Class : 12

Register Number | 1 | 2 | B | D | 2 |

UNIT TEST -1, JULY - 2024
MATHEMATICS

Time Allowed : 1.30 Hours]

[Max. Marks : 50

Part - I

1. Answer all the questions by choosing the correct answer from the given 4 alternatives. 10x1=10
2. Write question number, correct option and corresponding answer.
3. Each question carries 1 mark.

1. If A is a 3x3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$
   a) A   b) B   c) I_3 d) B^T

2. If A, B and C are invertible matrices of some order, then which one of the following is not true?
   a) $adj\ A = |A|\ A^{-1}$ c) $adj(AB) = (adj\ A)(adj\ B)$
   c) $A^{-1} = (det\ A)^{-1}$ d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

3. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, be such that $\lambda A^{-1} = A$, then λ is
   a) 17   b) 14   c) 19   d) 21

4. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adj A) is
   a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$
   b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$
   c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$
   d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

5. The value of $\sum_{i=1}^{13}(i^n + i^{n-1})$ is
   a) $1+i$ b) i c) 1   d) 0

6. If $|z| = 1$, then the value of $\dfrac{1+z}{1+\bar{z}}$ is
   a) z b) $\bar{z}$ c) $1/z$ d) 1

7. The principal argument of $\dfrac{3}{-1+i}$ is
   a) $\dfrac{-5\pi}{6}$ b) $\dfrac{-2\pi}{3}$ c) $\dfrac{-3\pi}{4}$ d) $\dfrac{-\pi}{2}$

8. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
   a) mn   b) m + n   c) m^n d) n^m

9. A polynomial equation in x of degree n always has
   a) n distinct roots   b) n real roots   c) n imaginary roots   d) at most one root.

10. Product of all the roots of nth roots of unity is
    a) 1   b) 0   c) $(-1)^{n-1}$ d) $(-1)^n$

PART – II

1. Answer any 4 questions. 2. Each question carries 2 marks   4x2=8
3. Question number 16 is compulsory

11. Find a matrix A if adj(A)= $\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$

CH/12/ Mat / 1

12. Solve the following system of linear equations by matrix inversion method: $2x - y = 8$, $3x + 2y = -2$

13. If $\dfrac{z+3}{z-5i} = \dfrac{1+4i}{2}$, find the complex number z in the rectangular form

14. Which one of the points $10 - 8i$, $11 + 6i$ is closest to $1 + i$.

15. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of the two roots is 1 .

16. If A and B are non-singular matrices of order n , then prove that $adj\,(AB) = (adj\,B)(adj\,A)$

PART – III

1. Answer any 4 questions.   2. Each question carries 3 marks $4 \times 3 = 12$
3. Question number 22 is compulsory

17. Find the rank of the matrix by row reduction method: $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

18. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

19. Find the value of the real numbers x and y, if the complex number $(2+i)\,x + (1-i)\,y + 2i - 3$ and $x + (-1+2i)\,y + 1 + i$ are equal.

20. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\dfrac{pq'-p'q}{q-q'}$ or $\dfrac{q-q'}{p'-p}$

21. Find the square root of $6-8i$.

22. For $n \in N$, $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = (2)^{n+1}\cos^n\dfrac{\theta}{2}\cos\dfrac{n\theta}{2}$

PART – IV

1. Answer all the questions.   2. Each question carries 5 marks $4 \times 5 = 20$

23. a) Find the inverse of each of the following by Gauss-Jordan method: $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

(OR)

b) The upward speed v(t) of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times $t = 3$, $t = 6$, and $t = 9$ seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.)

24. a) Test for consistency and if possible, solve the following systems of equations by rank method.
$3x + y + z = 2$, $x - 3y + 2z = 1$, $7x - y + 4z = 5$

(OR)

b) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$

25. a) Show that the points 1, $\dfrac{-1}{2} + i\dfrac{\sqrt{3}}{2}$ and $\dfrac{-1}{2} - i\dfrac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

(OR)

b) If $z = x + iy$ is a complex number such that $Im\left(\dfrac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

26. a) If $\dfrac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.

(OR)

b) Find a polynomial equation of minimum degree with rational coefficients having $\sqrt{5} - \sqrt{3}$ as a root.

CH/12/ Mat /2