

[2 m]

(11) $A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj(A)$

$|adj A| = 9 ; \sqrt{|adj A|} = 3$

$\therefore A^{-1} = \pm \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

(12) $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$; $R=2$ $C=4$

$\rho(A) \leq 2$

2×2 சிற்றணிக் கோவை

$\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1+0 = -1 \neq 0$

$\therefore \rho(A) = 2$

(13) $\frac{1+i}{1-i} = i ; \frac{1-i}{1+i} = -i$

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = i^3 - (-i)^3 = -2i$

(14) $x(x^8+9x^6+7x^4+5x^2+3) = 0$

$x=0$ $x^8+9x^6+7x^4+5x^2+3=0$

மிகை தீர்வுகள் மற்றும் குறை மெய் தீர்வுகள் கிடையாது.

ஒரு தீர்வானது மெய்யானது.

மீதமுள்ள 8 தீர்வுகளும் கற்பனையானது.

(15) $x = 2-\sqrt{3}$

$x-2 = -\sqrt{3}$

SOBS, $(x-2)^2 = (-\sqrt{3})^2$

$x^2+4-4x-3 = 0$

$x^2-4x+1 = 0$

(16) $1+i\tan\alpha = r(\cos\theta+i\sin\theta)$

$r = \sqrt{1+\tan^2\alpha} = \sqrt{\sec^2\alpha} = \sec\alpha$

~~$\theta = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta$~~

$\theta = \tan^{-1}\left(\frac{\tan\alpha}{1}\right) = \alpha$

$\therefore 1+i\tan\alpha = \sec\alpha(\cos\alpha+i\sin\alpha)$

(3m)

(17) $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $|A| = 4$

$X = A^{-1}B$.

$adj A = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

$x = -1$ $y = 4$.

(18) $z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$

$\bar{z} = (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$

$= -\left[(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}\right] = -z$

$\therefore$ z முழுவதும் கற்பனையானது

(19) $LHS = \left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^5 + \left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^5$

$= \cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}+\cos\frac{5\pi}{6}-i\sin\frac{5\pi}{6}$

$= 2\cos\frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3} = RHS$

(20) $x^2 = y$ என்க

$y^2-14y+45 = 0$

$y=5$ $y=9$

$\Rightarrow x^2=5$ $x^2=9$

$x=\pm\sqrt{5}$ $x=\pm 3$

தீர்வு: $\pm\sqrt{5}, \pm 3$.

(21) $(2^2)^x - 3(2^x \times 2^2) + 2^5 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

$2^x = y$ என்க

$y^2-12y+32 = 0$

$y=8$ $y=4$

$2^x = 2^3$ $2^x = 2^2$

$\boxed{x=3}$ $\boxed{x=2}$

$\therefore$ தீர்வுகள்: 2, 3

(22) $adj A = \begin{bmatrix} 3 & 3 \\ 2 & 4 \end{bmatrix}$

$A(adj A) = \begin{bmatrix} 12-6 & 12-12 \\ -6+6 & -6+12 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(adj A)A = \begin{bmatrix} 12-6 & -9+9 \\ 8-8 & -6+12 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 12-6 = 6$.

$\therefore A(adj A) = (adj A)A = |A| I$

(5m)

(23) $c-3b+9a = 21$

$c+5b+25a = 61$

$c+b+a = 9$

$[A|B] = \left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 1 & 5 & 25 & 61 \\ 1 & 1 & 1 & 9 \end{array}\right]$

$\sim \left[\begin{array}{cccc} 1 & -3 & 9 & 21 \\ 0 & 8 & 16 & 40 \\ 0 & 4 & -8 & 12 \end{array}\right]$ $R_2 \to R_2 - R$ $R_3 \to R_3 -$

$\sim \left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & -2 & 3 \end{array}\right]$ $R_2 \to \frac{R_2}{8}$ $R_3 \to \frac{R_3}{4}$

$$\begin{array}{c} \begin{array}{ccc} c & b & a \end{array} \\ \sim\left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{array}\right] \end{array} R_3 \to R_3 - R_2$$

$-4a = -8$ $\boxed{a = 2}$

$b + 2a = 5$ $\boxed{b = 1}$

$c - 3b + 9a = 21$ $\boxed{c = 6}$

$$\frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy}$$

$$\arg\left(\frac{a+ib}{c+id}\right) = \tan^{-1}\left(\frac{bc-ad}{ac+bd}\right)$$

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

$$\tan^{-1}\left[\frac{(x+1)y-(x-1)y}{(x-1)(x+1)+y\cdot y}\right] = \frac{\pi}{2}$$

$$\frac{xy+y-xy+y}{x^2-1+y^2} = \frac{1}{0}$$

$x^2+y^2-1=0 \quad \therefore x^2+y^2=1$

(24) $[A|O] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & \lambda & 0 \end{array}\right] R_1 \leftrightarrow R_3$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda-12 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda-8 & 0 \end{array}\right] R_3 \to R_3 + R_2$$

1. $\lambda \neq 8$ எனில் $\rho(A) = 3$, $\rho(A|O) = 3$

$\rho(A) = \rho(A|O) = 3$ வெளிப்படைத் தீர்வை பெற்றிருக்கும்.

2. $\lambda = 8$ எனில் $\rho(A) = \rho(A|O) = 2 < 3$

வெளிப்படையற்ற தீர்வுகள்.

(b)

2	6	-35	+62	-35	+6	
	0	12	-46	32	-6	
3	6	-23	16	-3	0	
	0	18	-15	-3		
$\frac{1}{2}$	6	-5	1	0		
	0	3	-1			
$\frac{1}{3}$	6	-2	0			
	0	+2				
	6	0				

$\therefore$ மூலங்கள் $2, \frac{1}{2}, 3, \frac{1}{3}$.

(25) $3a - 4b - 2c = 1$

$1a + 2b + 1c = 2$

$2a - 5b - 4c = -1$

$\Delta = -15 \quad \Delta_a = -15$

$\Delta_b = -5 \quad \Delta_c = -5$

$a = 1 \quad b = 1/3 \quad c = 1/3$

$(x, y, z) = (1, 3, 3)$

(b)

-1	2	11	-9	-18
	0	-2	-9	+18
	2	9	-18	0

$2x^2+9x-18=0$

$(2x-3)(x+6)=0$

$x = 3/2 \quad x = -6$

தீர்வு: $\{-1, -6, 3/2\}$.

26 (a) $x = \cos\alpha + i\sin\alpha$

$y = \cos\beta + i\sin\beta$

i. $\dfrac{x^m}{y^n} = \dfrac{\cos m\alpha + i\sin m\alpha}{\cos n\beta + i\sin n\beta}$

$= \cos(m\alpha - n\beta) + i\sin(m\alpha - n\beta)$

$\dfrac{y^n}{x^m} = \cos(m\alpha - n\beta) - i\sin(m\alpha - n\beta)$

$\therefore \dfrac{x^m}{y^n} - \dfrac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

(ii) $x^m y^n = (\cos m\alpha + i\sin m\alpha)(\cos n\beta + i\sin n\beta)$

$= \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$

$\dfrac{1}{x^m y^n} = \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$

$x^m y^n + \dfrac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

(b) $x = \sqrt{5} - \sqrt{3}$

SOBS, $x^2 = 5 + 3 - 2\sqrt{15}$

$x^2 - 8 = -2\sqrt{15}$

SOBS, $(x^2-8)^2 = (-2\sqrt{15})^2$

$x^4 + 64 - 16x^2 = 60$

$x^4 - 16x^2 + 4 = 0$