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- 1) A person learnt 100 words for an English test. The number of words the person remembers in  $t$  days after learning is given by  $W(t) = 100 \times (1 - 0.1t)^2$ ,  $0 \leq t \leq 10$ . What is the rate at which the person forgets the words 2 days after learning?
- 2) If we blow air into a balloon of spherical shape at a rate of  $1000 \text{ cm}^3$  per second, at what rate the radius of the balloon changes when the radius is  $7 \text{ cm}$ ? Also compute the rate at which the surface area changes.
- 3) Salt is poured from a conveyor belt at a rate of  $30$  cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is  $10$  metre high?
- 4) A conical water tank with vertex down of  $12$  meters height has a radius of  $5$  meters at the top. If water flows into the tank at a rate  $10$  cubic m/min, how fast is the depth of the water increases when the water is  $8 \text{ m}$  deep?
- 5) Find the points on the curve  $y = x^3 - 3x^2 + x - 2$  at which the tangent is parallel to the line  $y = x$ .
- 6) Find the angle between  $y = x^2$  and  $y = (x-3)^2$ .
- 7) If the curves intersect each other orthogonally then  $ax^2 + by^2 = 1$  and  $cx^2 + dy^2 = 1$  show that  $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ .
- 8) Find the angle between the curve  $y = x^2$  and  $x = y^2$  at their points of intersection  $(0,0)$  and  $(1,1)$ .
- 9) Find the angle between the rectangular hyperbola  $xy = 2$  and the parabola  $x^2 + 4y = 0$ .
- 10) Find the equation of tangent and normal to the curve given by  $x = 7 \cos t$  and  $y = 2 \sin t$ ,  $t \in \mathbb{R}$  at any point on curve.
- 11) Find the value in the interval  $(\frac{1}{2}, 2)$  satisfied by the Rolle's theorem for the function  $f(x) = x + \frac{1}{x}$ ,  $x \in [\frac{1}{2}, 2]$ .
- 12) Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  there is zero of polynomial  $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots + a_1$ .



- 13) Find the values in the interval  $(1, 2)$  of the mean value theorem satisfied by the function  $f(x) = x - x^2$  for  $1 \leq x \leq 2$
- 14) Show that the value in the conclusion of the mean value theorem for  $f(x) = 1/x$  on a closed interval of positive numbers  $[a, b]$  is  $\sqrt{ab}$
- 15) Suppose that for a function  $f(x)$ ,  $f'(x) \leq 1$  for  $1 \leq x \leq 4$ , show that  $f(4) - f(1) \leq 3$
- 16) Expand  $\log(1+x)$  as a Maclaurin's series upto 4 non zero terms for  $-1 < x \leq 1$
- 17) Write the Taylor's series expansion of  $1/x$  about  $x=2$  by finding the first three non zero terms
- 18) Evaluate the limit  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x^2} \right)$
- 19) Evaluate  $\lim_{x \rightarrow \infty} (1+2x)^{1/2 \log x}$
- 20)  $\lim_{x \rightarrow \pi/2} (\sin x) \tan x$
- 21) Prove that the function  $f(x) = x^2 - 2x - 3$  is strictly increasing in  $(2, \infty)$
- 22) Find the intervals of monotonicity and hence find the local extrema for the function  $f(x) = x^2 - 4x + 4$
- 23) Find the local extrema of the function  $f(x) = x^4 + 32x$
- 24) Find the local maximum and minimum of the function  $x^2 y^2$  on the  $x+y=10$
- 25) Find the points on the unit circle  $x^2 + y^2 = 1$ , nearest and farthest from  $(1, 1)$
- 26) Find two positive numbers whose sum is 12 and their product is maximum
- 27) Find two positive numbers whose product is 20 and sum is maximum
- 28) The volume of cylinder is given by the formula  $V = \pi r^2 h$ . Find the greatest and least values of  $V$  if  $r+h=6$
- 29) A hollow cone with base radius  $a$  cm and height  $b$  cm placed on a table. Show that the volume of largest cylinder that can be hidden underneath is  $4/9$  times volume of cone
- 30) Find the slant asymptote for the function  $f(x) = \frac{x^2 - 6x + 7}{x+5}$

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