IMPORTANT 2 MARK QUESTIONS

1.SETS RELATIONS AND FUNCTIONS

1. Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k.
2. If $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$.
3. If $\mathcal{P}(A)$ denotes the power set of A, then find $n(\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))))$.
4. Justify the trueness of the statement:
   "An element of a set can never be a subset of itself."
5. If $n(\mathcal{P}(A)) = 1024, n(A \cup B) = 15$ and $n(\mathcal{P}(B)) = 32$, then find $n(A \cap B)$.
6. If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n(\mathcal{P}(A\Delta B))$.
7. Let $A = \{0,1,2,3\}$. Construct relations on A of the following types:
   (i) not reflexive, not symmetric, not transitive. (ii) not reflexive, not symmetric, transitive.
   (iii) not reflexive, symmetric, not transitive. (iv) not reflexive, symmetric, transitive.
   (v) reflexive, not symmetric, not transitive. (vi) reflexive, not symmetric, transitive.
   (vii) reflexive, symmetric, not transitive. (viii) reflexive, symmetric, transitive.
8. Discuss the following relations for reflexivity, symmetricity and transitivity:
   (i) The relation R defined on the set of all positive integers by "mRn if m divides n".
   (ii) Let P denote the set of all straight lines in a plane. The relation R defined by "ℓRm if ℓ is perpendicular to m".
   (iii) Let A be the set consisting of all the members of a family. The relation R defined by "aRb if a is not a sister of b ".
   (iv) Let A be the set consisting of all the female members of a family. The relation R defined by "aRb if a is not a sister of b".
   (v) On the set of natural numbers the relation R defined by "xRy if $x + 2y = 1$".

2. BASIC ALGEBRA

1. Find a positive number smaller than $\frac{1}{2^{1000}}$. Justify.
2. Solve $|x - 9| < 2$ for x.
3. Solve $\left|\frac{2}{x-4}\right| > 1, x \neq 4$.
4. Solve for x : (i) $|3 - x| < 7$. (ii) $|4x - 5| \geq -2$.
5. Solve $|5x - 12| < -2$.
6. Solve $3x - 5 \leq x + 1$ for x.
7. Write $f(x) = x^2 + 5x + 4$ in completed square form.
8. Find the logarithm of 1728 to the base $2\sqrt{3}$.

9. If the logarithm of 324 to base a is 4 , then find a.

10. Prove $\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243} = \log 2$.

11. Solve $x^{\log_3 x} = 9$.

12. Compute $\log_3 5\log_{25} 27$.

13. Compute $\log_9 27 - \log_{27} 9$.

14. Solve $\log_8 x + \log_4 x + \log_2 x = 11$.

15. Solve $\log_4 2^{8x} = 2^{\log_2 8}$.

16. Prove $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$.

17. Simplify: (i) $(125)^{\frac{2}{3}}$, (ii) $16^{\frac{-3}{4}}$ (iii) $(-1000)^{\frac{-2}{3}}$, (iv) $(3^{-6})^{\frac{1}{3}}$ (v) $\frac{27^{\frac{-2}{3}}}{27^{\frac{-1}{3}}}$.

3.TRIGONOMETRY

1. For each given angle, find a coterminal angle with measure of θ such that $0° \le \theta < 360°$

   (i) $395°$ (ii) $525°$ (iii) $1150°$ (iv) $-270°$ (v) $-450°$

2. Express each of the following angles in radian measure:

   (i) $30°$ (ii) $135°$ (iii) $-205°$ (iv) $150°$ (v) $330°$.

3. Find the degree measure corresponding to the following radian measures

   (i) $\frac{\pi}{3}$ (ii) $\frac{\pi}{9}$ (iii) $\frac{2\pi}{5}$ (iv) $\frac{7\pi}{3}$ (v) $\frac{10\pi}{9}$.

4. What must be the radius of a circular running path, around which an athlete must run 5 times in order to describe 1 km ?

5. What is the length of the arc intercepted by a central angle of measure $41°$ in a circle of radius 10ft ?

6. Find the value of: (i) $\sin(765°)$ (ii) $\operatorname{cosec}(-1410°)$ (iii) $\cot\left(\frac{-15\pi}{4}\right)$.

7. Find the values of (i) $\sin(480°)$ (ii) $\sin(-1110°)$ (iii) $\cos(300°)$

   (iv) $\tan(1050°)$ (v) $\cot(660°)$ (vi) $\tan\left(\frac{19\pi}{3}\right)$ (vii) $\sin\left(-\frac{11\pi}{3}\right)$.

8. Find the value of (i) $\cos 105°$ (ii) $\sin 105°$ (iii) $\tan \frac{7\pi}{12}$.

9. Prove that (i) $\sin(45° + \theta) - \sin(45° - \theta) = \sqrt{2}\sin\theta$.(ii) $\sin(30° + \theta) + \cos(60° + \theta) = \cos\theta$.

10. Prove that $\sin 105° + \cos 105° = \cos 45°$.

11. Prove that

    (i) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$

    (ii) $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

    (iii) $\sin^2(A + B) - \sin^2(A - B) = \sin 2A\sin 2B$

    (iv) $\cos 8\theta\cos 2\theta = \cos^2 5\theta - \sin^2 3\theta$

12. Find the value of $\sqrt{3}\operatorname{cosec} 20° - \sec 20°$

13. Find the value of $\cos 2A, A$ lies in the first quadrant, when

 (i) $\cos A = \frac{15}{17}$ (ii) $\sin A = \frac{4}{5}$ (iii) $\tan A = \frac{16}{63}$.

14. Fxpress each of the following as a sum or difference

 (i) $\sin 35°\cos 28°$ (ii) $\sin 4x\cos 2x$ (iii) $2\sin 10\theta\cos 2\theta$

 (iv) $\cos 5\theta\cos 2\theta$ (v) $\sin 5\theta\sin 4\theta$.

15. Express each of the following as a product

 (i) $\sin 75° - \sin 35°$ (ii) $\cos 65° + \cos 15°$

 (iii) $\sin 50° + \sin 40°$ (iv) $\cos 35° - \cos 75°$.

16. Find the principal solution and general solutions of the following:

 (i) $\sin \theta = -\frac{1}{\sqrt{2}}$ (ii) $\cot \theta = \sqrt{3}$ (iii) $\tan \theta = -\frac{1}{\sqrt{3}}$.

17. Find the area of the triangle whose sides are 13 cm, 14 cm and 15 cm.

18. If the sides of a $\triangle ABC$ are $a = 4, b = 6$ and $c = 8$, then show that $4\cos B + 3\cos C = 2$.

19. In any $\triangle ABC$, prove that the area $\triangle = \frac{b^2+c^2-a^2}{4\cot A}$.

20. In a $\triangle ABC$, if $a = 12$ cm, $b = 8$ cm and $C = 30°$, then show that its area is 24 sq.cm.

21. Find the principal value of (i) $\sin^{-1} \frac{1}{\sqrt{2}}$ (ii) $\cos^{-1} \frac{\sqrt{3}}{2}$ (iii) $\text{cosec}^{-1} (-1)$

 (iv) $\sec^{-1} (-\sqrt{2})$ (v) $\tan^{-1} (\sqrt{3})$

4. COMBINATORICS AND MATHEMATICAL INDUCTION

1. There are 10 bulbs in a room. Each one of them can be operated independently. Find the number of ways in which the room can be illuminated.

2. If $n! + (n-1)! = 30$, then find the value of n.

3. If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ then find the value of A.

4. A mobile phone has a passcode of 6 distinct digits. What is the maximum number of attempts one makes to retrieve the passcode?

5. Four children are running a race.

 (i) In how many ways can the first two places be filled?

 (ii) In how many different ways could they finish the race?

6. How many three-digit numbers are there with 3 in the unit place? (i) with repetition (ii) without repetition.

7. How many strings can be formed using the letters of the word LOTUS if the word

 (i) either starts with L or ends with S ? (ii) neither starts with L nor ends with S ?

8. (i) Count the total number of ways of answering 6 objective type questions, each question having 4 choices.
   (ii) In how many ways 10 pigeons can be placed in 3 different pigeon holes ?
   (iii) Find the number of ways of distributing 12 distinct prizes to 10 students?
9. Find the value of n if (i) $(n+1)! = 20(n-1)!$ (ii) $\frac{1}{8!}+\frac{1}{9!}=\frac{n}{10!}$.
10. (i) Suppose 8 people enter an event in a swimming meet. In how many ways could the gold, silver and bronze prizes be awarded?
    (ii) Three men have 4 coats, 5 waist coats and 6 caps. In how many ways can they wear them?
11. Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time?
12. A test consists of 10 multiple choice questions. In how many ways can the test be answered if
    (i) Each question has four choices?
    (ii) The first four questions have three choices and the remaining have five choices?
    (iii) Question number n has $n+1$ choices?
13. A student appears in an objective test which contain 5 multiple choice questions. Each question has four choices out of which one correct answer.
    (i) What is the maximum number of different answers can the students give?
    (ii) How will the answer change if each question may have more than one correct answers?
14. How many strings can be formed from the letters of the word ARTICLE, so that vowels occupy the even places?
15. Find the distinct permutations of the letters of the word MISSISSIPPI?
16. How many ways can the product $a^2b^3c^4$ be expressed without exponents?
17. In how many ways 4 mathematics books, 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of the same subjects are together.
18. In how many ways can the letters of the word SUCCESS be arranged so that all Ss are together?
19. A coin is tossed 8 times,
    (i) How many different sequences of heads and tails are possible?
    (ii) How many different sequences containing six heads and two tails are possible?
20. How many strings are there using the letters of the word INTERMEDIATE, if
    (i) The vowels and consonants are alternative (ii) All the vowels are together
    (iii) Vowels are never together (iv) No two vowels are together.
21. Prove that ${}^{24}C_4 + \Sigma_{r=0}^{4}\ {}^{(28-r)}C_3 = {}^{29}C_4$
22. A Mathematics club has 15 members. In that 8 are girls. 6 of the members are to be selected for a competition and half of them should be girls. How many ways of these selections are possible?
23. A box of one dozen apple contains a rotten apple. If we are choosing 3 apples simultaneously, in how many ways, one can get only one apples.
24. How many diagonals are there in a polygon with n sides?
25. If ${}^{n}C_{12} = {}^{n}C_{9}$ find ${}^{21}C_{n}$.

26. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r.
27. How many ways a committee of six persons from 10 persons can be chosen along with a chair person and a secretary?
28. How many different selections of 5 books can be made from 12 different books if,
    (i) Two particular books are always selected? (ii) Two particular books are never selected?
29. In an examination a student has to answer 5 questions, out of 9 questions in which 2 are compulsory. In how many ways a student can answer the questions?
30. How many triangles can be formed by joining 15 points on the plane, in which no line joining any three points?
31. A polygon has 90 diagonals. Find the number of its sides?

5. BINOMIAL THEOREM, SEQUENCES AND SERIES

1. Expand (i) $\left(2x^2 - \frac{3}{x}\right)^3$
2. Write the first 6 terms of the sequences whose n^{th} term a_n is given below.

   (i) $a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$ (ii) $a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$

   (iii) $a_n = \begin{cases} n & \text{if } n \text{ is } 1,2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$
3. If t_k is the k^{th} term of a GP, then show that t_{n-k}, t_n, t_{n+k} also form a GP for any positive integer k.
4. Expand $(1+x)^{\frac{2}{3}}$ up to four terms for $|x| < 1$.
5. Write the first 6 terms of the exponential series (i) e^{5x} (ii) e^{-2x} (iii) $e^{\frac{1}{2}x}$.
6. Write the first 4 terms of the logarithmic series (i) $\log(1+4x)$ (ii) $\log(1-2x)$ (iii) $\log\left(\frac{1+3x}{1-3x}\right)$ (iv) $\log\left(\frac{1-2x}{1+2x}\right)$. Find the intervals on which the expansions are valid.

6.TWO DIMENSIONAL ANALYTICAL GEOMETRY.

1. Find the locus of a point P moves such that its distances from two fixed points $A(1,0)$ and $B(5,0)$, are always equal.
2. Find the locus of P, if for all values of α, the co-ordinates of a moving point P is
   (i) $(9\cos\alpha, 9\sin\alpha)$ (ii) $(9\cos\alpha, 6\sin\alpha)$.
3. Find the locus of a point P that moves at a constant distant of (i) two units from the x-axis (ii) three units from the y-axis.
4. Find the value of k and b, if the points $P(-3,1)$ and $Q(2,b)$ lie on the locus of $x^2 - 5x + ky = 0$.
5. Find the equation of the straight line passing through $(-1,1)$ and cutting off equal intercepts, but opposite in signs with the two coordinate axes.
6. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle $150°$ with positive direction of the x-axis. Find the equation of the line.

7. Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form.

8. Find the equation of the lines passing through the point $(1,1)$

   (i) with y-intercept (-4) (ii) with slope 3 (iii) and $(-2,3)$

   (iv) and the perpendicular from the origin makes an angle $60°$ with x - axis.

9. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

10. Find the equation of the straight line parallel to $5x - 4y + 3 = 0$ and having x-intercept 3 .

11. Find the distance between the line $4x + 3y + 4 = 0$, and a point (i) $(-2,4)$ (ii) $(7,-3)$

12. Write the equation of the lines through the point $(1,-1)$

    (i) parallel to $x + 3y - 4 = 0$ (ii) perpendicular to $3x + 4y = 6$

13. If $(-4,7)$ is one vertex of a rhombus and if the equation of one diagonal is $5x - y + 7 = 0$, then find the equation of another diagonal.

14. Find the distance between the parallel lines

    (i) $12x + 5y = 7$ and $12x + 5y + 7 = 0$ (ii) $3x - 4y + 5 = 0$ and $6x - 8y - 15 = 0$.

15. If the line joining two points A(2,0) and B(3,1) is rotated about A in anticlockwise direction through an angle of $15°$, then find the equation of the line in new position.

16. Find the combined equation of the straight lines whose separate equations are $x - 2y - 3 = 0$ and $x + y + 5 = 0$.

17. Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.

18. Show that $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines.

7. MATRICES AND DETERMINANTS

1. Find the sum $A + B + C$ if A, B, C are given by $A = \begin{bmatrix} \sin^2\theta & 1 \\ \cot^2\theta & 0 \end{bmatrix}, B = \begin{bmatrix} \cos^2\theta & 0 \\ -\mathrm{cosec}^2\theta & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

2. Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

   (i) $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$ (ii) $a_{ij} = \frac{|3i-4j|}{4}$ with $m = 3, n = 4$

3. Express the matrices as the sum of a symmetric matrix and a skew-symmetric matrix: (i) $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ is a matrix such that $AA^T = 9I$, find the values of x and y.

5. Find $|A|$ if $A = \begin{bmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$.

6. Evaluate $\begin{vmatrix} 2014 & 2017 & 0 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 0 \end{vmatrix}$.

7. Without expanding the determinant, prove that $\begin{vmatrix} s & a^2 & b^2+c^2 \\ s & b^2 & c^2+a^2 \\ s & c^2 & a^2+b^2 \end{vmatrix} = 0$.

8. Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$.

9. Prove that $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix} = 0$

10. Write the general form of a 3×3 skew-symmetric matrix and prove that its determinant is 0 .

11. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, find the value of $|3AB|$.

12. Determine the roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$.

13. Find the area of the triangle whose vertices are $(-2,-3)$, $(3,2)$, and $(-1,-8)$.

14. Find the area of the triangle whose vertices are $(0,0)$, $(1,2)$ and $(4,3)$.

8.VECTOR ALGEBRA

1. Prove that the relation R defined on the set V of all vectors by ' $\vec{a}\ R\ \vec{b}$ if $\vec{a} = \vec{b}$' is an equivalence relation on V.

2. If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$

3. If $\vec{a}$ and $\vec{b}$ represent a side and a diagonal of a parallelogram, find the other sides and the other diagonal.

4. Show that the points whose position vectors are $2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and, $6\hat{\imath} - 5\hat{\jmath} + 7\hat{k}$ are collinear.

5. Verify whether the following ratios are direction cosines of some vector or not.

   (i) $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$ (ii) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ (iii) $\frac{4}{3}, 0, \frac{3}{4}$

6. Find the direction cosines of a vector whose direction ratios are (i) $1,2,3$ (ii) $3,-1,3$ (iii) $0,0,7$

7. Find the direction cosines and direction ratios for the following vectors.

   (i) $3\hat{\imath} - 4\hat{\jmath} + 8\hat{k}$ (ii) $3\hat{\imath} + \hat{\jmath} + \hat{k}$ (iii) $\hat{\jmath}$ (iv) $5\hat{\imath} - 3\hat{\jmath} - 48\hat{k}$ (v) $3\hat{\imath} - 3\hat{k} + 4\hat{\jmath}$ (vi) $\hat{\imath} - \hat{k}$

8. If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$ are the direction cosines of some vector, then find a.

9. Find the value of λ for which the vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$ are parallel.

10. Find the value or values of m for which $m(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector.

11. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ prove that $\vec{a}$ and $\vec{b}$ are perpendicular.

12. Find the value λ for which the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, where $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}$.

13. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 10$, $|\vec{b}| = 15$ and $\vec{a} \cdot \vec{b} = 75\sqrt{2}$, find the angle between $\vec{a}$ and $\vec{b}$.

14. Show that the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$, $\vec{b} = 6\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, and $\vec{c} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ are mutually orthogonal.

15. If $|\vec{a}| = 5, |\vec{b}| = 6, |\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

16. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

17. Find the magnitude of $\vec{a} \times \vec{b}$ if $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$.

18. Show that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$.

9.DIFFERENTIAL CALCULUS-LIMITS AND COUTINUITY

1. Evaluate the following limits :

   (1) $\lim_{x\to 1} \frac{x^m - 1}{x^n - 1}$, m and n are integers. (2) $\lim_{\sqrt{x}\to 3} \frac{x^2-8}{\sqrt{x}-3}$ (3) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}, x > 0$

2. Evaluate $\lim_{x\to 2} \frac{1}{(x-2)^3}$.

3. Calculate $\lim_{x\to \infty} \frac{1-x^3}{3x+2}$.

4. (a) Find the left and right limits of $f(x) = \frac{x^2-4}{(x^2+4x+4)(x+3)}$ at $x = -2$. (b) $f(x) = \tan x$ at $x = \frac{\pi}{2}$.

5. Evaluate the following limits: 9. $\lim_{x\to \pi} (1 + \sin\ x)^{2\text{cosec}\ x}$

   1. $\lim_{x\to 0} (1 + x)^{1/3x}$ 2. $\lim_{x\to \infty} \left(1 + \frac{k}{x}\right)^{\frac{m}{x}}$ 3. $\lim_{\alpha\to 0} \frac{\sin\ (\alpha^n)}{(\sin\ \alpha)^m}$ 4. $\lim_{x\to 0} \frac{2\arcsin\ x}{3x}$

   5. $\lim_{x\to 0} \frac{\sin\ (a+x) - \sin\ (a-x)}{x}$ 6. $\lim_{x\to 0} \frac{\tan\ 2x}{\sin\ 5x}$ 7. $\lim_{x\to 0} \frac{1-\cos\ x}{x^2}$ 8. $\lim_{x\to 0} \frac{2^x - 3^x}{x}$

6. Prove that $f(x) = 2x^2 + 3x - 5$ is continuous at all points in $\mathbb{R}$.

7. Examine the continuity of the following: (i) $x + \sin\ x$

10.DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

1. Differentiate the following

   1. $y = e^x \sin\ x$ 2. $y = \frac{\tan\ x}{x}$ 3. $y = \frac{\sin\ x}{1+\cos\ x}$ 4. $y = \frac{x}{\sin\ x + \cos\ x}$

   5. $y = \frac{\tan\ x - 1}{\sec\ x}$ 6. $y = \frac{\sin\ x}{x^2}$ 7. $y = \tan\ \theta(\sin\ \theta + \cos\ \theta)$ 8. $y = \text{cosec}\ x \cdot \cot\ x$

2. Find $\frac{dy}{dx}$ if $x = at^2; y = 2at,\ t \neq 0$.

3. Differentiate the following

   1. $y = \cos\ (\tan\ x)$ 2. $y = e^{\sqrt{x}}$ 3. $f(t) = \sqrt[3]{1 + \tan\ t}$ 4. $y = 4\sec\ 5x$

   5. $y = \tan\ (\cos\ x)$ 6. $y = \sqrt{1 + 2\tan\ x}$ 7. $y = \sin^3\ x + \cos^3\ x$ 8. $y = \sin^2\ (\cos\ kx)$

11.INTEGRAL CALCULUS

1. Integrate the following with respect to

(1) (i) x^{11} (ii) $\frac{1}{x^7}$ (iii) $\sqrt[3]{x^4}$ (iv) $(x^5)^{\frac{1}{8}}$

(2) (i) $\frac{1}{\sin^2 x}$ (ii) $\frac{\tan x}{\cos x}$ (iii) $\frac{\cos x}{\sin^2 x}$ (iv) $\frac{1}{\cos^2 x}$

(3) (i) 12^3 (ii) $\frac{x^{24}}{x^{25}}$ (iii) e^x

(4) (i) $(1+x^2)^{-1}$ (ii) $(1-x^2)^{-\frac{1}{2}}$

2. Integrate the following functions with respect to x

(1) (i) $(x+5)^6$ (ii) $\frac{1}{(2-3x)^4}$ (iii) $\sqrt{3x+2}$

(2) (i) $\sin 3x$ (ii) $\cos(5-11x)$ (iii) $\operatorname{cosec}^2(5x-7)$

(3) (i) e^{3x-6} (ii) e^{8-7x} (iii) $\frac{1}{6-4x}$

(4) (i) $\sec^2 \frac{x}{5}$ (ii) $\operatorname{cosec}(5x+3)\cot(5x+3)$ (iii) $\sec(2-15x)\tan(2-15x)$

(5) (i) $\frac{1}{\sqrt{1-(4x)^2}}$ (ii) $\frac{1}{\sqrt{1-81x^2}}$ (iii) $\frac{1}{1+36x^2}$

3. Integrate the following with respect to x

(i) $5x^4$ (ii) $5x^2-4+\frac{7}{x}+\frac{2}{\sqrt{x}}$ (iii) $2\cos x-4\sin x+5\sec^2 x+\operatorname{cosec}^2 x$ (iv) $\frac{\sin^2 x}{1+\cos x}$

4. Integrate the following with respect to x

(i) $e^{-3x}\cos x$ (ii) $e^{2x}\sin x$

12. INTRODUCTION TO PROBABILITY THEORY

1. A man has 2 ten rupee notes, 4 hundred rupee notes and 6 five hundred rupee notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred rupee denomination and also its probability?

2. What is the chance that (i) non-leap year (ii) leap year should have fifty three Sundays?

3. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer chosen is a prime or multiple of 8 ?

4. A single card is drawn from a pack of 52 cards. What is the probability that

(i) the card is an ace or a king (ii) the card will be 6 or smaller (iii) the card is either a queen or 9 ?

5. A cricket club has 16 members, of whom only 5 can bowl. What is the probability that in a team of 11 members at least 3 bowlers are selected?

6. (i) The odds that the event A occurs is 5 to 7 , find $P(A)$.

(ii) Suppose $P(B)=\frac{2}{5}$. Express the odds that the event B occurs

7. If A and B are two events associated with a random experiment for which $P(A)=0.35$,

$P(A \text{ or } B) = 0.85$, and $P(A \text{ and } B) = 0.15$. Find (i) $P(\text{ only } B)$ (ii) $P(\bar{B})$ (iii) $P($ only $)$

8. Can two events be mutually exclusive and independent simultaneously?
9. If A and B are two events such that $P(A \cup B) = 0.7, P(A \cap B) = 0.2$, and $P(B) = 0.5$, then show that A and B are independent.
10. If A and B are two independent events such that $P(A \cup B) = 0.6, P(A) = 0.2$, find $P(B)$.
11. If $P(A) = 0.5, P(B) = 0.8$ and $P(B/A) = 0.8$, find $P(A/B)$ and $P(A \cup B)$.
12. If for two events A and $B, P(A) = \frac{3}{4}, P(B) = \frac{2}{5}$ and $A \cup B = S$ (sample space), find the conditional probability $P(A/B)$.

IMPORTANT 3 MARK QUESTIONS.

1.SETS RELATIONS AND FUNCTIONS

1. Prove that $\big((A \cup B' \cup C) \cap (A \cap B' \cap C')\big) \cup \big((A \cup B \cup C') \cap (B' \cap C')\big) = B' \cap C'$.
2. If $X = \{1,2,3, \ldots 10\}$ and $A = \{1,2,3,4,5\}$, find the number of sets $B \subseteq X$ such that $A - B = \{4\}$
3. For a set $A, A \times A$ contains 16 elements and two of its elements are $(1,3)$ and $(0,2)$. Find the elements of A.
4. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B, where x, y, z are distinct elements.
5. In the set $\mathbb{Z}$ of integers, define mRn if $m - n$ is a multiple of 12 . Prove that R is an equivalence relation.
6. Let $X = \{a, b, c, d\}$ and $R = \{(a, a), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
7. Let $A = \{a, b, c\}$ and $R = \{(a, a), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
8. Let P be the set of all triangles in a plane and R be the relation defined on P as aRb if a is similar to b. Prove that R is an equivalence relation.
9. Find the domain of $f(x) = \frac{1}{1-2\cos x}$.
10. Let $f, g: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. Find $f \circ g$.
11. If $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.
12. Find the domain of $\frac{1}{1-2\sin x}$.
13. Find the range of the function $\frac{1}{2\cos x - 1}$.
14. If $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse.
15. The distance of an object falling is a function of time t and can be expressed as $s(t) = -16t^2$. Graph the function and determine if it is one-to-one.
16. The total cost of airfare on a given route is comprised of the base cost C and the fuel surcharge S in rupee. Both C and S are functions of the mileage m; $C(m) = 0.4m + 50$ and $S(m) = 0.03m$. Determine a function for the total cost of a ticket in terms of the mileage and find the airfare for flying 1600 miles.
17. A salesperson whose annual earnings can be represented by the function $A(x) = 30{,}000 + 0.04x$, where x is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function $S(x) = 25{,}000 + 0.05x$. Find $(A + S)(x)$ and determine the total family income if they each sell Rupees 1,50,00,000 worth of merchandise.
18. The function for exchanging American dollars for Singapore Dollar on a given day is $(x) = 1.23x$, where x represents the number of American dollars. On the same day the function for exchanging Singapore Dollar to Indian Rupee is $g(y) = 50.50y$, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of Indian rupee.

19. The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100 . He estimates that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function $D(x) = 200 - x$. Express his day revenue, total cost and profit on this meal as functions of x.

2.BASIC ALGEBRA

1. Prove that $\sqrt{3}$ is an irrational number.
2. Solve $\frac{1}{|2x-1|} < 6$ and express the solution using the interval notation.
3. Solve $-3|x| + 5 \leq -2$ and graph the solution set in a number line.
4. Solve $2|x+1| - 6 \leq 7$ and graph the solution set in a number line.
5. Solve $\frac{1}{5}|10x - 2| < 1$.
6. Solve the following system of linear inequalities. $3x - 9 \geq 0,\ 4x - 10 \leq 6$
7. A girl A is reading a book having 446 pages and she has already finished reading 271 pages. She wants to finish reading this book within a week. What is the minimum number of pages she should read per day to complete reading the book within a week?
8. Solve: (i) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$. (ii) $\frac{5-x}{3} < \frac{x}{2} - 4$.
9. To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84,87,95,91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A grade in the course?
10. Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40 .
11. A model rocket is launched from the ground. The height h reached by the rocket after t seconds from lift off is given by $h(t) = -5t^2 + 100t, 0 \leq t \leq 20$. At what time the rocket is 495 feet above the ground?
12. A plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the second scheme he will be paid rupees 120 per hour.
    If he works x hours, then for what value of x does the first scheme give better wages?
13. A and B are working on similar jobs but their monthly salaries differ by more than Rs 6000 . If B earns rupees 27000 per month, then what are the possibilities of A's salary per month?
14. A quadratic polynomial has one of its zeros $1 + \sqrt{5}$ and it satisfies $p(1) = 2$. Find the quadratic polynomial.
15. If α and β are the roots of the quadratic equation $x^2 + \sqrt{2}x + 3 = 0$, form a quadratic polynomial with zeroes $\frac{1}{\alpha}, \frac{1}{\beta}$.
16. If one root of $k(x-1)^2 = 5x - 7$ is double the other root, show that $k = 2$ or -25.

17. If the difference of the roots of the equation $2x^2 - (a+1)x + a - 1 = 0$ is equal to their product, then prove that $a = 2$.
18. Find the condition that one of the roots of $ax^2 + bx + c$ may be (i) negative of the other, (ii) thrice the other, (iii) reciprocal of the other.
19. If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots, then prove that $ae = 2(b + f)$.
20. Solve $\sqrt{x + 14} < x + 2$.
21. Solve $2x^2 + x - 15 \leq 0$.
22. Solve $-x^2 + 3x - 2 \geq 0$.
23. If $x = -2$ is one root of $x^3 - x^2 - 17x = 22$, then find the other roots of equation.
24. Find the real roots of $x^4 = 16$.
25. Factorize: $x^4 + 1$. (Hint: Try completing the square.)
26. If $x^2 + x + 1$ is a factor of the polynomial $3x^3 + 8x^2 + 8x + a$, then find the value of a.
27. Resolve into partial fractions: $\frac{x}{(x+3)(x-4)}$.
28. Resolve the following rational expressions into partial fractions.

    1. $\frac{1}{x^2 - a^2}$ 2. $\frac{3x+1}{(x-2)(x+1)}$

29. Find the square root of $7 - 4\sqrt{3}$.
30. Evaluate $\left(\left((256)^{-1/2}\right)^{\frac{-1}{4}}\right)^3$.
31. If $\left(x^{1/2} + x^{-1/2}\right)^2 = 9/2$, then find the value of $\left(x^{1/2} - x^{-1/2}\right)$ for $x > 1$,
32. Simplify and hence find the value of n: $3^{2n}9^2 3^{-n}/3^{3n} = 27$.
33. If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.
34. Given that $\log_{10} 2 = 0.30103, \log_{10} 3 = 0.47712$ (approximately), find the number of digits in $2^8 \cdot 3^{12}$.
35. If $a^2 + b^2 = 7ab$, show that $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$.
36. Prove that $\log 2 + 16\log \frac{16}{15} + 12\log \frac{25}{24} + 7\log \frac{81}{80} = 1$.
37. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that $xyz = 1$.
38. Solve $\log_2 x - 3\log_{\frac{1}{2}} x = 6$.
39. Solve $\log_{5-x}(x^2 - 6x + 65) = 2$.

3.TRIGONOMETRY

1. If $a\cos\theta - b\sin\theta = c$, show that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$.

2. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$, prove that

   (i) $\sin^4\alpha + \sin^4\beta = 2\sin^2\alpha\sin^2\beta$ (ii) $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha} = 1$.

3. If $x = \sum_{n=0}^{\infty}\cos^{2n}\theta, y = \sum_{n=0}^{\infty}\sin^{2n}\theta$ and $z = \sum_{n=0}^{\infty}\cos^{2n}\theta\sin^{2n}\theta, 0 < \theta < \frac{\pi}{2}$, then show that $xyz = x+y+z$.

   [Hint: Use the formula $1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$, where $|x| < 1$].

4. If $\cot\theta(1+\sin\theta) = 4m$ and $\cot\theta(1-\sin\theta) = 4n$, then prove that $(m^2-n^2)^2 = mn$.

5. If in two circles, arcs of the same length subtend angles $60°$ and $75°$ at the centre, find the ratio of their radii.

6. An airplane propeller rotates 1000 times per minute. Find the number of degrees that a point on the edge of the propeller will rotate in 1 second.

7. A train is moving on a circular track of 1500 m radius at the rate of 66 km/hr. What angle will it turn in 20 seconds?

8. A circular metallic plate of radius 8 cm and thickness 6 mm is melted and molded into a pie (a sector of the circle with thickness) of radius 16 cm and thickness 4 mm. Find the angle of the sector.

9. Prove that $\tan(315°)\cot(-405°) + \cot(495°)\tan(-585°) = 2$

10. Prove that $\frac{\cot(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\tan(-\theta)\operatorname{cosec}(360°+\theta)} = \cos^2\theta\cot\theta$.

11. Show that $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$.

12. Point $A(9,12)$ rotates around the origin O in a plane through $60°$ in the anticlockwise direction to a new position B. Find the coordinates of the point B.

13. A ripple tank demonstrates the effect of two water waves being added together. The two waves are described by $h = 8\cos t$ and $h = 6\sin t$, where $t \in [0,2\pi)$ is in seconds and h is the height in millimeters above still water. Find the maximum height of the resultant wave and the value of t at which it occurs.

14. Find $\cos(x-y)$, given that $\cos x = -\frac{4}{5}$ with $\pi < x < \frac{3\pi}{2}$ and $\sin y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$.

15. Expand $\cos(A+B+C)$. Hence prove that $\cos A\cos B\cos C = \sin A\sin B\cos C + \sin B\sin C\cos A + \sin C\sin A\cos B$ if $A+B+C = \frac{\pi}{2}$.

16. If $a\cos(x+y) = b\cos(x-y)$, show that $(a+b)\tan x = (a-b)\cot y$.

17. Show that $\tan 75° + \cot 75° = 4$.

18. If $\tan x = \frac{n}{n+1}$ and $\tan y = \frac{1}{2n+1}$, find $\tan(x+y)$.

19. Prove that $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right) = -1$.

20. Find the values of $\tan(\alpha+\beta)$, given that $\cot\alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec\beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$.

21. Find the values of (i) $\sin 18^\circ$ (ii) $\cos 18^\circ$ (iii) $\sin 72^\circ$ (iv) $\cos 36^\circ$ (v) $\sin 54^\circ$

22. If $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$, show that $\cos 3\theta = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

23. Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.

24. If $A+B = 45^\circ$, show that $(1+\tan A)(1+\tan B) = 2$.

25. Prove that $(1+\tan 1^\circ)(1+\tan 2^\circ)(1+\tan 3^\circ)\ldots(1+\tan 44^\circ)$ is a multiple of 4 .

26. Prove that $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right) = 2\tan 2\theta$.

27. Show that $\frac{\sin 8x\cos x - \sin 6x\cos 3x}{\cos 2x\cos x - \sin 3x\sin 4x} = \tan 2x$.

28. Show that $\frac{(\cos\theta-\cos 3\theta)(\sin 8\theta+\text{si}\ 2\theta)}{(\sin 5\theta-\sin\theta)(\cos 4\theta-\cos 6\theta)} = 1$.

29. Prove that $\frac{\sin 4x+\sin 2x}{\cos 4x+\cos 2x} = \tan 3x$.

30. Prove that $\cos(30^\circ-A)\cos(30^\circ+A)+\cos(45^\circ-A)\cos(45^\circ+A) = \cos 2A+\frac{1}{4}$.

31. Prove that $\frac{\sin(4A-2B)+\sin(4B-2A)}{\cos(4A-2B)+\cos(4B-2A)} = \tan(A+B)$.

32. Solve the following equations for which solutions lies in the interval $0^\circ \le \theta < 360^\circ$

(i) $\sin^4 x = \sin^2 x$ (ii) $2\cos^2 x + 1 = -3\cos x$

33. In a $\triangle ABC$, prove that $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$

34. In any $\triangle ABC$, prove that $a\cos A + b\cos B + c\cos C = \frac{8\triangle^2}{abc}$.

35. A researcher wants to determine the width of a pond from east to west, which cannot be done by actual measurement. From a point P, he finds the distance to the eastern-most point of the pond to be 8 km, while the distance to the western most point from P to be 6 km. If the angle between the two lines of sight is 60°, find the width of the pond.

36. Two Navy helicopters A and B are flying over the Bay of Bengal at same altitude from the sea level to search a missing boat. Pilots of both the helicopters sight the boat at the same time while they are apart 10 km from each other. If the distance of the boat from A is 6 km and if the line segment AB subtends 60° at the boat, find the distance of the boat from B.

37. A man starts his morning walk at a point A reaches two points B and C and finally back to A such that $\angle A = 60^\circ$ and $\angle B = 45^\circ, AC = 4$ km in the $\triangle ABC$. Find the total distance he covered during his morning walk.

38. Two vehicles leave the same place P at the same time moving along two different roads. One vehicle moves at an average speed of 60 km/hr and the other vehicle moves at an average speed of 80 km/hr. After half an hour the vehicle reach the destinations A and B. If AB subtends $60°$ at the initial point P, then find AB.

39. A man standing directly opposite to one side of a road of width x meter views a circular shaped traffic green signal of diameter a meter on the other side of the road. The bottom of the green signal is b meter height from the horizontal level of viewer's eye. If α denotes the angle subtended by the diameter of the green signal at the viewer's eye, then prove that

$$\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right).$$

4. COMBINATORICS AND MATHEMATICAL INDUCTION

1. Four children are running a race.
   (i) In how many ways can the first two places be filled?
   (ii) In how many different ways could they finish the race?
2. Count the numbers between 999 and 10000 subject to the condition that there are (i) no restriction. (ii) no digit is repeated. (iii) at least one of the digits is repeated.
3. To travel from a place A to place B, there are two different bus routes B_1, B_2, two different train routes T_1, T_2 and one air route A_1. From place B to place C there is one bus route say B_1', two different train routes say T_1', T_2' and one air route A_1'. Find the number of routes of commuting from place A to place C via place B without using similar mode of transportation.
4. How many numbers are there between 1 and 1000 (both inclusive) which are divisible neither by 2 nor by 5 ?
5. Prove that $\frac{(2n)!}{n!} = 2^n(1.3.5\cdots(2n-1))$
6. If the letters of the word TABLE are permuted in all possible ways and the words thus formed are arranged in the dictionary order (alphabetical order), find the ranks of the words (i) TABLE, (ii) BLEAT
7. Find the number of ways of arranging the letters of the word RAMANUJAN so that the relative positions of vowels and consonants are not changed.
8. How many paths are there from start to end on a 6 × 4 grid as shown in the picture?

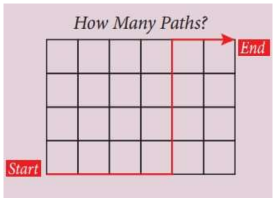


Figure 4.15

9. If ${}^{(n-1)}P_3 : {}^{n}P_4 = 1:10$, find n.
10. If ${}^{10}P_{r-1} = 2 \times {}^{6}P_r$, find r.
11. 8 women and 6 men are standing in a line.
    (i) How many arrangements are possible if any individual can stand in any position?
    (ii) In how many arrangements will all 6 men be standing next to one another?
    (iii) In how many arrangements will no two men be standing next to one another?
12. Prove that ${}^{35}C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4 = {}^{40}C_5$.

13. How many triangles can be formed by 15 points, in which 7 of them lie on one line and the remaining 8 on another parallel line?
14. There are 11 points in a plane. No three of these lies in the same straight line except 4 points, which are collinear. Find,
   (i) the number of straight lines that can be obtained from the pairs of these points?
   (ii) the number of triangles that can be formed for which the points are their vertices?
15. Each of the digits 1,1,2,3,3 and 4 is written on a separate card. The six cards are then laid out in a row to form a 6-digit number.
   (i) How many distinct 6-digit numbers are there?
   (ii) How many of these 6-digit numbers are even?
   (iii) How many of these 6-digit numbers are divisible by 4 ?
16. If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words (i) GARDEN (ii) DANGER.
17. Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary, what will be the 85^{th} string?
18. If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY.
19. Find the sum of all 4-digit numbers that can be formed using digits 1,2,3,4, and 5 repetitions not allowed?
20. Find the sum of all 4-digit numbers that can be formed using digits 0,2,5,7,8 without repetition?
21. If ${}^{n}P_{r} = 720$, and ${}^{n}C_{r} = 120$, find n, r.
22. Prove that if $1 \leq r \leq n$ then $n \times {}^{(n-1)}C_{r-1} = (n-r+1){}^{n}C_{r-1}$.
23. There are 5 teachers and 20 students. Out of them a committee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees
   (i) a particular teacher is included? (ii) a particular student is excluded?
24. Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans, so that always Indians will be the majority in the committee.
25. A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of
   (i) exactly 3 women? (ii) at least 3 women? (iii) at most 3 women?
26. 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife' s relatives?
27. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

5. BINOMIAL THEOREM, SEQUENCES AND SERIES

1. Compute (i) 102^4
2. If n is a positive integer, using Binomial theorem, show that, $9^{n+1}-8n-9$ is always divisible by 64.
3. If a and b are distinct integers, prove that $a-b$ is a factor of a^n-b^n, whenever n is a positive integer. [Hint: write $a^n=(a-b+b)^n$ and expand].
4. Prove that $C_0^2+C_1^2+C_2^2+\cdots+C_n^2=\frac{(2n)!}{(n!)^2}$.
5. If the 5th and 9th terms of a harmonic progression are $\frac{1}{19}$ and $\frac{1}{35}$, find the 12th term of the sequence.
6. Write the n^{th} term of the sequence $\frac{3}{1^2 2^2},\frac{5}{2^2 3^2},\frac{7}{3^2 4^2},\ldots$ as a difference of two terms.
7. If a,b,c are in geometric progression, and if $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$, then prove that x,y,z are in arithmetic progression.
8. Find $\sum_{k=1}^{n}\frac{1}{k(k+1)}$.
9. What will Rs.500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?
10. Find $\sum_{n=1}^{\infty}\frac{1}{n^2+5n+6}$.
11. Find $\sqrt[3]{65}$
12. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

    (i) $\frac{1}{5+x}$ (ii) $\frac{2}{(3+4x)^2}$ (iii) $(5+x^2)^{\frac{2}{3}}$ (iv) $(x+2)^{-\frac{2}{3}}$
13. If $y=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots$, then show that $x=y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+\cdots$.

6. TWO DIMENSIONAL ANALYTICAL GEOMETRY

1. A straight rod of the length 6 units, slides with its ends A and B always on the x and y axes respectively. If O is the origin, then find the locus of the centroid of $\triangle OAB$.
2. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $(a(\theta-\sin\theta),a(1-\cos\theta))$
3. A straight rod of length 8 units slides with its ends A and B always on the x and y axes respectively. Find the locus of the mid point of the line segment AB
4. Find the equation of the locus of the point P such that the line segment AB, joining the points $A(1,-6)$ and $B(4,-2)$, subtends a right angle at P.
5. If O is origin and R is a variable point on $y^2=4x$, then find the equation of the locus of the mid-point of the line segment OR.
6. The coordinates of a moving point P are $\left(\frac{a}{2}(\operatorname{cosec}\theta+\sin\theta),\frac{b}{2}(\operatorname{cosec}\theta-\sin\theta)\right)$, where θ is a variable parameter. Show that the equation of the locus P is $b^2x^2-a^2y^2=a^2b^2$.

7. If $P(2,-7)$ is a given point and Q is a point on $2x^2+9y^2=18$, then find the equations of the locus of the mid-point of PQ.
8. Find the equations of the straight lines, making the y-intercept of 7 and angle between the line and the y-axis is $30°$
9. A straight line L with negative slope passes through the point $(9,4)$ cuts the positive coordinate axes at the points P and Q. As L varies, find the minimum value of $|OP|+|OQ|$, where O is the origin.
10. Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of $45°$ with positive the x-axis.
11. Find the equation of the lines make an angle $60°$ with positive x-axis and at a distance $5\sqrt{2}$ units measured from the point $(4,7)$, along the line $x-y+3=0$.
12. If $P(r,c)$ is mid point of a line segment between the axes, then show that $\frac{x}{r}+\frac{y}{c}=2$.
13. Find the equation of the line passing through the point $(1,5)$ and also divides the co-ordinate axes in the ratio $3:10$.
14. Find the equation of the line, if the perpendicular drawn from the origin makes an angle $30°$ with x-axis and its length is 12 .
15. Find the equation of the straight lines passing through (8,3) and having intercepts whose sum is 1
16. Find the equations of a parallel line and a perpendicular line passing through the point $(1,2)$ to the line $3x+4y=7$.
17. Find the nearest point on the line $2x+y=5$ from the origin.
18. A straight line passes through a fixed point (6,8). Find the locus of the foot of the perpendicular drawn to it from the origin O.
19. If a line joining two points $(3,0)$ and $(5,2)$ is rotated about the point $(3,0)$ in counter clockwise direction through an angle $15°$, then find the equation of the line in the new position
20. Find the equations of two straight lines which are parallel to the line $12x+5y+2=0$ and at a unit distance from the point $(1,-1)$.
21. Find the equations of straight lines which are perpendicular to the line $3x+4y-6=0$ and are at a distance of 4 units from (2,1).
22. A ray of light coming from the point $(1,2)$ is reflected at a point A on the x-axis and it passes through the point $(5,3)$. Find the co-ordinates of the point A.
23. A line is drawn perpendicular to $5x=y+7$. Find the equation of the line if the area of the triangle formed by this line with co-ordinate axes is 10sq. units.
24. Find the equation of the pair of lines through the origin and perpendicular to the pair of lines $ax^2+2hxy+by^2=0$
25. Show that the straight lines $x^2-4xy+y^2=0$ and $x+y=3$ form an equilateral triangle.
26. Show that the straight lines joining the origin to the points of intersection of $3x-2y+2=0$ and $3x^2+5xy-2y^2+4x+5y=0$ are at right angles.

27. Find the equation of the pair of straight lines passing through the point (1,3) and perpendicular to the lines $2x-3y+1=0$ and $5x+y-3=0$
28. Find the separate equation of the following pair of straight lines (i) $2x^2-xy-3y^2-6x+19y-20=0$
29. The slope of one of the straight lines $ax^2+2hxy+by^2=0$ is twice that of the other, show that $8h^2=9ab$.
30. The slope of one of the straight lines $ax^2+2hxy+by^2=0$ is three times the other, show that $3h^2=4ab$.
31. Show that the equation $9x^2-24xy+16y^2-12x+16y-12=0$ represents a pair of parallel lines. Find the distance between them.
32. Prove that one of the straight lines given by $ax^2+2hxy+by^2=0$ will bisect the angle between the co-ordinate axes if $(a+b)^2=4h^2$
33. Prove that the straight lines joining the origin to the points of intersection of $3x^2+5xy-3y^2+2x+3y=0$ and $3x-2y-1=0$ are at right angles.

7. MATRICES AND DETERMINANTS

1. A fruit shop keeper prepares 3 different varieties of gift packages. Pack-I contains 6 apples, 3 oranges and 3 pomegranates. Pack-II contains 5 apples, 4 oranges and 4 pomegranates and Pack-III contains 6 apples, 6 oranges and 6 pomegranates. The cost of an apple, an orange and a pomegranate respectively are ₹30, ₹15 and ₹45. What is the cost of preparing each package of fruits?
2. Find the values of $p, q, r,$ and s if $\begin{bmatrix} p^2-1 & 0 & -31-q^3 \\ 7 & r+1 & 9 \\ -2 & 8 & s-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 \\ 7 & \frac{3}{2} & 9 \\ -2 & 8 & -\pi \end{bmatrix}$
3. Consider the matrix $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$
   (i) Show that $A_\alpha A_\beta = A_{(\alpha+\beta)}$.
   (ii) Find all possible real values of α satisfying the condition $A_\alpha + A_\alpha^T = I$.
4. If $A = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and such that $(A-2I)(A-3I)=O$, find the value of x.
5. Show that $f(x)f(y)=f(x+y)$, where $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
6. If A is a square matrix such that $A^2=A$, find the value of $7A-(I+A)^3$.
7. (i) For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric.
   (ii) If $\begin{bmatrix} 0 & p & 3 \\ 2 & q^2 & -1 \\ r & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of $p, q,$ and r.
8. If A and B are symmetric matrices of same order, prove that
   (i) $AB+BA$ is a symmetric matrix.
   (ii) $AB-BA$ is a skew-symmetric matrix.

9. Compute $|A|$ using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$.

10. If a, b, c and x are positive real numbers, then show that $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$ is zero.

11. Without expanding the determinants, show that $|B| = 2|A|$.

Where $B = \begin{bmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{bmatrix}$ and $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

12. Find the value of x if $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

13. Prove that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$.

14. If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$.

15. Using cofactors of elements of second row, evaluate $|A|$, where $A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

16. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$.

17. Show that $\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

18. Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1 - 2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2 - 2x \\ -x^2 & x^2 - 2x & -1 \end{vmatrix}$

19. If A_i, B_i, C_i are the cofactors of a_i, b_i, c_i, respectively, $i = 1$ to 3 in

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \text{ show that } \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = |A|^2$$

20. Show that the points $(a, b+c), (b, c+a)$, and $(c, a+b)$ are collinear.

21. If $(k, 2), (2,4)$ and $(3,2)$ are vertices of the triangle of area 4 square units then determine the value of k.

22. Determine the values of a and b so that the following matrices are singular: (i) $B = \begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

23. If $\cos 2\theta = 0$, determine $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$.

24. Find the value of the product; $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$.

25.8.VECTOR ALGEBRA

1. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.
2. Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.
3. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$.
4. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.
5. Prove that the points whose position vectors $2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}, 4\hat{\imath} + \hat{\jmath} + 9\hat{k}$ and $10\hat{\imath} - \hat{\jmath} + 6\hat{k}$ form a right angled triangle.
6. A triangle is formed by joining the points $(1,0,0), (0,1,0)$ and $(0,0,1)$. Find the direction cosines of the medians.
7. If $(a, a+b, a+b+c)$ is one set of direction ratios of the line joining $(1,0,0)$ and $(0,1,0)$, then find a set of values of a, b, c.
8. Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ form a right angled triangle.
9. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}, \vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, and $\vec{c} = -3\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, find the magnitude and direction cosines of (i) $\vec{a} + \vec{b} + \vec{c}$ (ii) $3\vec{a} - 2\vec{b} + 5\vec{c}$.
10. Find the unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$ if $\vec{a} = 3\hat{\imath} - \hat{\jmath} - 4\hat{k}, \vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$, and $\vec{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$
11. The position vectors of the points P, Q, R, S are $i + j + k, 2\hat{\imath} + 5\hat{\jmath}, 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, and $\hat{\imath} - 6\hat{\jmath} - \hat{k}$ respectively. Prove that the line PQ and RS are parallel.
12. Show that the points $A(1,1,1), B(1,2,3)$ and $C(2,-1,1)$ are vertices of an isosceles triangle.
13. For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}\cdot\hat{\imath})\hat{\imath} + (\vec{r}\cdot\hat{\jmath})\hat{\jmath} + (\vec{r}\cdot\hat{k})\hat{k}$
14. Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ where A, B, C, D are the points $(4,-3,0), (7,-5,-1), (-2,1,3), (0,2,5)$.
15. If $\vec{a}, \vec{b}$, and $\vec{c}$ are three unit vectors satisfying $\vec{a} - \sqrt{3}\vec{b} + \vec{c} = \vec{0}$ then find the angle between $\vec{a}$ and $\vec{c}$.
16. Show that the points $(4,-3,1), (2,-4,5)$ and $(1,-1,0)$ form a right angled triangle.
17. Find the angle between the vectors (i) $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ (ii) $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$.
18. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + 2\vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.
19. Show that the vectors $-\hat{\imath} - 2\hat{\jmath} - 6\hat{k}, 2\hat{\imath} - \hat{\jmath} + \hat{k}$, and $-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ form a right angled triangle.
20. Find the projection of the vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$
21. Find λ, when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.
22. Find the cosine and sine angle between the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

23. Find the area of the parallelogram whose adjacent sides are $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

24. Find the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane which contains $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 4\hat{k}$

25. Find the area of the parallelogram whose two adjacent sides are determined by the vectors $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

26. Find the area of the triangle whose vertices are $A(3,-1,2), B(1,-1,-3)$ and $C(4,-3,1)$.

27. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. Also deduce the condition for collinearity of the points A, B, and C.

28. For any vector $\vec{a}$ prove that $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$.

29. Find the angle between the vectors $2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

9.DIFFERENTIAL CALCULUS-LIMITS AND COUTINUITY

1. Evaluate : $\lim_{x\to 3} \frac{x^2-9}{x-3}$ if it exists by finding $f(3^-)$and $f(3^+)$.
2. Find $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.
3. Find $\lim_{x\to 0} \frac{(2+x)^5-2^5}{x}$.
4. Find the relation between a and b if $\lim_{x\to 3} f(x)$ exists where $f(x) = \begin{cases} ax+b & \text{if } x > 3 \\ 3ax-4b+1 & \text{if } x < 3 \end{cases}$.
5. Evaluate the following limits:

   1. $\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$
   2. $\lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5}$
6. Alcohol is removed from the body by the lungs, the kidneys, and by chemical processes in liver. At moderate concentration levels, the majority work of removing the alcohol is done by the liver; less than 5% of the alcohol is eliminated by the lungs and kidneys. The rate r at which the liver processes alcohol from the bloodstream is related to the blood alcohol concentration x by a rational function of the form $r(x) = \frac{\alpha x}{x+\beta}$ for some positive constants α and β. Find the maximum possible rate of removal.
7. The velocity in ft/sec of a falling object is modeled by $r(t) = -\sqrt{\frac{32}{k}\frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}}$, where k is a constant that depends upon the size and shape of the object and the density of the air. Find the limiting velocity of the object, that is, find $\lim_{t\to\infty} r(t)$.
8. Suppose that the diameter of an animal's pupils is given by $f(x) = \frac{160x^{-0.4}+90}{4x^{-0.4}+15}$, where x is the intensity of light and $f(x)$ is in mm. Find the diameter of the pupils with (a) minimum light (b) maximum light.
9. Evaluate the following limits:

   1. $\lim_{x\to\infty} \frac{3}{x-2} - \frac{2x+11}{x^2+x-6}$
   2. $\lim_{x\to\infty} \left(\frac{x^3}{2x^2-1} - \frac{x^2}{2x+1}\right)$

10. An important problem in fishery science is to estimate the number of fish presently spawning in streams and use this information to predict the number of mature fish or "recruits" that will return to the rivers during the reproductive period. If S is the number of spawners and R the number of recruits, "Beverton-Holt spawner recruit function" is $R(S)=\frac{S}{(\alpha S+\beta)}$ where α and β are positive constants. Show that this function predicts approximately constant recruitment when the number of spawners is sufficiently large.
11. A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water after t minutes (in grams per litre) is $C(t)=\frac{30}{200+t}$. What happens to the concentration as $\rightarrow \infty$?
12. Show that $\lim_{x\to 0^+} x\left[\left\lfloor\frac{1}{x}\right\rfloor+\left\lfloor\frac{2}{x}\right\rfloor+\cdots+\left\lfloor\frac{15}{x}\right\rfloor\right]=120.$
13. Evaluate : $\lim_{x\to\frac{\pi}{4}}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$
14. Evaluate the following limits:

1. $\lim_{x\to\infty}\left(\frac{2x^2+3}{2x^2+5}\right)^{8x^2+3}$ 2. $\lim_{x\to 0}\frac{\sin^3\left(\frac{x}{2}\right)}{x^3}$ 3. $\lim_{x\to 0}\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}$

4. $\lim_{x\to 0}\frac{3^x-1}{\sqrt{x+1}-1}$ 5. $\lim_{x\to\infty}\{x[\log(x+a)-\log(x)]\}$ 6. $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

7. $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}$ 8. $\lim_{x\to 0}\frac{\sin x(1-\cos x)}{x^3}$ 9. $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$

15. A tomato wholesaler finds that the price of a newly harvested tomatoes is ₹0.16 per kg if he purchases fewer than 100kgs each day. However, if he purchases at least 100kgs daily, the price drops to ₹ 0.14 per kg. Find the total cost function and discuss the cost when the purchase is 100kgs.
16. Examine the continuity of the following:

1. $e^x\tan x$ 2. $\frac{x^2-16}{x+4}$ 3. $|x+2|+|x-1$ 4. $\frac{|x-2|}{|x+1|}$

17. Find the constant b that makes g continuous on $(-\infty,\infty)$.

$$g(x)=\begin{cases}x^2-b^2 & \text{if } x<4\\ bx+20 & \text{if } x\geq 4\end{cases}$$

18. Consider the function $f(x)=x\sin\frac{\pi}{x}$. What value must we give $f(0)$ in order to make the function continuous everywhere?

10.DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

1. Differentiate the following with respect to x :

(i) $y=x^3+5x^2+3x+7$ (ii) $y=e^x+\sin x+2$

(iii) $y=4\operatorname{cosec} x-\log x-2e^x$ (iv) $y=\left(x-\frac{1}{x}\right)^2$

(v) $y=xe^x\log x$ (vi) $y=\frac{\cos x}{x^3}$

(vii) $y=\frac{\log x}{e^x}$ (viii) Find $f'(3)$ and $f'(5)$ if $f(x)=|x-4|$.

2. Differentiate the following

1 $y = x\sin x\cos x$ 2. $y = e^{-x}\cdot\log x$ 3.) $y = (x^2+5)\log(1+x)e^{-3x}$

4. $y = \sin x^{\circ}$ 5. $y = \log_{10} x$

3. Differentiate $(2x+1)^5(x^3-x+1)^4$.
4. Differentiate the following

1. $h(t) = \left(t-\frac{1}{t}\right)^{\frac{3}{2}}$ 2. $y = (2x-5)^4(8x^2-5)^{-3}$

3. $y = (x^2+1)\sqrt[3]{x^2+2}$ 4. $s(t) = \sqrt[4]{\frac{t^3+1}{t^3-1}}$

5. $f(x) = \frac{x}{\sqrt{7-3x}}$ 6. $y = \frac{\sin^2 x}{\cos x}$

7. $y = 5^{\frac{-1}{x}}$ 8. $y = \frac{e^{3x}}{1+e^x}$

9. $y = \sqrt{x+\sqrt{x}}$ 10. $y = e^{x\cos x}$

5. Differentiate : $y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$.
6. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y'.
7. Find the derivative of x^x with respect to $x\log x$.
8. Find y'' if $x^4+y^4 = 16$.
9. Find the second order derivative if x and y are given by $x = a\cos t$, $y = a\sin t$
10. Find $\frac{d^2y}{dx^2}$ if $x^2+y^2 = 4$
11. Find the derivatives of the following :

1. $y = x^{\cos x}$ 2. $x^y = y^x$ 3. $\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$ 4, $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

5. If $\cos(xy) = x$, show that $\frac{dy}{dx} = \frac{-(1+y\sin(xy))}{x\sin xy}$ 6. $\tan^{-1}\left(\frac{6x}{1-9x^2}\right)$

7. $x = a\cos^3 t; y = a\sin^3 t$ 8. $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$

9. $\sin^{-1}(3x-4x^3)$ 10. $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$.

11.INTEGRAL CALCULUS

1. Integrate the following with respect to x :

(1) $(x+4)^5+\frac{5}{(2-5x)^4}-\operatorname{cosec}^2(3x-1)$ (2) $\frac{8}{\sqrt{1-(4x)^2}}+\frac{27}{\sqrt{1-9x^2}}-\frac{15}{1+25x^2}$

(3) $\frac{6}{1+(3x+2)^2}-\frac{12}{\sqrt{1-(3-4x)^2}}$ (4) $\frac{1}{3}\cos\left(\frac{x}{3}-4\right)+\frac{7}{7x+9}+e^{\frac{x}{5}+3}$

2. If $f'(x) = 3x^2-4x+5$ and $f(1) = 3$, then find $f(x)$.

3. The rate of change of weight of person w in kg with respect to their height h in centimetres is given approximately by $\frac{dw}{dh} = 4.364 \times 10^{-5} h^2$. Find weight as a function of height. Also find the weight of a person whose height is 150 cm.

4. A tree is growing so that, after t - years its height is increasing at a rate of $\frac{18}{\sqrt{t}}$ cm per year. Assume that when $t = 0$, the height is 5 cm.

   (i) Find the height of the tree after 4 years. (ii) After how many years will the height be 149 cm ?

5. If $f'(x) = 9x^2 - 6x$ and $f(0) = -3$, find $f(x)$.

6. If $f''(x) = 12x - 6$ and $f(1) = 30, f'(1) = 5$ find $f(x)$.

7. Evaluate : $\int \frac{\sin x}{1+\sin x} dx$.

8. Evaluate: $\int (\tan x + \cot x)^2 dx$

9. Integrate the following functions with respect to x :

   1. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ 2. $\cos 3x \cos 2x$ 3. $e^{x \log a} e^x$ 4. $(3x + 4)\sqrt{3x + 7}$

   5. $\frac{8^{1+x} + 4^{1-x}}{2^x}$ 6. $\frac{1}{\sqrt{x+3} - \sqrt{x-4}}$ 7. $\frac{3x-9}{(x-1)(x+2)(x^2+1)}$

10. Integrate the following functions with respect to x :

   1. $\frac{\sin \sqrt{x}}{\sqrt{x}}$ 2. $\frac{\cot x}{\log(\sin x)}$ 3. $\frac{\operatorname{cosec} x}{\log\left(\tan\frac{x}{2}\right)}$ 4. $\frac{\sin 2x}{a^2 + b^2 \sin^2 x}$

   5. $\frac{\sqrt{x}}{1+\sqrt{x}}$ 6. $\frac{1}{x \log x \log(\log x)}$ 7. $x(1-x)^{17}$ 8. $\frac{\cos x}{\cos(x-a)}$

11. Integrate the following with respect to x.

   (i) $\int \tan x dx$ (ii) $\int \cot x dx$ (iii) $\int \operatorname{cosec} x dx$ (iv) $\int \sec x dx$

12. Integrate the following with respect to x.

   (i) $\int 9xe^{3x} dx$ (ii) $\int x \sec x \tan x \, dx$ (iii) $\int x^5 e^{x^2} dx$ (iv) $\sin^{-1} \int \left(\frac{2x}{1+x^2}\right) dx$

   (v) $\int e^x (\sin x + \cos x) dx$

13. Integrate the following with respect to x :

   (i) $e^x(\tan x + \log \sec x)$ (ii) $e^x \sec x(1 + \tan x)$ (iii) $e^{\tan^{-1} x}\left(\frac{1+x+x^2}{1+x^2}\right)$

   (iv) $\frac{\log x}{(1+\log x)^2}$

14. Integrate the following with respect to x :

   (i) $\frac{1}{9x^2-4}$ (ii) $\frac{1}{\sqrt{(2+x)^2-1}}$ (iii) $\frac{1}{\sqrt{9+8x-x^2}}$ (iv) $\frac{5x-2}{2+2x+x^2}$ (v) $\frac{3x+1}{2x^2-2x+3}$

   (vi) $\frac{x+2}{\sqrt{x^2-1}}$ (vii) $\sqrt{x^2 - 2x - 3}$ (viii) $\sqrt{(6-x)(x-4)}$ (ix) $\sqrt{9 - (2x+5)^2}$

   (x) $\sqrt{(x+1)^2 - 4}$

12. INTRODUCTION TO PROBABILITY THEORY

1. An integer is chosen at random from the first ten positive integers. Find the probability that it is (i) an even number (ii) multiple of three.
2. When a pair of fair dice is rolled, what are the probabilities of getting the sum
   (i) 7 (ii) 7 or 9 (iii) 7 or 12 ?
3. Three candidates X, Y, and Z are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. X is thrice as likely to win as Y and Y is twice as likely as to win Z. Find the respective probability of X, Y and Z to win the cup.
4. If A and B are mutually exclusive events $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{8}$, then find
   (i) $P(\bar{A})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap B)$ (iv) $P(\bar{A} \cup \bar{B})$
5. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards.
6. A town has 2 fire engines operating independently. The probability that a fire engine is available when needed is 0.96.
   (i) What is the probability that a fire engine is available when needed?
   (ii) What is the probability that neither is available when needed?
7. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then find the probability of
   (i) $P(A \cup B)$ (ii) $P(A \cap \bar{B})$ (iii) $P(\bar{A} \cap B)$.
8. A main road in a City has 4 crossroads with traffic lights. Each traffic light opens or closes the traffic with the probability of 0.4 and 0.6 respectively. Determine the probability of
   (i) a car crossing the first crossroad without stopping
   (ii) a car crossing first two crossroads without stopping
   (iii) a car crossing all the crossroads, stopping at third cross.
   (iv) a car crossing all the crossroads, stopping at exactly one cross.
9. One bag contains 5 white and 3 black balls. Another bag contains 4 white and 6 black balls. If one ball is drawn from each bag, find the probability that (i) both are white (ii) both are black (iii) one white and one black
10. Given $P(A) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(B)$ if
    (i) A and B are mutually exclusive (ii) A and B are independent events
    (iii) $P(A/B) = 0.4$ (iv) $P(B/A) = 0.5$
11. A year is selected at random. What is the probability that
    (i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays
12. Suppose ten coins are tossed. Find the probability to get (i) exactly two heads (ii) at most two heads
    (iii) at least two heads

IMPORTANT 5 MARK QUESTIONS

1.SETS, RELATIONS AND FUNCTIONS

1. If A and B are two sets so that $n(B-A)=2n(A-B)=4n(A\cap B)$ and if $n(A\cup B)=14$, then find $n(\mathcal{P}(A))$.
2. By taking suitable sets A, B, C, verify the following results:
   (i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$. (ii) $A\times(B\cup C)=(A\times B)\cup(A\times C)$.
   (iii) $(A\times B)\cap(B\times A)=(A\cap B)\times(B\cap A)$. (iv) $C-(B-A)=(C\cap A)\cup(C\cap B')$.
   (v) $(B-A)\cap C=(B\cap C)-A=B\cap(C-A)$. (vi) $(B-A)\cup C=(B\cup C)-(A-C)$.
3. On the set of natural numbers let R be the relation defined by aRb if $2a+3b=30$. Write down the relation by listing all the pairs. Check whether it is
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
4. On the set of natural numbers let R be the relation defined by aRb if $a+b\le 6$. Write down the relation by listing all the pairs. Check whether it is
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
5. In the set Z of integers, define mRn if $m-n$ is divisible by 7 . Prove that R is an equivalence relation.
6. Find the range of the function $f(x)=\frac{1}{1-3\cos x}$.
7. Find the largest possible domain for the real valued function given by $f(x)=\frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$.
8. Let $f,g:\mathbb{R}\to\mathbb{R}$ be defined as $f(x)=2x-|x|$ and $g(x)=2x+|x|$. Find $f\circ g$.
9. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=2x-3$ prove that f is a bijection and find its inverse.
10. Find the largest possible domain of the real valued function $f(x)=\frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.
11. If $f,g:\mathbb{R}\to\mathbb{R}$ are defined by $f(x)=|x|+x$ and $g(x)=|x|-x$, find $g\circ f$ and $f\circ g$.
12. The formula for converting from Fahrenheit to Celsius temperatures is $y=\frac{5x}{9}-\frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function.
13. A simple cipher takes a number and codes it, using the function $\int(x)=3x-4$. Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line $y=x$ (by drawing the lines).
14. Write the values of f at $-4,1,-2,7,0$ if

$$f(x)=\begin{cases}-x+4 & \text{if } -\infty<x\le -3\\ x+4 & \text{if } -3<x<-2\\ x^2-x & \text{if } -2\le x<1\\ x-x^2 & \text{if } 1\le x<7\\ 0 & \text{otherwise}\end{cases}$$

15. Write the values of f at $-3,5,2,-1,0$ if

$$f(x)=\begin{cases}x^2+x-5 & \text{if } x\in(-\infty,0)\\ x^2+3x-2 & \text{if } x\in(3,\infty)\\ x^2 & \text{if } x\in(0,2)\\ x^2-3 & \text{otherwise}\end{cases}$$

2.BASIC ALGEBRA

1. A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?
2. Find the number of solutions of $x^2 + |x - 1| = 1$.
3. Solve the equation $\sqrt{6 - 4x - x^2} = x + 4$.
4. Solve $(2x + 1)^2 - (3x + 2)^2 = 0$.
5. Find the values of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2 .
6. Solve $\frac{x+1}{x+3} < 3$.
7. Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$.
8. Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$.
9. Solve $\frac{x^2-4}{x^2-2x-15} \leq 0$.
10. Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$.
11. Resolve into partial fractions: $\frac{x+1}{x^2(x-1)}$.
12. Solve the linear inequalities and exhibit the solution set graphically: $x + y \geq 3, 2x - y \leq 5, -x + 2y \leq 3$.
13. Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$.
14. If $x = \sqrt{2} + \sqrt{3}$ find $\frac{x^2+1}{x^2-2}$.
15. Determine the region in the plane determined by the inequalities:

    (1) $2x + 3y \leq 35, y \geq 2, x \geq 5$. (2) $2x + 3y \leq 6, x + 4y \leq 4, x \geq 0, y \geq 0$.
16. Resolve the partial fractions

    i) $\frac{x}{(x^2+1)(x-1)(x+2)}$ ii) $\frac{x}{(x-1)^3}$ iii) $\frac{1}{x^4-1}$ iv) $\frac{(x-1)^2}{x^3+x}$

    v) $\frac{x^2+x+1}{x^2-5x+6}$ vi). $\frac{x^3+2x+1}{x^2+5x+6}$ vii) $\frac{x+12}{(x+1)^2(x-2)}$ viii) $\frac{6x^2-x+1}{x^3+x^2+x+1}$

    ix) $\frac{2x^2+5x-11}{x^2+2x-3}$ x) $\frac{7+x}{(1+x)(1+x^2)}$

3.TRIGONOMETRY

1. If $\sin\,\theta + \cos\,\theta = m$, show that $\cos^6\,\theta + \sin^6\,\theta = \frac{4-3(m^2-1)^2}{4}$, where $m^2 \leq 2$.
2. If $\tan^2\,\theta = 1 - k^2$, show that $\sec\,\theta + \tan^3\,\theta \text{cosec}\,\theta = (2 - k^2)^{3/2}$. Also, find the values of k for which this result holds.

3. Eliminate θ from the equations $a\sec\theta - c\tan\ \theta = b$ and $b\sec\theta + d\tan\ \theta = c$.

4. If $x\cos\ \theta = y\cos\ \left(\theta + \frac{2\pi}{3}\right) = z\cos\ \left(\theta + \frac{4\pi}{3}\right)$, find the value of $xy + yz + zx$.

5. If $\cos\ (\alpha - \beta) + \cos\ (\beta - \gamma) + \cos\ (\gamma - \alpha) = -\frac{3}{2}$, then prove that

   $\cos\ \alpha + \cos\ \beta + \cos\ \gamma = \sin\ \alpha + \sin\ \beta + \sin\ \gamma = 0$.

6. If $\theta + \phi = \alpha$ and $\tan\ \theta = k\tan\ \phi$, then prove that $\sin\ (\theta - \phi) = \frac{k-1}{k+1}\sin\ \alpha$.

7. Prove that $\sin x = 2^{10}\sin\left(\frac{x}{2^{10}}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\ldots\cos\left(\frac{x}{2^{10}}\right)$

8. Prove that $\cos\ A\cos\ 2A\cos\ 2^2A\cos\ 2^3A\ldots\cos\ 2^{n-1}A = \frac{\sin\ 2^nA}{2^n\sin\ A}$

9. Show that $\cot\ \left(7\frac{1}{2}^{\circ}\right) = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$.

10. Prove that $(1 + \sec\ 2\theta)(1 + \sec\ 4\theta)\ldots(1 + \sec\ 2^n\theta) = \tan\ 2^n\theta\cot\ \theta$.

11. Prove that $32(\sqrt{3})\sin\ \frac{\pi}{48}\cos\ \frac{\pi}{48}\cos\ \frac{\pi}{24}\cos\ \frac{\pi}{12}\cos\ \frac{\pi}{6} = 3$.

12. Show that $\cos\ 36^{\circ}\cos\ 72^{\circ}\cos\ 108^{\circ}\cos\ 144^{\circ} = \frac{1}{16}$.

13. Show that $\cos\ 10^{\circ}\cos\ 30^{\circ}\cos\ 50^{\circ}\cos\ 70^{\circ} = \frac{3}{16}$.

14. Show that $\sin\ 12^{\circ}\sin\ 48^{\circ}\sin\ 54^{\circ} = \frac{1}{8}$.

15. Show that $\cos\ \frac{\pi}{15}\cos\ \frac{2\pi}{15}\cos\ \frac{3\pi}{15}\cos\ \frac{4\pi}{15}\cos\ \frac{5\pi}{15}\cos\ \frac{6\pi}{15}\cos\ \frac{7\pi}{15} = \frac{1}{128}$.

16. Prove that $\frac{\sin\ x+\sin\ 3x+\sin\ 5x+\sin\ 7x}{\cos\ x+\cos\ 3x+\cos\ 5x+\cos\ 7x} = \tan\ 4x$.

17. Show that $\cot\ (A + 15^{\circ}) - \tan\ (A - 15^{\circ}) = \frac{4\cos\ 2A}{1+2\sin\ 2A}$.

18. If $A + B + C = \pi$, prove the following (i) $\cos\ A + \cos\ B + \cos\ C = 1 + 4\sin\ \left(\frac{A}{2}\right)\sin\ \left(\frac{B}{2}\right)\sin\ \left(\frac{C}{2}\right)$

19. Prove that $\sin\ \frac{A}{2} + \sin\ \frac{B}{2} + \sin\ \frac{C}{2} = 1 + 4\sin\ \left(\frac{\pi-A}{4}\right)\sin\ \left(\frac{\pi-B}{4}\right)\sin\ \left(\frac{\pi-C}{4}\right)$, if $A + B + C = \pi$.

20. If $A + B + C = \pi$, prove that $\cos^2\ A + \cos^2\ B + \cos^2\ C = 1 - 2\cos\ A\cos\ B\cos\ C$.

21. If $A + B + C = 180^{\circ}$, prove that

    (i) $\sin\ 2A + \sin\ 2B + \sin\ 2C = 4\sin\ A\sin\ B\sin\ C$

    (ii) $\cos\ A + \cos\ B - \cos\ C = -1 + 4\cos\ \frac{A}{2}\cos\ \frac{B}{2}\sin\ \frac{C}{2}$

22. If $A + B + C = \frac{\pi}{2}$, prove the following

    (i) $\sin\ 2A + \sin\ 2B + \sin\ 2C = 4\cos\ A\cos\ B\cos\ C$

    (ii) $\cos\ 2A + \cos\ 2B + \cos\ 2C = 1 + 4\sin\ A\sin\ B\sin\ C$.

23. Solve the following equations:

(i) $\sin\ 5x - \sin\ x = \cos\ 3x$ (ii) $\sin\ \theta + \cos\ \theta = \sqrt{2}$ (iii) $\sin\ \theta + \sqrt{3}\cos\ \theta = 1$

24. In a $\triangle ABC$, if $\cos\ C = \frac{\sin\ A}{2\sin\ B}$, show that the triangle is isosceles.

25. In a $\triangle ABC$, prove that $\frac{\sin\ B}{\sin\ C} = \frac{c-aco\ \ B}{b-aco\ \ C}$.

26. In a $\triangle ABC$, prove the following (i) $\frac{a+b}{a-b} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$.

27. Derive Projection formula from (i) Law of sines, (ii) Law of cosines.

28. State and prove Napier Formula.

29. Suppose two radar stations located 100 km apart, each detect a fighter aircraft between them. The angle of elevation measured by the first station is 30°, whereas the angle of elevation measured by the second station is 45°. Find the altitude of the aircraft at that instant.

30. A farmer wants to purchase a triangular shaped land with sides 120 feet and 60 feet and the angle included between these two sides is 60°. If the land costs ₹500 per sq.ft, find the amount he needed to purchase the land. Also find the perimeter of the land.

31. A fighter jet has to hit a small target by flying a horizontal distance. When the target is sighted, the pilot measures the angle of depression to be 30°. If after 100 km, the target has an angle of depression of 60°, how far is the target from the fighter jet at that instant?

32. Suppose that a satellite in space, an earth station and the centre of earth all lie in the same plane. Let r be the radius of earth and R be the distance from the centre of earth to the satellite. Let d be the distance from the earth station to the satellite. Let 30° be the angle of elevation from the earth station to the satellite. If the line segment connecting earth station and satellite subtends angle α at the centre of earth, then prove that

$d = R\sqrt{1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\ \alpha}$.

4.COMBINATORICS AND MATHEMATICAL INDUCTION

1. How many strings can be formed using the letters of the word LOTUS if the word
(i) either starts with L or ends with S ? (ii) neither starts with L nor ends with S ?

2. Prove that $nc_r + nc_{r-1} = n{+}1\ c_r$.

3. By the principle of mathematical induction, prove that, for all integers $n \geq 1$,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

4. Prove that the sum of first n positive odd numbers is n^2.

5. By the principle of mathematical induction, prove that, for all integers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

6. Prove that for any natural number $n, a^n - b^n$ is divisible by $a - b$, where $a > b$
7. Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.
8. By the principle of mathematical induction, prove that, for $n \geq 1$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$
9. By the principle of mathematical induction, prove that, for $n \geq 1$
$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$
10. Prove that the sum of the first n non-zero even numbers is $n^2 + n$.
11. By the principle of Mathematical induction, prove that, for $n \geq 1$
$$1.2 + 2.3 + 3.4 + \cdots + n.(n+1) = \frac{n(n+1)(n+2)}{3}.$$
12. Using the Mathematical induction, show that for any natural number $n \geq 2$,
$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$
13. Using the Mathematical induction, show that for any natural number $n \geq 2$,
$$\frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+n} = \frac{n-1}{n+1}.$$
14. Using the Mathematical induction, show that for any natural number n,
$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots + \frac{1}{n.(n+1).(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$
15. Using the Mathematical induction, show that for any natural number n,
$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}.$$
16. Prove by Mathematical Induction that
$$1! + (2 \times 2!) + (3 \times 3!) + \cdots + (n \times n!) = (n+1)! - 1.$$
17. Using the Mathematical induction, show that for any natural number $n, x^{2n} - y^{2n}$ is divisible by $x + y$.
18. By the principle of Mathematical induction, prove that, for $n \geq 1$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 > \frac{n^3}{3}.$$
19. Use induction to prove that $n^3 - 7n + 3$, is divisible by 3 , for all natural numbers n.
20. Use induction to prove that $5^{n+1} + 4 \times 6^n$ when divided by 20 leaves a remainder 9 , for all natural numbers n.
21. Use induction to prove that $10^n + 3 \times 4^{n+2} + 5$, is divisible by 9 , for all natural numbers n.
22. Prove that using the Mathematical induction

$$\sin(\alpha)+\sin\left(\alpha+\frac{\pi}{6}\right)+\sin\left(\alpha+\frac{2\pi}{6}\right)+\cdots+\sin\left(\alpha+\frac{(n-1)\pi}{6}\right)=\frac{\sin\left(\alpha+\frac{(n-1)\pi}{12}\right)\times\sin\left(\frac{n\pi}{12}\right)}{\sin\left(\frac{\pi}{12}\right)}.$$

5.BINOMIAL THEOREM, SEQUENCES AND SERIES

1. Find the coefficient of x^4 in the expansion of $(1+x^3)^{50}\left(x^2+\frac{1}{x}\right)^5$.
2. Find the constant term of $\left(2x^3-\frac{1}{3x^2}\right)^5$.
3. In the binomial expansion of $(a+b)^n$, if the coefficients of the 4th and 13th terms are equal then, find n.
4. If the binomial coefficients of three consecutive terms in the expansion of $(a+x)^n$ are in the ratio $1:7:42$, then find n.
5. In the binomial expansion of $(1+x)^n$, the coefficients of the 5th, 6th and 7th terms are in AP. Find all values of n.
6. Prove that if a, b, c are in HP, if and only if $\frac{a}{c}=\frac{a-b}{b-c}$.
7. Find seven numbers $A_1, A_2, \ldots, A_7$ so that the sequence $4, A_1, A_2, \ldots, A_7, 7$ is in arithmetic progression and also 4 numbers G_1, G_2, G_3, G_4 so that the sequence $12, G_1, G_2, G_3, G_4, \frac{3}{8}$ is in geometric progression.
8. If the product of the 4th, 5th and 6th terms of a geometric progression is 4096 and if the product of the 5th, 6th and 7th -terms of it is 32768, find the sum of first 8 terms of the geometric progression.
9. The product of three increasing numbers in GP is 5832. If we add 6 to the second number and 9 to the third number, then resulting numbers form an AP. Find the numbers in GP.
10. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.
11. If the roots of the equation $(q-r)x^2+(r-p)x+p-q=0$ are equal, then show that p, q and r are in AP.
12. If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a GP, show that $(q-r)\log a+(r-p)\log b+(p-q)\log c=0$.
13. Find the sum up to n terms of the series: $1+\frac{6}{7}+\frac{11}{49}+\frac{16}{343}+\cdots$.
14. Find the sum of the first n terms of the series $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots$
15. Find the sum of the first 20 -terms of the arithmetic progression having the sum of first 10 terms as 52 and the sum of the first 15 terms as 77.
16. Find the sum up to the 17th term of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\cdots$
17. Compute the sum of first n terms of the following series: $8+88+888+8888+\cdots$
18. Find the gencral term and sum to n terms of the scquence $1, \frac{4}{3}, \frac{7}{9}, \frac{10}{27}, \ldots$
19. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ term of an AP. is equal to twice the m^{th} term.

20. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?
21. Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.
22. Find $\sqrt[3]{1001}$ approximately (two decimal places).
23. Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.
24. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.
25. If $p - q$ is small compared to either p or q, then show that $\sqrt[n]{\frac{p}{q}} \simeq \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$. Hence find $\sqrt[8]{\frac{15}{16}}$.
26. Find the value of $\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{1}{9^{n-1}} + \frac{1}{9^{2n-1}}\right)$.

6.TWO DIMENSIONAL ANALYTICAL GEOMETRY

1. If the points $P(6,2)$ and $Q(-2,1)$ and R are the vertices of a $\triangle PQR$ and R is the point on the locus $y = x^2 - 3x + 4$, then find the equation of the locus of centroid of $\triangle PQR$.
2. Find the points on the locus of points that are 3 units from x-axis and 5 units from the point $(5,1)$.
3. The sum of the distance of a moving point from the points $(4,0)$ and $(-4,0)$ is always 10 units. Find the equation of the locus of the moving point.
4. The Pamban Sea Bridge is a railway bridge of length about 2065 m constructed on the PalkStrait, which connects the Island town of Rameswaram to Mandapam, the main land of India. The Bridge is restricted to a uniform speed of only 12.5 m/s. If a train of length 560 m starts at the entry point of the bridge from Mandapam, then

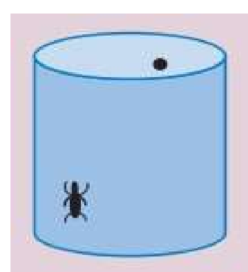
   (i) find an equation of the motion of the train.

   (ii) when does the engine touch island

   (iii) when does the last coach cross the entry point of the bridge

   (iv) what is the time taken by a train to cross the bridge.
5. Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form:

   (i) Slope and Intercept form (ii) Intercept form (iii) Normal form
6. Consider a hollow cylindrical vessel, with circumference 24 cm and height 10 cm. An ant is located on the outside of vessel 4 cm from the bottom. There is a drop of honey at the diagrammatically opposite inside of the vessel, 3 cm from the top. (i) What is the shortest distance the ant would need to crawl to get the honey drop? (ii) Equation of the path traced out by the ant. (iii) Where the ant enter in to the cylinder?. Here is a picture that illustrates the position of the ant and the honey.



7. An object was launched from a place P in constant speed to hit a target. At the 15 th second it was 1400 m away from the target and at the 18 th second 800 m away. Find (i) the distance between the place and the target (ii) the distance covered by it in 15 seconds.(iii) time taken to hit the target.

8. A 150 m long train is moving with constant velocity of 12.5 m/s. Find (i) the equation of the motion of the train, (ii) time taken to cross a pole. (iii) The time taken to cross the bridge of length 850 m is?
9. A spring was hung from a hook in the ceiling. A number of different weights were attached to the spring to make it stretch, and the total length of the spring was measured each time is shown in the following table.

Weight (kg)	2	4	5	8
Length (cm)	3	4	4.5	6

(i) Draw a graph showing the results.

(ii) Find the equation relating the length of the spring to the weight on it.

(iii) What is the actual length of the spring.

(iv) If the spring has to stretch to 9 cm long, how much weight should be added?

(v) How long will the spring be when 6 kilograms of weight on it?

10. A family is using Liquefied petroleum gas (LPG) of weight 14.2 kg for consumption. (Full weight 29.5 kg includes the empty cylinders tare weight of 15.3 kg.). If it is used with constant rate then it lasts for 24 days. Then the new cylinder is replaced (i) Find the equation relating the quantity of gas in the cylinder to the days. (ii) Draw the graph for first 96days.
11. In a shopping mall there is a hall of cuboid shape with dimension $800 \times 800 \times 720$ units, which needs to be added the facility of an escalator in the path as shown by the dotted line in the figure. Find (i)the minimum total length of the escalator. (ii) the heights at which the escalator changes its direction. (iii) the slopes of the escalator at the turning points.

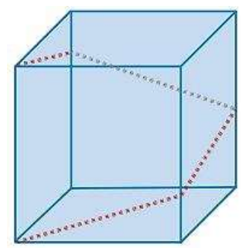


Figure 6.33

12. Find the distance

(i)between two points $(5,4)$ and $(2,0)$

(ii) from a point $(1,2)$ to the line $5x+12y-3=0$

(iii) between two parallel lines $3x+4y=12$ and $6x+8y+1=0$.

13. Show that the lines are $3x+2y+9=0$ and $12x+8y-15=0$ are parallel lines.
14. Find the equation of the straight line parallel to $5x-4y+3=0$ and having x-intercept 3 .
15. Find the distance between the line $4x+3y+4=0$, and a point (i) $(-2,4)$ (ii) $(7,-3)$
16. If $(-4,7)$ is one vertex of a rhombus and if the equation of one diagonal is $5x-y+7=0$, then find the equation of another diagonal.
17. Find the equations of two straight lines which are parallel to the line $12x+5y+2=0$ and at a unit distance from the point $(1,-1)$.
18. Find the length of the perpendicular and the co-ordinates of the foot of the perpendicular from $(-10,-2)$ to the line $x+y-2=0$

19. If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, find (i) the value of λ and the separate equations of the lines (ii) point of intersection of the lines (iii) angle between the lines

20. A student when walks from his house, at an average speed of 6kmph, reaches his school ten minutes before the school starts. When his average speed is 4kmph, he reaches his school five minutes late. If he starts to school every day at 8.00 A.M, then find (i) the distance between his house and the school (ii) the minimum average speed to reach the school on time and time taken to reach the school (iii) the time the school gate closes (iv) the pair of straight lines of his path of walk.

21. Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.

22. Prove that the equation to the straight lines through the origin, each of which makes an angle α with the straight line $y = x$ is $x^2 - 2xy \sec 2\alpha + y^2 = 0$

23. A $\triangle OPQ$ is formed by the pair of straight lines $x^2 - 4xy + y^2 = 0$ and the line PQ. The equation of PQ is $x + y - 2 = 0$. Find the equation of the median of the triangle $\triangle OPQ$ drawn from the origin O.

24. If the pair of straight lines $x^2 - 2kxy - y^2 = 0$ bisect the angle between the pair of straight lines $x^2 - 2lxy - y^2 = 0$, Show that the later pair also bisects the angle between the former.

7. MATRICES AND DETERMINANTS

1. Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.

2. Express the matrices as the sum of a symmetric matrix and a skew-symmetric matrix: $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.

3. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

4. Prove that $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.

5. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.

6. If $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$ prove that a, b, c are in G.P. or α is a root of $ax^2 + 2bx + c = 0$.

7. If a, b, c are p^{th}, q^{th} and r^{th} terms of an $A.P$, find the value of $\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$.

8. Show that $\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by x^4.

9. If a, b, c are all positive, and are p^{th}, q^{th} and r^{th} terms of a $G.P.$, show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

10. Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ if $x, y, z \neq 1$.

11. Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$.

12. Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

13. Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$.

14. In a triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A(1+\sin A) & \sin B(1+\sin B) & \sin C(1+\sin C) \end{vmatrix} = 0$,

prove that $\triangle ABC$ is an isosceles triangle.

15. Solve the following problems by using Factor Theorem :

(1) Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a)$.

(2) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$.

(3) Solve $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$.

(4) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$.

(5) Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$.

(6) Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.

8.VECTOR ALGEBRA

1. Let A, B, and C be the vertices of a triangle. Let D, E, and F be the midpoints of the sides BC, CA, and AB respectively. Show that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \vec{0}$.

2. If $ABCD$ is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}=4\overrightarrow{EF}$.

3. Show that the following vectors are coplanar
(i) $\hat{\imath}-2\hat{\jmath}+3\hat{k}, -2\hat{\imath}+3\hat{\jmath}-4\hat{k}, -\hat{\jmath}+2\hat{k}$ (ii) $2\hat{\imath}+3\hat{\jmath}+\hat{k}, \hat{\imath}-\hat{\jmath}, 7\hat{\imath}+3\hat{\jmath}+2\hat{k}$.

4. Show that the points whose position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}, -\hat{\jmath}-\hat{k}, 3\hat{\imath}+9\hat{\jmath}+4\hat{k}$ and $-4\hat{\imath}+4\hat{\jmath}+4\hat{k}$ are coplanar.

5. If $\vec{a},\vec{b}$ are unit vectors and θ is the angle between them, show that
(i) $\sin\frac{\theta}{2}=\frac{1}{2}|\vec{a}-\vec{b}|$ (ii) $\cos\frac{\theta}{2}=\frac{1}{2}|\vec{a}+\vec{b}|$ (iii) $\tan\frac{\theta}{2}=\frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$.

6. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

7. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.
Prove that $\vec{a}=\pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$.

9.DIFFERENTIAL CALCULUS-LIMITS AND COUTINUITY

1. Check if $\lim_{x\to-5} f(x)$ exists or not, where $f(x)=\begin{cases}\frac{|x+5|}{x+5}, & \text{for } x\neq-5\\ 0, & \text{for } x=-5\end{cases}$

2. Test the existence of the limit, $\lim_{x\to1}\frac{4|x-1|+x-1}{|x-1|}, x\neq1$.

3. $f(x)=\begin{cases}x^2, & x\leq2\\ 8-2x, & 2<x<4.\\ 4, & x\geq4\end{cases}$

4. Verify the existence of $\lim_{x\to1} f(x)$, where $f(x)=\begin{cases}\frac{|x-1|}{x-1}, & \text{for } x\neq1\\ 0, & \text{for } x=1\end{cases}$.

5. Evaluate the following limits:
1. $\lim_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$ 2. $\lim_{x\to2}\frac{2-\sqrt{x+2}}{\sqrt[3]{2}-\sqrt[3]{4-x}}$ 3. $\lim_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}\ (a>b)$

6. According to Einstein's theory of relativity, the mass m of a body moving with velocity v is $m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where m_0 is the initial mass and c is the speed of light. What happens to m as $v\to c^-$. Why is a left hand limit necessary?

7. $\lim_{x\to3}\frac{x^2-9}{x^2(x^2-6x+9)}$.

8. Show that (i) $\lim_{n\to\infty}\frac{1+2+3+\cdots+n}{3n^2+7n+2}=\frac{1}{6}$ (ii) $\lim_{n\to\infty}\frac{1^2+2^2+\cdots+(3n)^2}{(1+2+\cdots+5n)(2n+3)}=\frac{9}{25}$
(iii) $\lim_{n\to\infty}\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots+\frac{1}{n(n+1)}=1$.

9. Evaluate the following limits:

1. $\lim_{\alpha\to 0}\frac{\sin(\alpha^n)}{(\sin\alpha)^m}$ 2. $\lim_{x\to\infty} x\left[3^{\frac{1}{x}}+1-\cos\left(\frac{1}{x}\right)-e^{\frac{1}{x}}\right]$

3. $\lim_{x\to\pi}\frac{\sin 3x}{\sin 2x}$ 4. $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$

10. Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

(i) $f(x)=\begin{cases}2x+1, & \text{if } x\le -1\\ 3x & \text{if } -1<x<1\\ 2x-1, & \text{if } x\ge 1\end{cases}$ (ii) $f(x)=\begin{cases}(x-1)^3, & \text{if } x<0\\ (x+1)^3, & \text{if } x\ge 0\end{cases}$

11. A function f is defined as follows :

$$f(x)=\begin{cases}0 & \text{for } x<0\\ x & \text{for } 0\le x<1;\\ -x^2+4x-2 & \text{for } 1\le x<3\\ 4-x & \text{for } x\ge 3\end{cases}$$

Is the function continuous?

12. Which of the following functions f has a removable discontinuity at $= x_0$? If the discontinuity is removable, find a function g that agrees with f for $x\ne x_0$ and is continuous on $\mathbb{R}$.

(i) $f(x)=\frac{x^3+64}{x+4}, x_0=-4.$ (ii) $f(x)=\frac{3-\sqrt{x}}{9-x}, x_0=9.$

13. The function $f(x)=\frac{x^2-1}{x^3-1}$ is not defined at $x=1$. What value must we give $f(1)$ inorder to make $f(x)$ continuous at $=1$?

14. If $f(x)=|x+100|+x^2$, test whether $f'(-100)$ exists.

10.DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

1. Differentiate the following

1. $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$ 2. $y=\sin(\tan(\sqrt{\sin x}))$ 3. $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

2. Find the derivative of $\tan^{-1}(1+x^2)$ with respect to x^2+x+1.

3. Differentiate $\sin(ax^2+bx+c)$ with respect to $\cos(lx^2+mx+n)$.

4. Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x$.

5. If $u=\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ and $v=\tan^{-1} x$, find $\frac{du}{dv}$.

6. Find the derivative with $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ with respect to $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

7. If $y=\sin^{-1} x$ then find y''.

8. If $y=e^{\tan^{-1} x}$, show that $(1+x^2)y''+(2x-1)y'=0$.

9. If $y=\frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2-3xy_1-y=0$.

10. If $x = a(\theta + \sin\ \theta), y = a(1 - \cos\ \theta)$ then prove that at $\theta = \frac{\pi}{2}, y'' = \frac{1}{a}$.

11. If $\sin\ y = x\sin\ (a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2\ (a+y)}{\sin\ a}, a \neq n\pi$.

12. If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$.

11.INTEGRAL CALCULUS

1. A ball is thrown vertically upward from the ground with an initial velocity of 39.2 m/sec. If the only force considered is that attributed to the acceleration due to gravity, find
   (i) how long will it take for the ball to strike the ground?
   (ii) the speed with which will it strike the ground? and
   (iii) how high the ball will rise?
2. A wound is healing in such a way that t days since Sunday the area of the wound has been decreasing at a rate of $-\frac{6}{(t+2)^2}$ cm^2 per day where $0 < t \leq 8$. If on Monday the area of the wound was 1.4 cm^2
   (i) What was the area of the wound on Sunday?
   (ii) What is the anticipated area of the wound on Thursday if it continues to heal at the same rate?
3. Evaluate: (i) $\int \frac{3x+7}{x^2-3x+2}dx$ (ii) $\int \frac{x+3}{(x+2)^2(x+1)}dx$ (iii) $\frac{x+1}{(x+2)(x+3)}$
4. Integrate the following with respect to x: (i) $\frac{x\sin^{-1}\ x}{\sqrt{1-x^2}}$ (ii) $\tan^{-1}\left(\frac{8x}{1-16x^2}\right)$
5. Integrate the following with respect to x: (i) $e^{ax}\cos\ bx$ (ii) $e^{ax}\sin\ bx$
6. Evaluate the following integrals $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$.
7. Find the integrals of the following : (i) $\frac{1}{6x-7-x^2}$ (ii) $\frac{1}{(x+1)^2-25}$.
8. Integrate the following with respect to x
   (i) $\int \frac{3x+5}{x^2+4x+7}dx$ (ii) $\int \frac{2x+3}{\sqrt{x^2+x+1}}dx$ (iii) $\int \frac{2x-3}{x^2+4x-12}$ (iv) $\int \frac{2x+1}{\sqrt{9+4x-x^2}}$ (v) $\int \frac{x+2}{\sqrt{x^2-1}}$
   (vi) $\int \frac{3x+1}{2x^2-2x+3}$
9. Integrate the following functions with respect to x : (i) $\sqrt{x^2 + 2x + 10}$ (ii) $\sqrt{81 + (2x + 1)^2}$

12. INTRODUCTION TO PROBABILITY THEORY

1. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?
2. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

3. The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15.
   (i) If the oil had to be changed, what is the probability that a new oil filter is needed?
   (ii) If a new oil filter is needed, what is the probability that the oil has to be changed?
4. Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?
5. A factory has two machines I and II. Machine-I produces 40% of items of the output and Machine-II produces 60% of the items. Further 4% of items produced by Machine-I are defective and 5% produced by Machine-II are defective. If an item is drawn at random, find the probability that it is a defective item.
6. A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.
7. A construction company employs 2 executive engineers. Engineer- 1 does the work for 60% of jobs of the company. Engineer- 2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer- 2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?
8. The chances of X, Y and Z becoming managers of a certain company are 4: 2: 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?
9. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N ?
10. A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?
11. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?
12. A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past

experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,

(i) find the probability that the selected pipe is a defective one.

(ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y ?

13. The chances of A, B and C becoming manager of a certain company are $5:3:2$. The probabilities that the office canteen will be improved if A, B, and C become managers are 0.4,0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?
14. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.

STAGE I (50 /90)

2 MARKS	3 MARKS	5 MARKS
1,4,7,8,12	1,4,7,8,12	1,4,7,8,12

STAGE II (70 /90)

2 MARKS	3 MARKS	5 MARKS
1,2,4,7,8,10,12	1,2,4,7,8,10,12	1,2,4,7,8,10,12

STAGE III (80 /90)

2 MARKS	3 MARKS	5 MARKS
1,2,4,6,7,8,10,12	1,2,4,6,7,8,10,12	1,2,4,6,7,8,10,12