HALF YEARLY EXAMINATION - DECEMBER 2023

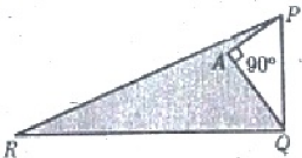Time: 3.00 Hrs.   X Standard – Mathematics   Maximum Marks - 100

PART– I (Marks - 14)

Note: i) Answer All the 14 questions

ii) Choose the most suitable answer from given the four alternatives and write the option code with the corresponding answers.   14 x 1 = 14

1. If $g = \{(1, 1),(2, 3),(3, 5),(4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are

   (A) $(-1, 2)$ (B) $(2, -1)$ (C) $(-1, -2)$ (D) $(1, 2)$

2. $n(A) = 4$, $n(b) = 2$ then the total number of relations that exist between set A and B is

   (A) 2^4 (B) 2^2 (C) 2^6 (D) 2^8

3. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is

   (A) 3   (B) 5   (C) 8   (D) 11

4. Common ratio of the Geometric Progression a^{m-n}, a^m, a^{m+n} is

   (A) a^m (B) a^{-m} (C) a^n (D) a^{-n}

5. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is

   (A) 0   (B) 1   (C) 0 or 1   (D) 2

6. If $A = \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$, then A^2 is

   (A) $\begin{pmatrix} 16 & 4 \\ 36 & 9 \end{pmatrix}$ (B) $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$ (C) $\begin{pmatrix} 8 & -4 \\ -12 & -6 \end{pmatrix}$ (D) $\begin{pmatrix} -4 & 2 \\ -6 & 3 \end{pmatrix}$

7. In the given figure, $PR = 26$ cm, $QR = 24$ cm, $\angle PAQ = 90°$, $PA = 6$ cm and $QA = 8$ cm. Find $\angle PQR$

   (A) 80°   (B) 85°   (C) 75°   (D) 90°

8. The point of intersection $3x - y = 4$ and $x + y = 8$ is

   (A) (5, 3)   (B) (2, 4)   (C) (3, 5)   (D) (4, 4)

9. The slope of the straight line $7y - 2x = 11$ is

(A) $-\dfrac{7}{2}$ (B) $\dfrac{7}{2}$ (C) $\dfrac{2}{7}$ (D) $-\dfrac{2}{7}$

10. The angle of elevation of a cloud from a point h meter above a lake is β. The angle of depression of its reflection in the lake is $45°$. The height of location of the cloud from the lake is

(A) $\dfrac{h(1+\tan\beta)}{1-\tan\beta}$ (B) $\dfrac{h(1-\tan\beta)}{1+\tan\beta}$ (C) $h\tan\left(45^0-\beta\right)$ (D) none of these

11. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(A) 60π cm^2 (B) 68π cm^2 (C) 120π cm^2 (D) 136π cm^2

12. A shuttle cock used for playing badminton has the shape of the combination of

(A) a cylinder and a sphere   (B) a hemisphere and a cone

(C) a sphere and a cone   (D) frustum of a cone and a hemisphere

13. The Variance of first 20 natural numbers is

(A) 32.25   (B) 44.25   (C) 33.25   (D) 30

14. If the sample space of the random experiment is S, then $P(S)$ is

(A) 0   (B) 0.5   (C) 1.2   (D) 1

PART – II (Marks - 20)

Note: Answer TEN questions. Question Number 28 is compulsory. $10 \times 2 = 20$

15. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

16. $f(x) = 2x - 1$ and $g(x) = \dfrac{x+1}{2}$ then, prove that $f \circ g = g \circ f = x$

17. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of 273 and 119

18. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

19. Determine the nature of the roots $15x^2 + 11x + 2 = 0$

20. If a matrix has 30 elements, what are the possible orders it can have?

21. What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

22. The line p passes through the points $(3, -2), (12, 4)$ and the line q passes through the points $(6, -2), (12, 2))$. Is p parallel to q ?

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23. Find the equation of a line whose intercepts on the x and y axes are 8 and 7 respectively.

24. Prove that $\sin^2 A \cos^2 B + \cos^2 A \sin^2 B + \cos^2 A \cos^2 B + \sin^2 A \sin^2 B = 1$

25. A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

26. What is the maximum volume of the cone if it was cut from the hemisphere of radius 21 cm.

27. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

28. Three dice are rolled once. What is the probability of getting the same number (triplet) in each face?

PART – III (Marks - 50)

Note: Answer TEN questions. Question Number 42 is compulsory.　　　10 x 5 = 50

29. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that $A \times (B - C) = (A \times B) - (A \times C)$

30. Let $A = \{2, 4, 6, 10, 12\}$ and $B = \{0, 1, 2, 4, 5, 9\}$ be two sets. Let $f : A \to B$ be a function given by $f(x) = \frac{x}{2} - 1$ Represent this function

   (i) as a set of ordered pairs　　　　　　(ii) by arrow diagram
   (iii) in a table form　　　　　　　　　　(iv) in a graphical form

31. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,...., 24 cm. How much area can be decorated with these colour papers?

32. Simplify:- $\dfrac{x}{x^2 - 3x + 2} + \dfrac{x}{x^2 - 5x + 6} - \dfrac{2x}{x^2 - 4x + 3}$

33. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

34. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ then, show that $A^2 - 5A + 7I_2 = 0$

35. Prove that Basic Proportionality theorem.

36. P and Q are the mid-points of the sides CA and CB respectively of a $\triangle ABC$, right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.

37. Find the area of the quadrilateral formed by the points $(-9, -2), (-8, -4), (2, 2)$ and $(1, -3)$.

38. Two ships are sailing in the sea on either side of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are $30°$ and $45°$ respectively. If the lighthouse is 200 m high, find the distance between the two ships. $(\sqrt{3} = 1.732)$

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39. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq cm is ₹ 2.

40. For the noon meal, food is kept ready in 3 cylindrical vessels of diameter $1.4\,m$ and height $56\,cm$. How many students can be served if the food is served twice with a hemispherical bowl of radius $14\,cm$?

41. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

42. If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

PART - IV (Marks- 16)

Note: Answer both questions. $2 \times 8 = 16$

43. **(A).** Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$).

OR

(B). Draw a triangle ABC of base $BC = 5.6$ cm, $\angle A = 40^0$ and the bisector of $\angle A$ meets BC at D such that $CD = 4$ cm.

44. **(A).** The following table shows the data about the number of pipes and the time taken to fill the same tank.

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used

(ii) Find the number of pipes when the time is 9 minutes.

OR

(B). Draw the graph of $y = x^2 + 4x + 3$ and use it to solve $x^2 + x + 1 = 0$.

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