Monthly Test - June, 2024

Standard XII

MATHEMATICS

Time: 1.30 hrs. Marks: 50

Section - A

I. **Choose and write the correct answer:** 10x1=10

1. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is

a) 17 b) 14 c) 19 d) 21

2. If $A=\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{bmatrix}$ and $A^T=A^{-1}$, then the value of x is

a) $\frac{-4}{5}$ b) $\frac{-3}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

3. If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|\text{adj}(AB)|=$

a) −40 b) −80 c) −60 d) −20

4. If $x^a y^b = e^m, x^c y^d = e^n$; $\Delta_1=\begin{vmatrix}m & b\\ n & d\end{vmatrix}$, $\Delta_2=\begin{vmatrix}a & m\\ c & n\end{vmatrix}$, $\Delta_3=\begin{vmatrix}a & b\\ c & d\end{vmatrix}$, then the values of x and y are respectively

a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ b) $\log\left(\frac{\Delta_1}{\Delta_3}\right), \log\left(\frac{\Delta_2}{\Delta_3}\right)$

c) $\log\left(\frac{\Delta_2}{\Delta_1}\right), \log\left(\frac{\Delta_3}{\Delta_1}\right)$ d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

14. Write $\overline{(5+9i)+(2-4i)}$ in rectangular form.

15. Show that $(2+i\sqrt{3})^{10}-(2-i\sqrt{3})^{10}$ is purely imaginary.

16. If $z=5-2i$ and $w=-1+3i$, evaluate $z^2+2zw+w^2$.

Section - C

III. Answer any four questions. Q.No.22 is compulsory: 4x3=12

17. If $A=\begin{bmatrix}3 & 2\\7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -3\\5 & 2\end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

18. Solve $\frac{3}{x}+2y=12$, $\frac{2}{x}+3y=13$ using Cramer's rule.

19. If $A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\4 & 4 & 7\\1 & -8 & 4\end{bmatrix}$, prove that $A^{-1}=A^{T}$.

20. Find the least value of the positive integer n for which $(\sqrt{3}+i$ is purely real.

21. If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$, sh that the locus of z is real axis.

22. If $z=(\cos\theta+i\sin\theta)$, show that $z^n-\frac{1}{z^n}=2i\sin\theta$.

Section - D

IV. Answer all the questions:

23. a) If $A=\begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$.

b) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8)$, $(-2, -12)$ and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will be meet his friend? (Use Gaussian elimination method).

24. a) If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

(OR)

b) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ using Gauss-Jordan method.

25. a) If $z = x + iy$, is a complex number such that $\text{Im}\left[\frac{2z+i}{iz+1}\right] = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

(OR)

b) Find the value of $\left[\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right]^{10}$.

26. a) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

(OR)

b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

5. If $A^T A^{-1}$ is symmetric, then A^2 =

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

6. The solution of the equation $|z| - z = 1 + 2i$ is

a) $\frac{3}{2} - 2i$ b) $-\frac{3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$

7. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

a) 0 b) 1 c) –1 d) i

8. If Z_1, Z_2 and Z_3 are complex numbers such that $Z_1 + Z_2 + Z_3 = 0$ and $|Z_1| = |Z_2| = |Z_3| = 1$ then $Z_1^2 + Z_2^2 + Z_3^2$ is

a) 3 b) 2 c) 1 d) 0

9. The product of all four values of $\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]^{\frac{3}{4}}$ is

a) –2 b) –1 c) 1 d) 2

10. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

a) (1, 0) b) (–1, 1) c) (0, 1) d) (1, 1)

Section - B

II. Answer any four questions. Q.No. 16 is compulsory: 4x2=8

11. By using Gaussian elimination method, balance the chemical equation $C_2H_6 + O_2 \rightarrow H_2O + CO_2$.

12. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

13. Find the rank of the matrix $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ in the row-echelon