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HIGHER SECONDARY SECOND YEAR

12th - Maths / UNIT 2 - Complex Numbers

Choose the correct or the most suitable answer from the given four alternatives :

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(1) 0 (2) 1 (3) -1 (4) i

2. The value of $\sum_{n=1}^{13}(i^n + i^{n-1})$ is

(1) $1+i$ (2) i (3) 1 (4) 0

3. The area of the triangle formed by the complex numbers z, iz, and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

(1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $\frac{-1}{i-2}$ (4) $\frac{1}{i-2}$

5. If $z = \frac{(\sqrt{3}+i)^3 (3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

6. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

7. If $|z-2+i| \le 2$, then the greatest value of $|z|$ is

(1) $\sqrt{3}-2$ (2) $\sqrt{3}+2$ (3) $\sqrt{5}-2$ (4) $\sqrt{5}+2$

8. If $\left|z-\frac{3}{z}\right| = 2$, then the least value of $|z|$ is

(1) 1 (2) 2 (3) 3 (4) 5

9. If $|z| - 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

10. The solution of the equation $|z| - z = 1 + 2i$ is

(1) $\frac{3}{2} - 2i$ (2) $-\frac{3}{2} + 2i$ (3) $2 - \frac{3}{2}i$ (4) $2 + \frac{3}{2}i$

11. If $|z_1| - 1$, $|z_2| - 2$, $|z_3| - 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

(1) 1 (2) 2 (3) 3 (4) 4

12. If z is a complex number such that $z \in \mathbb{C} \setminus \mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$, then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

13. z_1, z_2, and z_3 are complex numbers such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^2$ is

(1) 3 (2) 2 (3) 1 (4) 0

14. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

15. If $z=x+iy$ is a complex number such that $|z+2|=|z-2|$, then the locus of z is

(1) real axis (2) imaginary axis (3) ellipse (4) circle

16. The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

17. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

(1) -110° (2) -70° (3) 70° (4) 110°

18. If $(1+i)(1+2i)(1+3i)\cdots(1+ni)=x+iy$, then $2\cdot 5\cdot 10\cdots(1+n^2)$ is

(1) 1 (2) i (3) x^2+y^2 (4) $1+n^2$

19. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A+B\omega$, then (A,B) equals

(1) (1,0) (2) (−1,1) (3) (0,1) (4) (1,1)

20. The principal argument of the complex number $\frac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$ is

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

21. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(1) −2 (2) −1 (3) 1 (4) 2

22. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(1) −2 (2) −1 (3) 1 (4) 2

23. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

(1) 1 (2) −1 (3) $\sqrt{3}i$ (4) $-\sqrt{3}i$

24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(1) $cis\frac{2\pi}{3}$ (2) $cis\frac{4\pi}{3}$ (3) $-cis\frac{2\pi}{3}$ (4) $-cis\frac{4\pi}{3}$

25. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(1) 1 (2) 2 (3) 3 (4) 4

Properties of polar form

Property 1: If $z = r(\cos\theta + i\sin\theta)$, then $z^{-1} = \frac{1}{r}(\cos\theta - i\sin\theta)$.

Property 2: If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then $z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$.

Property3: If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$.

de Moivre's Theorem

(a) Given any complex number $\cos\theta + i\sin\theta$ and any integer n,

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

(b) If x is rational, then $\cos x\theta + i\sin x\theta$ in one of the values of $(\cos\theta + i\sin\theta)^x$

The n^{th} roots of complex number $z = r(\cos\theta + i\sin\theta)$ are

$$z^{1/n} = r^{1/n}\left(\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right),\ k = 0, 1, 2, 3, \ldots, n-1.$$

4. $i^{59} + \frac{1}{i^{59}}$ 5. $i i^2 i^3 \cdots i^{2000}$ 6. $\sum_{n=1}^{10} i^{n+50}$

Example 2.2

Find the value of the real numbers x and y, if the complex number $(2+i)x + (1-i)y + 2i - 3$ and $x + (-1+2i)y + 1 + i$ are equal

3. Find the values of the real numbers x and y, if the complex numbers $(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal.

3. If $z_1 = 2+5i$, $z_2 = -3-4i$, and $z_3 = 1+i$, find the additive and multiplicative inverse of z_1, z_2, and z_3.

Example 2.3

Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary parts.

Example 2.6

If $z_1 = 3-2i$ and $z_2 = 6+4i$, find $\frac{z_1}{z_2}$ in the rectangular form

Example 2.8

Show that (i) $\left(2+i\sqrt{3}\right)^{10} + \left(2-i\sqrt{3}\right)^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

3. If $z_1 = 2 - i$ and $z_2 = -4 + 3i$, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$.

4. The complex numbers u, v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$.

If $v = 3 - 4i$ and $w = 4 + 3i$, find u in rectangular form.

7. Show that (i) $\left(2 + i\sqrt{3}\right)^{10} - \left(2 - i\sqrt{3}\right)^{10}$ is purely imaginary

(ii) $\left(\frac{19 - 7i}{9 + i}\right)^{12} + \left(\frac{20 - 5i}{7 - 6i}\right)^{12}$ is real.

Example 2.9

If $z_1 = 3 + 4i$, $z_2 = 5 - 12i$, and $z_3 = 6 + 8i$, find $|z_1|$, $|z_2|$, $|z_3|$, $|z_1 + z_2|$, $|z_2 - z_3|$, and $|z_1 + z_3|$.

Example 2.10

Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

Example 2.12

If z_1, z_2, and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.

Example 2.14

Show that the points 1, $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

Example 2.17

Find the square root of $6 - 8i$.

7. If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$.

8. If the area of the triangle formed by the vertices z, iz, and $z + iz$ is 50 square units, find the value of $|z|$.

9. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

2. If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

5. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:

(i) $|z-4| = 16$ (ii) $|z-4|^2 - |z-1|^2 = 16$.

Example 2.24

Find the principal argument Arg z, when $z = \frac{-2}{1+i\sqrt{3}}$.

Example 2.26

Find the quotient $\frac{2\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

6. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

Example 2.30

Simplify $\left(\frac{1+\cos 2\theta + i\sin 2\theta}{1+\cos 2\theta - i\sin 2\theta}\right)^{30}$.

Example 2.35

Find all cube roots of $\sqrt{3} + i$.

7. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.

8. If $\omega \neq 1$ is a cube root of unity, show that

(i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$.

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots(1+\omega^{2^{11}}) = 1$.