FIRST MID TERM TEST - 2024

Standard XII

Reg.No.

MATHEMATICS

Time : 1.30 hrs **Part - I** ***Marks : 50***

I. Choose the correct answer: **10 x 1 = 10**

1. If $|adj(adj\,A)| = |A|^9$, then the order of the square matrix A is
   a) 3 b) 4 c) 2 d) 5

2. If A^TA^{-1} is symmetric, then $A^2 =$
   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3. If $adj\,A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adj\,B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, then adj(AB) is
   a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

4. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adj A) is
   a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

5. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
   a) 0 b) 1 c) -1 d) i

6. The solution of the equation $|z| - z = 1 + 2i$ is
   a) $\frac{3}{2} - 2i$ b) $\frac{-3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$

7. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is
   a) -2 b) -1 c) 1 d) 2

8. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is
   a) $\sqrt{3} - 2$ b) $\sqrt{3} + 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

9. A zero of $x^3 + 64$ is
   a) 0 b) 4 c) 4i d) -4

10. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4x - 2\sin^2x + 1 = 0$ is
    a) 2 b) 4 c) 1 d) ∞

Part - II

II. Answer any 4 questions. (Q.No.16 is compulsory) **4 x 2 = 8**

11. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}

12. Find the rank of the following matrix: $\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
13. Simplify $\sum_{n=1}^{12} i^n$
14. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.
15. Construct a cubic equation with roots 1, 1 and -2
16. Find z^{-1}, if $z = (2 + 3i)(1 - i)$

Part - III

III. Answer any 4 questions: (Q.No.22 is compulsory) **4 x 3 = 12**

17. Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$
18. Find the value of $\left(\dfrac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$
19. Find the square root of $6-8i$
20. If α and β are the roots of quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2
21. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has 6 imaginary solutions.
22. Determine the values of λ for which the following system of equations. $x + y + 3z = 0$, $2x + y + 2z = 0$, $4x + 3y + \lambda z = 0$ has a non-trivial solution.

Part - IV

IV. Answer all the questions. **4 x 5 = 20**

23. a) If $z = x + iy$ is a complex numbers such that $\text{Im}\left(\dfrac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$ *(OR)*

    b) Solve the equation : $(x-2)(x-7)(x-3)(x+2) + 19 = 0$

24. a) Solve the system of linear equations by Cramer's rule:

    $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ *(OR)*

    b) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$

25. a) Investigate for what values of λ and μ the system of linear equations. $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions *(OR)*

    b) If $z = x + iy$ and $\arg\left(\dfrac{z-1}{z+1}\right) = \dfrac{\pi}{2}$, show that $x^2 + y^2 = 1$

26. a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

    (OR)

    b) Solve the following system of linear equations by matrix inversion method : $2x + 3y - z = 9$, $x + y + z = 9$, $3x - y - z = -1$

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STD - 12 MATHEMATICS - KEY MARKS : 50

C. SELVAM, M.Sc., M.Ed.,

1	b	4
2	b	$(A^T)^2$
3	b	$\begin{bmatrix}-6 & 5\\-2 & -10\end{bmatrix}$
4	a	$\begin{bmatrix}3 & -3 & 4\\2 & -3 & 4\\0 & -1 & 1\end{bmatrix}$
5	a	0
6	a	$\frac{3}{2}-2i$
7	b	-1
8	d	$\sqrt{5}+2$
9	d	-4
10	a	2

II 11. $|adj A| = 36$, $A^{-1} = \pm\frac{1}{\sqrt{|adj A|}}(adj A)$

$A^{-1} = \pm\frac{1}{6}\begin{bmatrix}0 & -2 & 0\\6 & 2 & -6\\-3 & 0 & 6\end{bmatrix}$

12. $\rho(A) = 3$

13. $\sum_{n=1}^{12} i^n = 0$

14. $\alpha+\beta = 4$, $\alpha\beta = 1 \Rightarrow x^2-4x+1=0$

15. $S_1 = 0$, $S_2 = -3$, $S_3 = -2$

$x^3-3x+2=0$

16. $z = 5+i$, $z^{-1} = \frac{1}{z} = \frac{1}{5+i}$

$z^{-1} = \frac{5-i}{(5+i)(5-i)} = \frac{5}{26} - i\frac{1}{26}$

III 17. $|A| = 5 \Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix}7 & -9\\-1 & 2\end{bmatrix} \Rightarrow (A^{-1})^T = \frac{1}{5}\begin{bmatrix}7 & -1\\-9 & 2\end{bmatrix}$

$|A^T| = 5$, $(A^T)^{-1} = \frac{1}{5}\begin{bmatrix}7 & -1\\-9 & 2\end{bmatrix}$

$(A^{-1})^T = (A^T)^{-1}$

18. $z = \sin\frac{\pi}{10} + i\cos\frac{\pi}{10}$, $|z| = 1$

$\bar{z} = \frac{1}{z} = \sin\frac{\pi}{10} - i\cos\frac{\pi}{10}$

$= \left(\frac{1+z}{1+\frac{1}{z}}\right)^{10} = z^{10} = \left(\sin\frac{\pi}{10} + i\cos\frac{\pi}{10}\right)^{10}$

$= i^{10}\left(\cos\frac{\pi}{10} - i\sin\frac{\pi}{10}\right)^{10} = (-1)(-1) = 1$

19. $|6-8i| = 10$

$\sqrt{6-8i} = \pm\left(\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right) = \pm\sqrt{8} - i\sqrt{2}$

$= \pm(2\sqrt{2} - i\sqrt{2})$

20. $\alpha+\beta = \frac{7}{2}$ $\alpha\beta = \frac{13}{2}$

sum of the roots $= \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = -\frac{3}{4}$

product of the roots $= \alpha^2\beta^2 = \frac{169}{4}$

$x^2 + \frac{3}{4}x + \frac{169}{4} = 0 \Rightarrow 4x^2+3x+169=0$

21. $f(x) = + - + + +$ sign changes $= 2$

Positive roots $= 2$

$f(-x) = - + + + +$ sign changes $= 1$

max real roots $= 3$

minimum no. of imaginary roots $= 6$

22. $[A|B] = \begin{bmatrix}1 & 1 & 3 & 0\\2 & 1 & 2 & 0\\4 & 3 & \lambda & 0\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 3 & 0\\0 & -1 & -4 & 0\\0 & -1 & \lambda-12 & 0\end{bmatrix}$ $R_2 - 2R_1$, $R_3 - 4R_1$

$\sim \begin{bmatrix}1 & 1 & 3 & 0\\0 & -1 & -4 & 0\\0 & 0 & \lambda-8 & 0\end{bmatrix}$ $R_3 - R_2$

(i) Trivial solution $\lambda \neq 8$

(ii) non trivial solution $\lambda = 8$

V 23 a. $\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$

$Im\left(\frac{2z+1}{iz+1}\right) = 0 \Rightarrow \frac{-x(2x+1)+2y(1-y)}{(1-y)^2+x^2} = 0$

$2x^2+2y^2+x-2y=0$

b. $(x-2)(x-3)(x-7)(x+2)+19=0$

$(x^2-5x+6)(x^2-5x-14)+19=0$

$x^2-5x = y \Rightarrow (y+6)(y-14)+19=0$

$y^2-8y-65=0 \Rightarrow y=13, y=-5$

$x^2-5x-13=0$, $x^2-5x+5=0$

24. a. $X = \frac{1}{x}$ $Y = \frac{1}{y}$ $Z = \frac{1}{z}$

$3X-4Y-2Z=1$

$X+2Y+Z=2$

$2X-5Y-4Z=-1$

$\Delta = -15$, $\Delta x = -15$, $\Delta y = -5$, $\Delta z = -5$

$X = \frac{\Delta x}{\Delta} = \frac{-15}{-15} \Rightarrow x = 1$

$Y = \frac{\Delta y}{\Delta} = \frac{-5}{-15} \Rightarrow y = 3$

$Z = \frac{\Delta z}{\Delta} = \frac{-5}{-15} \Rightarrow z = 3$

b. $z^3 = -8i \Rightarrow z^3 = 8(-i)$

$z = 2\left(\cos\frac{-\pi+4k\pi}{6} + i\sin\frac{-\pi+4k\pi}{6}\right)$, $k = 0, 1, 2$

$k=0 \Rightarrow z = \sqrt{3}-i$

$k=1 \Rightarrow z = 2i$

$k=2 \Rightarrow z = -\sqrt{3}-i$

25. a. $(A|B) = \begin{bmatrix} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 5 \\ 1 & 1 & \lambda & \mu \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - R_1} \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & -1 & \lambda-1 & \mu-7 \end{bmatrix}$

$\xrightarrow{R_3 + R_2} \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & \lambda-7 & \mu-9 \end{bmatrix}$

(i) $\lambda = 7$, $\mu \neq 9$, $\rho(A) \neq \rho(A|B)$
no solution

(ii) $\lambda \neq 7$ $\rho(A) = \rho(A|B) = 3$
unique solution

(iii) $\lambda = 7$, $\mu = 9$ $\rho(A) = \rho(A|B) = 2$
infinite number of solution.

b. $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$\frac{z-1}{z+1} = \frac{(x^2+y^2-1)+i2y}{(x+1)^2+y^2}$

$\arg\left(\frac{z-1}{z+1}\right) = \pi/2 \Rightarrow \tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) = \pi/2$

$\frac{2y}{x^2+y^2-1} = \tan \pi/2 \Rightarrow x^2+y^2-1 = 0$

$\Rightarrow \boxed{x^2+y^2=1}$

26. a.

$$\begin{array}{c|ccccc} \frac{1}{3} & 6 & -5 & -38 & -5 & 6 \\ & 0 & 2 & -1 & -13 & -6 \\ \hline & 6 & -3 & -39 & -18 & \boxed{0} \end{array}$$

$6x^3 - 3x^2 - 39x - 18 = 0$

$\div 3$ $2x^3 - x^2 - 13x - 6 = 0$

$$\begin{array}{c|cccc} 3 & 2 & -1 & -13 & -6 \\ & 0 & 6 & 15 & 6 \\ \hline & 2 & 5 & 2 & \boxed{0} \end{array}$$

$2x^2 + 5x + 2 = 0$

$(x+2)(x+\tfrac{1}{2}) = 0 \Rightarrow x = -2, -\tfrac{1}{2}$

roots: $\tfrac{1}{3}, 3, -2, -\tfrac{1}{2}$

26. b. $|A| = 16$

$\text{adj}A = \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$

$X = A^{-1}B \Rightarrow X = \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$

$= \frac{1}{16}\begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix}$

$x = 2, y = 3, z = 4$

C. SELVAM, M.Sc., M.Ed.,
P.G. ASST (MATHS),
ST. JOSEPH'S HR. SEC. SCHOOL,
CHENGALPATTU - 603002