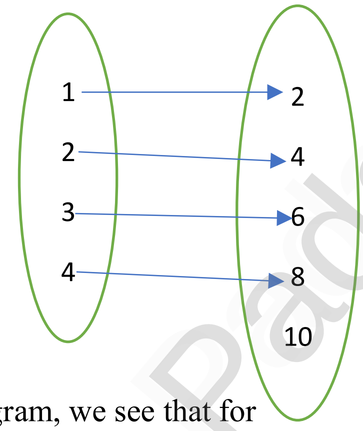
# *2 Marks*

## RELATIONS AND FUNCTIONS

1. If AXB = {(3,2), (3,4), (5,2), (5,4)} then find A and B.
   Solution:
   A = Set of all first coordinates of elements of AXB = {3,5}
   B = Set of all second coordinates of elements of AXB = {2,4}.
2. If BXA = {(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)} then find A and B.
   Solution:
   A = Set of all second coordinates of elements of AXB = {3,4}
   B = Set of all first coordinates of elements of AXB = {-2,0,3}.
3. Let A = {1,2,3} and B = {x/x is a prime number less than 10}. Find AXB and BXA.
   Solution:
   AXB = {1,2,3} X {2,3,5,7}
   = {(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7)}.
   BXA = {2,3,5,7} X {1,2,3}
   = {(2,1), (2,2), (2,3), (3,1), (3,2) (3,3), (5,1), (5,2), (5,3), (7,1), (7,2), (7,3)}.
4. Let X = {1,2,3,4} and Y = {2,4,6,8,10} and R = {(1,2), (2,4), (3,6), (4,8)}. Show that R is a function and find its domain, co-domain and range?
   Solution:




   From the diagram, we see that for each x∈X, there exists only one y∈ $Y$.
   Therefore R is a function. Domain X = {1,2,3,4}, Co-domain Y = {2,4,6,8,10} &
   Range = {2,4,6,8}.
5. Let A = {1,2,3,4,…,45} and R be the relation defined as "is square of a number" on A. Write R as a subset of AXA. Also, find the domain and range of R.
   Solution:
   A = {1,2,3,4,..,45}
   R = {(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)}
   R ⊂ (AXA)
   Domain = {1,2,3,4,5,6}
   Range = {1,4,9,16,25,36}.
6. A relation R is given by {(x,y)/y=x+3, x∈{0,1,2,3,4,5}}. Determine its domain and range.
   Solution:
   Given x∈{0,1,2,3,4,5} and f(x) = x+3.
   f(0) = 0+3 =3, f(1) = 1+3 =4, f(2) = 2 +3 = 5, f(3) = 3+3 = 6,
   f(4) = 4+3 = 7, f(5) = 5+3 = 8.

Hence R = {(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)}.
Domain = {0,1,2,3,4,5}
Range = {3,4,5,6,7,8}.

7. A relation $f:X \to Y$ is defined by f(x) = $x^2$-2 where X={-2,-1,0,3} and Y = R. (i) List the elements of f (ii) Is f a function?
   Solution:
   Given f(x) = $x^2$-2 and X = {-2,-1,0,3}.
   f(-2) = $(-2)^2$-2 = 4-2 = 2, f(-1) = $(-1)^2$-2 = 1-2 = -1, f(0) = -2, f(3)=$(3)^2$-2=7.
   Hence f = {(-2,2), (-1,-1), (0,-2), (3,7)}
   (ii) We note that each element in domain of f has a unique image. Therefore, f is a function.
8. A function f is defined by f(x) = 3 – 2x. Find x such that f($x^2$)=$(f(x))^2$.
   Solution:
   f($x^2$) = $(f(x))^2$. Then 3-2$x^2$ = $(3-2x)^2$.
   3 – 2$x^2$ = 9 – 12x + 4$x^2$.
   3 – 2$x^2$ – 9 -12x + 4$x^2$ = 0
   $x^2$-2x+1=0 then $(x-1)^2$=0. Therefore x = 1,1.
9. Let f be a function from R to R defined by f(x) = 3x-5. Find the values of a and b given that (a,4) and (1,b) belong to f.
   Solution:
   Given f(x) = 3x – 5. (a,4) means the image of a is 4, i.e) f(a) =4
   3a-5 = 4 then 3a = 9 and hence a=3.
   (1,b) means the image of 1 is b, i.e) f(1) = b, then 3(1) – 5 = b and hence b = -2.
10. Represent the function f(x) = $\sqrt{2x^2 - 5x + 3}$ as a composition of two functions.
    Solution:
    Let $f_1(x) = \sqrt{x}$ *and* $f_2(x) = 2x^2 - 5x + 3$.
    f(x) = $\sqrt{2x^2 - 5x + 3} = \sqrt{f_2(x)} = f_1[f_2(x) = f_1 f_2(x)$.
11. Find k if fof(k)=5 where f(k) = 2k-1.
    Solution:
    fof(k) = f(f(k) = f(2k-1) = 2(2k-1)-1 = 4k-2-1=4k-3.
    Given, fof(k)=5 then 4k-3 = 5 and hence 4k = 8, k=2.
12. If f(x) = $x^2$-1, g(x) = x-2 find a if gof(a) =1.
    Solution:
    gof(a) = g(f(a) = g($a^2$-1) = $a^2$-1-2 = $a^2$-3.
    Given, gof(a) =1 then $a^2$ – 3 = 1 and hence $a^2$ = 4. Therefore $a = \pm 2$.

## NUMBERS AND SEQUENCES

13. A man has 532 flower pots. He wants to arrange them in rows such that each rows contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.
    Solution:
    Total number of flower pots = 532.
    Each row contains 21 flower pots. Hence number of completed rows = 25 and remaining flower pots = 7.
14. 'a' and 'b' are two positive integers such that $a^b$ x $b^a$ = 800. Find 'a' and 'b'.
    Solution:

800 = 2x2x2x2x2x5x5 = $2^5$ x $5^2$.
Hence a = 5, b = 2 (or) a = 2, b = 5.

15. If 13824 = $2^a$ x $3^b$ then find 'a' and 'b'.
Solution:
13824 = 2x2x2x2x2x2x2x2x2x3x3x3 = $2^9$ x $3^3$.
Hence a = 9 and b = 3.

16. Find the least number that is divisible by the first ten natural numbers.
1 = 1, 2 = 2x1, 3 = 3x1, 4 = 2x2 = $2^2$, 5 = 5x1, 6 = 2x3,
7 = 7x1, 8 = 2x2x2=$2^3$, 9 = 3x3=$3^2$, 10 = 2x5,
Hence, the least number = LCM of the above 10 numbers = $2^3$x$3^2$x5x7 = 2520.

17. Find the HCF of 252525 and 363636.
Solution:
363636 = $2^2$x$3^3$x7x13x37
252525 = 3x$5^2$x7x13x37.
Hence HCF = 3x7x13x37 = 10101.

18. If ${p_1}^{x_1}\ x\ \ {p_2}^{x_2}\ x\ {p_3}^{x_3}\ x\ {p_4}^{x_4} = 113400$ where $p_1$,$p_2$,$p_3$,$p_4$ are primes in ascending order and $x_1$,$x_2$,$x_3$,$x_4$ are integers, find the value of $p_1$,$p_2$,$p_3$,$p_4$ and $x_1$,$x_2$,$x_3$,$x_4$.
Solution:
113400 = 2x2x2x3x3x3x3x5x5x7 = $2^3$x$3^4$x$5^2$x$7^1$.
Hence, $p_1$=2, $p_2$=3, $p_3$=5, $p_4$=7, $x_1$=3, $x_2$=4, $x_3$=2 and $x_4$=1.

19. Find the remainder when 70004 and 778 is divided by 7.
Solution:
Since 70000 is divisible by 7, $70000 \equiv 0\ (mod 7)$
$70000 + 4 \equiv 0 + 4\ (mod 7),\quad hence\ 70004 \equiv 4 (mod 7).$
Therefore, the remainder when 70004 is divided by 7 is 4.
Similarly, since 777 is divisible by 7, $777 \equiv 0\ (mod 7)$.
$777 + 1 \equiv 0 + 1\ (mod 7),\quad hence\ 778 \equiv 1\ (mod 7).$
Hence, the remainder when 778 is divisible by 7 is 1.

20. Find the least positive value of x such that $67 + x \equiv 1\ (mod 4)$.
Solution:
67 + x≡1 (mod 4)
67+x-1 = the multiple of 4 = 4n, for some integer n.
66+x = 4n = the multiple of 4.
Since 68 is the nearest multiple of 4 more than 68, hence, the least positive value of x must be 2.

21. Compute x, such that $10^4 \equiv x$ (mod 19).
Solution:
$10^2$ = 100 ≡ 5 (mod 19).
$10^4$ = $(10^2)^2$ ≡ $5^2$ (mod 19)
$10^4$ ≡ 25 (mod 19)
$10^4$ ≡ 6 (mod 19).
Hence x = 6.

22. Find the number of integer solutions of 3x ≡ 1 (mod 15).
Solution:
3x ≡ 1 (mod 15) can be written as 3x – 1 = 15k for some integer k.

3x = 15k+1, hence $x = \frac{15k+1}{3}$.

i.e) $x = 5k + \frac{1}{3}$. Here 5k is an integer but 5k + 1/3 cannot be an integer.

So there is no integer solution.

23. The general term of a sequence is defined as $a_n = \begin{cases} n(n+3), & n \in N \text{ is odd} \\ n^2+1, & n \in \text{is even} \end{cases}$. Find the eleventh and eighteenth terms.

Solution:

To find $a_{11}$, since 11 is odd, we put n = 11 in $a_n = n(n+3)$.

$a_{11} = 11(11+3) = 11 \times 14 = 154$.

To find $a_{18}$, since 18 is even, we put n = 18 in $a_{18} = n^2+1$.

Thus, $a_{18} = 18^2+1 = 324+1 = 325$.

24. Find the number of terms in the A.P. 3,6,9,12,…,111.

Solution:

First term a = 3, common difference d = $t_2 - t_1$ = 6 – 3 = 3, last term l = 111.

We know that $n = \frac{l-a}{d} + 1 = \frac{111-3}{3} + 1 = 36 + 1 = 37$.

25. Find the 19th term of an A.P. -11,-15,-19,…

Solution:

First term a = -11, common difference d = $t_2 - t_1$ = -15-(-11) = -15+11 = -4.

Hence 19th term = $t_{19}$ = a + (n-1)d = -11 + (19-1)(-4) = -11 – 72 = -83.

26. Which term of an A.P. 16,11,6,1,… is (-54)?

Solution:

First term a = 16, common difference d = $t_2 - t_1$ = 11-16 = -5, last term l = -54

We know that $n = \frac{l-a}{d} + 1, n = \frac{-54-16}{-5} + 1$.

Then, $n = \frac{-70}{-5} + 1 = 14 + 1 = 15$.

So, 54 is the 15th term.

27. If 3+k, 18-k, 5k+1 are in A.P, then find k.

Solution:

Since 3+k, 18-k and 5k+1 are in A.P, $t_2 - t_1 = t_3 - t_2$.

So, 18-k – (3+k) = 5k+1 – (18-k)

18 –k – 3 – k = 5k + 1- 18 + k

15 -2k = 6k -17

15+17 = 6k+2k

8k = 32, hence k = 4.

28. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution:

First term a = 20, common difference d = 2.

Number of seats in the last row = $t_n$ = a+(n-1)d = 20 + (30-1)(2) = 20 + 29x2 = 78.

29. In a G.P. 729, 243, 81,… find $t_7$.

Solution:

First term a = 729, common ratio $r = \frac{t_2}{t_1} = \frac{243}{729} = \frac{1}{3}$.

We know that $t_n = ar^{n-1}$.

Then, $t_7 = 729(\frac{1}{3})^6 = 729\ x\ \frac{1}{729} = 1.$

30. Find the 8th term of the G.P. 9,3,1,..

Solution:

First term a = 9, common difference r = $\frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$.

We know that, nth term $t_n = ar^{n-1}$.

Hence, $t_8 = 9(\frac{1}{3})^7 = 9\ x\ \frac{1}{9\ x\ 243} = \frac{1}{243}$.

31. Find the value of 1+2+3+…+50

Solution:

We know that 1+2+3+…+n = $\frac{n(n+1)}{2}$.

Here n = 50. There fore, 1+2+3+…+50 = $\frac{50x51}{2} = 25x51 = 1275.$

32. Find the value of 16+17+18+…+75

Solution:

We know that 1+2+3+…+n = $\frac{n(n+1)}{2}$.

Now, 16+17+18+…+75 = (**1+2+3+…+15**+16+17+…+75) – (**1+2+3+…+15**)

$= \frac{75x76}{2} - \frac{15x16}{2} = 75\ x\ 38 - 15\ x\ 8 = 2850 - 120 = 2730.$

33. Find the sum of 1+3+5+… to 40 terms

Solution:

We know that 1+3+5+… n terms = $n^2$.

Here, 1+3+5+… to 40 terms = $40^2$=1600.

34. Find the sum of 1+3+5+…+55

Solution:

We know that 1+3+5+…+$l$ = $(\frac{l+1}{2})^2 = \left(\frac{55+1}{2}\right)^2 = 28^2 = 784.$

35. If 1+2+3+…+n = 666 then find 'n'.

Solution:

We know that 1+2+3+…+ n =$\left(\frac{n(n+1)}{2}\right)$.

Here, $\frac{n(n+1)}{2} = 666.$

Now, $n^2$+n-1332=0,

Then, (n+37)(n-36)=0

Hence, n=-37 (or) n = 36.

But n = -37 is not possible, since number of terms is always positive.

36. If $1^3+2^3+3^3+…+k^3$ = 44100, then find 1+2+3…+k.

Solution:

We know that 1+2+3+…+k = $\frac{k(k+1)}{2}$.

Also, $1^3 + 2^3 + 3^3 + \cdots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$. Given, $1^3+2^3+3^3+…+k^3$ = 44100

Therefore, $\left(\frac{k(k+1)}{2}\right)^2 = 44100 = (210)^2 \Rightarrow \frac{k(k+1)}{2} = 210.$

Hence, 1+2+3+…+k = 210.

ALGEBRA

37. Find the LCM of $8x^4y^2$, $48x^2y^4$.

Solution:

LCM of 8 and 48 = 48
LCM of $x^4y^2$, $x^2y^4$ = $x^4y^4$.
Hence LCM = $48x^4y^4$.

38. Find the excluded value of $\frac{7p+2}{8p^2+13p+5}$.
Solution:
Given expression is undefined for $8p^2+13p+5 = 0$,
That is, $(8p+5)(p+1) = 0$. Hence excluded values are p = -5/8 and p = -1.

39. Find the excluded value of $\frac{y}{y^2-25}$.
Solution:
Given expression is undefined for $y^2-25 = 0$, that is $(y+5)(y-5) = 0$.
There fore, excluded values are y = -5 and y = 5.

40. Find the square root of $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$.
Solution:
$$\sqrt{\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}} = \sqrt{\frac{4y^8z^{12}}{x^4}} = 2\left|\frac{y^4z^6}{x^2}\right|.$$

41. Find the square root of $\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$.
Solution:
$$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}} = \sqrt{\frac{11^2(a+b)^8(x+y)^8}{9^2(a-b)^{12}}} = \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$$

42. Determine the quadratic equation whose sum and product of roots are -9, 20.
Solution:
General form of quadratic equation is $x^2$-(sum of the roots)x+product of the roots=0.
So, here $x^2-(-9)x+20=0$, that is, $x^2+9x+20=0$.

43. Determine the quadratic equation whose sum and product of roots are 5/3 and 4.
Solution:
General form of quadratic equation is $x^2$-(sum of the roots)x+product of the roots=0.
So, here $x^2-(5/3)x + 4 = 0$
Multiply each term by 3, we get, $3x^2-5x+12=0$.

44. Solve: $x^2-3x-2=0$.
Solution:
We know that $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Here, a = 1, b = -3, c= -2. So, $x = \frac{3\pm\sqrt{(-3)^2-4(1)(-2)}}{2(1)} = \frac{3\pm\sqrt{17}}{2}$.

45. Solve: $2x^2-3x-3=0$.
Solution:
We know that $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Here, a = 2, b = -3, c= -3. So, $x = \frac{3\pm\sqrt{(-3)^2-4(2)(-3)}}{2(2)} = \frac{3\pm\sqrt{33}}{4}$.

46. Determine the nature of the roots for $9x^2-24x+16=0$.
Solution:
Here a = 9, b = -24, c= 16.

Now, $\Delta = b^2 - 4ac = (-24)^2 - 4(9)(16) = 576 - 576 = 0$.
Roots are real and equal.

47. If a matrix has 16 elements, what are the possible order it can have?
Solution:
Possible orders are 1x16, 16x1, 4x4, 2x8, 8x2.

48. Construct a 3x3 matrix whose elements are $a_{ij} = i^2j^2$.
Solution:
The general 3x3 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ *where* $a_{ij} = i^2j^2$
$a_{11} = 1^2x1^2 = 1x1 = 1$; $a_{12} = 1^2x2^2 = 1x4 = 4$; $a_{13} = 1^2x3^2 = 1x9 = 9$; $a_{21} = 2^2x1^2 = 4x1 = 4$;
$a_{22} = 2^2x2^2 = 4x4 = 16$; $a_{23} = 2^2x3^2 = 4x9 = 36$; $a_{31} = 3^2x1^2 = 9x1 = 9$; $a_{32} = 3^2x2^2 = 9x4 = 36$;
$a_{33} = 3^2x3^2 = 9x9 = 81$;
Hence the required matric is $A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$.

49. Construct a 3x3 matrix whose elements are given by $a_{ij} = |i - 2j|$.
Solution:
The general 3x3 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ *where* $a_{ij} = |i - 2j|$.
$a_{11} = |1-2| = |-1| = 1$; $a_{12} = |1-4| = |-3| = 3$; $a_{13} = |1-6| = |-5| = 5$.
$a_{21} = |2-2| = 0$; $a_{22} = |2-4| = |-2| = 2$; $a_{23} = |2-6| = |-4| = 4$.
$a_{31} = |3-2| = |1| = 1$; $a_{32} = |3-4| = |-1| = 1$; $a_{33} = |3-6| = |-3| = 3$.
Hence the required matrix is $A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$.

50. If $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.
Solution:
$A^T = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$.

51. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of –A.
Solution:
Transpose of $-A = (-A)^T = -\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$.

52. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find A+B..
Solution:

$$A+B=\begin{pmatrix}1&2&3\\4&5&6\\7&8&9\end{pmatrix}+\begin{pmatrix}1&7&0\\1&3&1\\2&4&0\end{pmatrix}=\begin{pmatrix}1+1&2+7&3+0\\4+1&5+3&6+1\\7+2&8+4&9+0\end{pmatrix}=\begin{pmatrix}2&9&3\\5&8&7\\9&12&9\end{pmatrix}.$$

53. If $A=\begin{pmatrix}1&3&-2\\5&-4&6\\-3&2&9\end{pmatrix}, B=\begin{pmatrix}1&8\\3&4\\9&6\end{pmatrix}$, find A+B.

Solution:

It is not possible to add A and B because they have different orders.

54. If $A=\begin{pmatrix}7&8&6\\1&3&9\\-4&3&-1\end{pmatrix}, B=\begin{pmatrix}4&11&-3\\-1&2&4\\7&5&0\end{pmatrix}$ then find 2A+B.

Solution:

$$2A+B=2\begin{pmatrix}7&8&6\\1&3&9\\-4&3&-1\end{pmatrix}+\begin{pmatrix}4&11&-3\\-1&2&4\\7&5&0\end{pmatrix}.$$

$$=\begin{pmatrix}14&16&12\\2&6&18\\-8&6&-2\end{pmatrix}+\begin{pmatrix}4&11&-3\\-1&2&4\\7&5&0\end{pmatrix}$$

$$=\begin{pmatrix}18&27&9\\1&8&22\\-1&11&-2\end{pmatrix}.$$

55. Find the values of x,y,z if (i) $\begin{pmatrix}x-3&3x-z\\x+y+7&x+y+z\end{pmatrix}=\begin{pmatrix}1&0\\1&6\end{pmatrix}$.

Solution:

Since they are equal matrices, corresponding elements are equal.

x-3=0, hence x=3.

3x-z=0, then 3x3-z=0, z=9.

x+y+7=1, hence 3+y+7=1, then y = -9.

56. If $A=\begin{pmatrix}2&1\\1&3\end{pmatrix}, B=\begin{pmatrix}2&0\\1&3\end{pmatrix}$ find AB and BA. Verify AB = BA?

Solution:

Since A is of order 2x2 and B is of order 2x2, AB is defined.

$$AB=\begin{pmatrix}2&1\\1&3\end{pmatrix}\begin{pmatrix}2&0\\1&3\end{pmatrix}=\begin{pmatrix}4+1&0+3\\2+3&0+9\end{pmatrix}=\begin{pmatrix}5&3\\5&9\end{pmatrix}.$$

$$BA=\begin{pmatrix}2&0\\1&3\end{pmatrix}\begin{pmatrix}2&1\\1&3\end{pmatrix}=\begin{pmatrix}4+0&2+0\\2+3&1+9\end{pmatrix}=\begin{pmatrix}4&2\\5&10\end{pmatrix}.$$

Therefore, $AB \neq BA$.

57. Solve $\begin{pmatrix}2&1\\1&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\5\end{pmatrix}$.

Solution:

By matrix multiplication $\begin{pmatrix}2&1\\1&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\5\end{pmatrix}$

Then, $\begin{pmatrix}2x+y\\x+2y\end{pmatrix}=\begin{pmatrix}4\\5\end{pmatrix}$.

Rewritting, 2x+y = 4 and x+2y = 5

By using elimination method, we get x = 1 and y = 2.

58. If A is of order pxq and B is of order qxr what is the order of AB and BA?

Solution:

A is of order pxq and B is of order qxr then AB is of order pxr.

B is of order qxr and A is of order pxq then BA is not defined.

59. A has ‘a’ rows and ‘a+3’ columns, B has ‘b’ rows and’17-b’ columns, and if both products AB and BA exist find ‘a’ and ‘b’.
Solution:
A is of order a x (a+3) and B is of order b x (17-b). If AB exists then a+3 = b ---(1)
B is of order b x (17-b) and A is of order a x (a+3). If BA exists then 17-b=a ---(2)
Using (2) in (1), we get, 17-b+3 = b, that is, 2b=20 then **b=10**.
Also, b=10 in (1) then, we get **a=7**.

60. If A = $\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ then prove that $AA^T = I$.
Solution:

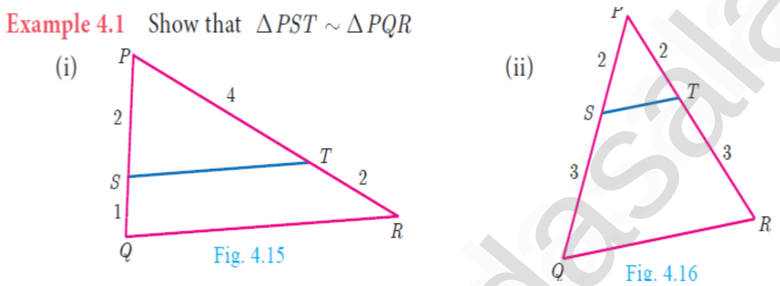
$$AA^T = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}^T = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} cos^2\theta + sin^2\theta & -cos\theta sin\theta + sin\theta cos\theta \\ -sin\theta cos\theta + cos\theta sin\theta & cos^2\theta + sin^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Hence $\mathbf{AA^T = I}$.

# GEOMETRY

61.




Solution:

(i) In $\Delta PST$ and $\Delta PQR$, $\frac{PS}{PQ} = \frac{PT}{PR}$; $\frac{2}{2+1} = \frac{4}{4+2}$; $\frac{2}{3} = \frac{2}{3}$. And $\angle P$ is common.
Hence $\Delta PST \sim \Delta PQR$.

(ii) In $\Delta PST$ and $\Delta PQR$, $\frac{PS}{PQ} = \frac{PT}{PR}$; $\frac{2}{2+3} = \frac{2}{2+3}$; $\frac{2}{5} = \frac{2}{5}$. And $\angle P$ is common.
Hence $\boldsymbol{\Delta PST \sim \Delta PQR}$.

62. The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10cm, find AB.
Solution:
The ratio of the corresponding sides of similar triangle is same as the ratio of their perimeters. Since $\Delta ABC \sim \Delta PQR$,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24} = \frac{3}{2}$.

$\frac{AB}{PQ} = \frac{3}{2} \Rightarrow \frac{AB}{10} = \frac{3}{2}$ then $\boldsymbol{AB = 15cm}$.

63. If $\Delta ABC$ is similar to $\Delta DEF$ such that BC=3 cm, EF = 4 cm and area of ΔABC = 54cm². Find the area of ΔDEF.
Solution:
Since the ratio of area of two similar triangles is equal to the squaresof any two corresponding sides, we have $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{Area\ (\Delta DEF)} = \frac{3^2}{4^2}$.

Hence, **Area** $(\Delta DEF) = \frac{16 \times 54}{9} = $ **96 cm²**.

64. In $\Delta ABC, if\ DE \parallel BC$, AD = x, DB = x-2, AE = x+2 and EC = x-1then find the lengths of the sides AB and AC.

Solution:

In $\Delta ABC\ we\ have\ DE \parallel BC$.

By thales theorem, we have $\frac{AD}{DB} = \frac{AE}{EC}$ then $\frac{x}{x-2} = \frac{x+2}{x-1}$ gives x(x-1)=(x-2)(x+2)

Hence, $x^2 - x = x^2 - 4$, so, x = 4.

**AB** = AD+DB = 4+4-2 = **6cm.**

**AC** = AE+EC = 4+2+4-1 = **9 cm**.

65. D and E are respectively the points on the sides AB and AC of a $\Delta ABC$ such that AB=5.6cm, AD=1.4cm, AC=7.2cm and AE=1.8cm, show that $DE \parallel BC$.

Solution:

Given AB=5.6cm, AD=1.4cm, AC=7.2cm and AE=1.8cm.

BD=AB-AD=5.6-1.4 = 4.2 cm EC=AC-AE=7.2-1.8 = 5.4cm

Also, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$.

Hence, $\frac{AD}{DB} = \frac{AE}{EC}$. Therefore by converse of Basic Proportionality theorem, we have DE is parallel to BC. Hence proved.

66. AD is the bisector of angle A. If BD=4cm, DC=3cm and AB=6cm find AC.

Solution:

In $\Delta ABC$, AD is the bisector of angle A.

By angle bisector theorem $\frac{BD}{DC} = \frac{AB}{AC}$ $then$ $\frac{4}{3} = \frac{6}{AC}$ which gives AC = 18cm. Hence **AC=9/2 = 4.5cm**.

67. What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Solution:

Let 'x' be the length of the ladder. BC=4ft, AC=7ft.

By Pythagoras theorem we have $AB^2 = AC^2 + BC^2$.

Hence, $x^2 = 7^2 + 4^2$ then $x^2 = 65$

Hence $\boldsymbol{x = \sqrt{65} = 8.1\ cm}$.

68. Find the length of the tangent drawn from a point whose distance from the centre of the circle is 5 cm and radius of the circle is 3 cm.

Solution:

Given, OP=5 cm, radius OT= 3cm.

In a right angled triangle OTP, $OP^2 = OT^2 + PT^2$.

Hence, $5^2 = 3^2 + PT^2$.

$PT^2 = 25 - 9 = 16$, therefore **length of the tangent PT = 4 cm**.

In fig4.65, $\Delta ABC$ is circumscribing a circle. Find the length of BC.

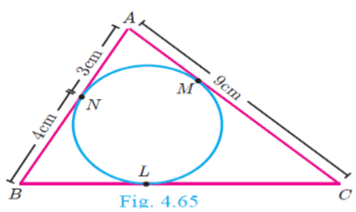


Fig. 4.65

Solution:
Since Tangents drawn from same external point are equal, AN=AM=3 cm, BN=BL=4cm and CL=CM=AC-AM=9-3 = 6 cm.
Then BC=BL+CL = 4+6 = 10cm.

69. If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.
Solution:
OA=4 cm, OB = 5 cm also OA perpendicular to BC.
By Pythagoras theorem, $OB^2 = OA^2+AB^2$.
$5^2 = 4^2 + AB^2$ then $AB^2 = 9$ cm.
Hence AB = 3 cm, Since BC = 2AB then BC = 2x3 = 6cm.

# COORDINATE GEOMETRY

70. Find the area of the triangle whose vertices are (-3,5), (5,6) and (5,-2).
Solution:
Let A(-3,5), B(5,6) and C(5,-2).
↓ ↓ ↓
$(x_1,y_1)$ $(x_2,y_2)$ $(x_3,y_3)$
Area of the Δ ABC = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$ sq.units.
$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$
= (1/2){(6+30+25)-(25-10-18)}
= (1/2) {61+3}
= 32 sq.units.

71. (i) what is the slope of a line whose inclination is 30°?
(iii) What is the inclination of a line whose slope is $\sqrt{3}$ ?
Solution:
(i) Slope m = $tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$.e points
(ii) Given $m = \sqrt{3}, \tan\theta = \sqrt{3}$ hence $tan\theta = tan60°, therefore\ \theta = 60°$.

72. The line r passes through the points (-2,2) and (5,8) and the line s passes through the points (-8,7) and (-2,0). Is the line r perpendicular to s?
Solution:
The slope of the line r is $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{8-2}{5+2} = \frac{6}{7}$.
The slope of the line s is $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{0-7}{-2+8} = -\frac{7}{6}$.
Here, the product of the slope $m_1xm_2 = \frac{6}{7}x - \frac{7}{6} = -1$.
That is, the product of the slopes = -1, the line r is perpendicular to line s.

73. The line p passes through the points (3,-2), (12,4) and the line q passes through the points (6,-2) and (12,2). Is p parallel to q?
Solution:
The slope of the line p is $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{4+2}{12-3} = \frac{6}{9} = \frac{2}{3}$.
The slope of the line q is $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{2+2}{12-6} = \frac{4}{6} = \frac{2}{3}$.
Thus, slope of p = slope of q, that is slopes are equal.
Hence line p is parallel to the line q.

74. Find the slope of a line joining $(5, \sqrt{5})$with the origin.
Solution:
Slope of a line $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sqrt{5}-0}{5-0} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$.
75. Calculate the slope and y intercept of the straight line 8x-7y+6 =0.
Solution:
Given straight line is 8x-7y+6 =0.
Hence 7y = 8x+6, that is y = $\frac{8}{7}x + \frac{6}{7}$. Compare with y = mx+c, we get slope m = 8/7 and y-intercept c = 6/7.
76. Find the equation of the line passing through the point (3,-4) and having slope (-5/7).
Solution:
Given $(x_1,y_1)$ = (3,-4) and slope m = (-5/7).
Hence, the equation of the line is $y-y_1=m(x-x_1)$.
y – (-4) = (-5/7)(x - 3)
7(y + 4) = -5 (x – 3 )
5x + 7y + 13 =0.
77. Find the equation of a line whose intercepts on the x and y axes are 4 and (-6).
Solution:
Given x-intercept a = 4 and y-intercept b = -6.
Hence, the equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$.
Here, $\frac{x}{4} + \frac{y}{-6} = 1$.
Multiply by 12 on both sides, 3x-2y=12.
78. Find the intercepts made by the following lines on the coordinate axes: 4x+3y+12=0.
Solution:
Given equation is 4x+3y = -12
Divided by (-12), we get, $\frac{x}{-3} + \frac{y}{-4} = 1$.
Compare with $\frac{x}{a} + \frac{y}{b} = 1$, X-intercept a = -3 and y-intercept b = -4.
79. Find the slope of the line which is (i) parallel to 3x-7y=11 (ii) perpendicular to 2x-3y+8=0.
Solution:
Given straight line is 3x-7y = 11.
Slope of the straight line ax+by+c=0 is $m = -\frac{a}{b}$.
So, here slope $= -\frac{3}{-7} = \frac{3}{7}$ . Since parallel line have same slopes, slope of any line parallel to 3x-7y=11 is 3/7.
(ii) Given straight line is 2x-3y+8=0.
Slope of the straight line ax+by+c=0 is $m = -\frac{a}{b}$.
So, here slope m = $-\frac{2}{-3} = \frac{2}{3}$. Since product of slopes is (-1) for perpendicular lines, slope any line perpendicular to 2x-3y+8=0 is $\frac{-1}{(2/3)} = -\frac{3}{2}$.
80. Show that the straight lines x-2y+3=0 and 6x+3y+8=0 are perpendicular.
Solution:
Slope of the straight line x-2y+3=0 is $m_1 = -\frac{a}{b} = \frac{-1}{-2} = \frac{1}{2}$.

Slope of the straight line 6x+3y+8=0 is $m_2 = -\frac{a}{b} = -\frac{6}{3} = -2$.

Now, $m_1 x m_2 = \left(\frac{1}{2}\right) X(-2) = -1$. Hence given two straight lines are perpendicular.

81. Find the equation of a straight line which is parallel to the line 3x-7y = 12 and passing through the point (6,4).

Solution:

Since parallel lines are differ only in constant terms, equation of the line parallel to 3x-7y=12 is 3x-7y+k=0 -----(1)

Since (1) passes through (6,4), 3(6)-7(4)+k=0

K = 10.

Therefore, equation of the required straight line is 3x-7y+10=0.

## TRIGONOMETRY

82. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = cosec\,\theta + cot\theta$.

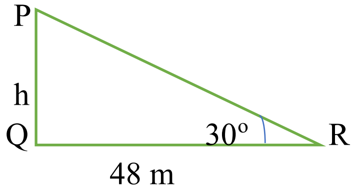
Solution:

$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta} X \frac{1+\cos\theta}{1+\cos\theta}}$ [multiply numerator and denominator by conjugate]

$= \sqrt{\frac{(1+\cos\theta)^2}{1-cos^2\theta}} = \sqrt{\frac{(1+cos\theta)^2}{sin^2\theta}}$

$= \frac{1+cos\theta}{sin\theta} = \frac{1}{sin\theta} + \frac{cos\theta}{sin\theta} = cosec\theta + cot\theta.$

83. A tower stands vertically on the ground. From a point on the ground, which is 48m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.




Solution:

Let PQ be the height of the tower and let PQ = h.

QR is the distance between the tower and the point R.

In the right angled triangle PQR,

$\tan\theta = \frac{PQ}{QR}$.

$\tan 30° = \frac{h}{48} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{48} \Rightarrow h = \frac{48}{\sqrt{3}} = 16\sqrt{3}$ m.

84. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. find the length of the string, assuming that there is no slack in the string.

Solution:

Let AB be the height of the kite above the ground. Then AB=75 m. Let AC be the length of the string.

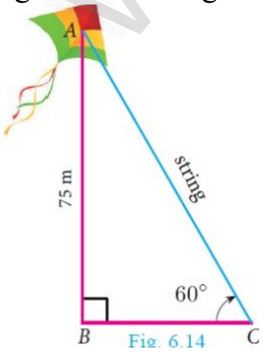


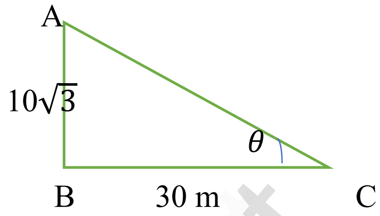

Fig. 6.14

In the right angled triangle ABC, $\sin\theta = \frac{AB}{AC}$.

$$\sin 60° = \frac{75}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{AC} \Rightarrow AC = \frac{150}{\sqrt{3}} \Rightarrow AC = 50\sqrt{3}.$$

Hence, the length of the string is $50\sqrt{3}$m.

85. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$m.




Solution:

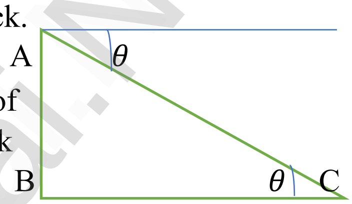
Let AB be the tower and whose height AB=$10\sqrt{3}$ m.

Let BC be the distance between the tower and C.

In a right angled triangle ABC, $tan\theta = \frac{AB}{AC} = \frac{10\sqrt{3}}{30}$

Hence $tan\theta = \frac{1}{\sqrt{3}}, that\ is\ \theta = 30°$.

86. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.




Solution:

Let AB be the rock and AB=$50\sqrt{3}$ m, let C be the position of the car and whose angle of depression from the top of a rock is 30°.

In a right angled triangle ABC, $\tan\theta = \frac{AB}{BC}$

Hence, $\tan 30° = \frac{50\sqrt{3}}{BC}$

Then, $\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}, BC = 50x3 = 150\ m.$

## MENSURATION

87. The curved surface area of a right circular cylinder of height 14 cm is 88 $cm^2$. Find the diameter of the cylinder.

Solution:

Given that, curved surface area of the cylinder = $2\pi rh = 88$ $cm^2$.

$$2 \text{ X } \frac{22}{7} X\ r\ X\ 14 = 88$$

$$2r = \frac{88\ x\ 7}{22\ x\ 14} \text{ cm} = 2 \text{ cm}.$$

88. If the total surface area of a cone of radius 7 cm is 704 $cm^2$, then find its slant height.

Solution:

Given that, radius r = 7 cm

Total surface area of the cone = $\pi r(l + r) = 704$ $cm^2$.

Hence, $\frac{22}{7}\ x\ 7(l + 7) = 704$.

Hence, $l + 7 = \frac{704}{22} = 32$, then $l = 32 - 7 = 25$ cm.

89. Find the diameter of a sphere whose surface area is 154 $m^2$.

Solution:

Let 'r' be the radius of the sphere.

Given that, surface area of the sphere = $4\pi r^2 = 154m^2$.

Hence, $4\ x\ \frac{22}{7} x\ r^2 = 154$.

Then, $r^2 = \frac{154\ x\ 7}{4\ x\ 22} = \frac{49}{4} = \left(\frac{7}{2}\right)^2$. Radius = 7/2 cm

Therefore, diameter is 7 cm.

90. If the base area of a hemispherical solid is 1386 $m^2$, then find its total surface area?
Solution:
Let 'r' be the radius of the hemi sphere.
Given that, base area = $\pi r^2 = 1386$ sq. m
Total surface area of the hemi sphere = $3\pi r^2 = 3 \; x \; 1386 = 4158$ sq.m

91. A sphere, a cylinder and a cone are of the same radius 'r' where as cone and cylinder are of same height. Find the ratio of their curved surface areas.
Solution:
Given, radius of a sphere = radius of a cylinder = radius of a cone = r units
Height of the cone = height of the cylinder = h = r units.
Slant height of the cone $l = \sqrt{r^2 + h^2} = \sqrt{r^2 + r^2} = \sqrt{2r^2} = r\sqrt{2}$.
Required ratio = CSA of a sphere : CSA of the cylinder : CSA of the cone
$= 4\pi r^2 : 2\pi rh : \pi rl$
$= 4\pi r^2 : 2\pi r^2 : \sqrt{2}\pi r^2$.
$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$.

92. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4cm and 1cm. Find its curved surface area.
Solution:
Let l,R and r be the slant height, top radius and bottom radius of a frustum.
Given that l = 5cm, R = 4cm, r = 1cm.
CSA of the frustum = $\pi(R + r)l = \frac{22}{7} \; x \; (4 + 1) \; x \; 5 = \frac{550}{7} = 78.57$ $cm^2$.

93. The volume of a solid right circular cone is 11088 $cm^3$. If its height is 24 cm then find the radius of the cone.
Solution:
Let r and h be the radius and height of the cone respectively.
Given that, volume of the cone = $\frac{1}{3}\pi r^2 h = 11088$.
Hence, $\frac{1}{3} \; x \; \frac{22}{7} \; x \; r^2 \; x \; 24 = 11088$.
Then, $r^2 = \frac{11088 \; x \; 3 \; x \; 7}{22 \; x \; 24} = 441$.
Therefore, radius of the cone r = 21 cm.

## STATISTICS AND PROBABILITY

94. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.
Solution: largest value $L$ = 67; Smallest value $S$ =18
Range R= $L - S = 67 - 18 = 49$
Coefficient of range = $\frac{L-S}{L+S} = \frac{67-18}{67+18} = \frac{49}{85}$ =0.576.

95. The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.
Solution:
Range R = L-S
13.67 = 70.08 – S
Therefore, S = 70.08-13.67 = 56.41.

96. Find the range and coefficient range of 63,89,98,125,79,108,117,68.
    Solution:
    Smallest value S = 63
    Largest value L = 125
    Range R = L-S = 125-63 = 62
    Coefficient of range = $\frac{L-S}{L+S} = \frac{125-62}{125+62} = \frac{63}{187} = 0.33.$
97. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
    Solution:
    Range R = 36.8
    Smallest value S = 13.4
    Range R = L-S
    36.8 = L – 13.4
    Therefore, L = 36.8 + 13.4 = 50.2.
98. Find the standard deviation of first 21 natural numbers.
    Solution:
    Standard deviation of first 'n' natural number = $\sqrt{\frac{n^2-1}{12}}$.
    Standard deviation of first 21 natural number = $\sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}} =$ $\sqrt{36.67}$=6.05.
99. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standatd deviation.
    Solution:
    Mean $\bar{x}$ = 25.6, Coefficient of variation C.V. = 18.75
    C.V. = $\frac{\sigma}{\bar{x}}\ X\ 100\%$
    18.75 = $\frac{\sigma}{25.6}X100$ then σ = 4.8.
100. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.
    Solution:
    Mean $\bar{x}$ = 12.5 and Standard deviation σ = 6.5.
    C.V. = $\frac{\sigma}{\bar{x}}\ X\ 100\% = \frac{6.5}{12.5}X100 = \frac{6500}{125} = 52\%.$
101. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.
    Solution:
    Standard deviation σ = 1.2, Coefficient of variation C.V.=25.6
    C.V. = $\frac{\sigma}{\bar{x}}\ X\ 100\%$
    25.6 = $\frac{1.2}{\bar{x}}\ X\ 100$ then $\bar{x} = \frac{120}{25.6} = 4.69.$
102. If mean and coefficient of variation of a data are 15 and 48 then find the value of standard deviation.
    Solution:
    Mean $\bar{x}$ = 15, Coefficient of variation C.V. = 48
    C.V. = $\frac{\sigma}{\bar{x}}\ X\ 100\%$
    48 = $\frac{\sigma}{15}X100$ then σ = $\frac{48\ x\ 15}{100}$=7.2.

103. Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution:

When two coins are tossed, S = {HH,HT,TH,TT}; n(S) = 4.

Let A be the event of getting different faces on the coins then A = {HT,TH}, n(A)=2

Hence $P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$.

104. What is the probability that a leap year selected at random will contains 53 saturdays?

Solution:

A leap year has 366 days = 52 weeks + 2 days.

52 saturdays must be in 52 full weeks.

The possible chances for the remaining 2 days will be the sample space S.

S = {Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun}; n(S) = 7.

A is the event of getting $53^{rd}$ Saturday, Then A = {Fri-Sat, Sat-Sun}; n(A) = 2

Then $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$.

105. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:

When a coin is tossed thrice, S={HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}; n(S)=8.

Let A be the event of getting two consecutive tails then A={HTT,TTH,TTT}; n(A)=3

Then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$.

106. If P(A) = 0.37, P(B) = 0.42, P(A∩B) = 0.09 then find P(A∪B).

Solution:

P(A∪B) = P(A) + P(B) - P(A∩B)

Then, P(A∪B) = 0.37 + 0.42 - 0.09 = 0.7

107. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A\cup B) = \frac{1}{3}$ then find P(A∩B).

Solution:

P(A∪B) = P(A) + P(B) - P(A∩B)

Then, $\frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A\cap B)$, hence $P(A\cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{11}{15}$.

108. If A and B are two mutually exclusive events of a random experiment and P(notA) = 0.45, P(A∪B)=0.65, then find P(B).

Solution:

$P(notA) = P(\bar{A}) = 1 - P(A) = 0.45$, hence P(A) = 0.55, P(AUB) = 0.65.

Since A and B are mutually exclusive events, P(AUB) = P(A) + P(B).

P(B) = P(AUB) - P(A) = 0.65 - 0.55 = 0.10.

109. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.

Solution:

Given, P(AUB) = 0.6 and $P(A \cap B) = 0.2$.

P(A∪B) = P(A) + P(B) - P(A∩B)

0.6 = P(A) + P(B) - 0.2

P(A) + P(B) = 0.6+0.2 = 0.8.

Hence, $P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B) = 2 - 0.8 = 1.2$.