SRI VINAYAGA TUITION CENTRE

ANAIMALAI

UNIT TEST (CHAPTER 1)

MATHEMATICS

Total Marks: 100 Marks Class: 12

Duration: 3 Hrs

PART A

CHOOSE THE CORRECT ANSWER 20 X 1 = 20

1. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is
   a) $\frac{-4}{5}$ b) $\frac{-3}{5}$
   c) $\frac{3}{5}$ d) $\frac{4}{5}$

2. If adj A = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is
   a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$
   c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

3. If ρ (A) = ρ ([A| B]) , then the system AX = B of linear equations is
   a) consistent and has a unique solution b) consistent
   c) consistent and has infinitely many solution d) inconsistent

4. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =
   a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$
   c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

5. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A , then $\frac{|adjB|}{|C|}$ =
   a) $\frac{1}{3}$ b) $\frac{1}{9}$
   c) $\frac{1}{4}$ d) 1

6. Which of the following is/are correct?
   (i) Adjoint of a symmetric matrix is also a symmetric matrix.
   (ii) Adjoint of a diagonal matrix is also a diagonal matrix.
   (iii) If A is a square matrix of order n and λ is a scalar, then adj(λA) = λ^n adj(A) .
   (iv) A(adjA) = (adjA)A = $|A|\, I$
   a) Only (i) b) (ii) and (iii)
   c) (iii) and (iv) d) (i), (ii) and (iv)

7. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,
   a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ b) $log\,(\Delta_1/\Delta_3)\,, log\,(\Delta_2/\Delta_3)$
   c) $log\,(\Delta_2/\Delta_1)\,, log\,(\Delta_3/\Delta_1)$ d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

8. If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is
   a) 17 b) 14 c) 19 d) 21

9. If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ and A(adj A) = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =

a) 0 b) sin θ

c) cos θ d) 1

10. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$, The system has infinitely many solutions if

a) λ = 7, μ ≠ - 5 b) λ = - 7, μ = 5

c) λ ≠ 7, μ ≠ - 5 d) λ = 7, μ = - 5

11. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2$ - A =

a) A^{-1} b) $\frac{A^{-1}}{2}$

c) 3 A^{-1} d) 2 A^{-1}

12. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$, If B is the inverse of A, then the value of x is

a) 2 b) 4 c) 3 d) 1

13. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0, (\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is.

a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$

c) $\frac{5\pi}{6}$ d) $\frac{\pi}{4}$

14. If A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{11} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

a) 0 b) -2 c) -3 d) -1

15. If $A^T A^{-1}$ is symmetric, then A^2 =

a) A^{-1} b) $(A^T)^2$

c) A^T d) $(A^{-1})^2$

16. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is :

a) 1 b) 2 c) 4 d) 3

17. If A = $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =

a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

18. If A,B and C are invertible matrices of some order, then which one of the following is not true?

a) adj A = |A| A^{-1} b) adj(AB) = (adj A)(adj B)

c) det A^{-1} = (det A)$^{-1}$ d) $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

19. If A = $\begin{bmatrix} 1 & tan\frac{\theta}{2} \\ -tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I_2, then B =

a) $(cos^2\frac{\theta}{2})A$ b) $(cos^2\frac{\theta}{2})A^T$

c) $(\cos^2\theta)I$ d) $(sin^2\frac{\theta}{2})A$

20. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adj A) is

a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$

c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

PART B

Answer any 7 questions (Q.no 30 compulsory) 7 X 2 = 14

21. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

22. Find the adjoint of the following: (i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

23. Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.

24. Find the rank of the following matrices by minor method: (i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

25. Find the inverse of each of the following by Gauss-Jordan method:
(i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

26. Solve the following system of linear equations, using matrix inversion method:
$5x + 2y = 3$, $3x + 2y = 5$.

27. Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$.

28. Solve the following systems of linear equations by Cramer's rule:
(i) $3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$

29. If A is a non-singular matrix of odd order, prove that $|adjA|$ is positive

30. If A= $\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$ verify B = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$,adj (AB) = (adj B).

PART C

Answer any 7 questions (Q.no 40 compulsory) 7 X 3 = 21

31. Find the inverse of the matrix : $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$

32. If A is symmetric, prove that adj A is also symmetric.

33. Proove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.

34. Solve the following system:
$x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 12z = 0$.

35. Solve the following system of homogenous equations.
(i) $3x + 2y + 7z = 0$, $4x - 3y - 2z = 0$, $5x + 9y + 23z = 0$

36. Find adj(adj(A)) if adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

37. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A = $|A| I_2$.

38. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with A = $\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, B = $\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

39. Solve the system: x + 3y − 2z = 0, 2x − y + 4z = 0, x −11y +14z = 0.

40. Verify that $(A^{-1})T = (A^T)^{-1}$ for A = $\begin{bmatrix} -2 & -3 \\ 5 & -6 \end{bmatrix}$.

PART D

Answer the following questions 7 X 5 = 35

41. a) Investigate for what values of λ and μ the system of linear equations x + 2y + z = 7, x + y +∟ z = μ, x + 3y − 5z = 5 has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(Or)

b) Test for consistency of the following system of linear equations and if possible solve:
x − y + z = −9, 2x − 2y + 2z = −18, 3x − 3y + 3z + 27 = 0.

42. a) If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

(Or)

b) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A) A = $|A| I_3$.

43. a) If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

(Or)

b) If $F(\alpha) = \begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$ .show that $[F(\alpha)]^{-1} = F(-\alpha)$.

44. a) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z =1,3x + 2y + z = 7,2x + y + 3z = 2.

(Or)

b) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

45. a) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 womencan finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

(Or)

b) If adj (A) = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A

46. a) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy -coordinate system in the vertical plane and the ball traversed through the points (10,8),(20,16),(30,18) , can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0).)

(Or)

b) A man is appointed in a job with a monthly salary of certain amount and a fixed amount ofannual increment. If his salary was 19,800 per month at the end of the first month after 3years of service and 23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

47. a) By using Gaussian elimination method, balance the chemical reaction equation:
$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

(Or)

b) Discuss the solutions of the system of equations for all values of λ.
$x + y + z = 2$, $2x + y - 2z = 2$, $\lambda x + y + 4z = 2$

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