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10th - MATHEMATICS

10 days study questions

Multiple choice questions

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is
(A) 1 (B) 2 (C) 3 (D) 6

2. $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is
(A) 8 (B) 20 (C) 12 (D) 16

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true.
(A) $(A \times C) \subset (B \times D)$ (B) $(B \times D) \subset (A \times C)$
(C) $(A \times B) \subset (A \times D)$ (D) $(D \times A) \subset (B \times A)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is
(A) 3 (B) 2 (C) 4 (D) 8

5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is
(A) $\{2,3,5,7\}$ (B) $\{2,3,5,7,11\}$
(C) $\{4,9,25,49,121\}$ (D) $\{1,4,9,25,49,121\}$

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
(A) $(2, -2)$ (B) $(5,1)$ (C) $(2,3)$ (D) $(3, -2)$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is
(A) m^n (B) n^m (C) $2^{mn} - 1$ (D) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively
(A) (8,6) (B) (8,8) (C) (6,8) (D) (6,6)

9. Let $A = \{1,2,3,4\}$ and $B = \{4,8,9,10\}$. A function $f : A \to B$ given by $f = \{(1,4), (2,8), (3,9), (4,10)\}$ is a
(A) Many-one function (B) Identity function
(C) One-to-one function (D) Into function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is
(A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ (C) $\frac{2}{9x^2}$ (D) $\frac{1}{6x^2}$

11. If $f : A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to
(A) 7 (B) 49 (C) 1 (D) 14

12. Let f and g be two functions given by
$f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$
$g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$ then the range of $f \circ g$ is
(A) $\{0,2,3,4,5\}$ (B) $\{-4,1,0,2,7\}$ (C) $\{1,2,3,4,5\}$ (D) $\{0,1,2\}$

13. Let $f(x) = \sqrt{1 + x^2}$ then
(A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$
(C) $f(xy) \leq f(x).f(y)$ (D) None of these

14. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
(A) $(-1,2)$ (B) $(2, -1)$ (C) $(-1, -2)$ (D) $(1,2)$

15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is
(A) linear (B) cubic (C) reciprocal (D) quadratic

Example 1.1 If $A = \{1,3,5\}$ and $B = \{2,3\}$ then (i) find $A \times B$ and $B \times A$. (ii) Is $A \times B = B \times A$? If not why? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$

Solution Given that $A = \{1,3,5\}$ and $B = \{2,3\}$

(i) $A \times B = \{1,3,5\} \times \{2,3\} = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$ ...(1)

$B \times A = \{2,3\} \times \{1,3,5\} = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$ ...(2)

(ii) From (1) and (2) we conclude that $A \times B \neq B \times A$ as $(1,2) \neq (2,1)$ and $(1,3) \neq (3,1)$, etc.

(iii) $n(A)=3$; $n(B) = 2$.

From (1) and (2) we observe that, $n(A \times B) = n(B \times A) = 6$;

we see that, $n(A) \times n(B) = 3 \times 2 = 6$ and $n(B) \times n(A) = 2 \times 3 = 6$

Hence, $n(A \times B) = n(B \times A) = n(A) \times n(B) = 6$.

Thus, $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

5. Given A=$\{1,2,3\}$, $B = \{2,3,5\}$, $C = \{3,4\}$ and $D = \{1,3,5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ (ii) $A \times (B - C) = (A \times B) - (A \times C)$

'Null relation'

Let us consider the following example. Suppose $A = \{-3,-2,-1\}$ and $B = \{1,2,3,4\}$. A relation from A to B is defined as $a - b = 8$ i.e., there is no pair(a,b) such that $a - b = 8$. Thus R contain no element and so $R = \phi$.

A relation which contains no element is called a "Null relation".

Example 1.5 The arrow diagram shows (Fig.1.10) a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.

Solution

(i) Set builder form of R $= \{(x,y) \mid y = x - 2,\ x \in P, y \in Q\}$

(ii) Roster form R $= \{(5,3),(6,4),(7,5)\}$

(iii) Domain of R $= \{5,6,7\}$ and range of R $= \{3,4,5\}$

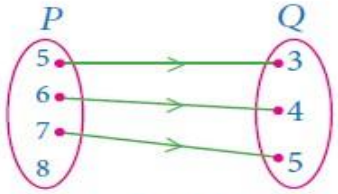

Fig. 1.10

3. A Relation R is given by the set $\{(x,y)/y=x+3,\ x\in\{0,1,2,3,4,5\}\}$. Determine its domain and range.

Example 1.9 Given $f(x)=2x-x^2$,

find (i) $f(1)$ (ii) $f(x+1)$ (iii) $f(x)+f(1)$

Solution (i) $x=1$, we get

$$f(1)=2(1)-(1)^2=2-1=1$$

(ii) $x=x+1$, we get

$$f(x+1)=2(x+1)-(x+1)^2=2x+2-(x^2+2x+1)=-x^2+1$$

(iii) $f(x)+f(1)=(2x-x^2)+1=-x^2+2x+1$

[Note that $f(x)+f(1)\neq f(x+1)$. In general $f(a+b)$ is not equal to $f(a)+f(b)$]

8. A function f is defined by $f(x)=3-2x$. Find x such that $f(x^2)=(f(x))^2$.

9. A plane is flying at a speed of 500 km per hour. Express the distance 'd' travelled by the plane as function of time t in hours.

Example 1.11 Let $A=\{1,2,3,4\}$ and $B=\{2,5,8,11,14\}$ be two sets. Let $f:A\to B$ be a function given by $f(x)=3x-1$. Represent this function

(i) by arrow diagram (ii) in a table form

(iii) as a set of ordered pairs (iv) in a graphical form

Example 1.16 Forensic scientists can determine the height (in cm) of a person based on the length of the thigh bone. They usually do so using the function $h(b) = 2 \cdot 47b + 54 \cdot 10$ where b is the length of the thigh bone.

(i) Verify the function h is one – one or not.

(ii) Also find the height of a person if the length of his thigh bone is 50 cm.

(iii) Find the length of the thigh bone if the height of a person is $147 \cdot 96$ cm.

Solution (i) To check if h is one – one, we assume that $h(b_1) = h(b_2)$.

Then we get, $2 \cdot 47b_1 + 54 \cdot 10 = 2 \cdot 47b_2 + 54 \cdot 10$

$$2 \cdot 47b_1 = 2 \cdot 47b_2$$

$$\Rightarrow b_1 = b_2$$

Thus, $h(b_1) = h(b_2) \Rightarrow b_1 = b_2$. So, the function h is one – one.

(ii) If the length of the thigh bone $b = 50$, then the height is $h(50) = (2 \cdot 47 \times 50) + 54 \cdot 10 = 177 \cdot 6$ cm.

(iii) If the height of a person is $147 \cdot 96$ cm, then $h(b) = 147 \cdot 96$ and so the length of the thigh bone is given by

$$2 \cdot 47b + 54 \cdot 10 = 147 \cdot 96$$

$$\Rightarrow 2 \cdot 47b = 147 \cdot 96 - 54 \cdot 10 = 93 \cdot 86$$

$$b = \frac{93 \cdot 86}{2 \cdot 47} = 38$$

Therefore, the length of the thigh bone is 38 cm.

5. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one-one function.

10. A function $f : [-5, 9] \to \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} 6x + 1; & -5 \le x < 2 \\ 5x^2 - 1; & 2 \le x < 6 \\ 3x - 4; & 6 \le x \le 9 \end{cases}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$ (iii) $2f(4) + f(8)$ (iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

11. The distance S an object travels under the influence of gravity in time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), a, b are constants. Verify wheather the function $S(t)$ is one-one or not.

Example 1.22 Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Solution $f \circ f(k) = f(f(k))$

$= 2(2k-1) - 1 = 4k - 3$.

$f \circ f(k) = 4k - 3$

But, $f \circ f(k) = 5$

$\therefore 4k - 3 = 5 \Rightarrow k = 2$.

Example 1.24 Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$.

Solution $gff(x) = g[f\{f(x)\}]$ (This means "g of f of f of x")

$= g[f(3x+1)] = g[3(3x+1)+1] = g(9x+4)$

$g(9x+4) = [(9x+4) + 3] = 9x + 7$

$fgg(x) = f[g\{g(x)\}]$ (This means "f of g of g of x")

$= f[g(x+3)] = f[(x+3)+3] = f(x+6)$

$f(x+6) = [3(x+6) + 1] = 3x + 19$

These two quantities being equal, we get $9x + 7 = 3x + 19$. Solving this equation we obtain $x = 2$.

6. Let $f(x) = x^2 - 1$. Find (i) $f \circ f$ (ii) $f \circ f \circ f$

7. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?

10. In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit $C(t) = 3t$ is linear.