IMPORTANT 2 MARK QUESTIONS

1.APPLICATIONS OF MATRICES AND DETERMINANTS

1. Find the adjoint of the following:

   (i) $\begin{bmatrix}-3 & 4\\ 6 & 2\end{bmatrix}$ (ii) $\begin{bmatrix}2 & 3 & 1\\ 3 & 4 & 1\\ 3 & 7 & 2\end{bmatrix}$ (iii) $\frac{1}{3}\begin{bmatrix}2 & 2 & 1\\ -2 & 1 & 2\\ 1 & -2 & 2\end{bmatrix}$
2. Find the inverse (if it exists) of the following:

   (i) $\begin{bmatrix}-2 & 4\\ 1 & -3\end{bmatrix}$ (ii) $\begin{bmatrix}5 & 1 & 1\\ 1 & 5 & 1\\ 1 & 1 & 5\end{bmatrix}$ (iii) $\begin{bmatrix}2 & 3 & 1\\ 3 & 4 & 1\\ 3 & 7 & 2\end{bmatrix}$
3. If $A = \begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_2$.
4. If $\text{adj }(A) = \begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.
5. Find the matrix A for which $A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix} = \begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$.
6. Find the rank of the following matrices by minor method:(i) $\begin{bmatrix}2 & -4\\ -1 & 2\end{bmatrix}$ (ii) $\begin{bmatrix}1 & -2 & -1 & 0\\ 3 & -6 & -3 & 1\end{bmatrix}$
7. Solve the following system of linear equations by matrix inversion method:(i) $2x + 5y = -2, x + 2y = -3$.
8. Solve the following systems of linear equations by Cramer's rule:

   (i) $5x - 2y + 16 = 0, x + 3y - 7 = 0$ (ii) $\frac{3}{x} + 2y = 12,\ \frac{2}{x} + 3y = 13$
9. If $A = \begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_3$.
10. If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.
11. If A is symmetric, prove that adj A is also symmetric.
12. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.
13. Reduce the matrix $\begin{bmatrix}3 & -1 & 2\\ -6 & 2 & 4\\ -3 & 1 & 2\end{bmatrix}$ to a row-echelon form.
14. Solve the following system of linear equations, using matrix inversion method: $5x + 2y = 3, 3x + 2y = 5$.
15. Solve, by Cramer's rule, the system of equations $x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7$.

2.COMPLEX NUMBERS

1. Simplify the following: 1. $i^{59} + \frac{1}{i^{59}}$ 2. $\sum_{n=1}^{10} i^{n+50}$
2. Simplify the following (i) i^7 (ii) i^{1729} (iii) $i^{-1924} + i^{2018}$ (iv) $\sum_{n=1}^{102} i^n$ (v) $i i^2 i^3 \cdots i^{40}$
3. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$ (i) $z^2 + 2zw + w^2$.
4. If $z_1 = 3,\ z_2 = -7i$, and $z_3 = 5 + 4i$, show that (i) $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$ (ii) $(z_1 + z_2)z_3 = z_1z_3 + z_2z_3$.
5. If $z_1 = 2 + 5i,\ z_2 = -3 - 4i$, and $z_3 = 1 + i$, find the additive and multiplicative inverse of $Z_1,\ Z_2$, and Z_3.
6. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. into rectangular form.
7. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form.
8. If $z = x + iy$, find the following in rectangular form. i) Re $(i\bar{z})$ (ii) Im $(3z + 4\bar{z} - 4i)$
9. If $z_1 = 2 - i$ and $z_2 = -4 + 3i$, find the inverse of z_1z_2 and $\frac{z_1}{z_2}$.
10. Prove the following properties: (i) z is real if and only if $z = \bar{z}$ (ii) Re $(z) = \frac{z+\bar{z}}{2}$ and Im $(z) = \frac{z-\bar{z}}{2i}$
11. Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$
12. Find the square root of $6 - 8i$.
13. Find the modulus of the complex numbers(i) $\frac{2i}{3+4i}$ (ii) $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$ (iii) $(1-i)^{10}$ (iv) $2i(3-4i)(4-3i)$.
14. Which one of the points $10 - 8i, 11 + 6i$ is closest to $1 + i$.

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15. If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$.
16. If $|z| = 1$, show that $2 \leq |z^2 - 3| \leq 4$.
17. Find the square roots of (i) $4 + 3i$ (ii) $-6 + 8i$ (iii) $-5 - 12i$.
18. Show that $|3z - 5 + i| = 4$ represents a circle, and, find its centre and radius.
19. Obtain the Cartesian form of the locus of $z = x + iy$ in each of the following cases: (i) $[\text{Re}\,(iz)]^2 = 3$.
20. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases: (i) $|z - 4| = 16$.
21. Write in polar form of the following complex numbers (i) $2 + i2\sqrt{3}$ (ii) $3 - i\sqrt{3}$ (iii) $-2 - i2$
22. If $\omega \neq 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\quad ^2}{c+a\omega+b\quad ^2} = -1$.

3.THEORY OF EQUATIONS

1. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.
2. Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0, a \neq 0$.
3. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was left standing.
4. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.
5. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root.
6. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.
7. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$. if sum of two of its roots vanishes.
8. Solve the equation: $x^4 - 14x^2 + 45 = 0$.
9. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.
10. Discuss the nature of the roots of the following polynomials: (i) $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$ (ii) $x^5 - 19x^4 + 2x^3 + 5x^2 + 11$.
11. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.
12. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.
13. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

4.INVERSE TRIGONOMETRIC FUNCTIONS

1. Find all the values of x such that (i) $-10\pi \leq x \leq 10\pi$ and $\sin\, x = 0$ (ii) $-3\pi \leq x \leq 3\pi$ and $\sin\, x = -1$.
2. Find the period and amplitude of (i) $y = \sin\, 7x$ (ii) $y = -\sin\left(\frac{1}{3}x\right)$ (iii) $y = 4\sin\,(-2x)$.
3. Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$ for $0 \leq x < 6\pi$.
4. For what value of x does $\sin\, x = \sin^{-1}\, x$?.
5. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.
6. Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$.
7. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$.
8. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.
9. Find the value of (i) $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$ (iii) $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

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10. Find (i) $\tan^{-1}(-\sqrt{3})$ (ii) $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ (iii) $\tan(\tan^{-1}(2019))$.
11. Find the domain of the following functions: (i) $\tan^{-1}\left(\sqrt{9-x^2}\right)$ (ii) $\frac{1}{2}\tan^{-1}(1-x^2)-\frac{\pi}{4}$.
12. Find the value of (i) $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ (ii) $\tan(\tan^{-1}(1947))$ (iii) $\tan(\tan^{-1}(-0.2021))$.
13. Find the principal value of (i) $\operatorname{cosec}^{-1}(-1)$ (ii) $\sec^{-1}(-2)$.
14. Find the principal value of (i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (ii) $\cot^{-1}(\sqrt{3})$ (iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$.
15. Find the value, if it exists. If not, give the reason for non-existence.
    (i) $\sin^{-1}(\cos\pi)$ (ii) $\tan^{-1}\left(\sin\left(-\frac{5\pi}{2}\right)\right)$ (iii) $\sin^{-1}[\sin 5]$.
16. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, show that $x+y+z=xyz$.

5.TWO DIMENSIONAL ANALYTICAL GEOMETRY

1. Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.
2. If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c.
3. Find the equation of the circle with centre $(2,-1)$ and passing through the point $(3,6)$ in standard form.
4. If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c.
5. Find centre and radius of the following circles.
   (i) $x^2+(y+2)^2=0$ (ii) $x^2+y^2+6x-4y+4=0$
   (iii) $x^2+y^2-x+2y-3=0$ (iv) $2x^2+2y^2-6x+4y+2=0$
6. Find the length of Latus rectum of the parabola $y^2=4ax$.
7. The orbit of Halley's Comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity.
8. Find the equation of the parabola in each of the cases given below:
   (i) passes through $(2,-3)$ and symmetric about y-axis. (ii) end points of latus rectum $(4,-8)$ and $(4,8)$.
9. Find the equation of the ellipse in each of the cases given below:
   (i) foci $(\pm3,0)$, $e=\frac{1}{2}$. (ii) foci $(0,\pm4)$ and end points of major axis are $(0,\pm5)$.
10. Find the equation of the hyperbola in each of the cases given below: (i) foci $(\pm2,0)$, eccentricity $=\frac{3}{2}$.
11. Identify the type of conic section for each of the equations.
12. Find the equations of tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{64}=1$ which are parallel to $10x-3y+9=0$.
13. Find the equation of the tangent to the parabola $y^2=16x$ perpendicular to $2x+2y+3=0$.
14. Find the equations of the tangent and normal to hyperbola $12x^2-9y^2=108$ at $\theta=\frac{\pi}{3}$. (Hint: use parametric form).

6.APPLICATIONS OF VECTOR ALGEBRA.

1. Prove by vector method that an angle in a semi-circle is a right angle.
2. Prove by vector method that the diagonals of a rhombus bisect each other at right angles.
3. The volume of the parallelepiped whose coterminous edges are $7\hat{\imath}+\lambda\hat{\jmath}-3\hat{k}, \hat{\imath}+2\hat{\jmath}-\hat{k}, -3\hat{\imath}+7\hat{\jmath}+5\hat{k}$ is 90 cubic units. Find the value of λ.
4. Determine whether the three vectors $2\hat{\imath}+3\hat{\jmath}+\hat{k}$, $\hat{\imath}-2\hat{\jmath}+2\hat{k}$ and $3\hat{\imath}+\hat{\jmath}+3\hat{k}$ are coplanar.
5. Prove that $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}]=0$.
6. Find the angle between the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.
7. Find the acute angle between the lines $\vec{r}=(\hat{\imath}+2\hat{\jmath}+4\hat{k})+t(2\hat{\imath}+2\hat{\jmath}+\hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.
8. Find the acute angle between the straight lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$ and state whether they are parallel or perpendicular.

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9. Find the non-parametric form of vector equation and Cartesian equations of the straight line passing through the point with position vector $4\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$ and parallel to the vector $2\hat{\imath} - 6\hat{\jmath} + 7\hat{k}$

10. Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4} = \frac{y+3}{5} = \frac{8-z}{6}$.

11. Find the acute angle between the following lines.
(i) $\vec{r} = (4\hat{\imath} - \hat{\jmath}) + t(\hat{\imath} + 2\hat{\jmath} - 2\hat{k}), \vec{r} = (\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) + s(-\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$
(ii) $\frac{x+4}{3} = \frac{y-7}{4} = \frac{z+5}{5}$, $\vec{r} = 4\hat{k} + t(2\hat{\imath} + \hat{\jmath} + \hat{k})$.
(iii) $2x = 3y = -z$ and $6x = -y = -4z$.

12. Find the angle between the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + t(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ and the plane $\vec{r} \cdot (6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}) = 8$

13. Find the angle between the planes $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - 2\hat{k}) = 3$ and $2x - 2y + z = 2$.

14. Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x - 3y + 5z + 7 = 0$. Also, find the distance between the two planes.

15. Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x - y + z = 5$.

7. APPLICATIONS OF DIFFERENTIAL CALCULUS

1. The temperature T in celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.
2. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t)^2, 0 \le t \le 10$. What is the rate at which the person forgets the words 2 days after learning?
3. A particle moves along a straight line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ metres.
(i) Find the average velocity between $t = 3$ and $t = 6$ seconds.
(ii) Find the instantaneous velocities at $t = 3$ and $t = 6$ seconds.

4. Find the slope of the tangent to the following curves at the respective given points.
(i) $y = x^4 + 2x^2 - x$ at $x = 1$ (ii) $x = a\cos^3 t,\ y = b\sin^3 t$ at $t = \frac{\pi}{2}$.

5. Find the tangent and normal to the following curves at the given points on the curve.
(i) $y = x^2 - x^4$ at $(1,0)$ (ii) $y = x^4 + 2e^x$ at $(0,2)$
(iii) $y = x\sin x$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ (iv) $x = \cos t, y = 2\sin^2 t$ at $t = \frac{\pi}{3}$

6. Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$.

7. Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line $x + 2y = 6$.

8. Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t, t \in \mathbb{R}$ at any point on the curve.

9. Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$.

10. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally.

11. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.
(i) $f(x) = \left|\frac{1}{x}\right|, x \in [-1,1]$ (ii) $f(x) = \tan x, x \in [0,\pi]$ (iii) $f(x) = x - 2\log x, x \in [2,7]$
12. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals :
(i) $f(x) = \frac{x+1}{x}, x \in [-1,2]$ (ii) $f(x) = |3x + 1|, x \in [-1,3]$
13. Write the Maclaurin series expansion of the following functions:(i) e^x.

14. Compute the limit $\lim_{x\to a}\left(\frac{x^n - a^n}{x-a}\right)$.

15. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

16. $\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3}$
17. $\lim_{x\to\infty} \frac{x}{\log x}$
18. Prove that the function $f(x) = x - \sin x$ is increasing on the real line. Also discuss for the existence of local extrema.
19. Find the absolute extrema of the following functions on the given closed interval.
    (i) $f(x) = x^2 - 12x + 10;\ [1,2]$

8.DIFFERENTIALS AND PARTIAL DERIVATIVES

1. Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate $f(3.2)$.
2. Find a linear approximation for the following functions at the indicated points.
   (i) $f(x) = x^3 - 5x + 12, x_0 = 2$ (ii) $g(x) = \sqrt{x^2+9}, x_0 = -4$ (iii) $h(x) = \frac{x}{x+1}, x_0 = 1$
3. Find differential dy for each of the following functions:
   (i) $y = \frac{(1-2x)^3}{3-4x}$ (ii) $y = (3 + \sin(2x))^{2/3}$ (iii) $y = e^{x^2-5x+7}\cos(x^2-1)$
4. Find df for $f(x) = x^2 + 3x$ and evaluate it for (i) $x = 2$ and $dx = 0.1$ (ii) $x = 3$ and $dx = 0.02$
5. Find Δf and df for the function f for the indicated values of $x, \Delta x$ and compare
   (i) $f(x) = x^3 - 2x^2;\ x = 2, \Delta x = dx = 0.5$ (ii) $f(x) = x^2 + 2x + 3;\ x = -0.5, \Delta x = dx = 0.1$
6. Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y) = \frac{3x^2-xy}{x^2+y^2+3}$.
7. Let $f(x,y) = \frac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ for $(x,y) \neq (0,0)$. Show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.
8. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{e^x \sin y}{y}\right)$, if the limit exists.
9. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.
   (i) $f(x,y) = x^2y + 6x^3 + 7$ (ii) $h(x,y) = \frac{6x^2y^3 - \pi y^5 + 9x^4y}{2020x^2 + 2019y^2}$
   (iii) $g(x,y,z) = \frac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$ (iv) $U(x,y,z) = xy + \sin\left(\frac{y^2-2z^2}{xy}\right)$.

9.APPLICATIONS OF INTEGRATION

1. Evaluate : $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x} dx$.
2. Evaluate : $\int_0^9 \frac{1}{x+\sqrt{x}} dx$.
3. Evaluate : $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$.
4. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x\, dx$.
5. Evaluate the following integrals using properties of integration :
   (i) $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right) dx$ (ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x\cos x + \tan^3 x + 1) dx$
6. Evaluate the following: (i) $\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$ (ii) $\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$ (iii) $\int_0^1 x^2(1-x)^3 dx$
7. Evaluate the following (i) $\int_0^\infty x^5 e^{-3x} dx$

10.ORIANARY DIFFERENTIAL CALCULUS

1. For each of the following differential equations, determine its order, degree (if exists)
   (i) $\frac{dy}{dx} + xy = \cot x$ (ii) $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$
   (iii) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$ (iv) $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

(v) $y\left(\frac{dy}{dx}\right) = \frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$ (vi) $x^2\frac{d^2y}{dx^2} + \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} = 0$

(vii) $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)}$ (viii) $\frac{d^2y}{dx^2} = xy + \cos\left(\frac{dy}{dx}\right)$

(ix) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + \int ydx = x^3$ (x) $x = e^{xy\left(\frac{dy}{dx}\right)}$

2. Express each of the following physical statements in the form of differential equation.
   (i) Radium decays at a rate proportional to the amount Q present.
   (ii) The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population.
   (iii) For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature.
   (iv) A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ₹ 400 per year.
3. Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.
4. Form the differential equation of all straight lines touching the circle $x^2 + y^2 = r^2$.
5. Find value of m so that the function $y = e^{mx}$ is a solution of the given differential equation.
   (i) $y' + 2y = 0$ (ii) $y'' - 5y' + 6y = 0$

11.PROBABILITY DISTRIBUTIONS

1. Suppose two coins are tossed once. If X denotes the number of tails, (i) write down the sample space (ii) find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images.
2. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.
3. In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.
4. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.
5. The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$. Find the value of k.
6. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3) = 10$ and $E(X+3)^2 = 116$, find μ and σ^2.
7. Compute $P(X = k)$ for the binomial distribution, $B(n,p)$ where
   (i) $n = 6, p = \frac{1}{3}, k = 3$ (ii) $n = 10, p = \frac{1}{5}, k = 4$ (iii) $n = 9, p = \frac{1}{2}, k = 7$

12. DISCRETE MATHEMATICS

1. Verify the (i) closure property, (ii) commutative property, (iii) associative property
   (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}_o$ = the set of all odd integers.
2. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.
3. Determine whether $*$ is a binary operation on the sets given below.
   (i) $a * b = a \cdot |b|$ on $\mathbb{R}$ (ii) $a * b = \min(a,b)$ on $A = \{1,2,3,4,5\}$ (iii) $(a * b) = a\sqrt{b}$ is binary on $\mathbb{R}$.
4. On $\mathbb{Z}$, define $*$ by $(m * n) = m^n + n^m : \forall m,n \in \mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?
5. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$.
6. Let $A = \{a + \sqrt{5}b : a,b \in \mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A.
7. Let p : Jupiter is a planet and q : India is an island be any two simple statements. Give verbal sentence describing each of the following statements.
   (i) $\neg p$ (ii) $p \wedge \neg q$ (iii) $\neg p \vee q$ (iv) $p \rightarrow \neg q$ (v) $p \leftrightarrow q$

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8. Write each of the following sentences in symbolic form using statement variables p and q.
   (i) 19 is not a prime number and all the angles of a triangle are equal.
   (ii) 19 is a prime number or all the angles of a triangle are not equal
   (iii) 19 is a prime number and all the angles of a triangle are equal
   (iv) 19 is not a prime number

9. Determine the truth value of each of the following statements
   (i) If $6+2=5$, then the milk is white.
   (ii) China is in Europe or $\sqrt{3}$ is an integer
   (iii) It is not true that $5+5=9$ or Earth is a planet
   (iv) 11 is a prime number and all the sides of a rectangle are equal

10. Which one of the following sentences is a proposition?
    (i) $4+7=12$ (ii) What are you doing?
    (iii) $3^n \leq 81, n \in \mathbb{N}$ (iv) Peacock is our national bird
    (v) How tall this mountain is!

11. Write the converse, inverse, and contrapositive of each of the following implication.
    (i) If x and y are numbers such that $x=y$, then $x^2=y^2$
    (ii) If a quadrilateral is a square then it is a rectangle

12. Construct the truth table for the following statements.
    (i) $\neg p \wedge \neg q$ (ii) $\neg(p \wedge \neg q)$ (iii) $(p \vee q) \vee \neg q$ (iv) $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$

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IMPORTANT 3 MARK QUESTIONS.

1.APPLICATIONS OF MATRICES AND DETERMINANTS

16. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.
17. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.
18. If adj $(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.
19. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.
20. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $AXB = C$.
21. Find the rank of the following matrices by minor method: (i) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$
22. Find the rank of the following matrices by row reduction method:
    (i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$
23. Find the inverse of each of the following by Gauss-Jordan method: (i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$
24. Solve the following system of linear equations by matrix inversion method: (i) $2x - y = 8,\ 3x + 2y = -2$
    (ii) $2x + 3y - z = 9,\ x + y + z = 9,\ 3x - y - z = -1$
    (iii) $x + y + z - 2 = 0,\ \ 6x - 4y + 5z - 31 = 0,\ \ 5x + 2y + 2z = 13$
25. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)
26. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.
27. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).
28. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem).
29. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.
30. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.
31. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

2.COMPLEX NUMBERS

1. Find the value of the real numbers x and y, if the complex number $(2 + i)x + (1 - i)y + 2i - 3$ and $x + (-1 + 2i)y + 1 + i$ are equal.
2. Given the complex number $z = 2 + 3i$, represent the complex numbers in Argand diagram.
   (i) z, iz, and $z + iz$ (ii) $z, -iz$, and $z - iz$.

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3. Find the values of the real numbers x and y, if the complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal.
4. The complex numbers u, v, and w are related by $\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$. If $v=3-4i$ and $w=4+3i$, find u in rectangular form.
5. State and prove Triangular Inequality.
6. If Z is imaginary if and only iff $z=-\bar{z}$.
7. Find the least value of the positive integer n for which $(\sqrt{3}+i)^n$ (i) real (ii) purely imaginary.
8. If z_1, z_2, and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$, find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$.
9. If $|z|=2$ show that $3\leq|z+3+4i|\leq 7$
10. Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.
11. Show that the equation $z^2=\bar{z}$ has four solutions.
12. If the area of the triangle formed by the vertices z, iz, and $z+iz$ is 50 square units, find the value of $|z|$.
13. If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$ show that the locus of z is real axis.
14. Obtain the Cartesian form of the locus of $z=x+iy$ in each of the following cases: (i) Im $[(1-i)z+1]=0$
15. Show that the following equations represent a circle, and, find its centre and radius. i) $|3z-6+12i|=8$.
16. Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the foiiowing (i) $|z-4|^2-|z-1|^2=16$.
17. Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.
18. Find the rectangular form of the complex numbers (i) $\frac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$.
19. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\cdots(x_n+iy_n)=a+ib$, show that
    (i) $(x_1^2+y_1^2)(x_2^2+y_2{}^2)(x_3^2+y_3^2)\cdots(x_n{}^2+y_n^2)=a^2+b^2$ (ii) $\sum_{r=1}^{n}\tan^{-1}\left(\frac{y_r}{x_r}\right)=\tan^{-1}\left(\frac{b}{a}\right)+2k\pi, k\in\mathbb{Z}$.
20. If $\frac{1+z}{1-z}=\cos 2\theta+i\sin 2\theta$, show that $z=i\tan\theta$.
21. If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$, show that
    (i) $\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$ and (ii) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$.
22. If $z=(\cos\theta+i\sin\theta)$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$.
23. Simplify $\left(\frac{1+\cos 2\theta+i\sin 2\theta}{1+\cos 2\theta-i\sin 2\theta}\right)^{30}$
24. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$.
25. Solve the equation $z^3+27=0$.
26. If $\omega\neq 1$ is a cube root of unity, show that
    (i) $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6=128$. (ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots\left(1+\omega^{2^{11}}\right)=1$.

3.THEORY OF EQUATIONS

1. Find the condition that the roots of cubic equation $x^3+ax^2+bx+c=0$ are in the ratio $p:q:r$.
2. If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of p.
3. If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.
4. Construct a cubic equation with roots (i) 1,2, and 3 (ii) 1,1 , and -2 (iii) $2, \frac{1}{2}$ and 1 .
5. Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio $3:2$.

6. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

7. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$

8. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

9. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.

10. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.

11. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.

12. Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$.

13. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

14. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

15. Solve the cubic equations : (i) $2x^3 - 9x^2 + 10x = 3$, (ii) $8x^3 - 2x^2 - 7x + 3 = 0$.

16. Solve the equation $(x-2)(x-7)(x-3)(x+2) + 19 = 0$.

17. Solve the equation $7x^3 - 43x^2 = 43x - 7$.

18. Solve the following equations (i) $\sin^2 x - 5\sin x + 4 = 0$ (ii) $12x^3 + 8x = 29x^2 - 4$

19. Examine for the rational roots of (i) $2x^3 - x^2 - 1 = 0$ (ii) $x^8 - 3x + 1 = 0$.

20. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$.

21. Discuss the maximum possible number of positive and negative zeros of the polynomials $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.

22. Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

4.INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the value of (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

2. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

3. Find the value of (i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$ (ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

4. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

5. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$.

6. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$.

7. Find the value of (i) $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

8. Find the value of the expression in terms of x, with the help of a reference triangle. (i) $\sin(\cos^{-1}(1-x))$.

9. Find the value of (i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (ii) $\cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$ (iii) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

10. Prove that (i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$ (ii) $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$.

11. Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

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5.TWO DIMENSIONAL ANALYTICAL GEOMETRY

1. Examine the position of the point (2,3) with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$.
2. A line $3x + 4y + 10 = 0$ cuts a chord of length 6 units on a circle with centre of the circle (2,1) . Find the equation of the circle in general form.
3. A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form.
4. A road bridge over an irrigation canal have two semi circular vents each with a span of 20 m and the supporting pillars of width 2 m. Use Fig.5.16 to write the equations that represent the semi-verticular vents.

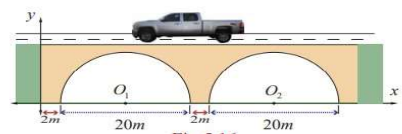

Fig.5.16

5. Find the equation of circles that touch both the axes and pass through $(-4,-2)$ in general form.
6. Find the equation of the circle with centre (2,3) and passing through the intersection of the lines $3x - 2y - 1 = 0$ and $4x + y - 27 = 0$.
7. Determine whether the points $(-2,1), (0,0)$ and $(-4,-3)$ lie outside, on or inside the circle $x^2 + y^2 - 5x + 2y - 5 = 0$.
8. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.
9. Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is (2,3) and a directrix is $x = 7$. Also find the length of the major and minor axes of the ellipse.
10. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.
11. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.
12. Find the equation of the ellipse in each of the cases given below:
    (i) length of latus rectum 8 , eccentricity $= \frac{3}{5}$, centre (0,0) and major axis on x-axis.
    (ii) length of latus rectum 4 , distance between foci $4\sqrt{2}$, centre (0,0) and major axis as y-axis.
13. Find the equation of the hyperbola in each of the cases given below:
    (i) passing through $(5,-2)$ and length of the transverse axis along x axis and of length 8 units.
14. Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1,-3)$.
15. Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$.
16. Find the equations of the two tangents that can be drawn from (5,2) to the ellipse $2x^2 + 7y^2 = 14$.
17. Prove that the point of intersection of the tangents at ' t_1 ' and ' t_2 ' on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$.
18. If the normal at the point ' t_1 ' on the parabola $y^2 = 4ax$ meets the parabola again at the point ' t_2, then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.
19. A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabolic bowl is 40 cm wide from rim to rim and 30 cm deep. The bulb is located at the focus.
    (1) What is the equation of the parabola used for reflector?
    (2) How far from the vertex is the bulb to be placed so that the maximum distance covered?
20. An equation of the elliptical part of an optical lens system is $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The parabolic part of the system has a focus in common with the right focus of the ellipse. The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.
21. A room 34 m long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in Fig. 5.64. If the maximum height of the ceiling is 8 m, determine where the foci are located.

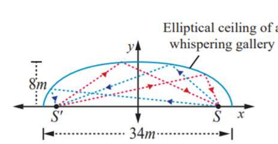

Fig. 5.64

22. If the equation of the ellipse is $\frac{(x-11)^2}{484} + \frac{y^2}{64} = 1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

6.APPLICATIONS OF VECTOR ALGEBRA.

1. With usual notations, in any triangle ABC, prove the following by vector method.
   (i) $a^2 = b^2 + c^2 - 2bc\cos A$ (ii) $b^2 = c^2 + a^2 - 2ca\cos B$ (iii) $c^2 = a^2 + b^2 - 2ab\cos C$.
2. With usual notations, in any triangle ABC, prove the following by vector method.
   (i) $a = b\cos C + c\cos B$ (ii) $b = c\cos A + a\cos C$ (iii) $c = a\cos B + b\cos A$.
3. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
4. If D is the midpoint of the side BC of a triangle ABC, show by vector method that $|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2)$.
5. A particle is acted upon by the forces $3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $2\hat{\imath} + \hat{\jmath} - \hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.
6. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$
7. If G is the centroid of a $\triangle ABC$, prove that (area of $\triangle GAB$) = (area of $\triangle GBC$) = (area of $\triangle GCA$) = $\frac{1}{3}$(area of $\triangle ABC$).
8. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $10\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ to the point with position vector $6\hat{\imath} + \hat{\jmath} - 3\hat{k}$. Find the work done by the forces.
9. Find the magnitude and direction cosines of the torque of a force represented by $3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ about the point with position vector $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ acting through a point whose position vector is $4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.
10. Find the torque of the resultant of the three forces represented by $-3\hat{\imath} + 6\hat{\jmath} - 3\hat{k}, 4\hat{\imath} - 10\hat{\jmath} + 12\hat{k}$ and $4\hat{\imath} + 7\hat{\jmath}$ acting at the point with position vector $8\hat{\imath} - 6\hat{\jmath} - 4\hat{k}$, about the point with position vector $18\hat{\imath} + 3\hat{\jmath} - 9\hat{k}$.
11. Show that the four points $(6,-7,0), (16,-19,-4), (0,3,-6), (2,-5,10)$ lie on a same plane.
12. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$, $\hat{b} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{c} = -3\vec{\imath} + \vec{\jmath} + 4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.
13. Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.
14. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}, \vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$, find (i) $(\vec{a} \times \vec{b}) \times \vec{c}$ (ii) $\vec{a} \times (\vec{b} \times \vec{c})$.
15. For any vector $\vec{a}$, prove that $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.
16. $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$, $\vec{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$ then find the value of $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$.
17. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$.
18. The vector equation in parametric form of a line is $\vec{r} = (3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}) + t(2\hat{\imath} - \hat{\jmath} + 3\hat{k})$. Find (i) the direction cosines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equations of the line.
19. Find the points where the straight line passes through (6,7,4) and (8,4,9) cuts the xz and yz planes.
20. Find the direction cosines of the straight line passing through the points (5,6,7) and (7,9,13). Also, find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points.
21. If the straight line joining the points (2,1,4) and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b.
22. If the straight lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, find the value of m.
23. Show that the points (2,3,4), (−1,4,5) and (8,1,2) are collinear.
24. Find the coordinate of the foot of the perpendicular drawn from the point (−1,2,3) to the straight line $\vec{r} = (\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$. Also, find the shortest distance from the given point to the straight line.

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25. Find the parametric form of vector equation and Cartesian equations of a straight line passing through (5,2,8) and is perpendicular to the straight lines $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + s(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) + t(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$.

26. If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of m.

27. Find the vector equation of a plane which is at a distance of 7 units from the origin having $3, -4, 5$ as direction ratios of a normal to it.

28. Find the direction cosines of the normal to the plane $12x + 3y - 4z = 65$. Also, find the non-parametric form of vector equation of a plane and the length of the perpendicular to the plane from the origin.

29. Find the vector and Cartesian equations of the plane passing through the point with position vector $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ and normal to the vector $\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$.

30. A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane.

31. Find the intercepts cut off by the plane $\vec{r} \cdot (6\hat{\imath} + 4\hat{\jmath} - 3\hat{k}) = 12$ on the coordinate axes.

32. If a plane meets the coordinate axes at A, B, C such that the centriod of the triangle ABC is the point (u, v, w), find the equation of the plane.

33. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1), (1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.

34. Show that the straight lines $\vec{r} = (5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}) + s(4\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ and $\vec{r} = (8\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + t(7\hat{\imath} + \hat{\jmath} + 3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.

7.APPLICATIONS OF DIFFERENTIALS CALCULUS

1. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$ (i) Compute the maximum height of the particle reached.
   (ii) What is the velocity when the particle hits the ground?
2. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds?
   (i) At what time the particle is at rest?
   (ii) At what time the particle changes its direction?
   (iii) Find the total distance travelled by the particle in the first 2 seconds.
3. The price of a product is related to the number of units available (supply) by the equation $Px + 3P - 16x = 234$, where P is the price of the product per unit in Rupees(₹) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week.
4. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   (iii) What is the instantaneous velocity of the camera when it hits the ground?
5. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
   (i) At what times the particle changes direction?
   (ii) Find the total distance travelled by the particle in the first 4 seconds.
   (iii) Find the particle's acceleration each time the velocity is zero.
6. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x =$ 5 units.
7. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

8. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point $(1,2)$.
9. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.
10. Find the equation of the tangent and normal at any point to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t, t \in \mathbb{R}$.
11. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$.
12. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$.
13. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.
14. A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from $-10°\text{C}$ to $100°\text{C}$. Show that the rate of change of temperature at some time t is $5°\text{C}$ per second.
15. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :
    (i) $f(x) = x^2 - x, x \in [0,1]$ (ii) $f(x) = \frac{x^2-2x}{x+2}, x \in [-1,6]$ (iii) $f(x) = \sqrt{x} - \frac{x}{3}, x \in [0,9]$
16. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
    (i) $f(x) = x^3 - 3x + 2, x \in [-2,2]$ (ii) $f(x) = (x-2)(x-7), x \in [3,11]$
17. Show that the value in the conclusion of the mean value theorem for
    (i) $f(x) = \frac{1}{x}$ on a closed interval of positive numbers $[a,b]$ is $\sqrt{ab}$
    (ii) $f(x) = Ax^2 + Bx + C$ on any interval $[a,b]$ is $\frac{a+b}{2}$.
18. A race car driver is kilometer stone 20. If his speed never exceeds 150 km/hr, what is the maximum kilometer he can reach in the next two hours.
19. Suppose that for a function $f(x), f'(x) \le 1$ for all $1 \le x \le 4$. Show that $f(4) - f(1) \le 3$.
20. Does there exist a differentiable function $f(x)$ such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \le 2$ for all x. Justify your answer.
21. Show that there lies a point on the curve $f(x) = x(x+3)e^{-\frac{\pi}{2}}$, $-3 \le x \le 0$ where tangent drawn is parallel to the x-axis.
22. Using mean value theorem prove that for, $a > 0$, $b > 0$, $|e^{-a} - e^{-b}| < |a - b|$.
23. Expand $\log(1 + x)$ as a Maclaurin's series upto 4 non-zero terms for $-1 < x \le 1$.
24. Write the Taylor series expansion of $\frac{1}{x}$ about $x = 2$ by finding the first three non-zero terms.
25. Write the Maclaurin series expansion of the following functions: (i) $\sin x$
26. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$.
27. Expand $\sin x$ in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.
28. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of $x - 1$.
29. Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)$.
30. If $\lim_{\theta\to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$.
31. Evaluate : $\lim_{x\to 1^-}\left(\frac{\log(1-x)}{\cot(\pi x)}\right)$.
32. Evaluate : $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right), m \in N$.
33. Evaluate : $\lim_{x\to 1} x^{\frac{1}{1-x}}$.
34. $\lim_{x\to\frac{\pi}{2}^-}\frac{\sec x}{\tan x}$
35. $\lim_{x\to\infty} e^{-x}\sqrt{x}$
36. $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$
37. $\lim_{x\to 1^+}\left(\frac{2}{x^2-1} - \frac{x}{x-1}\right)$
38. $\lim_{x\to 0^+} x^x$
39. $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$
40. $\lim_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}$

41. Discuss the monotonicity and local extrema of the function
$f(x) = \log(1+x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $\log(1+x) > \frac{x}{1+x}$.
42. Find the intervals of monotonicity and local extrema of the function $f(x) = x\log x + 3x$.
43. Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$.
44. Find the absolute extrema of the following functions on the given closed interval.
(i) $f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}$; $[-1,1]$ (ii) $f(x) = 2\cos x + \sin 2x$; $\left[0, \frac{\pi}{2}\right]$
45. Find the intervals of monotonicities and hence find the local extremum for the following functions:
(i) $f(x) = 2x^3 + 3x^2 - 12x$ (ii) $f(x) = \frac{x^3}{3} - \log x$
46. Find intervals of concavity and points of inflexion for the following functions:(i) $f(x) = x(x-4)^3$.
47. Find the local extrema for the following functions using second derivative test :
(i) $f(x) = -3x^5 + 5x^3$ (ii) $f(x) = x\log x$
48. Find two positive numbers whose sum is 12 and their product is maximum.
49. Find two positive numbers whose product is 20 and their sum is minimum.
50. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$.
51. Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2-6x+7}{x+5}$.
52. Find the asymptotes of the following curves: (i) $f(x) = \frac{x^2}{x^2-1}$ (iv) $f(x) = \frac{x^2-6x-1}{x+3}$

8.DIFFERENTIALS AND PARTIAL DERIVATIVES

1. Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at $x = 27$. Use the linear approximation to approximate $\sqrt[3]{27.2}$.
2. Use the linear approximation to find approximate values of (i) $(123)^{\frac{2}{3}}$ (ii) $\sqrt[4]{15}$ (iii) $\sqrt[3]{26}$
3. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?
4. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.
5. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?
6. In a newly developed city, it is estimated that the voting population (in thousands) will increase according to $V(t) = 30 + 12t^2 - t^3, 0 \le t \le 8$ where t is the time in years. Find the approximate change in voters for the time change from 4 to $4\frac{1}{6}$ year.
7. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.
8. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.
9. Show that $f(x,y) = \frac{x^2-y^2}{y^2+1}$ is continuous at every $(x,y) \in \mathbb{R}^2$.
10. Let $g(x,y) = \frac{e^y \sin x}{x}$, for $x \neq 0$ and $g(0,0) = 1$. Show that g is continuous at (0,0).
11. Let $w(x,y) = xy + \frac{e^y}{y^2+1}$ for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial^2 w}{\partial y\,\partial x}$ and $\frac{\partial^2 w}{\partial x\,\partial y}$.
12. For each of the following functions find the f_x, f_y, and show that $f_{xy} = f_{yx}$. (ii) $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$
13. If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.
14. If $w(x,y) = xy + \sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y}$.

15. Let $U(x,y,z) = x^2 - xy + 3\sin z, x,y,z \in \mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$.
16. If $w(x,y) = x^3 - 3xy + 2y^2,\ x,y \in \mathbb{R}$, find the linear approximation for w at $(1,-1)$.
17. Let $z(x,y) = x^2y + 3xy^4,\ x,y \in \mathbb{R}$. Find the linear approximation for z at $(2,-1)$.
18. If $w(x,y,z) = x^2 + y^2 + z^2, x = e^t, y = e^t\sin t$ and $z = e^t\cos t$, find $\frac{dw}{dt}$.
19. If $z(x,y) = x\tan^{-1}(xy), x = t^2, y = se^t,\ s,t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at $s = t = 1$.
20. Let $z(x,y) = x^3 - 3x^2y^3$, where $x = se^t,\ y = se^{-t},\ s,t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

9.APPLICATIONS OF INTEGRATION

1. Find an approximate value of $\int_1^{1.5} x\,dx$ by applying the left-end rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$
2. Find an approximate value of $\int_1^{1.5} x^2\,dx$ by applying the right-end rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$
3. Find an approximate value of $\int_1^{1.5} (2-x)\,dx$ by applying the mid-point rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$.
4. Evaluate $\int_{-4}^{4} |x+3|\,dx$
5. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$.
6. Show that $\int_0^1 (\tan^{-1} x + \tan^{-1}(1-x))\,dx = \frac{\pi}{2} - \log_e 2$.
7. Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$.
8. Evaluate $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$.
9. Evaluate the following: i) $\int_0^1 x^3 e^{-2x}\,dx$ ii) $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}\,dx$
10. Evaluate the following: $\int_0^{\frac{\pi}{2}} \frac{dx}{1+5co^{2}x}$.
11. Evaluate the following: i) $\int_0^{\frac{\pi}{6}} \sin^5 3x\,dx$ ii) $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^5\theta\,d\theta$
12. Show that $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$.
13. Evaluate $\int_0^\infty \frac{x^n}{n^x}\,dx$, where n is a positive integer ≥ 2.
14. Evaluate the following (i) $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x}\,dx$
15. If $\int_0^\infty e^{-\alpha x^2} x^3\,dx = 32,\ \alpha > 0$, find α
16. Find the area of the region bounded by $3x - 2y + 6 = 0,\ x = -3,\ x = 1$ and x-axis.
17. Find the area of the region bounded by $2x - y + 1 = 0,\ y = -1,\ y = 3$ and y-axis.
18. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by

    $y = 2x^2, y = 0$ and $x = 1$.
19. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by

    $y = e^{-2x}y = 0, x = 0$ and $x = 1$
20. Find, by integration, the volume of the solid generated by revolving about the y-axis, the region enclosed by

    $x^2 = 1 + y$ and $y = 3$.

10. ORDIANRY DIFFERENTIALS EQUATIONS.

1. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop.
2. Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.
3. Find the differential equation of the family of all the parabolas with latus rectum $4a$ and whose axes are parallel to the x-axis.

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4. Find the differential equation of the family of parabolas with vertex at $(0,-1)$ and having axis along the y-axis.
5. Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.
6. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.
7. Find the differential equation of the curve represented by $xy = ae^{x} + be^{-x} + x^2$.
8. Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$.
9. Show that $y = a\cos(\log x) + b\sin(\log x), x > 0$ is a solution of the differential equation $x^2y'' + xy' + y = 0$
10. The slope of the tangent to the curve at any point is the reciprocal of four times the ordinate at that point. The curve passes through $(2,5)$. Find the equation of the curve.
11. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.
12. Show that $y = ax + \frac{b}{x}, x \neq 0$ is a solution of the differential equation $x^2y'' + xy' - y = 0$.
13. Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$
14. Show that the differential equation representing the family of curves $y^2 = 2a\left(x + a^{\frac{2}{3}}\right)$, where a is a positive parameter, is $\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$.
15. Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.
16. Solve: $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$.
17. Solve: $\left[x + y\cos\left(\frac{y}{x}\right)\right]dx = x\cos\left(\frac{y}{x}\right)dy$
18. Solve: $2xydx + (x^2 + 2y^2)dy = 0$

11.PROBABILITY DISTRIBUTIONS

1. Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10 , and (iv) the number of elements in inverse image of X.
2. An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images.
3. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.
4. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹ 15 for each red ball selected and we lose ₹ 10 for each black ball selected. If X denotes the winning amount, find the values of X and number of points in its inverse images.
5. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.
6. If X is the random variable with distribution function $F(x)$ given by,
$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & 1 \le x \end{cases}$$
then find (i) the probability density function $f(x)$ (ii) $P(0.2 \le X \le 0.7)$.
7. The probability density function of X is $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$.
Find (i) $P(0.2 \le X < 0.6)$ (ii) $P(1.2 \le X < 1.8)$ (iii) $P(0.5 \le X < 1.5)$

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8. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.

9. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

10. A lottery with 600 tickets gives one prize of ₹200, four prizes of ₹ 100 , and six prizes of ₹ 50. If the ticket costs is ₹2, find the expected winning amount of a ticket.

11. Using binomial distribution find the mean and variance of X for the following experiments
(i) A fair coin is tossed 100 times, and X denote the number of heads.
(ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.

12. DISCRETE MATHEMATICS

1. Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b; \forall a, b \in \mathbb{N}$ (exponentiation property)

2. Fill in the following table so that the binary operation $*$ on $A = \{a, b, c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

3. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.
Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A \vee B) \wedge C$ (iv) $(A \wedge B) \vee C$.

4. Consider the binary operation $*$ defined on the set $A = \{a, b, c, d\}$ by the following table:

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Is it commutative and associative?

5. Verify whether the following compound propositions are tautologies or contradictions or contingency
(i) $(p \wedge q) \wedge \neg(p \vee q)$ (ii) $((p \vee q) \wedge \neg p) \rightarrow q$

6. Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \rightarrow q) \equiv p \wedge \neg q$.

7. Prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$.

8. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.

9. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

IMPORTANT 5 MARK QUESTIONS

1.APPLICATIONS OF MATRICES AND DETERMINANTS

32. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.
33. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.
34. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.
35. Find the inverse of each of the following by Gauss-Jordan method:(i) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$
36. If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x + y + 2z = 1,\ 3x + 2y + z = 7,\ 2x + y + 3z = 2$.
37. The prices of three commodities A, B and C are ₹x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.)
38. Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0,\ \ \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0,\ \ \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$
39. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem).
40. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had?
41. Solve the following systems of linear equations by Gaussian elimination method:
(i) $2x + 4y + 6z = 22,\ 3x + 8y + 5z = 27,\ \ -x + y + 2z = 2$
42. If $ax^2 + bx + c$ is divided by $x + 3, x - 5$, and $x - 1$, the remainders are 21,61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)
43. An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is ₹4,800. The income from the third bond is ₹600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)
44. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.)
45. Test for consistency and if possible, solve the following systems of equations by rank method.
(i) $x - y + 2z = 2,\ 2x + y + 4z = 7,\ 4x - y + z = 4$
(ii) $3x + y + z = 2,\ x - 3y + 2z = 1,\ 7x - y + 4z = 5$
(iii) $2x + 2y + z = 5,\ x - y + z = 1,\ 3x + y + 2z = 4$
(iv) $2x - y + z = 2,\ 6x - 3y + 3z = 6,\ 4x - 2y + 2z = 4$
46. Find the value of k for which the equations $kx - 2y + z = 1,\ x - 2ky + z = -2,\ x - 2y + kz = 1$ have
(i) no solution (ii) unique solution (iii) infinitely many solution
47. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

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48. If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}.

49. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4,\ x - 2y - 2z = 9, 2x + y + 3z = 1$.

50. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7,\ x + y + \lambda z = \mu,\ x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

51. Solve the system: $x + y - 2z = 0, 2x - 3y + z = 0, 3x - 7y + 10z = 0, 6x - 9y + 10z = 0$.

52. Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$ has a non-trivial solution.

53. By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$.

(The above is the reaction that is taking place in the burning of organic compound called isoprene.)

54. In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points $(10,8), (20,16), (40,22)$, can you conclude that the team won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is $(70,0)$.)

55. The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \leq t \leq 100$ where a, b, and c are constants. It has been found that the speed at times $t - 3, t - 6$, and $t - 9$ seconds are respectively, 64, 133 , and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.)

56. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have
(i) no solution (ii) a unique solution (iii) an infinite number of solutions.

57. Solve the following system of homogenous equations.
(i) $3x + 2y + 7z = 0,\ 4x - 3y - 2z = 0,\ 5x + 9y + 23z = 0$
(ii) $2x + 3y - z = 0,\ x - y - 2z = 0,\ 3x + y + 3z = 0$

58. Determine the values of λ for which the following system of equations $x + y + 3z = 0, 4x + 3y + \lambda z = 0,\ 2x + y + 2z = 0$ has
(i) a unique solution
(ii) a non-trivial solution.

59. By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$.

2.COMPLEX NUMBERS

1. Show that (i) $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

2. Show that (i) $(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10}$ is purely imaginary (ii) $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

3. Let z_1, z_2, and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$.
Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$.

4. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1z_2 \neq -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number.

5. If $|z| = 2$, show that $8 \leq |z + 6 + 8i| \leq 12$.

6. If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$.

7. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

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8. If $z = x + iy$ is a complex number such that Im $\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$
9. If $z = x + iy$ and arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.
10. If $z = x + iy$ and arg $\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.
11. If $z = x + iy$ and arg $\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.
12. Find the cube roots of unity.
13. Find the fourth roots of unity.
14. Find all cube roots of $\sqrt{3} + i$.
15. Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$.
16. If $2\cos\,\alpha = x + \frac{1}{x}$ and $2\cos\,\beta = y + \frac{1}{y}$, show that (i) $\frac{x}{y} + \frac{y}{x} = 2\cos\,(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin\,(\alpha + \beta)$ (iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin\,(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos\,(m\alpha + n\beta)$.

3.THEORY OF EQUATIONS

1. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1 - 2\omega, 1 - 2\omega^2$.
2. Form the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$.
3. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are (i) $2\alpha, 2\beta, 2\gamma$ (ii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ (iii) $-\alpha, -\beta, -\gamma$
4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.
5. Prove that a line cannot intersect a circle at more than two points.
6. Prove that a straight line and parabola cannot intersect at more than two points.
7. If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.
8. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.
9. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.
10. Solve: (i) $(x-5)(x-7)(x+6)(x+4) = 504$ (ii) $(x-4)(x-7)(x-2)(x+1) = 16$
11. Solve: $(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$
12. Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.
13. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$.
14. Solve the equations (i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ (ii) $x^4 + 3x^3 - 3x - 1 = 0$
15. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.
16. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

4.INVERSE TRIGONOMERTIC FUNCTIONS

1. Find the domain of $\sin^{-1}(2 - 3x^2)$.
2. Find the domain of the following (i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (ii) $g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$.
3. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.
4. Find the domain of (i) $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$ (ii) $g(x) = \sin^{-1} x + \cos^{-1} x$
5. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}, -1 < x < 1$.
6. Find the value of (i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$. (iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$.

7. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

8. If $a_1, a_2, a_3, \dots a_n$ is an arithmetic progression with common difference d, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$.

9. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

10. Prove that $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$.

11. Simplify: $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$.

12. Solve: (i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$ (ii) $2\tan^{-1} x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}, a > 0, b > 0$.
(iii) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ (iv) $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}, x > 0$.

13. Find the number of solutions of the equation $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1}(3x)$.

5.TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

2. Find the equation of the circle through the points $(1,0)$, $(-1,0)$, and $(0,1)$.

3. For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2 .

4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
(i) $x^2 - 2x + 8y + 17 = 0$ (ii) $y^2 - 4y - 8x + 12 = 0$

5. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
(i) $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ (ii) $9x^2 - y^2 - 36x - 6y + 18 = 0$

6. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

7. A semielliptical archway over a one-way road has a height of 3 m and a width of 12 m. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?

8. A room 34 m long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in Fig. 5.64. If the maximum height of the ceiling is 8 m, determine where the foci are located.

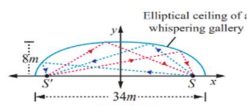


Fig. 5.64

9. If the equation of the ellipse is $\frac{(x-11)^2}{484} + \frac{y^2}{64} = 1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

10. A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides.

11. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16 m, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be?

12. At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

13. An engineer designs a satellite dish with a parabolic cross section. The dish is 5 m wide at the opening, and the focus is placed 1.2 m from the vertex
(a) Position a coordinate system with the origin at the vertex and the x-axis on the parabola's axis of symmetry and find an equation of the parabola.
(b) Find the depth of the satellite dish at the vertex.

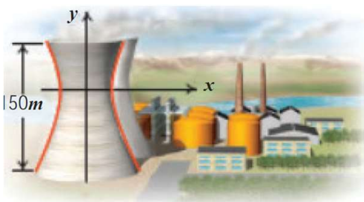
14. A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

15. Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

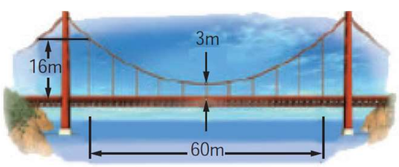
16. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

17. Points A and B are 10 km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

18. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



19. Parabolic cable of a 60 m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6 m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



6.APPLICATIONS OF VECTOR ALGEBRA

1. By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

2. Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

3. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

4. Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

5. Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

6. If $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}, \vec{c} = 3\hat{\jmath} - \hat{k}$ and $\vec{d} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$, verify that
(i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$ (ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

7. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}, \vec{b} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}, \vec{c} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, verify that
(i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$ (ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

8. Show that the lines $\vec{r} = (6\hat{\imath} + \hat{\jmath} + 2\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) + t(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ are skew lines and hence find the shortest distance between them.

9. Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}$, $z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}$, $y - 2 = 0$ intersect. Also find the point of intersection.

10. Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are skew and hence find the shortest distance between them.

11. Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + t(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and hence find the shortest distance between the lines.

12. Find the foot of the perpendicular drawn from the point (5,4,2) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular.
13. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{\imath}+2\hat{\jmath}-4\hat{k})+s(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=(\hat{\imath}-3\hat{\jmath}+5\hat{k})+t(\hat{\imath}+\hat{\jmath}-\hat{k})$.
14. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0),(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.
15. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$
16. Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1),(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.
17. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.
18. Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.
19. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\hat{\imath}-\hat{\jmath}+3\hat{k})+t(2\hat{\imath}-\hat{\jmath}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+\hat{k})=8$.
20. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3,6,-2),(-1,-2,6)$, and $(6,4,-2)$.
21. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=(6\hat{\imath}-\hat{\jmath}+\hat{k})+s(-\hat{\imath}+2\hat{\jmath}+\hat{k})+t(-5\hat{\imath}-4\hat{\jmath}-5\hat{k})$.
22. Show that the straight lines $\vec{r}=(5\hat{\imath}+7\hat{\jmath}-3\hat{k})+s(4\hat{\imath}+4\hat{\jmath}-5\hat{k})$ and $\vec{r}=(8\hat{\imath}+4\hat{\jmath}+5\hat{k})+t(7\hat{\imath}+\hat{\jmath}+3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.
23. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines.
24. Find the equation of the plane passing through the intersection of the planes $2x+3y-z+7=0$ and $x+y-2z+5=0$ and is perpendicular to the plane $x+y-3z-5=0$.
25. Find the image of the point whose position vector is $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ in the plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+4\hat{k})=38$.
26. Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.
27. Find the point of intersection of the line $x-1=\frac{y}{2}=z+1$ with the plane $2x-y+2z=2$. Also, find the angle between the line and the plane.

7.APPLICATIONS OF DIFFERENTIALS CALCULUS

1. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second, at what rate the radius of the baloon changes when the radius is 7 cm ? Also compute the rate at which the surface area changes.
2. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?
3. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

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4. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of $5\ \text{m/s}$. When the base of the ladder is 8 metres from the wall,
(i) how fast is the top of the ladder moving down the wall?
(ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?
5. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?
6. Find the angle between $y = x^2$ and $y = (x-3)^2$.
7. Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection $(0,0)$ and $(1,1)$.
8. If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.
9. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.
10. Expand $\tan x$ in ascending powers of x upto 5th power for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
11. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$
12. Find the intervals of monotonicities and hence find the local extremum for the following functions:
(i) $f(x) = \sin x \cos x + 5,\ x \in (0, 2\pi)$
13. Find the local extrema of the function $f(x) = 4x^6 - 6x^4$.
14. Find the local maximum and minimum of the function x^2y^2 on the line $x + y = 10$.
15. Find intervals of concavity and points of inflexion for the following functions:
(i) $f(x) = x(x-4)^3$ (ii) $f(x) = \frac{1}{2}(e^x - e^{-x})$
16. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.
17. We have a 12 unit square piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?
18. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from $(1,1)$.
19. A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40-5x}{10-x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.
20. Prove that among all the rectangles of the given area square has the least perimeter.
21. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.
22. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.
23. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?
24. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.
25. Prove that among all the rectangles of the given perimeter, the square has the maximum area.
26. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.
27. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.
28. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if $r + h = 6$.

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29. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

30. Sketch the curve $y = f(x) = x^2 - x - 6$.
31. Sketch the curve $y = f(x) = x^3 - 6x - 9$.

32. Sketch the graphs of the following functions:
(i) $y = -\frac{1}{3}(x^3 - 3x + 2)$ (ii) $y = x\sqrt{4 - x}$
(iii) $y = \frac{x^2+1}{x^2-4}$ (iv) $y = \frac{1}{1+e^{-x}}$

8.DIFFERENTIALS AND PARTIAL DERIVATIVES.

1. Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.
2. A right circular cylinder has radius = 10 cm. and height $h = 20$ cm. Suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.
3. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate:
(i) Absolute error (ii) Relative error (iii) Percentage error

4. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following:
(i) change in the volume (ii) change in the surface area

5. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

6. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

7. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

8. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm
(i) Approximately, how much did the tree's diameter grow?
(ii) What is the percentage increase in area of the tree's cross-section?

9. The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}, 0 \le x \le 9$. What is the approximate number of words learned when x changes from
(i) 1 to 1.1 hour? (ii) 4 to 4.1 hour?

10. Consider $f(x,y) = \frac{xy}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is not continuous at (0,0) and continuous at all other points of $\mathbb{R}^2$.

11. Let $g(x,y) = \frac{x^2y}{x^4+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$.
(i) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = 0$ along every line $y = mx, m \in \mathbb{R}$.
(ii) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = \frac{k}{1+k^2}$ along every parabola $y = kx^2, k \in \mathbb{R} \setminus \{0\}$.

12. Let $F(x,y) = x^3y + y^2x + 7$ for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial F}{\partial x}(-1,3)$ and $\frac{\partial F}{\partial y}(-2,1)$.

13. Let $f(x,y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y\,\partial x}$ and $\frac{\partial^2 f}{\partial x\,\partial y}$.

14. Let $w(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}, (x,y,z) \neq (0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$.

15. If $V(x,y) = e^x(x\cos y - y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$.

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16. If $w(x,y) = xy + \sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y}$.
17. Let $U(x,y) = e^x \sin y$, where $x = st^2, y = s^2t, s,t \in \mathbb{R}$. Find $\frac{\partial U}{\partial s}, \frac{\partial U}{\partial t}$ and evaluate them at $s = t = 1$.
18. $W(x,y,z) = xy + yz + zx,\ x = u - v,\ y = uv,\ z = u + v,\ \ u,v \in \mathbb{R}$. Find $\frac{\partial W}{\partial u}, \frac{\partial W}{\partial v}$, and evaluate them at $\left(\frac{1}{2}, 1\right)$
19. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.
20. Prove that $f(x,y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous; what is the degree? Verify Euler's Theorem for f.
21. Prove that $g(x,y) = x\log\left(\frac{y}{x}\right)$ is homogeneous; what is the degree? Verify Euler's Theorem for g.
22. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.
23. If $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$
24. If $w(x,y,z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$.

9.APPLICATIONS OF INTEGRATION

1. Evaluate $\int_0^1 x^3 dx$, as the limit of a sum.
2. Evaluate $\int_1^4 (2x^2 + 3)dx$, as the limit of a sum.
3. Evaluate the following integrals as the limits of sums: (i) $\int_0^1 (5x + 4)dx$ (ii) $\int_1^2 (4x^2 - 1)dx$
4. Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin x + \cos x} dx$
5. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$.
6. Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)dx = \frac{\pi}{8}\log 2$.
7. Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$
8. Evaluate the following integrals using properties of integration :
   (i) $\int_0^{2\pi} x\log\left(\frac{3+\cos x}{3-\cos x}\right)dx$ (ii) $\int_0^{2\pi} \sin^4 x\cos^3 x dx$ (iii) $\int_0^1 |5x - 3|dx$
   (iv) $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$ (v) $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ (vi) $\int_0^{\pi} \frac{x\sin x}{1+\sin x}dx$
   (vii) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx$ (viii) $\int_0^{\pi} x[\sin^2(\sin x) + \cos^2(\cos x)]dx$
9. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$.
10. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}dx$.
11. Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.
12. Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.
13. Find the area of the region bounded by $y = \cos x, y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.
14. Find, by integration, the area of the region bounded by the lines $5x - 2y = 15, x + y + 4 = 0$ and the x-axis.
15. Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are $(-1,1), (3,2)$, and $(0,5)$ respectively.
16. Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x-axis, $x = -3$ and $x = 3$.
17. Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$.
18. Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

19. Find the area of the region bounded by $y = \tan x$, $y = \cot x$ and the lines $x = 0$, $x = \frac{\pi}{2}$, $y = 0$.
20. Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.
21. Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.
22. The curve $y = (x-2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2 . Find the area bounded by the curve and the chord PQ.
23. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.
24. Find the volume of a sphere of radius a.
25. Find the volume of a right-circular cone of base radius r and height h.
26. Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ about the major axis.
27. The region enclosed between the graphs of $y = x$ and $y = x^2$ is denoted by R, Find the volume generated when R is rotated through 360° about x-axis.
28. Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in the Fig 9.46

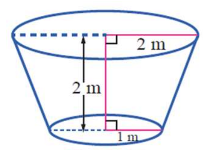

Fig. 9.46

29. A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration.

10.ORDINARY DIFFERENTIAL CALCULUS.

1. Find the particular solution of $(1 + x^3)dy - x^2 y dx = 0$ satisfying the condition $y(1) = 2$.
2. If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Express V in terms of t given that $V = 0$ when $t = 0$.
3. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1,0).
4. Solve the following differential equations:
   (i) $ydx + (1 + x^2)\tan^{-1} xdy = 0$ (ii) $(e^y + 1)\cos x\, dx + e^y \sin xdy = 0$
   (iii) $(ydx - xdy)\cot\left(\frac{x}{y}\right) = ny^2 dx$ (iv) $x\cos y\, dy = e^x(x\log x + 1)dx$
   (v) $\tan y\frac{dy}{dx} = \cos(x+y) + \cos(x-y)$ (vi) $\frac{dy}{dx} = \tan^2(x+y)$
5. Solve $(2x + 3y)dx + (y - x)dy = 0$.
6. Solve $\left(1 + 2e^{\frac{x}{y}}\right)dx + 2e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$.
7. Solve : $(x^3 + y^3)dy - x^2 y dx = 0$
8. Solve : $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y\right)dy$
9. Solve : $(y^2 - 2xy)dx = (x^2 - 2xy)dy$
10. Solve : $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$
11. Solve : $\left(1 + 3e^{\frac{y}{x}}\right)dy + 3e^{\frac{y}{x}}\left(1 - \frac{y}{x}\right)dx = 0$, given that $y = 0$ when $x = 1$
12. Solve : $(x^2 + y^2)dy = xy\, dx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0.
13. Solve $(1 + x^3)\frac{dy}{dx} + 6x^2 y = 1 + x^2$.
14. Solve : $\cos x\frac{dy}{dx} + y\sin x = 1$.
15. Solve : $(1 - x^2)\frac{dy}{dx} - xy = 1$
16. Solve : $(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$

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17. Solve : $x\sin x\frac{dy}{dx}+(x\cos x+\sin x)y=\sin x$
18. Solve : $\left(y-e^{\sin^{-1}x}\right)\frac{dx}{dy}+\sqrt{1-x^2}=0$
19. Solve : $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$
20. Solve : $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\sin 2x}{\log x}$
21. Solve : $\frac{dy}{dx}=\frac{\sin^2 x}{1+x^3}-\frac{3x^2}{1+x^3}y$
22. Solve : $x\frac{dy}{dx}+2y-x^2\log x=0$
23. Solve: $\frac{dy}{dx}+\frac{3y}{x}=\frac{1}{x^2}$, given that $y=2$ when $x=1$
24. The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?
25. A radioactive isotope has an initial mass 200mg, which two years later is 50mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).
26. In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?
    [log (2.43) = 0.88789; log (0.5) = −0.69315]
27. A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.
28. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?
29. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.
30. The equation of electromotive force for an electric circuit containing resistance and self inductance is $E=Ri+L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E=0$.
31. The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.
32. Suppose a person deposits ₹10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?
33. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?
34. Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C. Find
    (i) The temperature of water after 20 minutes
    (ii) The time when the temperature is 40°C $\left[\log_e \frac{11}{15}=-0.3101; \log_e 5=1.6094\right]$
35. At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was 180°F, and 10 minutes later it was 160°F. Assume that constant temperature of the kitchen was 70°F.
    (i) What was the temperature of the coffee at 10.15A.M.? $\left[\log\frac{9}{11}=-0.6061\right]$
    (ii) The woman likes to drink coffee when its temperature is between 130°F and 140°F. between what times should she have drunk the coffee? $\left[\log\frac{6}{11}=-0.2006\right]$

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36. A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C, and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

37. A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.

11.PROBABILITY DISTRIBUTIONS

1. A six sided die is marked ' 2 ' on one face, ' 3 ' on two of its faces, and ' 4 ' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.
2. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.
3. If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find (ii) $P(X \leq 3)$ and, (iii) $P(X \geq 2)$

4. A six sided die is marked ' 1 ' on one face, ' 2 ' on two of its faces, and ' 3 ' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.
 (i) Find the probability mass function.
 (ii) Find the cumulative distribution function.
 (iii) Find $P(3 \leq X < 6)$ (iv) Find $P(X \geq 4)$.

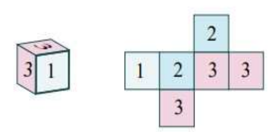

Fig. 11.7

5. Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

 Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$.

6. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

 Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X < 5)$
 (iii) $P(X \leq 4)$ (iv) $P(3 < X)$

7. A six sided die is marked ' 1 ' on one face, ' 3 ' on two of its faces, and ' 5 ' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find
 (i) the probability mass function (ii) the cumulative distribution function
 (iii) $P(4 \leq X < 10)$ (iv) $P(X \geq 6)$

8. Suppose a discrete random variable can only take the values 0,1 , and 2.The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

 Find (i) the value of k (ii) cumulative distribution function (iii) $P(X \geq 1)$.

9. The cumulative distribution function of a discrete random variable is given by

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$$F(x)=\begin{cases}0 & -\infty < x < -1\\ 0.15 & -1 \le x < 0\\ 0.35 & 0 \le x < 1\\ 0.60 & 1 \le x < 2\\ 0.85 & 2 \le x < 3\\ 1 & 3 \le x < \infty\end{cases}$$

Find (i) the probability mass function (ii) $P(X < 1)$ and (iii) $P(X \ge 2)$.

10. A random variable X has the following probability mass function

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$

11. The cumulative distribution function of a discrete random variable is given by

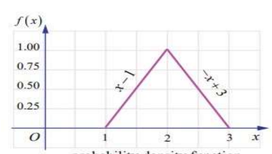
$$F(x)=\begin{cases}0 & \text{for } -\infty < x < 0\\ \frac{1}{2} & \text{for } 0 \le x < 1\\ \frac{3}{5} & \text{for } 1 \le x < 2\\ \frac{4}{5} & \text{for } 2 \le x < 3\\ \frac{9}{10} & \text{for } 3 \le x < 4\\ 1 & \text{for } 4 \le x < \infty\end{cases}$$

Find (i) the probability mass function (ii) $P(X < 3)$ and (iii) $P(X \ge 2)$.

12. Find the constant C such that the function $f(x)=\begin{cases}Cx^2 & 1 < x < 4\\ 0 & \text{Otherwise}\end{cases}$ is a density function, and compute
(i) $P(1.5 < X < 3.5)$ (ii) $P(X \le 2)$ (iii) $P(3 < X)$.

13. If X is the random variable with probability density function $f(x)$ given by,

$$f(x)=\begin{cases}x-1, & 1 \le x < 2\\ -x+3, & 2 \le x < 3\\ 0 & \text{otherwise}\end{cases}$$



probability density function
Fig. 11.16

find (i) the distribution function $F(x)$ (ii) $P(1.5 \le X \le 2.5)$

14. The probability density function of random variable X is given by $f(x)=\begin{cases}k & 1 \le x \le 5\\ 0 & \text{otherwise}\end{cases}$ Find
(i) Distribution function (ii) $P(X < 3)$ (iii) $P(2 < X < 4)$ (iv) $P(3 \le X)$

15. Let X be a random variable denoting the life time of an electrical equipment having probability density function

$$f(x)=\begin{cases}ke^{-2x} & \text{for } x > 0\\ 0 & \text{for } x \le 0\end{cases}$$

Find (i) the value of k (ii) Distribution function (iii) $P(X < 2)$
(iv) calculate the probability that X is at least for four unit of time (v) $P(X = 3)$.

16. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$$f(x)=\begin{cases}k & 200 \le x \le 600\\ 0 & \text{otherwise}\end{cases}$$

Find (i) the value of k (ii) the distribution function
(iii) the probability that daily sales will fall between 300 litres and 500 litres?

17. The probability density function of X is given by $f(x)=\begin{cases}ke^{-\frac{x}{3}} & \text{for } x > 0\\ 0 & \text{for } x \le 0\end{cases}$

Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv) $P(5 \le X)$ (v) $P(X \le 4)$.

18. If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \le X \le 0.5)$

19. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.3 \le X \le 0.6)$

20. Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance.

21. For the random variable X with the given probability mass function as below, find the mean and variance.

(i) $f(x) = \begin{cases} \frac{1}{10} & x = 2,5 \\ \frac{1}{5} & x = 0,1,3,4 \end{cases}$ (ii) $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$

22. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected life of this electronic equipment.

23. The probability density function of the random variable X is given by

$$f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$$

find the mean and variance of X.

24. The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

25. A retailer purchases a certain kind of electronic device from a manufacturer.
The manufacturer indicates that the defective rate of the device is 5%.
The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items?

26. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
(i) exactly 10 will have a useful life of at least 600 hours;
(ii) at least 11 will have a useful life of at least 600 hours;
(iii) at least 2 will not have a useful life of at least 600 hours.

12.DISCRETE MATHEMATICS

1. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}_e$ = the set of all even integers.

2. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set.

$$m * n = m + n - mn; m, n \in \mathbb{Z}$$

3. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

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4. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.
5. (i) Define an operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right); a, b \in \mathbb{Q}$. Examine the closure, commutative, and associative properties satisfied by $*$ on $\mathbb{Q}$.
   (ii) Define an operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right); a, b \in \mathbb{Q}$. Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.
6. (i) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M.
   (ii) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the existence of identity, existence of inverse properties for the operation $*$ on M.
7. (i) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A ? If so, examine the commutative and associative properties satisfied by $*$ on A.
   (ii) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is * binary on A ? If so, examine the existence of identity, existence of inverse properties for the operation $*$ on A
8. Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.
9. Verify whether the following compound propositions are tautologies or contradictions or contingency
   (i) $(p \to q) \leftrightarrow (\neg p \to q)$ (ii) $((p \to q) \wedge (q \to r)) \to (p \to r)$
10. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table.
11. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.
12. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.
13. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

STAGE – I (50 /90)

2 MARKS	3 MARKS	5 MARKS
1,2,3,11,12	1,2,3,11,12	5,6,12

STAGE – II (75 /90)

2 MARKS	3 MARKS	5 MARKS
1,2,3,4,5,11,12	1,2,3,4,5,11,12	2,3,5,6,7,12

STAGE – III (85 /90)

2 MARKS	3 MARKS	5 MARKS
1,2,3,4,5,8,11,12	1,2,3,4,5,8,11,12	2,3,5,6,7,8,11,12

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