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HIGHER SECONDARY SECOND YEAR

12^{th} - Maths / 10 days study questions

Choose the Correct or the most suitable answer from the given four alternatives:

1. If $|\text{adj}(\text{adj}\, A)| = |A|^9$, then the order of the square matrix A is
   (1) 3 (2) 4 (3) 2 (4) 5

2. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$
   (1) A (2) B (3) I_3 (4) B^T

3. If $A=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}$, $B = \text{adj}\, A$ and $C = 3A$, then $\frac{|\text{adj}\, B|}{|C|} =$
   (1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4) 1

4. If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$, then $A =$
   (1) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$ (3) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ (4) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$

5. If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I_2 - A =$
   (1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

6. If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|\text{adj}\,(AB)| =$
   (1) −40 (2) −80 (3) −60 (4) −20

7. If $P=\begin{bmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is
   (1) 15 (2) 12 (3) 14 (4) 11

8. If $A=\begin{bmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ then the value of a_{23} is
   (1) 0 (2) −2 (3) −3 (4) −1

9. If A, B and C are invertible matrices of some order, then which one of the following is not true?
   (1) $\text{adj}\, A = |A|A^{-1}$ (2) $\text{adj}(AB) = (\text{adj}\, A)(\text{adj}\, B)$
   (3) $\det A^{-1} = (\det A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

10. If $(AB)^{-1}=\begin{bmatrix}12 & -17\\ -19 & 27\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$, then $B^{-1} =$
    (1) $\begin{bmatrix}2 & -5\\ -3 & 8\end{bmatrix}$ (2) $\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}$ (3) $\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$ (4) $\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}$

11. If $A^T A^{-1}$ is symmetric, then $A^2 =$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

12. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

(1) $\frac{-4}{5}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

14. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$

(1) $\left(\cos^2\frac{\theta}{2}\right)A$ (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (3) $(\cos^2\theta)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

15. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj} A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(1) 17 (2) 14 (3) 19 (4) 21

17. If $\text{adj} A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $\text{adj} B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $\text{adj}(AB)$ is

(1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

19. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

(1) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4)) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $\operatorname{adj}(\lambda A)=\lambda^n \operatorname{adj}(A)$.

(iv) $A(\operatorname{adj}A)=(\operatorname{adj}A)A=|A|I$

(1) Only (i) (2) (ii) and (iii) (3) (iii) and (iv) (4) (i), (ii) and (iv)

21. If $\rho(A)=\rho([A\,|\,B])$, then the system $AX=B$ of linear equations is

(1) consistent and has a unique solution (2) consistent

(3) consistent and has infinitely many solution (4) inconsistent

22. If $0\le\theta\le\pi$ and the system of equations $x+(\sin\theta)y-(\cos\theta)z=0$, $(\cos\theta)x-y+z=0$, $(\sin\theta)x+y-z=0$ has a non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix}1 & 2 & 7 & 3\\ 0 & 1 & 4 & 6\\ 0 & 0 & \lambda-7 & \mu+5\end{bmatrix}$. The system has infinitely many solutions if

(1) $\lambda=7,\mu\ne-5$ (2) $\lambda=-7,\mu=5$ (3) $\lambda\ne7,\mu\ne-5$ (4) $\lambda=7,\mu=-5$

24. Let $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and $4B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & x\\ -1 & 1 & 3\end{bmatrix}$. If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

25. If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then $\operatorname{adj}(\operatorname{adj}A)$ is

(1) $\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ (2) $\begin{bmatrix}6 & -6 & 8\\ 4 & -6 & 8\\ 0 & -2 & 2\end{bmatrix}$ (3) $\begin{bmatrix}-3 & 3 & -4\\ -2 & 3 & -4\\ 0 & 1 & -1\end{bmatrix}$ (4) $\begin{bmatrix}3 & -3 & 4\\ 0 & -1 & 1\\ 2 & -3 & 4\end{bmatrix}$

Example 1.5

Find a matrix A if $\operatorname{adj}(A)=\begin{bmatrix}7 & 7 & -7\\ -1 & 11 & 7\\ 11 & 5 & 7\end{bmatrix}$.

Solution

First, we find $|\operatorname{adj}(A)|=\begin{vmatrix}7 & 7 & -7\\ -1 & 11 & 7\\ 11 & 5 & 7\end{vmatrix}=7(77-35)-7(-7-77)-7(-5-121)=1764>0.$

So, we get

$$A=\pm\frac{1}{\sqrt{|\operatorname{adj} A|}}\operatorname{adj}(\operatorname{adj} A)=\pm\frac{1}{\sqrt{1764}}\begin{bmatrix}+(77-35) & -(-7-77) & +(-5-121)\\ -(49+35) & +(49+77) & -(35-77)\\ +(49+77) & -(49-7) & +(77+7)\end{bmatrix}^T$$

$$=\pm\frac{1}{42}\begin{bmatrix}42 & 84 & -126\\ -84 & 126 & 42\\ 126 & -42 & 84\end{bmatrix}^T=\pm\begin{bmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ -3 & 1 & 2\end{bmatrix}.$$

Example1.7

If A is symmetric, prove that adj A is also symmetric.

Solution

Suppose A is symmetric. Then, $A^T = A$ and so, by theorem 1.9 (vi), we get

$\text{adj}(A^T) = (\text{adj } A)^T \Rightarrow \text{adj } A = (\text{adj } A)^T \Rightarrow \text{adj } A$ is symmetric.

Theorem 1.10

If A and B are any two non-singular square matrices of order n, then

$$\text{adj}(AB) = (\text{adj } B)(\text{adj } A).$$

Proof

Replacing A by AB in $\text{adj}(A) = |A|A^{-1}$, we get

$$\text{adj}(AB) = |AB|(AB)^{-1} = \left(|B|B^{-1}\right)\left(|A|A^{-1}\right) = \text{adj}(B)\,\text{adj}(A).$$

Example 1.9

Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

Solution

We get

$$AB = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+3 \\ -2+0 & -3-4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -7 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{(0+6)}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \quad \ldots (1)$$

$$A^{-1} = \frac{1}{(0+3)}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$$

$$B^{-1} = \frac{1}{(2-0)}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}. \quad \ldots (2)$$

As the matrices in (1) and (2) are same, $(AB)^{-1} = B^{-1}A^{-1}$ is verified. ■

Example 1.11

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Solution

Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Then, $A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

So, we get

$$AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2.$$

Similarly, we get $A^TA = I_2$. Hence $AA^T = A^TA = I_2 \Rightarrow A$ is orthogonal.

4. If $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$, show that $A^2-3A-7I_2=O_2$. Hence find A^{-1}.

11. $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$, show that $A^TA^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$.

12. Find the matrix A for which $A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$.

14. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$.

Example 1.14

Reduce the matrix $\begin{bmatrix}0 & 3 & 1 & 6\\ -1 & 0 & 2 & 5\\ 4 & 2 & 0 & 0\end{bmatrix}$ to a row-echelon form.

Solution

$$\begin{bmatrix}0 & 3 & 1 & 6\\ -1 & 0 & 2 & 5\\ 4 & 2 & 0 & 0\end{bmatrix}\xrightarrow{R_1\leftrightarrow R_2}\begin{bmatrix}-1 & 0 & 2 & 5\\ 0 & 3 & 1 & 6\\ 4 & 2 & 0 & 0\end{bmatrix}\xrightarrow{R_3\to R_3+4R_1}\begin{bmatrix}-1 & 0 & 2 & 5\\ 0 & 3 & 1 & 6\\ 0 & 2 & 8 & 20\end{bmatrix}$$

$$\xrightarrow{R_3\to R_3-\frac{2}{3}R_2}\begin{bmatrix}-1 & 0 & 2 & 5\\ 0 & 3 & 1 & 6\\ 0 & 0 & \frac{22}{3} & 16\end{bmatrix}\xrightarrow{R_3\to 3R_3}\begin{bmatrix}-1 & 0 & 2 & 5\\ 0 & 3 & 1 & 6\\ 0 & 0 & 22 & 48\end{bmatrix}.$$

Example 1.17

Find the rank of the matrix $\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 3 & 0 & 5\end{bmatrix}$ by reducing it to a row-echelon form.

Example 1.19

Show that the matrix $\begin{bmatrix}3 & 1 & 4\\ 2 & 0 & -1\\ 5 & 2 & 1\end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

3. Find the inverse of each of the following by Gauss-Jordan method:

(i) $\begin{bmatrix}2 & -1\\ 5 & -2\end{bmatrix}$ (ii) $\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix}$ (iii) $\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$

Example 1.22

Solve the following system of linear equations, using matrix inversion method: $5x+2y=3,\ 3x+2y=5$.

Solution

The matrix form of the system is $AX = B$, where $A=\begin{bmatrix}5 & 2\\3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}3\\5\end{bmatrix}$

We find $|A| = \begin{vmatrix}5 & 2\\3 & 2\end{vmatrix} = 10-6=4\neq 0$. So, A^{-1} exists and $A^{-1}=\frac{1}{4}\begin{bmatrix}2 & -2\\-3 & 5\end{bmatrix}$.

Then, applying the formula $X=A^{-1}B$, we get

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}2 & -2\\-3 & 5\end{bmatrix}\begin{bmatrix}3\\5\end{bmatrix}=\frac{1}{4}\begin{bmatrix}6-10\\-9+25\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-4\\16\end{bmatrix}=\begin{bmatrix}\frac{-4}{4}\\\frac{16}{4}\end{bmatrix}=\begin{bmatrix}-1\\4\end{bmatrix}.$$

So the solution is $(x=-1, y=4)$.

5. The prices of three commodities A, B and C are ₹ x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.)

Example 1.26

In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y=ax^2+bx+c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points $(10,8), (20,16), (40,22)$, can you conclude that the team won the match?

Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is $(70,0)$.)

3. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

4. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ? (Use Cramer's rule to solve the problem).

4. A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.)

2. Find the value of k for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have

(i) no solution (ii) unique solution (iii) infinitely many solution

Example 1.40

If the system of equations $px+by+cz=0$, $ax+qy+cz=0$, $ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.

Solution