UG - TRB MATHEMATICS

Vector Analysis

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Algebra

2005 - 2006

TRB - 2

Vector calculus:

1) Unit Normal vector:

The unit normal to the surface $\phi(x,y,z)$ is $\vec{n} = \frac{\nabla\phi}{|\nabla\phi|}$

Problems:

1. Find the unit normal vector to the sphere $x^2+y^2+z^2=9$ at $(0,0,3)$

Soln

$$\phi = x^2+y^2+z^2-9$$

$$\nabla\phi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$$

$$\nabla\phi \text{ at } (0,0,3) = 6\vec{k}$$

The unit normal vector $\vec{n} = \frac{6\vec{k}}{\sqrt{6^2}} = \frac{6\vec{k}}{6} = \vec{k}$.

2) Find the unit normal to the surface

1) $x^2+3y^2+2z^2=6$ at $(2,0,1)$

2) $x^3-xyz+z^2=1$ at $(1,1,1)$

3) $x^2+y^2+z^2=3$ at $(1,1,1)$

4) $x^2+y^2+z^2=5$ at $(0,1,2)$

5) $xy^3z^2=4$ at $(-1,-1,2)$

6) $xy+yz+zx=7$ at $(1,1,3)$

7) $x^2+y^2-z=1$ at $(1,1,1)$

8) $x^2+2y^2+z^2=7$ at $(1,-1,2)$

9) $x^2+y^2-z^2=1$ at $(1,1,1)$

10) $x^2y+2xz^2=8$ at $(1,0,2)$

11) $z^2=4(x^2+y^2)$ at $(1,0,2)$

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11) $z^2=4(x^2+y^2)$ at $(1,0,2)$

1) $x^2+3y^2+2z^2=6$ at $(2,0,1)$

$$\phi = x^2+3y^2+2z^2-6$$

$$\nabla\phi = 2x\vec{i}+6y\vec{j}+4z\vec{k}$$

At $(2,0,1)$ $\nabla\phi = 4\vec{i}+0\vec{j}+4\vec{k}$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{4(\vec{i}+\vec{k})}{4\sqrt{1+1}} = \frac{\vec{i}+\vec{j}}{\sqrt{2}}$$

2) $x^3-xyz+z^2=1$ at $(1,1,1)$

$$\nabla\phi = (3x^2-yz)\vec{i}+(-xz)\vec{j}+(2z-xy)\vec{k}$$

$\nabla\phi$ at $(1,1,1) = 2\vec{i}-\vec{j}+\vec{k}$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2\vec{i}-\vec{j}+\vec{k}}{\sqrt{4+1+1}} = \frac{2\vec{i}-\vec{j}+\vec{k}}{\sqrt{6}}.$$

3) $x^2+y^2+z^2=3$ at $(1,1,1)$

$$\nabla\phi = 2x\vec{i}+2y\vec{j}+2z\vec{k}$$

At $(1,1,1)$ $\nabla\phi = 2\vec{i}+2\vec{j}+2\vec{k}$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

$$= \frac{2(\vec{i}+\vec{j}+\vec{k})}{2\sqrt{3}} = \frac{\vec{i}+\vec{j}+\vec{k}}{\sqrt{3}}$$

4) $x^2+y^2+z^2=5$ at $(0,1,2)$

$$\nabla\phi = 2x\vec{i}+2y\vec{j}+2z\vec{k}$$

At $(0,1,2)$ $\nabla\phi = 2\vec{j}+4\vec{k}$

$$= 2(\vec{j}+2\vec{k})$$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2(\vec{j}+2\vec{k})}{2\sqrt{1+4}} = \frac{\vec{j}+2\vec{k}}{\sqrt{5}}$$

5) $xy^3z^2=4$ at $(-1,-1,2)$

$$\nabla\phi = (y^3z^2)\vec{i}+(3xy^2z^2)\vec{j}+2xy^3z\vec{k}$$

At $(-1,-1,2)$ $\nabla\phi = -4\vec{i}-12\vec{j}+4\vec{k}$

$$= 4(-\vec{i}-3\vec{j}+\vec{k})$$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{4(-\vec{i}-3\vec{j}+\vec{k})}{4\sqrt{1+1+9}}$$

$$= -\vec{i}-3\vec{j}+\vec{k}$$

6) $xy+yz+zx=7$ at $(1,1,3)$

$$\nabla\phi = (y+z)\vec{i} + (x+z)\vec{j} + (y+z)\vec{k}$$

At $(1,1,3)$ $\quad \nabla\phi = 4\vec{i}+4\vec{j}+2\vec{k} = 2(2\vec{i}+2\vec{j}+\vec{k})$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2(2\vec{i}+2\vec{j}+\vec{k})}{2\sqrt{4+4+1}} = \frac{2\vec{i}+2\vec{j}+\vec{k}}{3}$$

7) $x^2+y^2-z=1$ at $(1,1,1)$

$$\nabla\phi = 2x\vec{i}+2y\vec{j}-\vec{k}$$

At $(1,1,1)$ $\quad \nabla\phi = 2\vec{i}+2\vec{j}-\vec{k}$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2\vec{i}+2\vec{j}-\vec{k}}{3}$$

8) $x^2+2y^2+z^2=7$ at $(1,-1,2)$

$$\nabla\phi = 2x\vec{i}+4y\vec{j}+2z\vec{k}$$

At $(1,-1,2)$ $\quad \nabla\phi = 2\vec{i}-4\vec{j}+4\vec{k} = 2(\vec{i}-2\vec{j}+2\vec{k})$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\vec{i}-2\vec{j}+2\vec{k}}{3}$$

9) $x^2+y^2-z^2=1$ at $(1,1,1)$.

$$\nabla\phi = 2x\vec{i}+2y\vec{j}-2z\vec{k}$$

At $(1,1,1)$ $\quad \nabla\phi = 2\vec{i}+2\vec{j}-2\vec{k}$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} \qquad \frac{2\vec{i}+2\vec{j}-2\vec{k}}{\sqrt{4+4+4}}$$

$$= \frac{\vec{i}+\vec{j}-\vec{k}}{\sqrt{3}}$$

10) $x^2y+2xz^2=8$ at $(1,0,2)$

$$\nabla\phi = (2xy+2z^2)\vec{i} + x^2\vec{j} + 4xz\vec{k}$$

At $(1,0,2)$ $\quad \nabla\phi = 8\vec{i}+\vec{j}+8\vec{k}$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{8\vec{i}+\vec{j}+8\vec{k}}{\sqrt{129}}$$

11) $z^2=4(x^2+y^2)$ at $(1,0,2)$

$$\nabla\phi = 8x\vec{i}+8y\vec{j}-2z\vec{k}$$

At $(1,0,2)$ $\quad \nabla\phi = 8\vec{i}-4\vec{j}$

$$= 4(2\vec{i}-\vec{j})$$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2\vec{i}-\vec{j}}{\sqrt{5}}$$

Directional derivatives

The directional derivative of ϕ in the direction $\vec{a}$ is given by

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|}$$

(or) $\nabla\phi \cdot \hat{n}$ where $\hat{n} = \dfrac{\vec{a}}{|\vec{a}|}$

Problems:

Find the directional derivative of

1) $\phi = x^2+2xy$ at $(1,-1,3)$ in the direction $\vec{i}+2\vec{j}+2\vec{k}$
2) $\phi = xy+yz+zx$ at $(1,1,1)$ " $\vec{i}+\vec{j}$.
3) $\phi = 3x^2+2y-3z$ at $(1,1,1)$ " $2\vec{i}+2\vec{j}-\vec{k}$
4) $\phi = xy+yz+zx$ at $(3,1,2)$ " $2\vec{i}+3\vec{j}+6\vec{k}$.
5) $\phi = xy^2+yz^3$ at $(2,-1,1)$ " $\vec{i}+2\vec{j}+2\vec{k}$
6) $\phi = 2x^2+3y^2+z^2$ at $(2,1,3)$ " $\vec{i}-2\vec{k}$
7) $\phi = xyz$ at $(3,2,-1)$ " $3\vec{i}+2\vec{j}-2\vec{k}$
8) $\phi = xy+y^2-z^2$ at $(1,2,1)$ " $3\vec{i}-2\vec{j}+\vec{k}$
9) $\phi = x^2yz^2+4xz^2$ at $(1,-2,1)$ " $2\vec{i}-\vec{j}-2\vec{k}$
10) $\phi = x^2yz+4xz^2$ at $(1,-2,-1)$ " $2\vec{i}-\vec{j}-2\vec{k}$
11) $\phi = xy+yz+zx$ at $(1,1,3)$ " $\vec{i}+2\vec{j}+2\vec{k}$
12) $\phi = 4xz^2+2xyz$ at $(1,2,3)$ " $2\vec{i}+\vec{j}-\vec{k}$
13) $\phi = x^2y^2z$ at $(2,1,4)$ " $\vec{i}+2\vec{j}+2\vec{k}$
14) $\phi = 2xy+5yz+zx$ at $(1,2,3)$ " $3\vec{i}-5\vec{j}+4\vec{k}$
15) $\phi = x^2+y^2+z^2$ at $(1,2,3)$ " $3\vec{i}+2\vec{j}+5\vec{k}$
16) $\phi = 3x^2-x^2yz$ at $(1,2,3)$ " $\vec{i}-2\vec{j}+2\vec{k}$
17) $\phi = xyz-xy^2z^3$ at $(1,2,-1)$ " $\vec{i}-\vec{j}-3\vec{k}$
18) $\phi = x^2y^2z$ at $(2,1,4)$ " $\vec{i}+2\vec{j}+2\vec{k}$
19) $\phi = x^2+2xy$ at $(1,-1,3)$ " $\vec{i}+2\vec{j}+2\vec{k}$.

1. $\phi = x^2+2xy$ at $(1,-1,3)$ d.. $\vec{i}+2\vec{j}+2\vec{k}$

$\nabla\phi = (2x+2y)\vec{i} + 2x\vec{j}$ $\qquad \vec{a} = \vec{i}+2\vec{j}+2\vec{k}$

$|\vec{a}| = 3$

At $(1,-1,3)$ $\nabla\phi = 2\vec{j}$

Directional derivative $= \nabla\phi \cdot \vec{n}$

$$= (2\vec{j}) \cdot \frac{(\vec{i}+2\vec{j}+2\vec{k})}{3}$$

$$= \frac{4}{3}.$$

2) $\phi = xy \neq yz + zx$ at $(1,1,1)$ in $\vec{i}+\vec{j}$.

$\nabla\phi = (y+z)\vec{i} + (x+z)\vec{j} + (x+y)\vec{k}$

$\nabla\phi = 2\vec{i}+2y\vec{j}+2z\vec{k}$

$\vec{a} = \vec{i}+0\vec{j}+\vec{k}$ $\qquad |\vec{a}| = \sqrt{2}$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{2+0+2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

3) $\phi = 3x^2+2y-3z$ at $(1,1,1)$ " $2\vec{i}+2\vec{j}-\vec{k}$.

$\nabla\phi = 6x\vec{i}+2\vec{j}-3\vec{k}$

At $(1,1,1)$ $\nabla\phi = 6\vec{i}+2\vec{j}-3\vec{k}$

$\vec{a} = 2\vec{i}+2\vec{j}-\vec{k}$ $\qquad |\vec{a}| = 3$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{12+4+3}{3} = 19/3$$

4) $\phi = xy+yz+zx$ at $(3,1,2)$ " $2\vec{i}+3\vec{j}+6\vec{k}$.

$\nabla\phi = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$

at $(3,1,2)$ $\nabla\phi = 3\vec{i}+5\vec{j}+4\vec{k}$

$\vec{a} = 2\vec{i}+3\vec{j}+6\vec{k}$ $\qquad |\vec{a}| = \sqrt{4+9+36} = 7$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{6+15+24}{7} = \frac{45}{7} =$$

5) $\phi = xy^2+yz^3$ at $(2,-1,1)$; $\vec{i}+2\vec{j}+2\vec{k}$.

$\nabla\phi = (y^2\vec{i} + (2xy+z^3)\vec{j} + 3yz^2\vec{k}$

At $(2,-1,1)$ $\nabla\phi = \vec{i}-3\vec{j}-3\vec{k}$

$\vec{a} = \vec{i}+2\vec{j}+2\vec{k}$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{1-6-6}{3} = \frac{-11}{3}$$

6) $\phi = 2x^2+3y^2+z^2$ at $(2,1,3)$ in the

$$\nabla\phi = 4x\vec{i}+6y\vec{j}+2z\vec{k}$$

At $(2,1,3)$ $\nabla\phi = 8\vec{i}+6\vec{j}+6\vec{k}$

$\vec{a} = \vec{i}+0\vec{j}-2\vec{k}$ $\quad |\vec{a}| = \sqrt{5}$

$$\nabla\phi\cdot\frac{\vec{a}}{|\vec{a}|} = \frac{8-12}{\sqrt{5}} = \frac{-4}{\sqrt{5}}$$

7) $\phi = xyz$ at $(3,2,-1)$ in the $3\vec{i}+2\vec{j}-2\vec{k}$.

$$\nabla\phi = yz\vec{i}+zx\vec{j}+xy\vec{k}$$

At $(3,2,-1)$ $\nabla\phi = -2\vec{i}-3\vec{j}+6\vec{k}$

$\vec{a} = 3\vec{i}+2\vec{j}-2\vec{k}$ $\quad |\vec{a}| = \sqrt{9+4+4} = \sqrt{17}$

$$\nabla\phi\cdot\frac{\vec{a}}{|\vec{a}|} = \frac{-6-6-12}{\sqrt{17}} = \frac{-24}{\sqrt{17}}$$

8) $\phi = xy+y^2-z^2$ at $(1,2,1)$ " $3\vec{i}-2\vec{j}+\vec{k}$

$$\nabla\phi = y\vec{i}+(x+2y)\vec{j}-2z\vec{k}$$

At $(1,2,1)$ $\nabla\phi = 2\vec{i}+5\vec{j}-2\vec{k}$

$\vec{a} = 3\vec{i}-2\vec{j}+\vec{k}$ $\quad |\vec{a}| = \sqrt{9+4+1} = \sqrt{14}$

$$\nabla\phi\cdot\frac{\vec{a}}{|\vec{a}|} = \frac{6-10-2}{\sqrt{14}} = \frac{-6}{\sqrt{14}}.$$

10) $\phi = x^2yz^2+4xz^2$ at $(1,-2,1)$ in the $2\vec{i}-\vec{j}-2\vec{k}$.

$$\nabla\phi = (2xyz^2+4z^2)\vec{i}+(x^2z^2)\vec{j}+2z(x^2y+4x)\vec{k}$$

At $(1,-2,1)$ $\nabla\phi = 0\vec{i}+\vec{j}+4\vec{k}$

$\vec{a} = 2\vec{i}-\vec{j}-2\vec{k}$

$$\nabla\phi\cdot\frac{\vec{a}}{|\vec{a}|} = \frac{0-1-8}{3} = -3.$$

11) $\phi = x^2yz+4xz^2$ at $(1,-2,-1)$ in $2\vec{i}-\vec{j}-2\vec{k}$.

$\cancel{\nabla\phi = 0\vec{i}+\vec{j}}+4\vec{k}$

$\cancel{\vec{a} = 2\vec{i}-\vec{j}}-2\vec{k}$

$$\nabla\phi = (2xyz+4z^2)\vec{i}+(x^2z)\vec{j}+(x^2y+8xz)\vec{k}$$

At $(1,-2,-1)$ $\nabla\phi = 8\vec{i}-\vec{j}-10\vec{k}$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{16+1+22}{3} = \frac{37}{3}$$

12) $\phi = xy + yz + zx$ at $(1,1,3)$ " $\vec{i} + 2\vec{j} + 2\vec{k}$.

$$\nabla\phi = 4\vec{i} + 4\vec{j} + 2\vec{k}$$

$$\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k} \qquad |\vec{a}| = 3$$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{4+8+4}{3} = \frac{16}{3}$$

13) $\phi = 4xz^2 + 2xyz$ at $(1,2,3)$ " $2\vec{i} + \vec{j} - \vec{k}$,

$$\nabla\phi = (4z^2 + 2yz)\vec{i} + 2xz\vec{j} + (8xz + 2xy)\vec{k}$$

At $(1,2,3)$ $\nabla\phi = 48\vec{i} + 4\vec{j} + 28\vec{k}$

$$\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{96+4-28}{\sqrt{6}} = \frac{72}{\sqrt{6}}$$

14) $\phi = x^2y^2z$ at $(2,1,4)$ " $\vec{i} + 2\vec{j} + 2\vec{k}$

$$\nabla\phi = 2xy^2z\,\vec{i} + 2x^2yz\,\vec{j} + x^2y^2\vec{k}$$

At $(2,1,4)$ $\nabla\phi = 16\vec{i} + 32\vec{j} + 4\vec{k}$

$$\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{16+64+8}{3} = \frac{88}{3}$$

15) $\phi = 2xy + 5yz + zx$ at $(1,2,3)$ " $3\vec{i} - 5\vec{j} + 4\vec{k}$.

$$\nabla\phi = (2y+z)\vec{i} + (2x+5z)\vec{j} + (5y+x)\vec{k}$$

At $(1,2,3)$ $\nabla\phi = 7\vec{i} + 17\vec{j} + 11\vec{k}$

$$\vec{a} = 3\vec{i} - 5\vec{j} + 4\vec{k} \qquad |\vec{a}| = \sqrt{9+25+16} = 5\sqrt{2}$$

$$\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{21-85+44}{5\sqrt{2}} = \frac{-20}{5\sqrt{2}}$$

16) $\phi = x^2 + y^2 + z^2$ at $(1,2,3)$ in $\frac{x}{3} = \frac{y}{2} = \frac{z}{5}$

$$\nabla\phi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$$

At $(1,2,3)$ $\nabla\phi = 2\vec{i} + 4\vec{j} + 6\vec{k} \qquad |\vec{a}| = \sqrt{9+4+25}$

$$\vec{a} = 3\vec{i} + 2\vec{j} + 5\vec{k}$$

Maximum directional derivatives.

Maximum directional derivative at P is

$|\nabla\phi|$

Problems

1. The maximum directional derivatives at of $\phi(x,y,z) = xyz$ at $(1,2,3)$ is.

$\phi = xyz$

$\nabla\phi = yz\vec{i} + zx\vec{j} + xy\vec{k}$

At $(1,2,3)$ $\nabla\phi = 6\vec{i} + 3\vec{j} + 2\vec{k}$

$|\nabla\phi| = \sqrt{36+9+4} = 7$

2) $\phi = x^3y^2z$ at $(1,4,1)$

$\nabla\phi = 3x^2y^2z\vec{i} + 2x^3yz\vec{j} + x^3y^2\vec{k}$

At $(1,4,1)$ $\nabla\phi = 48\vec{i} + 8\vec{j} + 16\vec{k}$

$= 8(6\vec{i} + \vec{j} + 2\vec{k})$

$|\nabla\phi = 8\sqrt{36+1+4} = 8\sqrt{41}$

3) $\phi = 2xy + z^2 - xy^2$ at $(1,1,0)$

$\nabla\phi = (2y - y^2)\vec{i} + (2x - 2xy)\vec{j} + 2z\vec{k}$

At $(1,1,0)$ $\nabla\phi = \vec{i} + 0\vec{j} + 0\vec{k}$

$|\nabla\phi = 1.$

3) $\phi = x^3y^2z$ at $(1,1,1)$

$\nabla\phi = 3x^2y^2z\vec{i} + 2x^3yz\vec{j} + x^3y^2\vec{k}$

At $(1,1,1)$ $\nabla\phi = 3\vec{i} + 2\vec{j} + \vec{k}$

$|\nabla\phi| = \sqrt{9+4+1} = \sqrt{14}$

5) $\phi = 2x^2 + 3y^2 + 5z^2$ at $(1,1,-4)$

$\nabla\phi = 4x\vec{i} + 6y\vec{j} + 10z\vec{k}$

At $(1,1,-4)$ $\nabla\phi = 4\vec{i} + 6\vec{j} - 40\vec{k}$

$|\nabla\phi| = \sqrt{16+36+1600} = \sqrt{1652}$

6) $\phi = x^3yz$ at $(1,4,1)$

$$\nabla\phi = 3x^2yz\,\vec{i} + x^3z\,\vec{j} + x^3y\,\vec{k}$$

At $(1,4,1)$ $\nabla\phi = 12\vec{i} + \vec{j} + 4\vec{k}$

$$|\nabla\phi| = \sqrt{144+1+16}$$

$$= \sqrt{161}$$

7) $\phi = xyz^2$ at $(1,0,3)$

$$\nabla\phi = yz^2\,\vec{i} + xz^2\,\vec{j} + 2xyz\,\vec{k}$$

at $(1,0,3)$ $\nabla\phi = 0\vec{i} + 9\vec{j} + 0\vec{k}$

$$|\nabla\phi = 9$$

8) $\phi = x^2+y^2+z^2$ at $(1,1,1)$

$$\nabla\phi = 2\vec{i} + 2\vec{j} + 2\vec{k}$$

$$|\nabla\phi| = 2\sqrt{3}$$

9) $\phi = 2xz^4 - x^2y$ at $(2,-2,-1)$

$$\nabla\phi = (2z^4 - 2xy)\vec{j} - x^2\vec{j} + 8xz^3\,\vec{k}$$

At $(2,-2,-1)$ $\nabla\phi = 10\vec{i} - 4\vec{j} - 16\vec{k}$

$$|\nabla\phi| = \sqrt{100+16+256} = \sqrt{372}$$

10) $\phi = xy+yz+zx$ at $(1,1,3)$

$$\nabla\phi = (y+z)\,\vec{i} + (z+x)\,\vec{j} + (x+y)\,\vec{k}$$

$$\nabla\phi = 4\vec{i} + 4\vec{j} + 2\vec{k}$$

$$|\nabla\phi = \sqrt{16+16+4} = 6.$$

Solenoidial vector:

A vector $\vec{F}$ is said to be solenoidal vector if $\nabla\cdot\vec{F}=0$ (ie) div $\vec{F}=0$.

Irrotational vector:

A vector $\vec{F}$ is said to be irrotational vector if $\nabla\times\vec{F}=0$ (ie) curl $\vec{F}=0$.

problems:

1. If $\vec{F}=(z+3y)\vec{i}+(x-2z)\vec{j}+(x+az)\vec{k}$ is Solenoidal. Find a.

$$\nabla\cdot\vec{F}=0$$
$$0+0+a=0$$
$$a=0.$$

2) If $\vec{F}=3x\vec{i}+(x+y)\vec{j}-az\vec{k}$ is solenoidal Find a

$$\nabla\cdot\vec{F}=0$$
$$3+1-a=0$$
$$a=4$$

3) If $\vec{F}=(ax+3y+4z)\vec{i}+(2x-3y+3z)\vec{j}+(3x+2y-z)\vec{k}$ Find a

$$\nabla\cdot\vec{F}=0$$
$$a+3-1=0$$
$$a=4.$$

4) If $\vec{F}=2xy^2z\,\vec{i}+ay^2z\,\vec{j}$ is solenoidal Find a,

$$\nabla\cdot\vec{F}=0$$
$$2y^2z+3ay^2z+2y^2z=0$$
$$4y^2z+3ay^2z=0$$
$$4+3a=0$$
$$a=-4/3$$

5) Find a if $\vec{F}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+az)\vec{k}$ is solenoidal.

$$\nabla\cdot\vec{F}=0$$
$$1+1+a=0$$

6) Find λ if $\vec{F} = \lambda y^4 z^2 \vec{i} + 4x^3 z^2 \vec{j} + 5x^2 y^2 \vec{k}$ is irrotational solenoidal.

λ - For any real.

7) Find λ if $\vec{F} = (2x-5y)\vec{i} + (x+\lambda y) + (3x-z)\vec{k}$ is solenodial.

$$\nabla \cdot \vec{F} = 0$$

$$2 + \lambda - 1 = 0$$

$$\boxed{\lambda = -1}$$

6) Find a so that $\vec{F} = (x+z)\vec{i} + (3x+ay)\vec{j} + (x-5z)\vec{k}$ is solenoidal.

$$\nabla \cdot \vec{F} = 0$$

$$1 + a + (-5) = 0$$

$$a = 4.$$

Irrotational

1. If $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational find a, b, c.

$$\nabla \times \vec{F} = 0$$

$$(c+1)\vec{i} + (a-4)\vec{j} + (b-2) = 0$$

$$c = -1, \quad a = 4, \quad b = 2.$$

2) If $\vec{F} = (x+az+y)\vec{i} + (bx+2y-z)\vec{j} + (x+cy+2z)$ is irrotational find a, b, c.

$$\nabla \times \vec{F} = 0$$

$$(c+1)\vec{i} + (1-a)\vec{j} + (b-1)\vec{k} = 0$$

$$\therefore a = 1-1$$

$$b = 1$$

$$c = -1.$$

4) If $\vec{F} = (axy - z^3)\vec{i} + (a-2)x^2\vec{j} + (1-a)xz^2\vec{k}$ is irrotational find a.

$$\nabla \times \vec{F} = 0$$

$$(az^2 - 4z^2)\vec{j} + (ax - 4x)\vec{k} = 0$$

$$a - 4 = 0$$

$$a = 4.$$

5) If $\vec{F} = (x+2y+az)\vec{i} + (bx-3$

is irrotational find a, b, c.

$$\nabla \times \vec{F} = 0$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix} = 0$$

$$\vec{i}(c+1) - \vec{j}(4-a) + \vec{k}(b-2) = 0$$

$$c = -1, \quad a = 4, \quad b = 2.$$

6) Find a, b, c if $\vec{F} = (2x+3y+az)\vec{i} + (bx+2y+3z)\vec{j} + (2x+cy+3z)\vec{k}$

is irrotational.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+3y+az & bx+2y+3z & 2x+cy+3z \end{vmatrix} = 0$$

$$\vec{i}(c-3) - \vec{j}(2-a) + \vec{k}(b-3) = 0$$

$$c = 3, \quad a = 2, \quad b = 3.$$

7) Find a if $\vec{F} = (axy - z^2)\vec{i} + (x^2+2yz)\vec{j} + (y^2 - axz)\vec{k}$

is irrotational.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ axy - z^2 & x^2+2yz & y^2 - axz \end{vmatrix} = 0$$

$$(a-2)z\vec{j} + (2-a)x\vec{k} = 0$$

$$a = 2$$

8) Find a, b, c if $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ axy+bz^3 & 3x^2 - cz & 3xz^2 - y \end{vmatrix} = 0$$

$$\vec{i}(-1+c) - \vec{j}(3z^2 - 3z^2 b) + \vec{k}(6x - ax) = 0$$

$$c = 1, \quad b = 1 \quad a = 6.$$

9) Find a, b, c if $\vec{F} = (x+y+az)\vec{i} + (bx+2y-z)\vec{j} + (-x+cy+2z)\vec{k}$ is irrotational.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y+az & bx+2y-z & -x+cy+2z \end{vmatrix} = 0$$

$\vec{i}(c+1) - \vec{j}(-1-a) + \vec{k}(b-1) = 0$

$c = -1$

$b = 1$

$a = -1.$

10) Find a, b, c if $\vec{F} = (x+2ayz)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+2ayz & bx-3y-z & 4x+cy+2z \end{vmatrix} = 0$$

$\vec{i}(c+1) - \vec{j}(4-2ya) + \vec{k}(b-2za) = 0$

$c = -1$

$2ay = 4 \Rightarrow a = \frac{2}{y}$

$b - 2az = 0 \Rightarrow b = 2az = z/y.$

Irrotational & solenoidal vectors.

1. $(y^2-z^2+3yz-2x)\vec{i} + (3xz+2xy)\vec{j} + (3xy-2xz+2z)\vec{k}.$

Results

1) $\nabla r = \frac{\vec{r}}{r}$

2) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$

3) $\nabla r^n = n r^{n-2}\vec{r}$

4) $\nabla f(r) = f'(r)\frac{\vec{r}}{r} = f'(r)\nabla r$

5) $\nabla(\log r) = \frac{\vec{r}}{r^2}$

6) $\nabla f(r)\times\vec{r} = 0.$

8) $\text{grad}(\vec{r}\cdot\vec{a}) = \vec{a}$

9) $\text{grad}[\vec{r},\vec{a},\vec{b}] = \vec{a}\times\vec{b}$

10) $\text{div}\,\vec{r} = 3$; $\nabla\cdot\vec{r} = 3$

11) $\text{curl}\,\vec{r} = 0$ $\nabla\times\vec{r} = 0.$

12) $\nabla\times(\vec{a}\times\vec{r}) = 2\vec{a}$

13 If $\vec{v} = \vec{\omega}\times\vec{r}$ then $\vec{\omega} = \frac{1}{2}\,\text{curl}\,\vec{v}.$

14) $\nabla\times[f(r)\vec{r}] = 0$

15. $\nabla\times(r^n\vec{r}) = 0.$

16) $\text{div}\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$

17) $\nabla\cdot(r^n\vec{r}) = (n+3)r^n.$

18) $\text{div}(\vec{r}\times\vec{a}) = 0$

19) $\text{curl}(\vec{r}\times\vec{a}) = -2\vec{a}$

20) $\text{grad}(\vec{r}\cdot\vec{a}) = \vec{a}$

21. $\nabla(r^2\vec{c}) = 2\vec{c}\cdot r$

22) $\nabla[(\vec{a}\cdot\vec{r})\vec{r}] = 4(\vec{a}\cdot\vec{r})$

23) $\nabla\cdot[f(r)\vec{r}] = r f'(r) + 3f(r)$

24) If $\vec{A}$ & $\vec{B}$ are irrotational then $\vec{A}\times\vec{B}$ is solenoidal.

25) $(\vec{\nabla} \times \nabla) \times \vec{r} = -2\vec{v}$

26) curl curl curl $(\text{curl} \vec{F}) = \nabla^4 \vec{F}$.

27) $\vec{F} \cdot \text{curl} \vec{F} = 0$.

28) $\text{div}(\vec{A} \times \vec{r}) = \vec{A} \cdot \text{curl} \vec{r}$.

29 $\text{div}(r^n(\vec{a} \times \vec{r})) \doteq 0$.

30) $\nabla [(\vec{r} \times \vec{a}) \times \vec{b}] = -2(\vec{b} \cdot \vec{a})$

31.) $\nabla \times [(\vec{r} \times \vec{a}) \times \vec{b}] = \vec{b} \times \vec{a}$

32) $\nabla \cdot \left[\frac{f(r)}{r} \vec{r} \right] = \frac{1}{r^2} \frac{d}{dr}(r^2 f)$

33) $\nabla \cdot \left[r \nabla \left(\frac{1}{r^3} \right) \right] = \frac{3}{r^4}$

34) $\nabla \cdot [\nabla r^n] = n(n+1) r^{n-2}$

35) $\nabla \times (\nabla r^n) = 0$.

36) $\text{curl}(\vec{a} \times \vec{r}) r^n = (n+2) r^n \vec{a} - n r^{n-2} (\vec{r} \cdot \vec{a}) \vec{r}$.

37) $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}$

38. $\nabla^2 (e^r) = e^r + \frac{2}{r} e^r$

39) $\nabla \phi \times \nabla \psi$ is solenoidal.

40) $\text{curl}(\phi \, \text{grad} \, \phi) = 0$.

41) $\nabla \cdot (u \nabla v) = u \nabla^2 v + \nabla u \cdot \nabla v$.

42) $(\vec{v} \cdot \nabla) v = \frac{1}{2} \nabla v^2 - v + \text{curl} \, v$.

43) $\nabla^2 \left(\frac{1}{r} \right) = 0$

44) $\nabla^2 r^n = n(n+1) r^{n-2}$

45) $\text{curl}(\phi \nabla \phi) = 0$.