FML

FIRST MID TERM TEST - 2024

12 - Std **MATHEMATICS**

Time : 1.30 Hrs. MARKS : 50

PART -A

I Choose the correct answer. 10 X 1 = 10

1. If $|adj\ (adj\ A)| = |A|^9$, then the order of the square matrix A is
   a) 3 b) 4 c) 2 d) 5

2. Let $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $AB=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$, if B is the inverse of A, then the value of x is a) 2 b) 4 c) 3 d) 1

3. The value of $\sum_{i=1}^{13}\left(i^n+i^{n-1}\right)$ is
   a) 1 + i b) i c) 1 d) 0

4. If $\left|z-\frac{3}{z}\right|=2$, then the least value of |z| is
   a) 1 b) 2 c) 3 d) 5

5. The principal argument of $\frac{3}{-1+i}$ is
   a) $\frac{-5\pi}{6}$ b) $\frac{-2\pi}{3}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{2}$

6. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10^3 - 5$?
   a) - 1 b) $\frac{5}{4}$ c) $\frac{4}{5}$ d) 5

7. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k satisfies
   a) $|k| \leq 6$ b) k = 0 c) $|k| > 6$ d) $|k| \geq 6$

8. If the system of linear equations x + y + az = b, x + 5y + 2z = 6, x + 2y + 3z =3 has infinitely many solutions then the value of a and b is
   a) 3,7 b) 7,3 c) - 3, 7 d) 3,-7

9. If z = x + iy, then the argument θ is
   a) $[-\pi, \pi]$ b) $(-\pi, \pi)$ c) $(-\pi, \pi]$ d) $[-\pi, \pi)$

10. The sum of all the n^{th} root of unity is a) 0 b) 1 c) -1 d) n

PART - B

II Answer any four question. (Q.No. 16 is compulsory) 4 X 2 = 8

11. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

12. Solve the system of linear equations by matrix inversion method 2x + 5y = -2, x + 2y = -3

13. If $|z| = 2$ show that $3 \leq |z + 3 + 4i| \leq 7$.
14. Show that i) $\left(2+i\sqrt{3}\right)^{10} - \left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.
15. Find a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ as a root.
16. Find the square root of -7 + 24i.

PART - C

Answer any four questions. (Q.No. 22 is compulsory) 4 x 3 = 12

17. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$.
18. Test the consistency of the system of linear equations.
x - y + z = -9, 2x - y + z = 4, 3x - y + z = 6, 4x - y + 2z = 7.
19. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.
20. If $z = (\cos\theta + i \sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i \sin n\theta$.
21. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.
22. If $z_1 = r_1 (\cos\theta_1 + i \sin\theta_1)$ and $z_2 = r_2 (\cos\theta_2 + i \sin\theta_2)$, prove that $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$.

PART - D

Answer all the questions. 4 X 5 = 20

23. a) Solve the systems of linear equations by Cramer's rule :
$\frac{3}{4} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \ \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \ \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$. ***(OR)***
b) Given the complex number z = 3 + 2i, represent the complex numbers z, iz and z + iz on one argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.
24. a) Investigate for what values of λ and μ the system of linear equations x + 2y + z = 7, x + y + λz = μ, x + 3y - 5z = 5 has i) no solution ii) a unique solution iii) an infinite number of solution. ***(OR)***
b) If 2 + i and $3 - \sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ find all roots.
25. a) If z = x + iy and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$. ***(OR)***
b) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.
26. a) If n is a positive integer, prove that $\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\frac{n\pi}{6}$ ***(OR)***
b) Test the consistency of the system of linear equations 2x+5y+7z=52, x+y+z=9, 2x+y-z = 0.