z-55=0$.

$x^2+y^2+z^2-x-\frac{1}{2}y-z-\frac{55}{16}=0$

centre $\left(\frac{1}{2},\frac{1}{4},\frac{1}{2}\right)$

radius $=\sqrt{\frac{1}{4}+\frac{1}{16}+\frac{1}{4}+\frac{55}{16}}$

$=\sqrt{\frac{4+1+4+55}{16}}=\sqrt{\frac{64}{16}}=\sqrt{4}=2$

Plane

The distance between the parallel planes $ax+by+cz+d=0$ & $ax+by+cz+d_1=0$ is given by $\left|\frac{d_1-d}{\sqrt{a^2+b^2+c^2}}\right|$

1. $2x-3y+6z+12=0$, $6x-9y+18z-6=0$.

$2x-3y+6z-2=0$

$$\text{distance} = \left|\frac{12+2}{\sqrt{4+9+36}}\right|$$

$$= \left|\frac{14}{\sqrt{49}}\right| = \frac{14}{7} = 2.$$

2) $3x+6y+2z-22=0$ $3x+6y+2z-27=0$

$$\text{distance} = \left|\frac{-22+27}{\sqrt{9+36+4}}\right| = \left|\frac{5}{\sqrt{49}}\right| = \frac{5}{7}.$$

3) $2x+y+2z-8=0$ & $4x+2y+4z+5=0$

$2x+y+2z+5/2=0$

$$\text{distance} = \left|\frac{-5/2+8}{\sqrt{4+1+4}}\right| = \left|\frac{21/2}{3}\right| = \left|7/2\right| = 7/2$$

4) $x-2y+2-2=0$, $3x-6y+3z+1=0$

$x-2y+z+1/3=0$

$$\text{distance} = \left|\frac{1/3+2}{\sqrt{1+4+1}}\right| = \frac{7/3}{\sqrt{6}} = \frac{7}{3\sqrt{6}}.$$

5) $2x+y+2z+6=0$, $4x+2y+4z+9=0$

$2x+y+2z+9/2=0$

$$\text{distance} = \left|\frac{9/2-6}{\sqrt{4+1+4}}\right| = \left|\frac{-3/2}{3}\right| = 1/2$$

6) $x + y - 2z - 1 = 0$ & $4x + 2y - 4z + 1 = 0$.

$2x + y - 2z + \frac{1}{2} = 0$

$$\text{distane} = \left| \frac{\frac{1}{2}+1}{\sqrt{4+1+4}} \right| = \left| \frac{3/2}{3} \right| = \frac{1}{2}$$

7) $2x - 2y + z + 3 = 0$ & $4x - 4y + 2z + 5 = 0$

$2x - 2y + z + \frac{5}{2} = 0$

$$\text{distance} = \left| \frac{\frac{5}{2} - 3}{\sqrt{4+4+1}} \right| = \left| \frac{-\frac{1}{2}}{3} \right| = \frac{1}{6}.$$

8) $2x - y + z + 1 = 0$ & $4x - 2y + 2z + 3 = 0$.

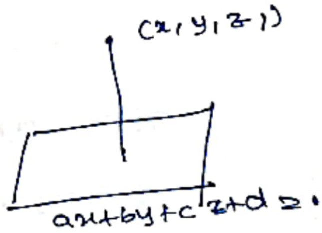
$2x - y + z + \frac{3}{2} = 0$.

$$\text{distane} = \left| \frac{\frac{3}{2} - 1}{\sqrt{4+1+1}} \right| = \left| \frac{-\frac{1}{2}}{\sqrt{6}} \right| = \frac{1}{2\sqrt{6}}$$

Formula:

1) The $\perp$r distance from (x_1, y_1, z_1) to the plane $ax + by + cz + d$ is given by

$\perp$r distane $= \left| \dfrac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2+b^2+c^2}} \right|$



2) The $\perp$r distance from the origine to the plane $ax + by + cz + d$ is given by

$$\text{distance} = \left| \frac{d}{\sqrt{a^2+b^2+c^2}} \right|$$

1. Find the ⊥r distance from (2,1,0) to the plane $2x+y+2z-7=0$

Soln

$$\perp r \text{ distance} = \left| \frac{4+1+0-7}{\sqrt{4+1+4}} \right| = \left| \frac{-2}{3} \right| = 2/3.$$

2) $(0,1,-1)$ to the plane. $2x-y+2z+3=0$

$$\perp r \text{ dstance} = \left| \frac{0-1-2+3}{\sqrt{4+1+4}} \right| = 0.$$

3) $(0,0,0)$ to the plane $x-2y+2z+5=0$

$$\perp r \text{ distane} = \left| \frac{0+0+0+5}{\sqrt{1+4+4}} \right| = 5/3.$$

4) $(0,0,0)$ to the plane $6x-3y+2z-14=0$

$$\perp r \text{ distance} = \left| \frac{0+0+0-14}{\sqrt{36+9+4}} \right| = \left| \frac{-14}{7} \right| = 2.$$

Angle between the planes.

The angle between the planes $a_1x+b_1y+c_1z+d_1$ & $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Formulas:

Problems:

1. $2x-y+z+7=0$, $x+y+2z-11=0$.

$$\cos\theta = \frac{2-1+2}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = 1/2$$

$$\cos\theta = 1/2$$

$$\theta = 60^\circ = \pi/3.$$

2) $2x-y+z=6$, $x+y+2z=3$

$$\cos\theta = \frac{2-1+2}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{3}{6} = 1/2$$

$\cos\theta = 1/2$

$\theta = 60^\circ = \pi/3$.

3) $x+2y+2z=0$, $2x+y-2z=0$

$$\cos\theta = \frac{2+2-4}{\sqrt{1+4+4}\sqrt{4+1+4}} = 0.$$

$\cos\theta = 0$

$\theta = 90^\circ = \pi/2$.

4) $x+2y+2z+7=0$, $4x-y-z+6=0$.

$$\cos\theta = \frac{4-2-2}{\sqrt{1+4+4}\sqrt{16+1+1}} = 0.$$

$\cos\theta = 0$

$\theta = 90^\circ = \pi/2$

5) $2x+y-z=9$, $x+2y+z=7$.

$$\cos\theta = \frac{2+2-1}{\sqrt{4+1+1}\sqrt{1+4+1}} = \frac{3}{\sqrt{6}\sqrt{6}} = 1/2$$

$\cos\theta = 1/2$

$\theta = 60 = \pi/3$

6) $2x-3y+4z=1$, $-x+y=4$.

$$\cos\theta = \frac{-2-3+0}{\sqrt{4+9+16}\sqrt{1+1+0}} = \frac{-5}{\sqrt{29}\sqrt{2}} = \frac{-5}{\sqrt{58}}$$

$\theta = \cos^{-1}\left(-5/\sqrt{58}\right)$

7). $3x+y-z=7$, $x+4y-2z=10$.

$$\cos\theta = \frac{3+4+2}{\sqrt{9+1+1}\sqrt{1+16+4}} = \frac{9}{\sqrt{11}\sqrt{21}} = \frac{9}{\sqrt{231}}$$

$\theta = \cos^{-1}\left(9/\sqrt{231}\right)$

8) $2x-y+z=15$, $-x-y-3z=3$.

$$\cos\theta = \frac{-2+1-3}{\sqrt{4+1+1}\sqrt{1+1+9}} = \frac{-4}{\sqrt{6}\sqrt{11}}$$

$$= -\frac{4}{\sqrt{66}}$$

$\theta = \cos^{-1}(-4/\sqrt{66})$

10) $2x-y+z=4$, $x+y+2z=4$.

$$\cos\theta = \frac{2-1+2}{\sqrt{4+1+1}\sqrt{1+4+1}} = \frac{3}{\sqrt{6}\sqrt{6}} = 1/2$$

$$\theta = \cos^{-1}(1/2) = 60^\circ = \pi/3.$$

Angle between the ~~planes~~ Lines

The angle between the lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \quad \& \quad \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

is given by

$$\theta = \cos^{-1}\left(\frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}\right)$$

Problems

1. $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+1}{2}$ & $\frac{x}{3} = \frac{y-5}{2} = \frac{z-2}{6}$.

$$\theta = \cos^{-1}\left(\frac{3+4+12}{\sqrt{9}\sqrt{9+4+36}}\right) = \cos^{-1}\left(\frac{19}{21}\right)$$

2) $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-4}{6}$ & $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-4}{2}$

$$\theta = \cos^{-1}\left(\frac{2+6+12}{\sqrt{4+9+36}\sqrt{1+4+4}}\right) = \cos^{-1}\left(\frac{20}{21}\right)$$

3) $\frac{x-1}{-1} = \frac{y+7}{4} = \frac{z}{2}$ & $\frac{x+2}{3} = \frac{y}{0} = \frac{z-1}{4}$

$$\theta = \cos^{-1}\left(\frac{-3+0+8}{\sqrt{1+16+4}\sqrt{9+16}}\right) = \cos^{-1}\left(\frac{5}{5\sqrt{21}}\right) = \cos^{-1}(1/\sqrt{21})$$

4) $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-1}{4}$ & $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{-1}$

$$\theta = \cos^{-1}\left(\frac{6-2-4}{\sqrt{4+1+16}\sqrt{9+4+1}}\right) = \cos^{-1}(0) = 90^\circ = \pi/2$$

5) $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-1}{4}$ & $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{-1}$

$$\theta = \cos^{-1}\left(\frac{6-2-4}{\sqrt{\cdot}\,\cdot\sqrt{}}\right) = \cos^{-1}(0) = 90^\circ = \pi/2$$

Angle between the St. line & plane.

The angle between the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax+by+cz+d=0$ is given by

$$\sin\theta = \left(\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\,\sqrt{l^2+m^2+n^2}}\right)$$

1. $x+2y+3z+4=0$ & $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{2}$

$$\theta = \sin^{-1}\left(\frac{2-2+6}{\sqrt{1+4+9}\,\sqrt{4+1+4}}\right) = \sin^{-1}\left(\frac{6}{\sqrt{14}\,(3)}\right) = \sin^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

2) $3x+4y+z+5=0$ & $\frac{x-1}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$.

$$\theta = \sin^{-1}\left(\frac{9-4-2}{\sqrt{9+16+1}\,\sqrt{9+1+4}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{26}\,\sqrt{14}}\right)$$

3) $x+y=1$ & $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-3}{-1}$

$$\theta = \sin^{-1}\left(\frac{2+1+0}{\sqrt{2}\,\sqrt{6}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{12}}\right)$$

4) $3x-2y+6z=0$ & $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+1}{2}$

$$\theta = \sin^{-1}\left(\frac{6-2+12}{\sqrt{9+4+36}\,\sqrt{4+1+4}}\right) = \sin^{-1}\left(\frac{16}{7\times 3}\right)$$

$$= \sin^{-1}\left(\frac{16}{21}\right)$$

5) $x+y+4=0$ & $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$

$$\theta = \sin^{-1}\left(\frac{2+1}{\sqrt{2}\,\sqrt{9}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi/4 = 45^\circ$$

6) $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ & $3x+y+z=7$

$$\theta = \sin^{-1}\left(\frac{6+3+6}{\sqrt{4+9+36}\sqrt{9+1+1}}\right) = \sin^{-1}\left(\frac{15}{7\times\sqrt{11}}\right)$$

7) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-3}{2}$ & $x+z-7=0$

$$\theta = \sin^{-1}\left(\frac{1+2}{3\sqrt{2}}\right) = \sin^{-1}\left(\sqrt{3/2}\right)$$

St. Line

1. Find the equ. of plane passing through $(3,2,-1)$ & $\perp$ to $5x-4y+7z-1=0$.

Soln

$$\frac{x-3}{5} = \frac{y-2}{-4} = \frac{z+1}{7}$$

2) $(1,-2,-1)$ $\perp$ $2x-3y+5z+4=0$.

$$\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z+1}{5}$$

3) $(3,2,-8)$ $\perp$ $3x-y-2z+2=0$

$$\frac{x-3}{3} = \frac{y-2}{-1} = \frac{z+8}{-2}.$$

4) $(0,0,0)$ $\perp$ to $2x-3y+5z+12=0$.

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{5}$$

5) $(1,-2,5)$ $\perp$ to $2x+3y-4z+7=0$.

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-4}.$$

6) $(2,3,5)$ & $(-1,2,-4)$

$$\frac{x-2}{3} = \frac{y-3}{1} = \frac{z-5}{9}.$$

7) $(1,-1,3)$ & $(3,3,1)$

$$\frac{x-1}{-2} = \frac{y+1}{-4} = \frac{z-3}{2}.$$

8) $(2,1,3)$ $(1,-2,4)$

$$\frac{x-2}{1} = \frac{y-1}{3} = \frac{z-3}{-1}$$

Plane:

1. If the foot of ⊥r from the origin to the plane is $(2,-1,1)$ & find its equation.

$$2x-y+z+d=0.$$

it is passing through $(2,-1,1)$

$$4+1+1+d=0$$

$$d=-6$$

From (1)

$$2x-y+z-6=0.$$

2) If the foot of ⊥r from the origin to the plane is $(2,3,-1)$ then find its equation.

$$2x+3y-z+d=0 \longrightarrow (1)$$

it is passing through $(2,3,-1)$

$$\therefore 4+9+1+d=0 \Rightarrow d=-14$$

From (1)

$$2x+3y-z+d=0$$

3) If the foot of ⊥r from the origin to the plane is $(3,1,2)$ then find its equ.

$$3x+y+2z+d=0 \qquad d=-(9+1+4)$$

$$3x+y+2z-14=0.$$

4) If the foot of ⊥r from the origin to the plane is $(12,-4-3)$. then find its equation.

$$12x-4y-3z+d=0 \qquad d=(144+16+9)$$

$$12x-4y-3z-169=0.$$

5) If the foot of ⊥r from the origin to the plane is $(2,1,3)$ find its equ.

$$2x+y+3z+d=0 \qquad d=-(4+1+9)$$

$$2x+y+3z-14=0.$$

6) Find the equ. of plane passing through $(2,-3,4)$ and parallel to $2x-5y-7z+15=0$.

$$2x-5y-7z+d=0$$

$$4+15-28+d=0$$

$$-9+d=0 \qquad \therefore 2x-5y-7z-9=$$

7) Find the equ. of plane passing through (0,0,0) ~~to~~ ||le to $2x-3y+5z+12=0$.

Soln

$$2x-3y+5z+d=0 \quad \Rightarrow d=0$$

$$2x-3y+5z=0.$$

8) (1,2,3) parallel to $4x+5y-3z+7=0$

$$4x+5y-3z+d=0 \quad \rightarrow (1)$$

$$4+10-9+d=0$$

$$d=-5$$

From (1) $4x+5y-3z-5=0$.

Extra problems

1. The angle between the two planes $2x-y+z=6$ & $x+y+2z=3$ is.

$$\theta = \cos^{-1}\left(\frac{2-1+2}{\sqrt{6}\sqrt{6}}\right) = \cos^{-1}(1/2) = 60^\circ = \pi/3.$$

2) The distance of the pt (2,3,-5) from the plane $x+2y-2z=9$ is.

$$\text{distance} = \left|\frac{2+6+10-9}{\sqrt{1+4+4}}\right| = \left|\frac{9}{3}\right| = 3.$$

3) The x,y,z intercept of the plane $4x-3y+2z=7$ are.

$$\frac{x}{(7/4)} + \frac{y}{(-7/3)} + \frac{z}{(7/2)} = 1$$

$\therefore$ Intercept are $7/4, -7/3, 7/2$.

4) The distance between the parallel planes $2x+y+2z=8$ & $4x+2y+4z+5=0$ is

$$\text{distance} = \left|\frac{5/2+8}{\sqrt{4+1+4}}\right| = \left|\frac{21/2}{3}\right| = 7/2$$

5) The equ. of plane $\parallel^{le}$ to $3x+4y-5z+2=0$ and passing through $(1,2,3)$ is

$$3x+4y-5z+d=0 \longrightarrow (1)$$

it is passing through $(1,2,3)$

$$3+8-15+d=0 \Rightarrow d=4$$

From (1)

$$3x+4y-5z+4=0.$$

6) The equation to the plane which cuts of intercepts $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ from the axis is.

$$\frac{x}{1/2}+\frac{y}{1/3}+\frac{z}{1/4}=1$$

$$2x+3y+4z=1.$$

8) Equation of plane passing through $(1,2,3)$ and drs of its normal are $(2,-1,3)$ is.

Soln $\quad 2x-y+3z+d=0 \longrightarrow (1)$

$(1,2,3) \quad 2-2+9+d=0 \Rightarrow d=-9$

$$\therefore 2x-y+3z-9=0.$$

9) Equation to the plane, on which the foot of the ⊥r from the origin is $(12,-4,-3)$ is.

$$12x-4y-3z+d=0 \qquad d=-(144+16+9)$$

$$12x-4y-3z-169=0.$$

10) Equ. to the plane passing through the pts $(a,0,0)$ $(0,b,0)$ $(0,0,c)$. is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$$

11) The equ. of plane passing thro. $(3,-3,1)$ and normal to the line joining $(3,4,-1)$ & $(2,-1,5)$

drs : $(3-2, 4+1, -1-5) = (1,5,-6)$

$$\therefore x+5y-6z+d=0$$

$(3,-3,1) \quad 3-15-6+d=0 \Rightarrow d=18$

12) Angle between the $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$ &

the plane $2x - y + z = 1$ is

$$\theta = \sin^{-1}\left(\frac{2+1+1}{\sqrt{3}\sqrt{6}}\right) = \sin^{-1}\left(\frac{4}{\sqrt{18}}\right) = \sin^{-1}\left(\frac{4}{3\sqrt{2}}\right) =$$

$$= \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

13) Angle between the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{0}$ &

$\frac{x+5}{-3} = \frac{y+6}{2} = \frac{z-1}{2}$ is.

$$\theta = \cos^{-1}(-6+6+0) = \cos^{-1}0 = 90^\circ = \pi/2$$

14) The pt at which the line $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z+2}{-2}$

meets the plane $3x + 4y + 5z = 5$ is.

pt: $(r-1, 3r-3, -2r+2)$

$$3r - 3 + 12r - 12 - 10r - 10 = 5$$

$$5r = 30$$

$$r = 6.$$

$\therefore$ pts is $(5, 15, -19)$

15) The equ. of the line passing through two pts $(1,2,3)$ & $(3,4,5)$ is

$$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{2}.$$

$$x-1 = y-2 = z-3.$$

16) The equ. of line through $(7,8,9)$ & $\perp$r to $2x + 3y + 4z + 5 = 0$ is.

$$\frac{x-7}{2} = \frac{y-8}{3} = \frac{z-9}{4}$$

17) The pt of intersection of the lines

$\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4}$ & $\frac{x-1}{5} = \frac{y-2}{8} = \frac{3-z}{7}$ is.

$(2a-1, 3a-1, 4a-1)$, $(5b+1, 8b+2, -7b+3)$

$2a - 1 = 5b + 1$ | $3a - 1 = 8b + 2$

$\Rightarrow 2a - 5b = 2$ | $3a - 8b = 3$

$6a - 15b = 6$

$6a - 16b = 6$

$\underline{- \quad + \quad -}$

$b = 0.$

pts is $(1,2,3)$

18) equ. of the sphere whose centre is $(1,-2,3)$ & passing through $(2,1,2)$ is

$$x^2+y^2+z^2-2x+4y-6z+d=0 \longrightarrow ①$$

$(2,1,2)$ $\cancel{4+1+4-4+4-12+d=0}$ $d = u^2+v^2+w^2-d^2$

$\cancel{d=9}$ $= 4+1+4-11$

$= -2$

from (1). $\cancel{x^2+y^2+z^2-2x+4y-4z+9=0}$

$$x^2+y^2+z^2-2x+4y-6z+3=0.$$

19) The volume of the sphere $x^2+y^2+z^2+2x-4y+8z-2=0$ is

$$c = (-1, 2, -4)$$

$$d = \sqrt{1+4+16+2} = \sqrt{23}.$$

$$\text{volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\sqrt{23})^3.$$

20) equ. of the sphere described on the line joining $(2,7,5)$ & $(8,-5,1)$ as diameter is

$$x^2+y^2+z^2-10x-2y-6z+d=0$$

$(2,7,5)$ $4+49+25-20-14-30+d=0$ $\quad$ $88 - 69 = 19$

$$14+d=0$$

$$d=-14.$$

21) The equ. to the sphere concentric with $x^2+y^2+z^2-2x-4y-6z+9=0$ and passing through $(1,-1,2)$ is.

$$x^2+y^2+z^2-2x-4y-6z+d=0$$

$(1,-1,2)$ $1+1+4-2+4-12+d=0$

$$d=4.$$

$$x^2+y^2+z^2-2x-4y-6z+4=0.$$

22) Equation of the sphere with centre $(1,-1,2)$ & touching the plane $2x-2y+z=3$ is.

$$x^2+y^2+z^2-2x+2y-4z+d=0. \longrightarrow (1)$$

$$d = 1+1+4-a^2$$