UNIT - 3

CLASSICAL MECHANICS

Introduction: classical mechanics deals with the motion of physical bodies at the macroscopic level. The mechanics based on Newton's law of motion and alternatively developed by Lagrange, Hamilton and others is called classical mechanics. The classical mechanics explains correctly the motion of celestial bodies like planets, stars and macroscopic as well as microscopic terrestrial bodies moving with non relativistic speeds (i.e $v << c$; c being speed of light)

Newton's Law of Motion

i) First law of motion (Law of Inertia): Every object continues in its state of rest or uniform motion in a straight line unless a net external force acts on it to change that state.

Momentum of a body proportional to its velocity.

$$P = mv$$

In the absence of an external force acting on a body

$$P = mv = \text{constant}$$

This is the law of conservation of momentum.

As per the special theory of relativity, mass is not a constant but varies with velocity

ii) Second Law of motion: The rate of change of momentum of an object is directly proportional to the force applied and takes place in the direction of the force.

$$F = \frac{dp}{dt}$$

$$F = m\frac{dv}{dt} = ma \qquad [\because p = mv]$$

or

$$F = m\frac{d^2r(t)}{dt^2} \qquad \left[\because v = \frac{dr(t)}{dt}\right]$$

This is also known as equation of motion of the particle.

iii) Third law of motion: Whenever a body exerts a force on a second body, the second exerts an equal and opposite force on the first.

(i.e) To every action there is an equal and opposite reaction.

$$F_{12} = -F_{21} \longrightarrow (1)$$

$\because$ Force is the rate of change of momentum

$$\frac{dP_1}{dt} = -\frac{dP_2}{dt}$$

$$m_1 a_1 = -m_2 a_2$$

$$\frac{m_2}{m_1} = \left|\frac{a_1}{a_2}\right| \longrightarrow (2)$$

Eq (2) used to determine the mass of particles.

Inertial reference frames : Newton's law of inertia holds good.

Non inertial reference frames : Newton's law of inertia does not hold.

Mechanics of a particle : conservation Laws

i) conservation of Linear Momentum

From Newton's 2nd law of motion,

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv)$$

If external force acting on the particle is zero

$$\frac{dP}{dt} = \frac{d}{dt}(mv) = 0$$

$$P = mv = \text{constant}$$

In absence of external force, the linear momentum of a particle is conserved. This is the principle of conservation of linear momentum of a system of particles.

ii) Conservation of angular momentum

Angular momentum $J = r \times p$

If the force on the particle is F, then the moment of force or torque is $\tau = r \times F$

$$\Rightarrow \frac{dJ}{dt} = \frac{d}{dt}(r \times p) = r \times \frac{dP}{dt} = r \times F$$

$$\tau = \frac{dJ}{dt}$$

(i.e) The time rate of change of angular momentum of a particle is equal to the torque acting on it.

If the torque acting on the particle is zero, (i.e) $\tau = 0$, then $\frac{dJ}{dt} = 0$ or $J =$ constant

$\therefore$ The angular momentum of a particle is constant of motion in absence of external torque. This is the conservation of angular momentum.

Conservation of energy:

Work done by an external force F is $W_{12} = \int_1^2 F \cdot dr$

Kinetic energy & Work energy theorem

$$W_{12} = \int_1^2 F \cdot dr = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

The scalar quantity $\frac{1}{2} m v^2$ is defined as the kinetic energy and denoted by T.

Thus work done by the force acting on the particle appears equal to the change in kinetic energy

$$W_{12} = \int_1^2 F \cdot dr = T_2 - T_1$$

This is known as work energy theorem

conservative force: If the work done by a force is independent of path, the force is said to be conservative. If the forces on the particle are conservative; its mechanical energy (potential energy + kinetic energy) remains constant.

In case of a non conservative force like friction, the amount of work done around different closed paths are different and not zero.

For non conservative force, the total energy of universe (mechanical energy + chemical energy + sound energy + light energy + heat energy, etc) remains constant.

The necessary and sufficient condition for a force to be conservative is $\nabla \times F = \text{curl}\, F = 0$

The curl of a vector is zero if it can be expressed as the gradient of a scalar function of position $\Rightarrow$ $F = -\nabla V(r)$ $\rightarrow (1)$

Scalar function $V(r)$ in eq (1) is called potential energy.

Conservation theorem:

$$W_{12} = \int_1^2 F \cdot dr = T_2 - T_1 = V_1 - V_2$$

$$\Rightarrow \quad T_2 - T_1 = V_1 - V_2 \quad \text{or} \quad T_1 + V_1 = T_2 + V_2 = \text{constant}$$

Thus the sum of kinetic and potential energies remains constant. ($T+V$ is constant)

$\Rightarrow$ $\boxed{T+V = E}$ $\Rightarrow$ conservation energy theorem

Constraints : The motion of a particle or system of particles is restricted by one or more conditions. The limitations on the motion of a system are called constraints and the motion is said to be constrained motion.

Ex: i) Motion of rigid bodies (distance between any 2 particles remains unchanged)

ii) Gas molecules within a container are constrained by the walls of the vessel to move only inside the container

iii) Motion of point mass of a simple pendulum

iv) Motion of a particle placed on the surface of a solid sphere

There are two main types of constraints,

i) holonomic

ii) Non holonomic

i) Holonomic constraints:

The conditions of constraint are expressible as equations connecting the coordinates and time, having the form of

$$f(r_1, r_2, r_3, \cdots r_n, t) = 0 \longrightarrow (1)$$

Examples

a) In rigid body, the distance between any two particles of the body remains constant.

$$|r_i - r_j|^2 = c_{ij}^2$$