TRB –UG – MATHEMATICS

Differential Equations

Topics :

- Solvable for p
- Clairaut's Equations
- Total differentiation
- Formation of a differential equation
- Integral factors
- homogeneous equations
- Variable separation
- Laplace transformation

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Differential equations

Solvable for p:

1) Solve $p^2-7p+10=0$.

$$p^2-7p+10=0 \qquad 10 \begin{array}{l} -5 \\ -2 \\ \hline -7 \end{array}$$

$$(p-5)(p-2)=0$$

$$(y-5x+c)(y-2x+c)=0.$$

2) Solution of $p^2-5p+6=0$ is.

$$p^2-5p+6=0$$

$$(p-3)(p-2)=0$$

$$(y-3x+c)(y-2x+c)=0.$$

3) Solution of $p^2-7p+12=0$ is.

$$p^2-7p+12=0$$

$$(p-4)(p-3)=0$$

$$(y-4x+c)(y-3x+c)=0.$$

4) Solution of $p^2-9p+18=0$ is.

$$p^2-9p+18=0$$

$$(p-6)(p-3)=0$$

$$(y-6x+c)(y-3x+c)=0.$$

5) Solution of $p^3-2p^2+3p=0$ is

p [illegible].

$$y=c$$

$$y-c=0.$$

6) Solution of $p^3-p^2-6p=0$

$$p(p-3)(p+2)=0$$

$$(y+c)(y-3x+c)(y+2x+c)=0.$$

7) Solution of $xyp^2+(x+y)p+1=0$ is

$$(xp+1)(yp+1)=0$$

$$(y+\log x+c)(y^2+x+c)=0.$$

8) Solution of $xyp^2+(y^2-x^2)p-xy=0$ is.

$$xyp^2+y^2p-x^2p-xy=0$$

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$yp(xp+y) - x(xp+y) = 0$

$(yp-x)(xp+y) = 0$

$(y^2-x^2+c)(xy-c) = 0.$

9) Solution of $xyp^2 + p(3x^2-2y^2) - 6xy = 0$ is.

$xyp^2 + 3x^2p - 2y^2p - 6xy = 0$

$xp(yp+3x) - 2y(yp+3x) = 0$

$(xp-2y)(yp+3x) = 0$

$(y/x^2 - c)(y^2+3x^2-c) = 0.$

10) Solution of $xp^2 - (2x+3y)p + 6y = 0$ is

$xp^2 - 2xp - 3yp + 6y = 0$

$xp(p-2) - 3y(p-2) = 0$

$(xp-3y)(p-2) = 0$

$(y/x^3 - c)(y-2x-c) = 0.$

11) Solution of $p^2y + p(x-y) - x = 0$ is

$p^2y + px - py - x = 0$

$p(py+x) - (py+x) = 0$

$(p-1)(py+x) = 0$

$(y-x-c)(y^2+x^2-c) = 0.$

12) Solution of $p^2 - (\cos x + \sec x)p + 1 = 0$ is.

$p(p-\cos x) - \sec x(p-\cos x) = 0$

$(p-\sec x)(p-\cos x) = 0$

$(y - \sec x \tan x - c)(y - \sin x - c) = 0.$

13) Solution of $xp^2 - (1+xy)p + y = 0$ is.

$xp^2 - p - xyp + y = 0$

$p(xp-1) - y(xp-1) = 0$

$(xp-1)(p-y) = 0$

$(y - \log x - c)(\log y - x - c) = 0.$

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14) Solution of $yp^2-(x-y^2)p-xy=0$ is.

$$yp^2-xp+y^2p-xy=0$$
$$p(yp-x)+y(yp-x)=0$$
$$(yp-x)(p+y)=0$$
$$(y^2-x^2-c)(\log y+x-c)=0.$$

15) Solution of $x^2p^2+3xyp+2y^2=0$ is.

$$(xp+y)(xp+2y)=0$$
$$(xy-c)(yx^2-c)=0.$$

16) Solution of $x^2p^2+5xyp+6y^2=0$ is.

$$(xp+2y)(px+3y)=0$$
$$(yx^2-c)(yx^3-c)=0.$$

17) Solution of $p^2-2py\cot x=y^2$

$$y=\sin x(\operatorname{cosec}x+\cot x)c_2$$

Clairaut's equation: $\boxed{y=px+f(p)}$

1. Solution of $y=px+a/p$ is $y=cx+a/c.$
2. Solution of $y=px+\sqrt{1+p^2}$ is $y=cx+\sqrt{1+c^2}$
3) Solution of $y=px+\sqrt{a^2+p^2}$ is $y=cx+\sqrt{a^2+c^2}$
4) Soln of $y=(x-a)p-p^2$ is $y=cx-ac-c^2$
5) Soln of $(y-px)(p-1)=p$ is $y=cx+c/(c-1)$
6) Soln of $p=\log(px-y)$ is $y=cx-e^c$
7) Solution of $p=\sin(y-px)$ is $y=cx+\sin^{-1}c$
8) Solution of $p=px+\frac{ap}{(1+p^2)^{1/2}}$ $(y-cx)^2(1+c^2)=a^2c^2$
9) Solution of $p=\tan(y-xp)$ is $y=cx+\tan^{-1}c$
10) Solution of $(p+1)(y-px)=p$ is $y=cx+\frac{c}{c+1}$

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Total differentiation.

1. Solution of $yz\,dx + xz\,dy + xy\,dz = 0$

$$xyz = c.$$

2. Solution of $y^2z^2 + z^2x^2\,dy + x^2y^2\,dz = 0$ is

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = c.$$

3. Solution of $(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$ is

$$xy + yz + zx = c.$$

4. Solution of $(z^2+2xy)\,dx + (x^2+2yz)\,dy + (y^2+2zx)\,dx$ is

$$xz^2 + yx^2 + zy^2 = c$$

5) Solution of $x\,dx + z\,dy + (y+2z)\,dz = 0$ is

$$\frac{x^2}{2} + yz + z^2 = c.$$

6) Solution of $(x+z)^2\,dy + y^2(dx+dy) = 0$ is

$$\frac{dy}{y^2} + \frac{d(x+y)}{(x+y)^2} = c$$

$$-\frac{1}{y} - \frac{1}{x+y} = c$$

$$-\frac{(x+y+z)}{y(x+c)} = \frac{1}{c}$$

$$c(x+y+z) + y(x+z) = c.$$

7) Solution of $(yz+2x)\,dx + (zx-2z)\,dy + (xy-2y)\,dz = 0$ is

$$d(xyz) + 2x\,dx - 2(d\,yz) = 0$$

$$xyz + x^2 - 2yz = c.$$

8) Solution of $(yz+xyz)\,dx + (zx+xyz)\,dy + (xy+xyz)\,dz = 0$ is

$$\frac{d(xyz)}{xyz} + (dx+dy+dz) = 0$$

$$\log(xyz) + x+y+z = c.$$

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9) Solution of $yz\log z\,dx - zx\log z\,dy + xy\,dz = 0$ is

$$\frac{dx}{x} - \frac{dy}{y} + \frac{dz}{z\log z} = 0$$

$$\log x - \log y + \log\log z = \log c$$

$$x\log z = cy.$$

10) Solution of $(2y-z)dx + 2(x-z)dy - (x+2y)dz = 0$ is

$$2d(xy) - d(xz) - 2d(yz) = 0$$

$$2xy - xz - 2yz = c$$

11) Solution of $(ye^x + e^z)dx + (ze^y + e^x)dy + (e^y - ye^x - ze^y)dz = 0$

$$d(ye^x) + d(ze^y) + d(xe^z) - (xe^z + ye^x + ze^y)dz = 0$$

$$\frac{d(xe^z + ye^x + ze^y)}{xe^z + ye^x + ze^y} - dz = 0$$

$$xe^z + ye^x + ze^y = ke^z$$

12) Solution of $dx + dy + (x+y)dz = 0$ is

$$\log(x+y) + z = c$$

13) Solution of $yz(x+y+z-x)dx + zx(x+y+z-y)dy + xy(x+y+z-z)dz = 0$

$$(x+y+z)\,d(xyz) - xyz\,d(x+y+z) = 0$$

$$\log xyz - \log(x+y+z) = \log c$$

$$xyz = c(x+y+z)$$

14) solution of $(y+z)dx + dy + dz = 0$ is

$$x + \log(y+z) = c$$

15) solution of $xdx + ydy + zdz = 0$ is

$$x^2 + y^2 + z^2 = c.$$

16) Solution of $dx + dy + (x+y+z+1)dz = 0$ is

$$d(x+y+z) + (x+y+z)dz = 0$$

$$\log(x+y+z) + z = c.$$

$$(x+y+z)e^z = c.$$

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17) $(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$

$d(xyz) +$ [illegible]

Formulation of a differential equation.

1. $y = mx + c$ where m & c are both constants.

$$\frac{d^2y}{dx^2} = 0$$

2) $y = mx + c$ where c is fixed.

$$y = x\left(\frac{dy}{dx}\right) + c$$

3) $y = mx + c$ where m is fixed.

$$\frac{dy}{dx} = m.$$

4) $y^2 = 4a(x-a)$

$$y^2 = 4a(x-a) \rightarrow (1)$$

$$2yy' = 4a$$

$\cancel{yy' = 2a}$ using this in (1)

$$y^2 = 2yy_1\left(x - \frac{yy_1'}{2}\right)$$

$$(yy')^2 - 2xyy' + y^2 = 0.$$

5) $x^2 + y^2 = a^2$

$$x\,dx + y\,dy = 0.$$

6) The differential equation of all st line in a plane is $\frac{d^2y}{dx^2} = 0$.

7) All circles of radius r whose centre lie on the x-axis is

$$y^2\left(1 + \left(\frac{dy}{dx}\right)^2\right) = r^2$$

8) all circles touching the axis of y at the origin and centres ~~lie~~ on the x axis is

$$2xy\frac{dy}{dx} + x^2 - y^2 = 0.$$

9) All circles of radius a is

$$\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$$

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10) All parabolas with x axis as the axis and (a,0) as focus $(y^2=4ax)$ is

$$y\frac{dy}{dx}=2a.$$

11) All conics whose axis coincide with the axes of coordinates.

$$xy\left(\frac{d^2y}{dx^2}\right)+x\left(\frac{dy}{dx}\right)^2=y\frac{dy}{dx}.$$

12) All St line passing through the origin is $y=\left(\frac{dy}{dx}\right)x.$

13) All circles of radius a with centred at origin is

$$x+y\frac{dy}{dx}=0.$$

14) $xy=c^2$

$$x\frac{dy}{dx}+y=0$$

15) diff equ. of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$x(yy''+y'^2)=yy'$$

18) Diff. equation of $y^2=4ax$ is

$$y=2x\frac{dy}{dx}$$

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Find the Integral factor

1. $\frac{dy}{dx} + y\cot x = \sin x$.

$$I.F = e^{\int p\,dx} = e^{\int \cot x} = e^{\log \sin x} = \sin x.$$

2) $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{(1-x^2)}$

$$I.F = e^{\int p\,dx} = e^{\int \frac{2x}{1-x^2}dx} = e^{-\log(1-x^2)} = \frac{1}{1-x^2}$$

3) $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$

$$I.F = e^{\int \frac{-1}{x+1}dx} = e^{-\log(x+1)} = \frac{1}{x+1}$$

4) $\frac{dy}{dx} + 2y\tan x = \sin x$

$$I.F = e^{\int 2\tan x\,dx} = e^{2\log\sec x} = \sec^2 x.$$

5) $\frac{dy}{dx} + y = x$

$$I.F = e^{\int dx} = e^x.$$

6) $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^2}$

$$I.F = e^{\int p\,dx} = e^{2\int \frac{2x}{x^2+1}dx} = e^{2\log(x^2+1)} = (x^2+1)^2$$

7) $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$.

$$I.F = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$$

8) $\frac{dy}{dx} + \frac{y}{x} = \sin^2(x^2)$

$$I.F = e^{\int 1/x\,dx} = e^{\log x} = x.$$

9) $\frac{dy}{dx} + xy = x$.

$$I.F = e^{\int x\,dx} = e^{x^2/2}$$

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10) $dx + x\,dy = e^{-y}\sec^2 y\,dy$

$$\frac{dx}{dy} + x = e^{-y}\sec^2 y$$

$$I.F = e^{\int dy} = e^{y}.$$

11) $\frac{1}{x}\frac{dy}{dx} + \frac{y}{x}\tan x = \cos x.$

$$I.F = e^{\int p\,dx} = e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$$

12) $\cos^2 x \frac{dy}{dx} + y = \tan x.$

$$I.F = e^{\int p\,dx} = e^{\int \sec^2 x\,dx} = e^{\tan x}.$$

13) $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x.$

$$I.F = e^{\int p\,dx} = e^{\int \cot x} = e^{\log \sin x} = \sin x.$$

14) $\frac{dy}{dx} - \sin x = y\cot x.$

$$I.F = e^{\int p\,dx} = e^{-\int \cot x\,dx} = e^{-\log \sin x} = \operatorname{cosec} x.$$

15) $(1+x^3)\frac{dy}{dx} + 3x^2 y = \sin^2 x.$

$$I.F = e^{\int p\,dx} = e^{\int \frac{3x^2}{1+x^3}dx} = e^{\log(1+x^3)} = 1+x^3$$

16) $\frac{dy}{dx} + \frac{3x^2 y}{5+x^3} = \frac{\cos^2 x}{5+x^3}$

$$I.F = e^{\int p\,dx} = e^{\int \frac{3x^2}{5+x^3}dx} = e^{\log(5+x^3)} = 5+x^3$$

17) $\frac{dy}{dx} + y\sin x = y^2 \sin 2x.$

$$I.F = e^{\int p\,dx} = e^{\int \sin x} = e^{\cos x}$$

18) $x\frac{dy}{dx} + y = y^2 \log x.$

$$I.F = e^{\int p\,dx} = e^{\int 1/x\,dx} = e^{\log x} = x.$$

19) $\frac{dy}{dx} - y\tan x = \frac{\sin x \cos^2 x}{y^2}$

$$I.F = e^{\int p\,dx} = e^{-\int \tan x} = e^{-\log \sec x}$$

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20) $(1-x^2)\frac{dy}{dx}+2xy = x\sqrt{1-x^2}$

$$I.F = e^{\int p\,dx} = e^{\int \frac{2x}{1-x^2}dx} = e^{-\log(1-x^2)} = \frac{1}{1-x^2}$$

21. $(1-x^2)\frac{dy}{dx}+xy = y^3\sin x.$

$$I.F = e^{\int \frac{x}{1-x^2}dx} = e^{-\frac{1}{2}\log(1-x^2)} = \frac{1}{\sqrt{1-x^2}}$$

22) $\frac{dy}{dx}+\frac{y}{x}\log x = e^x x^{-\frac{1}{2}\log x}.$

$$I.F = e^{\int \frac{\log x}{x}dx} = e^{\int t\,dt}$$

$\log x = t$

$\frac{1}{x}dx = dt$

$$= e^t$$
$$= e^{\log x}$$
$$= x.$$

23) $\frac{dy}{dx}+\frac{2y}{x} = e^{4x}$

$$I.F = e^{\int p\,dx} = e^{\int \frac{2}{x}dx} = e^{\log x^2} = x^2$$

23) If $\cos x$ is an integral factor of $\frac{dy}{dx}+Py=Q.$

then $P=$

$$e^{\int p\,dx} = \cos x.$$
$$\int p\,dx = \log\cos x$$
$$P = \frac{1}{\cos x}\cdot -\sin x = -\tan x.$$

24) $\frac{dy}{dx}+\frac{1}{x\log x}y = \frac{2}{x^2}$

$$I.F = e^{\int p\,dx} = e^{\int \frac{1}{x\log x}dx} = e^{\int \frac{1}{t}dt} = e^{\log t} = e^{\log\log x}$$
$$= \log x.$$

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Homogeneous Equations

1) On putting $y = vx$, the homogeneous equation $x^2 dy + y(x+y)\,dx = 0$ becomes.

Soln

$x^2(v\,dx + x\,dv) + y(x+y)\,dx = 0$

$y = vx$

$\frac{dy}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}$

$dy = v\,dx + x\,dv$

Find the C.F. of following D.E.

1. $(D^2 + 5D + 6) = 0$ — $Ae^{-2x} + Be^{-3x}$.
2. $(D^2 + 6D + 9)y = 0$ — $(Ax + B)e^{-3x}$
3. $(D^2 + D + 1)y = 0$ — $y = e^{-x/2}\left[A\cos \frac{\sqrt{3}}{2} + B\sin \frac{\sqrt{3}}{2}x\right]$
4. $(D^2 + 6D + 8)y = e^{-2x}$ — $Ae^{-4x} + Be^{-2x}$.
5. $(D^2 - 6D + 9)y = e^{3x}$ — $(Ax + B)e^{3x}$.
6. $(2D^2 + 5D + 2)y = e^{-\frac{1}{2}x}$ — $Ae^{-\frac{1}{2}x} + Be^{-2x}$.
7. $(D^2 + 4D + 13)y = \cos 3x$ — $e^{-2x}[A\cos 3x + B\sin 3x)$
8. $(D^2 - 3D + 2)y = x$ — $Ae^{2x} + Be^{2x}$.
9. $(D^2 - 4D + 1)y = x^2$ — $Ae^{(2+\sqrt{3}x)} + Be^{(2-\sqrt{3})x}$.
10) $(D^2 + 7D + 12)y = e^{2x}$ — $y = Ae^{-4x} + Be^{-3x}$
11) $(D^2 - 4D + 13)y = e^{-3x}$ — $e^{2x}(A\cos 3x + B\sin 3x)$
12) $(D^2 + 14D + 49)y = e^{-7x} + 4$ — $(Ax + B)e^{-7x}$
13) $(D^2 - 13D + 12)y = e^{-2x}$ — $Ae^{12x} + Be^{x}$
14) $D^2 + 1 = 0$ — $A\cos x + B\sin x$.
15) $(D^2 - 3D + 2)y = 2e^{3x}$
16) $(D^2 + 3D - 4)y = x^2$ — $Ae^{x} + Be^{-4x}$
17) $(D^2 - 2D - 3)y = \sin x \cos x$, — $y = Ae^{3x} + Be^{-x}$.

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Variable Seperation

1) Solution of $\frac{dy}{dx} = 1+x+y+xy$ is

Soln

$$\frac{dy}{dx} = (1+x)(1+y)$$

$$\log(1+y) = x + \frac{x^2}{2} + c$$

2) Sol. of $3e^x \tan y\, dx + (1+e^x)\sec^2 y\, dy = 0$ is

$$\frac{3e^x}{1+e^x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

$$\log(1+e^x)^3 + \log \tan y = \log c$$

$$(1+e^x)^3 \tan y = c.$$

3) Soln of $\frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{1/2} = 0$

$$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$$

~~$\sin^{-1}(x\sqrt{1-}$~~

$$\sin^{-1} y + \sin^{-1} x = c$$

$$\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = c$$

$$x\sqrt{1-y^2} + y\sqrt{1-x^2} = c$$

4) Soln of $e^x\sqrt{1-y^2}\, dx + \frac{y}{x} dy = 0$ is

Soln

$$xe^x dx = \frac{-y}{\sqrt{1-y^2}} dy$$

$$xe^x - e^x = \sqrt{1-y^2} + c.$$

5) Soln of $x\,dy = (y + 4x^5 e^{x^4}) dx$ is

$$\frac{x\,dy - y\,dx}{x^2} = 4x^3 e^{x^4} dx$$

$$\frac{y}{x} = e^{x^4} + c.$$

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6) Soln of $(x^2-y)dx+(y^2-x)dy=0$ is

$$\frac{x^3}{3}+\frac{y^3}{3}=xy+c$$

7) Soln. of $\sec 2x\,dy-\sin 5x \sec^2 y\,dx=0$ is

$$y+\frac{\sin 2y}{2}+\frac{\cos 7x}{7}+\frac{\cos 3x}{3}=c$$

8) Soln. of $\cos^2 x\,dy+y\,e^{\tan x}dx=0$ is

$$\frac{dy}{y}+\sec^2 x\,e^{\tan x}dx=0.$$

$$\log y+e^{\tan x}=c.$$

9) Soln. of $yx^2dx+e^{-x}dy=0$ is

$$e^{x}x^2dx+\frac{dy}{y}=0$$

$$x^2e^{x}-2xe^{x}+2e^{x}+\log y=c.$$

10) Soln. of $ydx+xdy=e^{-xy}dx$ is

$$d(xy)=e^{-xy}dx$$

$$e^{xy}d(xy)=dx$$

$$e^{xy}=x+c$$

11) Soln. of $\frac{dy}{dx}=e^{2x+3y}$ is

$$e^{-3y}dy=e^{2x}dx$$

$$\frac{e^{-3x}}{-3}=\frac{e^{2x}}{2}+c$$

$$3e^{2x}+2e^{3x}=c$$

12) Soln of $(1+x)y\,dx+(1+y)x\,dy=0$ is

$$\left(\frac{1}{x}+1\right)dx+\left(\frac{1}{y}+1\right)dy=0$$

$$x+y+\log xy=c.$$

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13) Soln. of $(x+y)(dx-dy) = dx+dy$ is

$$dx - dy = \frac{dx+dy}{x+y}$$

$$c + (x-y) = \log(x+y)$$

$$x+y = ce^{x-y}.$$

14) Soln. of $(1+x^2)\frac{dy}{dx} = 1+y^2$

$$\tan^{-1}y - \tan^{-1}x = c$$

$$\tan^{-1}\left(\frac{y-x}{1+xy}\right) = c$$

$$y - x = c(1+xy)$$

15) Soln. of $e^{y}(1+x^2)\frac{dy}{dx} - 2x(1+e^y) = 0$

$$\frac{e^y}{1+e^y}dy - \frac{2x}{1+x^2}dx = 0$$

$$\log(1+e^y) - \log(1+x^2) = \log c$$

$$1+e^y = c(1+x^2)$$

16) Soln. of $\sec^2x \tan y\, dx + \sec^2y \tan x\, dy = 0$ is

$$\log\tan x + \cancel{\tan}\ \cancel{\log}\ \tan y = \log c$$

$$\tan x \tan y = c.$$

17) Soln. of $(1+x^3)dy - x^2y\,dx = 0$ is

$$\frac{dy}{y} - \frac{1}{3}\frac{3x^2\,dx}{1+x^3} = 0$$

$$\log y - \log(1+x^3)^{1/3} = \log c$$

$$y = c(1+x^3)^{1/3}$$

$$y^3 = c(1+x^3)$$

18) Soln. of $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

$$e^y dy = (e^x + x^2)dx$$

$$e^y = e^x + \frac{x^3}{3} + c$$

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19) Soln. of $a(xdy + ydx) = xy\,dy$ is

$$a(d(xy)) = xy\,dy$$

$$a \log xy = y + c$$

20) Soln. of $(x^2 - yx^2)\frac{dy}{dx} + y^2 + x^2y = 0.$

$$\frac{(1-y)}{y^2}dy + \frac{(1+x)}{x^2}dx = 0$$

$$\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \left(\frac{1}{x^2} + \frac{1}{x}\right)dx = 0.$$

$$-\log y - \frac{1}{y} - \frac{1}{x} + \log x = \log c$$

$$\log(x/y) - \frac{1}{x} - \frac{1}{y} = c.$$

Homogenous equations.

1. When $y = vx$, the equation $\frac{dy}{dx} = \frac{y^3 + 3x^2y}{x^3 + 3xy^2}$ reduces to .

Results

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Soln

$$v + x\frac{dv}{dx} = \frac{v^3 + 3v}{1 + 3v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 + 3v - v - 3v^3}{1 + 3v^2}$$

$$x\frac{dv}{dx} = \frac{2v - 2v^3}{1 + 3v^2}$$

$$\frac{dx}{x} = \frac{1 + 3v^2}{2v - 2v^3}dv.$$

2) When $y = vx$, the equation $x^2dy + y(x+y)dx = 0$ reduces to .

$$x^2\frac{dy}{dx} + y(x+y) = 0$$

$$x^2\left(v + x\frac{dv}{dx}\right) + vx(x + vx) = 0$$

$$v + x\frac{dv}{dx} + v(1+v) = 0$$

$$x\frac{dv}{dx} + 2v + v^2 = 0$$

$$x\,dv + (2v + v^2)\,dv = 0.$$

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3) When $y = vx$, then the equ. $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ reduces to.

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v + \tan^{-1} v$$

$$x\frac{dv}{dx} = \tan v$$

$$x\,dv = \tan v\,dx.$$

$$\frac{dv}{\tan v} = \frac{dx}{x}.$$

4) When $y = vx$, the equ. $\frac{dy}{dx} = \frac{y}{x}\left(\frac{x-2y}{x-3y}\right)$ reduces to

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v\left(\frac{1-2v}{1-3v}\right)$$

$$x\frac{dv}{dx} = \frac{v - 2v^2 - v + 3v^2}{1-3v}$$

$$x\frac{dv}{dx} = \frac{v^2}{1-3v}$$

$$\frac{(1-3v)\,dv}{v^2} = \frac{dx}{x}$$

5) When $y = vx$, then the equ. ~~redus~~ $(x^2+y^2)dy = xy\,dx$ reduces to.

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(x^2 + x^2v^2)\left(v + x\frac{dv}{dx}\right) = x(vx)$$

$$(1+v^2)\left(v + x\frac{dv}{dx}\right) = v$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v - v - v^3}{1+v^2}$$

$$\left(\frac{1+v^2}{-v^3}\right)dv = \frac{dx}{x}.$$

6) When $y = vx$ $(x^2+y^2)dx + 3xy\,dy = 0$ reduces to

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx},$

$$(x^2 + v^2x^2) + 3xvx\left(v + x\frac{dv}{dx}\right) = 0$$

$$(1+v^2) + 3v\left(v + x\frac{dv}{dx}\right) = 0$$

$$1 + 4v^2 + 3vx\frac{dv}{dx} = 0$$

$$3vx\frac{dv}{dx} = -(1+4v^2)$$

$$\left(\frac{3v}{1+4v^2}\right)dv = -\frac{dx}{x}$$

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7) When $y = vx$, the equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ reduces to

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} + v = v^2$$

$$x\frac{dv}{dx} = v^2 - 2v$$

$$\left(\frac{1}{v^2-2v}\right)dv = \frac{dx}{x}.$$

8) When $y = vx$, the equ. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$ reduces to

$$x\left(v + x\frac{dv}{dx}\right) - vx = x\sqrt{1+v^2}$$

$$v + x\frac{dv}{dx} - v = \frac{1}{\sqrt{1+v^2}}\sqrt{1+v^2}$$

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}.$$

9) When $y = vx$, the equ. $(2\sqrt{xy} - x)dy + y\,dx = 0$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$2(x\sqrt{v} - x)\left(v + x\frac{dv}{dx}\right) + vx\,dx = 0$$

$$2(\sqrt{v} - 1)\left(v + x\frac{dv}{dx}\right) + v = 0$$

$$v + x\frac{dv}{dx} = \frac{-v}{2\sqrt{v}-1}$$

$$x\frac{dv}{dx} = \frac{-v - 2v\sqrt{v} + v}{\sqrt{2v} - 1}$$

$$x\frac{dv}{dx} = \frac{-2v\sqrt{v}}{2\sqrt{v} - 1}$$

$$\left(\frac{2\sqrt{v}-1}{2v\sqrt{v}}\right)dv = -\frac{dx}{x},$$

10) When $y = vx$ the equation $x^2\frac{dy}{dx} = y^2 + 2xy$ reduces to

$$v + x\frac{dv}{dx} = v^2 + 2v$$

$$x\frac{dv}{dx} = v^2 + v$$

$$\frac{1}{v(v+1)}\,dv = \frac{dx}{x}.$$

11) When $y = vx$, then the equ. $(x+y)\,dx + (y-x)dy = 0$ reduces to.

$$(x + vx) + (vx - x)\left(v + x\frac{dv}{dx}\right) = 0$$

$$(1+v) + (v-1)\left(v + x\frac{dv}{dx}\right) = 0$$

$$1 \cancel{+v+} \quad v + x\frac{dv}{dx} = \frac{-(1+v)}{v-1}$$

$$x\frac{dv}{dx} = -\frac{(1+v^2)}{v-1}$$

$$\left(\frac{v-1}{1+v^2}\right)dv = -\frac{dx}{x}.$$

12) When $y=vx$, the equ. $x\frac{dy}{dx}+\frac{y^2}{x}=y$ is reduces to

$$\left(v+x\frac{dv}{dx}\right)+v^2=v$$

$$x\frac{dv}{dx}+v^2=0$$

$$x\,dv+v^2dx=0.$$

13) When $y=vx$, the equ. $(x^2-y^2)dx=2xy\,dy$ is reduces to.

$$(x^2-v^2x^2) = 2x^2v\left(v+x\frac{dv}{dx}\right)$$

$$v+x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1-v^2-2v^2}{2v}$$

$$\frac{2v}{1-3v^2}dv = \frac{dx}{x}$$

14) When $y=vx$, the equ. $x\frac{dy}{dx}=y+x\cos^2 y/x$ is reduces to.

$$v+x\frac{dv}{dx} = v+\cos^2 v$$

$$x\,dv-\cos^2 v\,dx=0.$$

15) When $y=vx$, the equ. $x\frac{dy}{dx}=y(\log y-\log x)$ reduces to.

$$v+x\frac{dv}{dx} = v[\log x+\log v-\log x)$$

$$v+x\frac{dv}{dx} = v\log v$$

$$x\,dv = v(\log v-1)\,dx.$$

$$\frac{dv}{v(\log v-1)} = \frac{dx}{x}.$$

16) When $y=vx$, the equ. $(x^2+2y^2)dx-xy\,dy$ is reduces to.

$$(1+2v^2)-v\left(v+x\frac{dv}{dx}\right)=0$$

$$v+x\frac{dv}{dx} = \frac{1+2v}{v}$$

$$x\frac{dv}{dx} = \frac{1+2v-v^2}{v}$$

$$\left(\frac{v}{1+2v+v^2}\right)dv = \frac{dx}{x}$$

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Find the P.I of followings.

1. $(D^2+D)y = e^x$. $\quad \frac{e^x}{2}$

2. $(D^2+7D-8)y = e^{2x}$ $\quad \frac{e^{2x}}{10}$.

3) $(D^2+5D+6)y = e^x$ $\quad \frac{e^x}{12}$

4) $(D^2-5D+6)y = e^{4x}$. $\quad \frac{1}{42}e^{4x}$

5) $(D^2-6D+13)y = 5e^{2x}$. $\quad e^{2x}$

6) $(D^2+4D+6)y = 5e^{-2x}$. $\quad \frac{5}{2}e^{-2x}$

7) $(D^2+2D+1)y = 2e^{3x}$ $\quad \frac{1}{8}e^{3x}$

8) $(D^2-D-2)y = e^{2x}+e^x$ $\quad \frac{1}{3}xe^{2x} - \frac{1}{2}e^x$.

9) $(D^2-6D+9)y = e^{3x}$ $\quad \frac{x^2}{2}e^{3x}$

10) $(D^2+7D+12)y = e^{2x}$ $\quad \frac{1}{30}e^{2x}$

11) $(D^2-4D+13)y = e^{-3x}$ $\quad \frac{1}{34}e^{-3x}$

12) $(D^2+6D+8)y = e^{-2x}$ $\quad \frac{1}{2}xe^{-2x}$.

13) $(D^2-6D+9)y = e^{3x}$ $\quad \frac{1}{2}x^2e^{3x}$

14) $(2D^2+5D+2)y = e^{-\frac{1}{2}x}$ $\quad \frac{x}{3}e^{-\frac{1}{2}x}$.

15) $(D^2-13D+12)y = e^{-2x}+5e^x$. $\quad \frac{1}{42}e^{-2x} - \frac{5}{11}xe^x$

16) $(D^2+14D+49)y = e^{-7x}+4$ $\quad \frac{1}{2}x^2e^{-7x} + \frac{4}{49}$

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Laplace transformation.

Results:

1) $L\left(\frac{d^2y}{dx^2}\right) = s^2 L(y) - s\,y(0) - y'(0)$

2) $L\left(\frac{dy}{dx}\right) = sL(y) - y(0)$

problems:

1. If $y'' + 4y' + 3y = e^{-t}$ given $y(0) = 1$, $y'(0) = 0$ then $L(y) = ?$

Soln

$$L(y'' + 4y' + 3y) = L(e^{-t})$$

$$[s^2 L(y) - s\,y(0) - y'(0)] + 4(sL(y) - y(0)] + 3L(y) = \frac{1}{s+1}$$

$$L(y)(s^2 + 4s + 3) = \frac{1}{s+1}$$

$$L(y) = \frac{1}{(s+1)(s^2+4s+3)}$$

2) If $y'' - 3y' + 2y = \sin t$ given $y(0) = 0$, $y'(0) = 1$ then $L(y) = ?$

$$(s^2 L(y) - s(0) - 1) - 3[sL(y) - 0] + 2L(y) = L(\sin t)$$

$$L(y)[s^2 - 3s + 2) - 1 = \frac{1}{s^2+1}$$

$$L(y)(s^2 - 3s + 2) = \frac{s^2+2}{s^2+1)}$$

$$L(y) = \frac{(s^2+3)}{(s^2+1)(s^2-3s+2)}$$

3) If $(y'' - y' - 2y) = 0$ given $y(0) = -2$, $y'(0) = 5$ then $L(y) = ?$

$$[s^2 L(y) - s(-2) - 5] - [sL(y) + 2] - 2L(y) = 0$$

$$L(y)[s^2 - s - 2] + 2s - 5 - 2 = 0$$

$$L(y)(s^2 - s - 2) = \cancel{2+5}\ 7 \mp 2s$$

$$L(y) = \frac{-2s+7}{s^2-s-2}.$$

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4) If $(y''-4y'+5y) = 4e^{3t}$ given $y(0)=2$, $y'(0)=7$ then $L(y)=?$

$$[s^2 L(y) - 2s - 7] - 4[sL(y) - 2] + 5L(y) = 4L(e^{3t})$$

$$L(y)[s^2-4s+5] - 2s - 7 + 8 = \frac{4}{s-3}$$

$$L(y)(s^2-4s+5) - 2s + 1 = \frac{4}{s-3}$$

$$L(y)(s^2-4s+5) = \frac{4}{s-3} + 2s - 1$$

$$L(y)(s^2-4s+5) = \frac{4 + 2s^2 - 6s - s + 3}{s-3}$$

$$L(y) = \frac{4 + (2s-1)(s-3)}{(s-3)(s^2-4s+5)}$$

5) If $(y''-4y'-5)y = te^{t}$ given $y(0)=0$, $y'(0)=0$ then $L(y)=?$

$$[s^2 L(y) - s\,y(0) - y'(0)] - 4[sL(y) - y(0)] - 5L(y) = L(te^t)$$

$$L(y)[s^2-4s-5] = \frac{1}{(s-1)^2}$$

$$L(y) = \frac{1}{(s-1)^2(s^2-4s-5)}$$

6) If $y''+4y'+5y = 4e^{3t}$ given $y(0)=2$, $y'(0)=7$ then $L(y)=?$

$$[s^2 L(y) - 2s - 7] + 4[sL(y) - 2] + 5L(y) = \frac{4}{s-3}$$

$$(s^2+4s+5)L(y) - 2s - 7 - 8 = \frac{4}{s-3}$$

$$(s^2+4s+5)L(y) = \frac{4}{s-3} + (15+2s)$$

$$L(y) = \frac{4 + (s-3)(15+2s)}{(s-3)(s^2+4s+5)}$$

7) If $(y''-3y'+2y) = \sin t$ given $y(0)=0$, $y'(0)=1$ then $L(y)=?$

$$(s^2 L(y) - 0s - 1) - 3[sL(y) - 0] + 2L(y) = \frac{1}{s^2+1}$$

$$(s^2-3s+2)L(y) - 1 = \frac{1}{s^2+1}$$

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8) If $(y''+4y'+13)y = 2e^{-t}$ given $y(0)=0$, $y'(0)=-1$ then $L(y)=?$

$$(s^2L(y) - 0s + 1)\cancel{L(y)} + 4[sL(y) - 0] + 13L(y) = \frac{2}{s+1}$$

$$L(y)(s^2+4s+13) + 1 = \frac{2}{s+1}$$

$$L(y) = \frac{1-s}{(s+1)(s^2+4s+13)}$$

9) If $(y''+5y'+6)y = e^{-t}$ given $y(0)=0$ $y'(0)=1$ then ~~find~~ $L(y)=?$

$$[s^2L(y) - 0s - 1] + 5[sL(y) - 0] + 6L(y) = \frac{1}{s+1}$$

$$L(y)(s^2+5s+6) - 1 = \frac{1}{s+1}$$

$$L(y) = \frac{s+2}{(s+1)(s^2+5s+6)}$$

10) If $(y''+6y'+5)y = e^{-2t}$ given that $y(0)=0$, $y'(0)=1$ then $L(y)=?$

$$[s^2L(y) - 0 - 1] + 6[sL(y) - 0] + 5L(y) = \frac{1}{s+2}$$

$$(s^2+6s+5)L(y) - 1 = \frac{1}{s+2}$$

$$L(y) = \frac{s+3}{(s+2)(s^2+6s+5)}$$

11) If $(y''+6y'+9)y = 6$ given $y(0)=-1$, $y'(0)=1$ then $L(y)=?$

$$[s^2L(y) + s - 1]\cancel{L(y)} + 6[sL(y) + 1] + 9L(y) = \frac{6}{s}$$

$$(s^2+6s+9)L(y) + s + 5 = \frac{6}{s}$$

$$(s^2+6s+9)L(y) = \frac{6}{s} - (s+5)$$

$$L(y) = \frac{6 - s(s^2+5)}{s(s^2+6s+9)}$$

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12) $= t^2+2$ given $y(0)$

Then $L(y) = ?$

$$(s^2L(y) - s + 1) + L(y) = L(t^2+2)$$

$$(s^2+1)L(y) - s + 1 = \frac{2}{s^3} + \frac{2}{s}$$

$$(s^2+1)L(y) - s + 1 = \frac{2s + 2s^2}{s^3}$$

$$L(y) = \frac{(2+2s^3) - s^3(s-1)}{s^3(s^2+1)}.$$

13) If $(y'' - 4y' + 5y) = 4e^{3t}$ given $y(0) = 2,\ y'(0) = 7$

Then $L(y) =$

$$(s^2L(y) - 2s - 7) - 4[sL(y) - 2] + 5L(y) = \frac{4}{s-3}$$

$$(s^2 - 4s + 5)L(y) - 2s + 1 = \frac{4}{s-3}$$

$$L(y) = \frac{4 + (2s-1)(s-3)}{s^2-4s+5}.$$

14) If $(y'' + 5y' - 6y) = 21e^t$ given $y(0) = 0,\ y'(0) = 9$

Then $L(y) = ?$

$$(s^2L(y) - 0s - 9)L(y) + 5[sL(y) - 0] - 6L(y) = \frac{21}{s-1}$$

$$(s^2 + 5s - 6)L(y) - 9 = \frac{21}{s+1}$$

$$L(y) = \frac{21 + 9(s+1)}{s^2+5s-6}.$$

15) If $(y'' - 6y' + 5)y = e^t$ given $y(0) = 0,\ y'(0) = -1$

Then $L(y) = ?$

$$(s^2L(y) - 0 + 1) - 6[sL(y) - 0] + 5L(y) = \frac{1}{s-1}$$

$$(s^2 - 6s + 5)L(y) + 1 = \frac{1}{s-1}$$

$$L(y) = \frac{2-s}{(s-1)(s^2-6s+5)}$$

16) If $(y'' + y) = 0$ given $y(0) = 0,\ y'(0) = 0$

Then $L(y) = ?$

$$(s^2L(y) - 0s - 0) + L(y) = 0$$

$$(s^2+1)L(y) = 0$$

$$L(y) = 0.$$

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17) If $(y'' - 2y' + 2y) = 0$, $y(0) = 1$, $y'(0) = 1$ then $L(y) = ?$

$$(s^2 L(y) - s - 1) - 2(sL(y) - 1) + 2L(y) = 0$$

$$(s^2 - 2s + 2) L(y) - s - 2 + 2 = 0$$

$$L(y) = \frac{s}{s^2 - 2s + 2}$$

18) If $(y'' - y' - 2)y = 20 \sin 2t$, $y(0) = -1$, $y'(0) = 2$ then $L(y) = ?$

$$(s^2 L(y) + s - 2) - [sL(y) + 1] - 2L(y) = \frac{20(2)}{s^2+4}$$

$$(s^2 - s - 2) L(y) + s - 3 = \frac{40}{s^2+4}$$

$$L(y) = \frac{40 + (3-s)(s^2+4)}{(s^2 - s - 2)}$$

19) If $(y'' + 2y' + 1)y = t$ given $y(0) = -3$, $y'(0) = 2$, then $L(y) = ?$

$$(s^2 L(y) + 3s - 2) + 2(sL(y) + 3] + L(y) = L(t)$$

$$(s^2 + 2s + 1) L(y) + 3s + 4 = \frac{1}{s^2}$$

$$L(y) = \frac{1 - (3s+4)s^2}{(s^2 + 2s + 1)}$$

20) If $(y'' + 2y' - 3)y = \sin t$ $y(0) = y'(0) = 0$ then $L(y) = ?$

$$(s^2 L(y) - 0s - 0) + 2[sL(y) - 0] - 3L(y) = \frac{1}{s^2+1}$$

$$(s^2 + 2s - 3) L(y) = \frac{1}{(s^2+1)}$$

$$L(y) = \frac{1}{(s^2+1)(s^2+2s-3)}$$

21) If $(y'' - 10y' + 24)y = 24t$ given $y(0) = y'(0) = 0$

$$(s^2 - 10s + 24) L(y) = \frac{24}{s^2}$$

$$L(y) = \frac{24}{s^2(s^2 - 10s + 24)}$$

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Extra problems

1. solvable for p:

1. Solution of $p^2 + p(y-x) - xy = 0$ is

$$p^2 + py - xp - xy = 0$$
$$p(p+y) - x(p+y) = 0$$
$$p - x = 0 \quad | \quad p + y = 0$$
$$\frac{dy}{dx} - x = 0 \quad | \quad \frac{dy}{dx} + y = 0$$
$$(2y - x^2 + c) \quad (\log y + x + c) = 0$$

2. Solution of $p^2 - (\log x + e^x)p + e^x \log x = 0$ is.

$$p^2 - p\log x - e^x p + e^x \log x = 0$$
$$p(p - \log x) - e^x(p - \log x) = 0$$
$$(p - e^x)(p - \log x) = 0$$
$$\frac{dy}{dx} - e^x = 0 \quad | \quad \frac{dy}{dx} = \log x.$$
$$(y - e^x - c)\ (xy - c) = 0.$$

~~Charp~~ Clairauts equation:

1. C.F of $(D^2 - 3D + 2)y = e^{2x}$ is $Ae^x + Be^{2x}$

Equation	C.F.
1. $(D^2 - 2D + 1)y = \sin 2x$	$(Ax+B)e^{2x}$,
2. $(D^2 - 4D + 5)y = 3e^{5x}$	$e^{2x}(A\cos x + B\sin x)$
3. $(6D^2 - 5D + 1)y = \sin x$	$Ae^{\frac{1}{2}x} + Be^{\frac{1}{3}x}$
4. $(4D^2 + 8D + 1)y = 4\sin 3x$	$e^{x}(Ae^{\sqrt{3}/2\, x} + Be^{-\sqrt{3}/2\, x})$
5) $(D^2 + 3D + 2)y = 0$	$Ae^{-2x} + Be^{-x}$
6) $(D^2 + 8D + 6)y = 0$	$e^{-4x}(Ax+B)$
7) $(D^2 - 2D + 4)y = 0$	$e^x(A\cos\sqrt{3}x + B\sin\sqrt{3}x)$
8) $(D^2 - 2D - 2)y = 0$	$e^x(Ae^{\sqrt{3}x} + Be^{-\sqrt{3}x})$
9) $(3D^2 + 6D + 2)y = 0$	
10) $(D^2 - 1)y = 0$	$Ae^x + Be^{-x}$

Equation. P.I

1. $(D^2+5D+7)y = 5$ $\frac{5}{7}$

2. $(D^2+D-2)y = e^{2x}$ $\frac{e^{2x}}{2}$ $4+2-2 = 2$

3. $(D^2-2D-3)y = 3e^{-x}$ $\frac{3xe^{-x}}{-4}$ $1+2-3$ $2D-2 = -4$

4) $(D^2-10D+25)y = 6e^{5x}$ $3x^2e^{5x}$

5) $(6D^2-D-2)y = 3e^{2/3x}$

$$P.I. = \frac{1}{6D^2-D-2} 3e^{2/3x}$$ $6\frac{4}{9}-\frac{2}{3}-2$

$$= \frac{1}{12D-1} 3xe^{2/3x}$$

$$= \frac{3}{7} xe^{2/3x}$$

6) $(D^2+2D+1)y = \sin 2x$.

$$P.I = \frac{1}{-4+2D+1} \sin 2x$$

$$= \frac{2D+3}{4D^2-9} \sin 2x$$

$$= \frac{4\cos 2x + 3\sin 2x}{-25}.$$

7) $(3D^2+4D+1)y = \cos 3x$

$$P.I = \frac{1}{4D-26} \cos 3x$$

$$= \frac{1}{4} \frac{D+7}{D^2-49} \cos 3x$$

$$= \frac{1}{4}\left(\frac{-\sin 3x \cdot 3 + 2\cos 3x}{-13}\right)$$

$$= \frac{1}{2} \frac{1}{2D-13} \cos 3x$$

$$= \frac{1}{2}\left(\frac{2D+13}{4D^2-169} \cos 3x\right)$$

$$= \frac{1}{2}\left(\frac{-6\sin 3x + 13\cos 3x}{-36-169}\right)$$

$$= \frac{6\sin 3x - 13\cos 3x}{410}.$$

169
36
205

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8) $(D^2+9)y = \sin 3x$.

$$P.I = \frac{1}{D^2+9}\sin 3x$$

$$= \frac{x}{2}\int \sin 3x\, dx$$

$$= -\frac{x\cos 3x}{6}$$

9) $(D^2+4)y = \cos 2x$.

$$P.I = \frac{x}{2}\int \cos 2x\, dx$$

$$= \frac{x\sin 2x}{4}$$

10) $(D^2+2)y = \sin^2 x$.

$$P.I = \frac{1}{D^2+2}\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{D^2+2}(1-\cos 2x)\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{\cos 2x}{-2}\right)$$

$$= \frac{1}{4}(1+\cos 2x)$$

11) $(D^2+2D+3)y = \cancel{1}+x^2$

$$P.I = \frac{1}{3}\frac{1}{\left(1+\frac{2D+D^2}{3}\right)}(1+x^2)$$

$$= \frac{1}{3}\left(1-\left(\frac{2D+D^2}{3}\right)+\left(\frac{2D+D^2}{3}\right)^2\right)(1+x^2)$$

$$= \frac{1}{3}\left(1-\frac{2D}{3}-\frac{D^2}{3}+\frac{4D^2}{9}\right)(1+x^2)$$

$$= \frac{1}{3}\left(1-\frac{2D}{3}+\frac{D^2}{9}\right)(1+x^2)$$

$$= \frac{1}{3}\left(1+x^2-\frac{2}{3}(0+2x)+\frac{1}{9}(2)\right)$$

$$= \frac{1}{3}\left(1+x^2-\frac{4}{3}x+\frac{2}{9}\right)$$

$$= \cancel{\frac{1}{3}\left(\frac{25}{3}+4x+x^2\right)}$$

$$= \frac{1}{3}\left(\frac{10-12x+9x^2}{9}\right)$$

$$= \frac{1}{27}(10-12x+9x^2)$$

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12) $(D^2+4)y = x^3$

$D\ x^3$
$D^2\ 3x^2$
$6x$
6

$$P.I = \frac{1}{4+D^2} x^3$$

$$= \frac{1}{4} \frac{1}{1+\frac{D^2}{4}} x^3$$

$$= \frac{1}{4} \left[1 - \left(\frac{D^2}{4}\right) + \left(\frac{D^2}{4}\right)^2 - \left(\frac{D^2}{4}\right)^3 \right] x^3$$

$$= \frac{1}{4} \left(x^3 - \frac{6x}{4} \right)$$

$$= \frac{1}{4} \left(x^3 - \frac{3x}{2} \right)$$

$$= \frac{1}{8} (2x^3 - 3x)$$

13) $(D^2 - 3D + 2)y = x^2 + 2x + 3$

Total Diff equations

1. solution of $(x+z)^2\,dy + y^2\,(dx+dz) = 0$

$$\frac{dy}{y^2} + \frac{d(x+z)}{(x+z)^2} = 0$$

$$-\frac{1}{y} - \frac{1}{x+z} = c$$

$$\frac{1}{y} + \frac{1}{x+z} = c$$

$$\cancel{x+2y}$$

$$x+y+z = c\,(y(x+z))$$

2) Solution of $yz\,dx - z^2dy - xy\,dz = 0$

$$yz\,dx - xy\,dz - z^2dy = 0$$

$$y(z\,dx - x\,dz) - z^2dy = 0 \qquad d\left(\frac{x}{y}\right) = y\,dx.$$

$$\frac{z\,dx - x\,dz}{z^2} - \frac{dy}{y} = 0$$

$$d\left(\frac{x}{z}\right) + \frac{1}{y^2} = c$$

$$\frac{x}{z} + \cancel{\frac{1}{y^2}} = c \qquad \frac{x}{z} - \log y = c$$

$$xy^2 + z = czy^2 \qquad \boxed{x - z\log y = zc.}$$

3) Solution of $(y\,dx + x\,dy)(a-z) - 2z\,dz = 0$.

Soln

$$d(xy) - \frac{2z}{a-z}\,dz = 0$$

$$d(xy) + 2\left(\frac{a-z-a}{a-z}\right)dz = 0$$

$$xy + 2\left(1 - \frac{a}{a-z}\right)dz = 0$$

$$xy + 2(z + a\log(a-z)) = c.$$

4) Solution of $z\,dx + xz\cos y\,dy + x(1-z)(\log x + \sin y)\,dz = 0$

$$(z\,dx \cancel{+ x\,dz}) + \cancel{xz\cos y\,dy - xz\sin y\,dz}$$

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