FIRST MID TERM TEST - 2024

Standard XI

Reg.No. | | | | | | |

MATHEMATICS

Time : 1.30 hrs *Marks : 50*

Part - I

I. Choose the correct answer: **10 x 1 = 10**

1. If $n((A \times B) \cap (A \times C)) = 8$ and $n(B \cap C) = 2$, then $n(A)$ is
   a) 6 b) 4 c) 8 d) 16
2. The number of constant functions from a set containing m elements to a set containing n elements is
   a) mn b) m c) n d) m + n
3. Let $f : R \to R$ be defined by $f(x) = 1 - |x|$. Then the range of f is
   a) R b) $(1, \infty)$ c) $(-1, \infty)$ d) $(-\infty, 1]$
4. The number of relations on a set containing 3 elements is
   a) 9 b) 81 c) 512 d) 1024
5. If $|x + 2| = 9$, then x belongs to
   a) $(-\infty, -7)$ b) $[-11, 7]$ c) $(-\infty, -7) \cup [11, \infty)$ d) $(-11, 7)$
6. The number of roots of $(x + 3)^4 + (x + 5)^4 = 16$ is
   a) 4 b) 2 c) 3 d) 0
7. If 3 is the logarithm of 343, then the base is
   a) 5 b) 7 c) 6 d) 9
8. The solution of $5x - 1 < 24$ and $5x + 1 > -24$ is
   a) (4,5) b) (–5,–4) c) (–5,5) d) (–5,4)
9. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots\ldots\ldots + \cos 179^\circ =$
   a) 0 b) 1 c) –1 d) 89
10. In a triangle ABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is
    a) equilateral triangle b) isosceles triangle
    c) right triangle d) scalene triangle

Part - II

II. Answer any 4 questions. (Q.No.16 is compulsory) **4 x 2 = 8**

11. Find the number of subsets of A if $A = \{x : x = 4n+1, 2 \le n \le 5, n \in N\}$.
12. If $f : [-2,2] \to B$ is given by $f(x) = 2x^3$, then find B so that f is onto.
13. Construct a quadratic equation with root 7 and –3.
14. Express 30° angle in radian measure.
15. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$
16. Solve : $|2x - 17| = 3$ for x.

Part - III

III. Answer any 4 questions. (Q.No.22 is compulsory) **4 x 3 = 12**

17. If $n(P(A)) = 1024$, $n(A \cup B) = 15$ and $n(P(B)) = 32$, then find $n(A \cap B)$
18. Simplify : $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

19. Solve $23x < 100$ when
i) x is a natural number ii) x is an integer

20. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$

21. Find the value of cos 135°.

22. Draw the followings:
i) $y = |x|$ ii) $y = |x - 1|$ iii) $y = |x + 1|$

Part - IV

IV. Answer all the questions. 4 x 5 = 20

23. a) In ΔABC, prove that $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{c}{2}\right)$

(OR)

b) If $f : R \to R$ is defined by $f(x) = 2x - 3$, prove that f is bijection and find its inverse.

24. a) Write the values of f at –3, 5, 2, –1, 0 if $f(x)=\begin{cases} x^2+x-5 & \text{if } x\in(-\infty,0) \\ x^2+3x-2 & \text{if } x\in(3,\infty) \\ x^2 & \text{if } x\in(0,2) \\ x^2-3 & \text{otherwise} \end{cases}$

(OR)

b) Resolve into partial fractions : $\frac{2x}{(x^2+1)(x-1)}$

25. a) If $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$, then prove that xyz = 1

(OR)

b) Prove that $\frac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)}=\cos^2\theta\cot\theta$

26. a) On the set of natural numbers let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is
i) Reflexive ii) Symmetric
iii) Transitive iv) Equivalence

(OR)

b) Solve the equation $\sqrt{6-4x-x^2}=x+4$

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ST. JOSEPHS HR. SEC. SCHOOL - CHENGALPATTU

FIRST MID TERM TEST - 2024

7/8/24

STD - 11 MATHEMATICS - KEY MARKS: 50

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I

1. b 4
2. c n
3. d $(-\infty, 1]$
4. c 512
5. b $[-11, 7]$
6. a 4
7. b 7
8. c $(-5, 5)$
9. a 0
10. c right triangle

II

11. $A = \{9, 13, 17, 21\}$, $n(A) = 4$
$n(P(A)) = 2^4 = 16$

12. $f(-2) = -16$, $f(2) = 16$
$B = [-16, 16]$

13. $\alpha + \beta = 4$, $\alpha\beta = -21$
$x^2 - 4x - 21 = 0$

14. $30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$ radians

15. $\sin y = \frac{1}{\sqrt{2}}$
$\sin y = \sin \pi/4 \Rightarrow y = \pi/4$
$\sin^{-1}(1/\sqrt{2}) = \pi/4$

16. $2x - 17 = \pm 3 \Rightarrow x = 10, x = 7$

III

17. $n(P(A)) = 2^{10}$, $n(A) = 10$
$n(P(B)) = 2^5$, $n(B) = 5$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$15 = 10 + 5 - n(A \cap B)$
$n(A \cap B) = 0$

18. $\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$

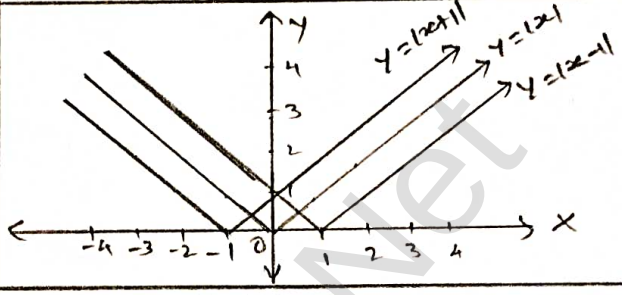
IV

19. $\frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8}-\sqrt{7}$, $\frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$
$\frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5}$, $\frac{1}{\sqrt{5}-2} = \sqrt{5}+2$
$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$
$= 5$

20. $= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$
$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$
$= \frac{1 + \sin\theta}{\cos\theta}$

21. $\cos(90^\circ + 45^\circ) = -\sin 45^\circ = -\frac{1}{\sqrt{2}}$

22.



IV

23. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

a. $\frac{a-b}{a+b}\cot C/2 = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}\cot C/2$
$= \frac{\sin A - \sin B}{\sin A + \sin B}\cot C/2$
$= \frac{2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})}{2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})}\cot C/2$
$= \cot\frac{A+B}{2}\tan\frac{A-B}{2}\cot C/2$
$= \cot(90 - C/2)\tan\frac{A-B}{2}\cot C/2$
$= \tan\frac{A-B}{2}$

b. one to one: $f(x) = f(y) \Rightarrow 2x-3 = 2y-3 \Rightarrow x = y$
onto: $x = \frac{y+3}{2} \Rightarrow f(x) = 2(\frac{y+3}{2}) - 3 = y$
f is onto
Inverse: $y = 2x-3$, $y+3 = 2x \Rightarrow x = \frac{y+3}{2}$
$f^{-1}(y) = \frac{y+3}{2}$, $f^{-1}(x) = \frac{x+3}{2}$

24. a. $f(-3) = 1$ $f(5) = 38$
$f(2) = 1$ $f(-1) = -5$
$f(0) = -3$

b. $= \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$
$2x = A(x^2+1) + (Bx+C)(x-1)$
$x = 1$ $A = 1$
$x = 0$ $A - C = 0 \Rightarrow A = C \Rightarrow C = 1$
$x = -1$ $2A - 2(C-B) = -2 \Rightarrow B = -1$
$\frac{2x}{(x^2+1)(x-1)} = \frac{1}{(x-1)} + \frac{1-x}{x^2+1}$

V

A (i) x is natural number: $x < \frac{100}{23} \Rightarrow x = \{1, 2, 3, 4\}$

(ii) x is an integer: $x < 4.347$
$x = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$

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25. a. $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

$\log x = k(y-z) = ky - kz$ — ①

$\log y = k(z-x) = kz - kx$ — ②

$\log z = k(x-y) = kx - ky$ — ③

① + ② + ③ $\Rightarrow \log x + \log y + \log z$

$= ky - kz + kz - kx + kx - ky$

$\log xyz = 0 = \log 1$

$xyz = 1$

b. $\cot(180° + \theta) = \cot\theta$, $\sin(270 + \theta) = -\cos\theta$

$\sin(90° - \theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$

$\cos(-\theta) = \cos\theta$, $\operatorname{cosec}(360 + \theta) = \operatorname{cosec}\theta$

LHS: $\frac{\cot\theta\cos\theta\cos\theta}{(-\cos\theta)(-\tan\theta)(\operatorname{cosec}\theta)} = \frac{\cot\theta\cos^2\theta}{\cos\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta}}$

$= \cot\theta\cos^2\theta$

26 a. $2a + 3b = 30 \Rightarrow a = \frac{30 - 3b}{2}$

a	12	9	6	3
b	2	4	6	8

i Reflexivity : $(2,2) \notin R$

R is not reflexive

ii Symmetricity : $(9,4) \in R \Rightarrow (4,9) \notin R$

$\therefore$ R is not symmetric

iii. Transitivity : clearly R is not transitive

iv. Equivalence :

R is not an equivalence relation

b. $\sqrt{6 - 4x - x^2} = x + 4$

$(x+4) \geq 0$, $6 - 4x - x^2 = (x+4)^2$

$x \geq -4$, $x^2 + 6x + 5 = 0$, $x = -1, -5$

$x = -1$ satisfies both the conditions

Hence $x = -1$