AVM INSTITUTE

Date:06/07/2024

AVM INSTITUTE

MODEL FIRST MIDTERM EXAMINATION-JULY-2024

12TH STANDARD

MATHEMATICS

Time:2 hours Max Marks:50

Part-1

Answer any 4 question (Question no 5 is compulsory) 4 x 2 = 8 marks

1) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

2) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form.

3)If α,β,γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of coefficients.

4) If |z| = 2 show that 3 ≤ |z+3+4i| ≤ 7.

5) Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.

==

Part-2

Answer any 4 question (Question no 10 is compulsory) 4 x 3 = 12 marks

6)In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem).

7)If z_1, z_2, z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$.

8)Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

9)If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

10) If z = x+iy and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

==

Part-3

Answer all the questions 5 x 5 = 25 marks

11) a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(or)

b)Find the cube roots of unity.

12) a) Solve $(2x - 3)(6x - 1)(3x - 2)(x - 2) - 5 = 0$.

(or)

b) If z = x+iy is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

13) a) A boy is walking along the path $y = ax^2 + bx + c$ through the points (-6,8),(-2,-12) and (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian elimination method.)

(or)

b)Find all the zeroes of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that 1+2i and √3 are two of its zeroes.

14) a)If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

(or)

b)Show that (i) $\left(2 + i\sqrt{3}\right)^{10} - \left(2 - i\sqrt{3}\right)^{10}$ is purely imaginary (ii) $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

15) a) Solve $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.

(or)

b)If $A = \begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}.

Mark summary:

2 x 4 = 8 marks
3 x 4 = 12 marks
5 x 5 = 25 marks
Submitting on Time = 3 marks
Handwriting = 1 mark
Presentation = 1 mark

Total = 50 marks

ALL THE BEST!!!

===================================XXXXXXXXXXXXX==============================

Prepared by
Monish Kannan
Co-Founder, Teaching staff
-AVM Institute.