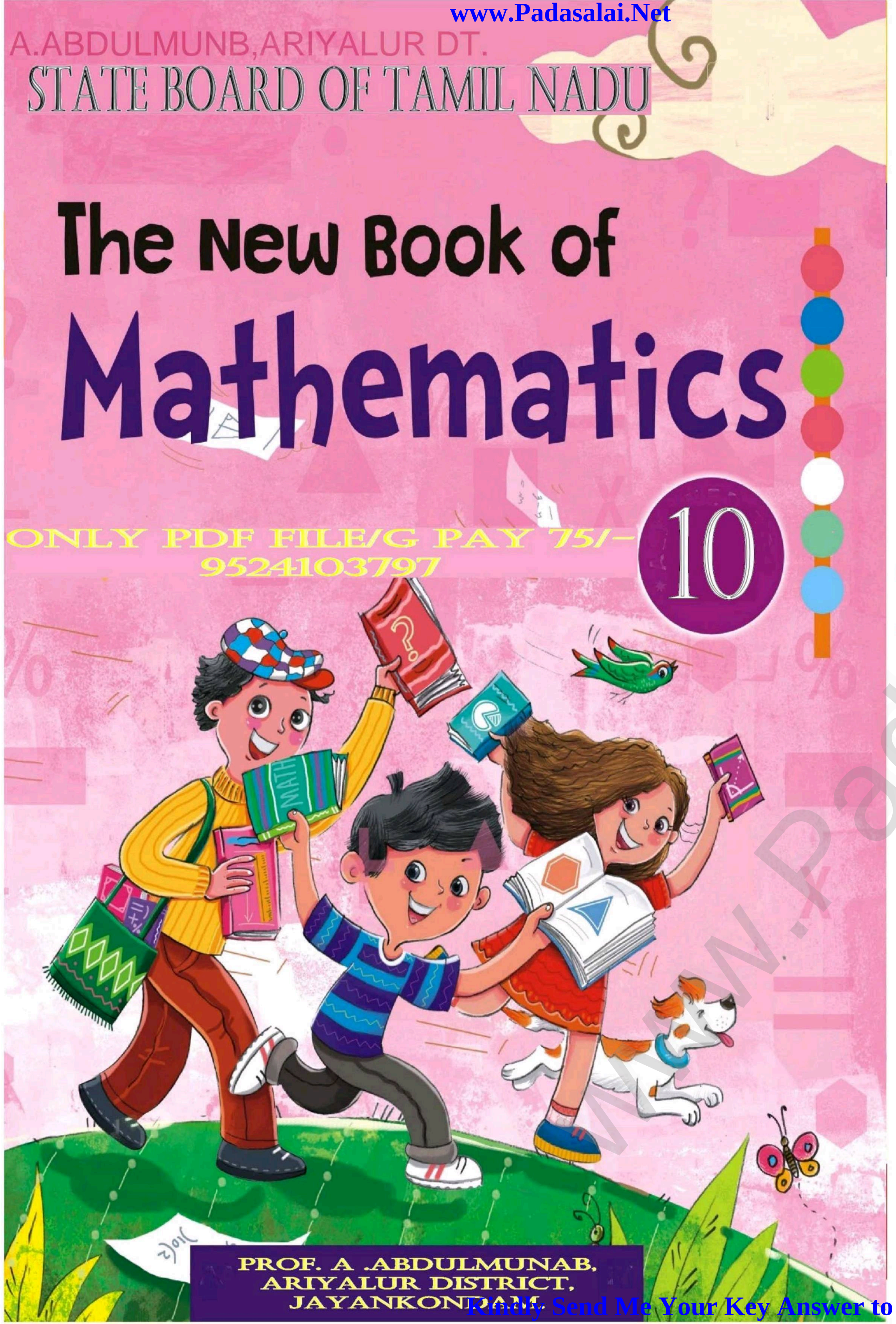


If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B.

Solution : $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

$A = \{3, 5\}$ and $B = \{2, 4\}$.

If $A = \{2, -2, 3\}$ and $B = \{1, -4\}$, Find $A \times B$ and $B \times A$

Solution: Given $A = \{2, -2, 3\}$, $B = \{1, -4\}$.

$A \times B = \{2, -2, 3\} \times \{1, -4\} = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$

$B \times A = \{1, -4\} \times \{2, -2, 3\} = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$

If $A = B = \{p, q\}$, Find $A \times A$ and $B \times A$

Solution: Given $A = B = \{p, q\}$

$A \times A = \{p, q\} \times \{p, q\} = \{(p, p), (p, q), (q, p), (q, q)\}$

$B \times A = \{p, q\} \times \{p, q\} = \{(p, p), (p, q), (q, p), (q, q)\}$

Let $A = \{1, 2, 3\}$ and $B = \{x \mid x$ is a prime number less than 10$\}$. Find $A \times B$ and $B \times A$.

Solution : Given $A = \{1, 2, 3\}$, $B = \{x \mid x$ is a prime number less than 10$\} \Rightarrow B = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

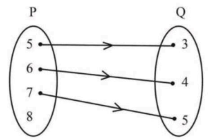
$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ find A and B.

Solution : $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$

$\therefore B = \{-2, 0, 3\}$, $A = \{3, 4\}$

The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



Solution : (i) Set builder form of $R = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain of $R = \{5, 6, 7\}$ and range of $R = \{3, 4, 5\}$

Let $A = \{1, 2, 3, 4 \ldots\ldots, 45\}$ and R be the relation defined as "is square of" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R.

Solution : **Let $A = \{1, 2, 3, 4 \ldots\ldots, 45\}$ and R be the relation defined as "is square of" on A.**

$R = \{(1,1), (2, 4), (3, 9), (4,16), (5,25), (6,36)\}$ $\therefore$ **R as a subset of $A \times A$.**

$\therefore$ Domain $= \{1, 2, 3, 4, 5, 6\}$ $\therefore$ Range $= \{1, 4, 9, 16, 25, 36\}$

A Relation R is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Solution : Given $R = \{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$

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Show that the square of an odd integer is of the form $4q+1$, for some integer q.

Solution : Let x be any odd integer. $x = 2k+1$,

$x^2 = (2k+1)^2 = 4k^2+4k+1 = 4k(k+1)+1 = 4q+1$, where $q = k(k+1)$ is some integer.

Find the sum of first 15 terms of the A.P. $8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, \ldots$

Solution :

$a = 8, \quad d = 7\frac{1}{4} - 8 = -\frac{3}{4},$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{15} = \frac{15}{2}\left[2\times 8+(15-1)(-\frac{3}{4})\right] = \frac{15}{2}\left[16+(14)(-\frac{3}{4})\right] = \frac{15}{2}\left[16-\frac{21}{2}\right] = \frac{165}{4}$

Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Solution : $445-4 = 441, 572-5 = 567.$

$567 = 441\times 1+126$

$441 = 126\times 3+63$

$126 = 63\times 2+0$

HCF of 441,567 = 63

The greatest number that will divide 445 and 572 leaving remainders 4 and 5 is 63.

Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Solution :

2	408
2	204
2	102
3	51
	17

2	170
5	85
	17

$\therefore 408 = 2^3\times 3\times 17$

$170 = 2\times 5\times 17$

$\therefore$ H.C.F $= 2\times 17 = 34$

L.C.M $= 2^3\times 17\times 5\times 3 = 2040$

What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?

Solution : The required number is the LCM of (35, 56, 91) + remainder 7

$35 = 7\times 5$

$56 = 7\times 2\times 2\times 2$

$91 = 7\times 13 \quad \therefore \quad$ L.C.M $= 7\times 5\times 13\times 8 = 3640$

$\therefore$ The required number is $3640+7 = 3647$

Find the least number that is divisible by the first ten natural numbers.

Solution : The required number is the LCM of (1, 2, 3, ... 10)

$1 = 1, \; 2 = 2, \; 4 = 2^2, \; 6 = 3\times 2,$

$7 = 7, \; 8 = 2^3, \; 9 = 3^2, \; 10 = 5\times 2.$

L.C.M $= 2^3\times 3^2\times 5\times 7 = 2520$

Prove that two consecutive positive integers are always coprime.

Solution : Let $x, x+1$ be two consecutive integers.

By Eculid's division lemma $a = bq+r$, where $0 \le r < |b|$

$x+1 = x\times 1+1$

G.C.D. of $(x, x+1) = 1$

x and $x+1$ are Co-prime.

In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution : $a = 20, d = 2, n = 30$

$t_{30} = a+29d = 20+29(2)$

$= 20+58 = 78$

$\therefore$ The no. of seats in 30th row = 78

Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Solution : HCF of 1230 - 12 and 1926 - 12

$1914 = 1218\times 1+696$ $\quad 696 = 522\times 1+174$

$1218 = 696\times 1+522$ $\quad 522 = (174)\times 3+0$

$\therefore$ HCF = 174

The greatest number that will divide 1230 and 1926 leaving remainders 12 is 174

Can the number 6^n, n being a natural number end with the digit 5 ? Give reason for your answer.

Solution : $6^n = (2\times 3)^n = 2^n\times 3^n$,

2 is a factor of 6^n. So, 6^n is always even.

But any number whose last digit is 5 is always odd. Hence, 6^n cannot end with the digit 5.

Find the 8th term of the G.P. 9, 3, 1, ...

Solution : First term $a = 9$, common ratio $r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$, $\quad t_n = ar^{n-1}$

$t_8 = 9\times\left(\frac{1}{3}\right)^{8-1} = 9\times\left(\frac{1}{3}\right)^7 = 9\times\frac{1}{2187} = \frac{1}{243}$

Is $7\times 5\times 3\times 2+3$ a composite number ? Justify your answer.

Solution : Yes, the given number is a composite number, because

$7\times 5\times 3\times 2+3 = 3\times(7\times 5\times 2+1) = 3\times 71$

Since the given number can be factorized in terms of two primes, it is a composite number.

Show that 107 is of the form $4q+3$ for any integer q.

Solution : $107 = 4(26)+3$

This is of the form $107 = 4q+3$ for q = 26.

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Find the remainder when 2^{81} is divided by 17.

Solution :

$\therefore\ 2^4 = 16 = -1 \pmod{17} \Rightarrow 2^{80} = (2^4)^{20} = (-1)^{20} = 1$

$\Rightarrow \therefore\ 2^{81} = 2^{80} \times 2^1 = 1 \times 2 = 2$

Solve $3x - 2 \equiv 0 \pmod{11}$

Solution : $3x - 2 \equiv 0 \pmod{11}$

$\therefore$ 3x – 2 is divisible by 11

$\therefore$ The possible values of x are 8, 19, 30, ...

Find the least positive value of x such that $5x \equiv 4 \pmod 6$

Solution : $5x \equiv 4 \pmod 6 \Rightarrow \therefore\ 5x - 4$ is a multiple of 6

$\therefore$ The least +ve value of x is 2 $(\because 5(2) - 4 = 6$ is a multiple of 6)

Find the least positive integer n such that $1 + 6 + 6^2 + ... + 6^n > 5000$.

Solution :

$$S_n > 5000 \Rightarrow \frac{a(r^n-1)}{r-1} > 5000 \Rightarrow \frac{1(6^n-1)}{6-1} > 5000$$

$$\frac{6^n-1}{5} > 5000 \Rightarrow 6^n - 1 > 25000 \Rightarrow 6^n > 25001$$

Since, $6^5 = 7776$ and $6^6 = 46656$

The least positive value of n is 6 such that $1 + 6 + 62 + ... + 6^n > 5000$.

A person saved money every year, half as much as he could in the previous year. If he had totally saved ₹ 7875 in 6 years then how much did he save in the first year ?

Solution : Total amount saved in 6 years is $S_6 = 7875$ We have $r = \frac{1}{2} < 1$

$$\frac{a\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = 7875 \Rightarrow \frac{a\left(1-\frac{1}{64}\right)}{\frac{1}{2}} = 7875 \Rightarrow a \times \frac{63}{32} = 7875 \Rightarrow a = \frac{7875 \times 32}{63} \Rightarrow a = 4000$$

The amount saved in the first year is ₹4000.

Find the sum of 8 terms of the G.P. 1, –3, 9, –27, ...

Solution : Here the first term $a = 1$, common ratio $r = \frac{-3}{1} = -3 < 1$, Here $n = 8$.

Sum to n terms of a G.P. is $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

$$S_8 = \frac{1((-3)^8 - 1)}{1-(-3)}$$

$$= \frac{1-6561}{4}$$

$$= -1640$$

Reduce the given Rational expression to its lowest form $\frac{x^{3a}-8}{x^2a+2xa+4}$

Solution :

$$\frac{x^{3a}-8}{x^2a+2xa+4} = \frac{(x^a)^3 - 2^3}{x^{2a}+2x^a+4} = \frac{(x^a-2)(x^{2a}+2x^a+4)}{x^{2a}+2x^a+4} = x^a - 2$$

Find the LCM of $x^4 - 1,\ x^2 - 2x + 1$

$$x^4 - 1 = (x^2)^2 - 1 = (x^2+1)(x^2-1) = (x^2+1)(x+1)(x-1)$$

$$x^2 - 2x + 1 = (x-1)^2$$

$$\text{LCM } [(x^4-1), (x^2-2x+1)] = (x^2+1)(x+1)(x-1)^2$$

Find the sum and product of the roots $x^2 + 8x - 65 = 0$

Solution : Let α and β be the roots of the given quadratic equation $x^2 + 8x - 65 = 0$

$a = 1, b = 8, c = -65$; $\alpha + \beta = -\frac{b}{a} = -8$ and $\alpha\beta = \frac{c}{a} = -65$

Find the sum and product of the roots $kx^2 - k^2x - 2k^3 = 0$

Solution : Let α and β be the roots of the given quadratic equation $kx^2 - k^2x - 2k^3 = 0$

$a = k, b = -k^2, c = -2k^3$; $\alpha + \beta = -\frac{b}{a} = \frac{-(-k^2)}{k}$ and $\alpha\beta = \frac{c}{a} = \frac{-2k^3}{k} = -2k^2$

Find the sum and product of the roots $x^2 + 3x = 0$

Solution : $a = 1, b = 3, c = 0$

$\therefore\ \alpha + \beta = \frac{-b}{a} = \frac{-3}{1} = -3$ $\alpha\beta = \frac{c}{a} = \frac{0}{1} = 0$

Write down the quadratic equation which sum and product of the roots are $-\frac{3}{5}, -\frac{1}{2}$

Solution :

x^2 – (sum of the roots) x + product of the roots = 0

$$x^2 - \left(-\frac{3}{5}\right)x + \left(-\frac{1}{2}\right) = 0 \Rightarrow \frac{10x^2+6x-5}{10} = 0 \Rightarrow 10x^2 + 6x - 5 = 0$$

Determine the quadratic equations whose sum and product of roots are $-9, 20$

Solution : The required quadratic equation is $x^2 - (SOR)\,x + (POR) = 0$

$$x^2 - (-9)x + 20 = 0$$

$$x^2 + 9x + 20 = 0$$

Find the LCM of $5x - 10,\ 5x^2 - 20$

Solution : $5x - 10 = 5(x-2)$ $5x^2 - 20 = 5(x^2 - 4) = 5(x+2)(x-2)$

$\text{LCM } [(5x-10), (5x^2-20)] = 5(x+2)(x-2)$

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Solve **$36y^2 - 12ay + (a^2 - b^2) = 0$** by formula method

Solution : $A = 36,\ B = -12a,\ C = a^2 - b^2$

$$\therefore\ y = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{12a \pm \sqrt{144a^2 - 4(36)(a^2 - b^2)}}{2(36)} = \frac{12a \pm \sqrt{144a^2 - 144(a^2 - b^2)}}{72}$$

$$= \frac{12a \pm \sqrt{144b^2}}{72} = \frac{12a \pm 12b}{72} = \frac{a \pm b}{6} = \frac{a+b}{6}, \frac{a-b}{6}$$

Determine the nature of roots for $x^2 - x - 20 = 0$

Solution : Here, $a = 1,\ b = -1,\ c = -20$

$$\Delta = b^2 - 4ac = (-1)^2 - 4(1)(-20) = 81 > 0$$

So, the equation will have real and unequal roots

Determine the nature of roots for $2x^2 - 2x + 9 = 0$

Solution : Here, $a = 2,\ b = -2,\ c = 9$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(2)(8) = -68 < 0$$

So, the equation will have no real roots

Determine the nature of the roots $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

Solution : $a = \sqrt{2},\ b = -3,\ c = 3\sqrt{2}$

$\therefore\ \Delta = b^2 - 4ac = 9 - 4(\sqrt{2})(3\sqrt{2}) = 9 - 24 = -15 < 0$ $\therefore$ The roots are unreal.

Find the value(s) of 'k' $(5k - 6)x^2 + 2kx + 1 = 0$ roots of the equations are real and equal.

Solution : $a = 5k - 6,\ b = 2k,\ c = 1$

$\therefore\ \Delta = b^2 - 4ac = 0$

$4k^2 - 4(5k - 6)(1) = 0$

$4k^2 - 20k + 24 = 0$

$k^2 - 5k + 6 = 0$

$(k - 3)(k - 2) = 0$

$\therefore\ k = 3, 2$

If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 find k.

Solution : Let α, β be the roots of the equation. $a = 1,\ b = -13,\ c = k$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-13)}{1} = 13 \qquad \alpha\beta = \frac{c}{a} = \frac{k}{1} = k$$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$17^2 = (13)^2 - 4k$

$289 = 169 - 4k$

$4k = 169 - 289$

$4k = -120$

$k = -30$

If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution : $a = 3,\ b = 7,\ c = -2$

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{3}; \qquad \alpha\beta = \frac{c}{a} = \frac{-2}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-7}{3}\right)^2 - 2\left(\frac{-2}{3}\right)}{\frac{-2}{3}} = \frac{-61}{6}$$

If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution : $a = 3,\ b = 7,\ c = -2$

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{3}; \qquad \alpha\beta = \frac{c}{a} = \frac{-2}{3}$$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{\left(-\frac{7}{3}\right)^3 - 3\left(-\frac{2}{3}\right)\left(-\frac{7}{3}\right)}{-\frac{7}{3}} = \frac{67}{9}$$

The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. find $\frac{1}{\alpha} + \frac{1}{\beta}$

Solution : $a = 2,\ b = -7,\ c = 5$

$$\alpha + \beta = -b/a = 7/2 \qquad \alpha\beta = c/a = 5/2 \qquad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7/2}{5/2} = 7/5$$

The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution : $a = 2,\ b = -7,\ c = 5$

$$\alpha + \beta = -b/a = 7/2 \qquad \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{49/4 - 5}{5/2}$$

$$= 29/4 \times 2/5 = 29/10$$

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26. If $A = \begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}$, then Find $\frac{1}{2}A - \frac{3}{2}B$

Solution :

$$\frac{1}{2}A - \frac{3}{2}B = \frac{1}{2}(A - 3B) = \frac{1}{2}\left(\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} - 3\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} + \begin{pmatrix} -24 & 18 & 12 \\ -6 & -33 & 9 \\ 0 & -3 & -15 \end{pmatrix}\right)$$

$$= \frac{1}{2}\begin{pmatrix} -23 & 26 & 15 \\ -3 & -28 & 9 \\ 8 & 4 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{23}{2} & 13 & \frac{15}{2} \\ -\frac{3}{2} & -14 & \frac{9}{2} \\ 4 & 2 & -\frac{9}{2} \end{pmatrix}$$

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If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes.

Solution : Given ratio of radii of 2 spheres = 4 : 7 $\Rightarrow \frac{r_1}{r_2} = \frac{4}{7}$

$$\therefore \text{Ratio of their volumes} = \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{7}\right)^3 = \frac{64}{343} = 64:343$$

A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$: 4.

Solution : TSA of a solid sphere = TSA of a solid hemisphere

$$4\pi R^2 = 3\pi r^2$$

$$\therefore \frac{R^2}{r^2} = \frac{3}{4} \Rightarrow \frac{R}{r} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{Ratio of their volumes} = \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{2R^3}{r^3} = 2\left[\frac{R}{r}\right]^3 = 2\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{4} = 3\sqrt{3}:4$$

A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Solution : Let h_1 and h_2 be the heights of a cone and cylinder respectively.

A child reshapes it in the form of a cylinder of same radius as cone.

Volume of cylinder = Volume of cone

$$\pi^2 h_2 = \frac{1}{3}\pi r^2 h_1 \Rightarrow h_2 = \frac{1}{3}\times h_1 \Rightarrow h_2 = \frac{1}{3}\times 24 = 8$$

The height of cylinder is 8 cm.

A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm.

Solution : Given r = 5 cm, h = 12 cm Given, area of sheet of paper = 5720 cm²

$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{144+25} = \sqrt{169} = 13$$

$$\therefore \text{CSA of cone} = \pi r l = \frac{22}{7}\times 5\times 13 = \frac{110\times 13}{7}\text{ cm}^2$$

$$\therefore \text{Number of caps} = \frac{5720\times 7}{110\times 3} = 28 \text{ caps}$$

A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Solution : In a hollow hemisphere, Volume = $\frac{436\pi}{3}$ cm³ D = 14 cm, R = 7 cm, t = ?

$$\frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3} \Rightarrow 7^3 - r^3 = 218 \Rightarrow 343 - r^3 = 218$$

$$r^3 = 125 \Rightarrow r = 5\text{ cm}$$

$$\therefore \text{thickness}, t = R - r = 7 - 5 = 2\text{ cm}$$

Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution : Largest value = 67 ; Smallest value = 18

Range R= L − S = 67 −18 = 49

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution : Largest value L = 28

Smallest value S = 18

Range R = L − S = 28 − 18 = 10Years.

The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution : Range R = 13.67 and Largest value L = 70.08

Range R = L − S

13.67 = 70.08 − S

S = 70.08 − 13.67 = 56.41

The smallest value is 56.41.

Find the range and coefficient of range of the following data. 63, 89, 98, 125, 79, 108, 117, 68

Solution: Largest value = 125 ; Smallest value = 63

Range = L − S = 125 − 63 = 62

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{185} = 0.33$$

Find the range and coefficient of range of the following data. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution: Largest value = 125 ; Smallest value = 63

Range = L − S = 61.4 − 13.6 = 47.8

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{61.4-13.6}{61.4+13.6} = \frac{47.8}{75} = 0.64$$

If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution : Range R = 36.8

Range R = L − S

36.8 = L − 13.4

L = 36.8 + 13.4 = 50.2

The Largest value is 50.2

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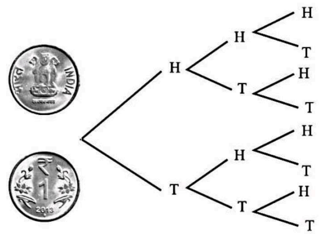
Write the sample space for tossing three coins using tree diagram.

Solution :



Sample space = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$, find the number of defective bulbs.

Solution : Let x be the number of defective bulbs. $\therefore$ n(S) = $x + 20$

Let A be the event of selecting defective balls $\therefore$ n(A) = x

$$P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+20} \Rightarrow \frac{x}{x+20} = \frac{3}{8}$$

$$8x = 3x + 60$$

$$5x = 60 \Rightarrow x = \frac{60}{5} = 12$$

$\therefore$ Number of defective balls = 12.

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution : S = {1H,1T,2H,2T,3H,3T,4H,4T,5H,5T,6H,6T}; n (S) = 12

Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H}; n (A)= 3 ; $P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution : Number of green balls is n (G) = 6 Let number of red balls is n (R) = x

Number of black balls is n (B) = $2x$

Total number of balls n(S) = $6 + x + 2x = 6 + 3x$

P(G) = 3 × P(R)

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

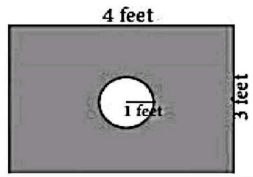
$$3x = 6$$

$$x = 2.$$

(i) Number of black balls = 2×2 = 4

(ii) Total number of balls = 6 +(3×2) = 12

Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



Solution : Area of the rectangular region = 4 × 3 $\Rightarrow$ n(S) = 12 $feet^2$

Area of the circular region = πr^2 $\Rightarrow$ n(A) = $\pi \times 1^2 = \pi \, feet^2$

$\therefore$ Probability to win the game = $\frac{\pi}{12} = \frac{3.14}{12} = \frac{314}{1200} = \frac{157}{600}$

If P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09 then find P(A∪B) .

Solution : P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09

P(A∪B) = P(A)+P(B)−P(A∩B) = 0.37 + 0.42 − 0.09 = 0.7

If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B) = \frac{1}{2}$ and P (A and B) = $\frac{1}{8}$, find P(A or B)

Solution : P(A or B) = P(A∪B) = P(A) + P(B) − P(A∩B) = $\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$

If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B) = \frac{1}{2}$ and P (A and B) = $\frac{1}{8}$, find P (not A and not B).

Solution : P(A∪B) = P(A) + P(B) − P(A∩B) = $\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$

P (not A and not B) = $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B) = 1 - \frac{5}{8} = \frac{3}{8}$

If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A\cup B) = \frac{1}{3}$ then find P(A∩B).

Solution : P(A∩B) = P(A) + P(B) − P(A∪B) = $\frac{2}{3}+\frac{2}{5}-\frac{1}{3} = \frac{10+6-5}{15} = \frac{11}{15}$

A and B are two events such that, P(A) = 0.42, P(B) = 0.48 , and P(A∩B) = 0.16 . Find P (A or B) and P(not B)

Solution : P (A or B) = P(A∪B) = P(A) + P(B) + P(A∩B) = 0.42 + 0.48 − 0.16 = 0.74

A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

Solution: P(A) = 0.5 , P(A∩B) = 0.3   P(A) + P(B) − P(A∩B) ≤ 1   P(B) ≤ 1 − 0.2

P(A∪B) ≤ 1   0.5 + P(B) − 0.3 ≤ 1   P(B) ≤ 0.8

$\therefore$ The probability of B getting selected is atmost 0.8.

In a two children family, find the probability that there is at least one girl in a family.

Solution : S = {(BB), (BG), (GB), (GG)} $\Rightarrow$ n(S) = 4

Let A be the event of getting atleast one girl. $\Rightarrow$ A = {(BG), (GB), (GG)} $\therefore$ n(A) = 3

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

If $A=\begin{pmatrix}1&2&0\\3&1&5\end{pmatrix}, B=\begin{pmatrix}8&3&1\\2&4&1\\5&3&1\end{pmatrix}$ find AB.

Solution :

$$AB=\begin{pmatrix}1&2&0\\3&1&5\end{pmatrix}\times\begin{pmatrix}8&3&1\\2&4&1\\5&3&1\end{pmatrix}=\begin{pmatrix}8+4+0&3+8+0&1+2+0\\24+2+25&9+4+15&3+1+5\end{pmatrix}=\begin{pmatrix}12&11&3\\51&28&9\end{pmatrix}$$

Find x and y if $x\begin{pmatrix}4\\-3\end{pmatrix}+y\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}4\\6\end{pmatrix}$

Solution :

$$x\begin{pmatrix}4\\-3\end{pmatrix}+y\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}4\\6\end{pmatrix}\Rightarrow 4x-2y=4$$
$$\Rightarrow -3x+3y=6$$

$2x-y=2$ .... (1)

$-x+y=2$ .... (2)

$x=4$

Sub x = 4 in (2)

$-4+y=2 \Rightarrow y=6$

$\therefore x=4, y=6$

If $A=\begin{pmatrix}1&9\\3&4\\8&-3\end{pmatrix}, B=\begin{pmatrix}5&7\\3&3\\1&0\end{pmatrix}$ **then verify that** $A+(-A)=(-A)+A=O$

Solution :

$$A+(-A)=\begin{pmatrix}1&9\\3&4\\8&-3\end{pmatrix}+\begin{pmatrix}-1&-9\\-3&-4\\-8&3\end{pmatrix}=\begin{pmatrix}0&0\\0&0\\0&0\end{pmatrix}=O$$

$$(-A)+A=\begin{pmatrix}-1&-9\\-3&-4\\-8&3\end{pmatrix}+\begin{pmatrix}1&9\\3&4\\8&-3\end{pmatrix}=\begin{pmatrix}0&0\\0&0\\0&0\end{pmatrix}=O$$

$\therefore$ **A + (−A) = (−A) + A = O**

If $A=\begin{pmatrix}0&4&9\\8&3&7\end{pmatrix}, B=\begin{pmatrix}7&3&8\\1&4&9\end{pmatrix}$ **find the value of B − 5A**

Solution :

Given $A=\begin{pmatrix}0&4&9\\8&3&7\end{pmatrix}, B=\begin{pmatrix}7&3&8\\1&4&9\end{pmatrix}$

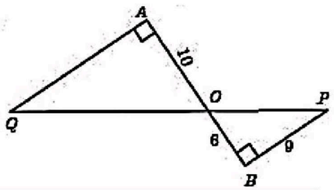
$$B-5A=\begin{pmatrix}7&3&8\\1&4&9\end{pmatrix}-5\begin{pmatrix}0&4&9\\8&3&7\end{pmatrix}=\begin{pmatrix}7&3&8\\1&4&9\end{pmatrix}-\begin{pmatrix}0&20&45\\40&15&35\end{pmatrix}=\begin{pmatrix}7&-17&-37\\-39&-11&-26\end{pmatrix}$$

If $A=\begin{pmatrix}0&4&9\\8&3&7\end{pmatrix}, B=\begin{pmatrix}7&3&8\\1&4&9\end{pmatrix}$ **find the value of 3A − 9B**

Solution :

$$3A-9B=3\begin{pmatrix}0&4&9\\8&3&7\end{pmatrix}-9\begin{pmatrix}7&3&8\\1&4&9\end{pmatrix}=\begin{pmatrix}0&12&27\\24&9&21\end{pmatrix}-\begin{pmatrix}63&27&72\\9&36&81\end{pmatrix}=\begin{pmatrix}-63&-65&-45\\15&-27&-60\end{pmatrix}$$

In Fig. QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.



Solution : In Δ AOQ and Δ BOP, ∠OAQ = ∠OBP = 90°

∠AOQ = ∠BOP (Vertically opposite angles)

By AA similarity, $\Delta AOQ \sim \Delta BOP$

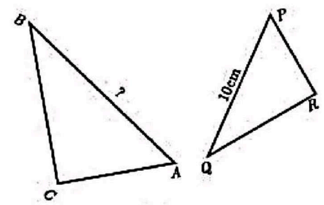
$$\frac{AO}{BO}=\frac{OQ}{OP}=\frac{AQ}{BP} \quad \frac{10}{6}=\frac{AQ}{9} \Rightarrow AQ=\frac{10\times 9}{6}=15cm$$

If ΔABC is similar to ΔDEF such that BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Solution : ΔABC is similar to ΔDEF, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)}=\frac{BC^2}{EF^2} \Rightarrow \frac{54}{Area\ (\Delta DEF)}=\frac{3^2}{4^2}$

$$Area\ (\Delta DEF)=\frac{16\times 54}{9}=96\ cm^2$$

The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ =10 cm, find AB.

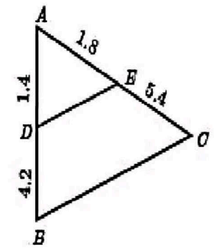


Solution : $\Delta ABC \sim \Delta PQR$,

$$\frac{\text{Perimeters of}(\Delta ABC)}{\text{Perimeters of}(\Delta PQR)}=\frac{AB}{PQ} \Rightarrow \frac{36}{24}=\frac{AB}{PQ} \Rightarrow \frac{36}{24}=\frac{AB}{10}$$

$$AB=\frac{36\times 10}{24}=15cm$$

D and E are respectively the points on the sides AB and AC of a Δ ABCsuch that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE | | BC.



Solution : AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

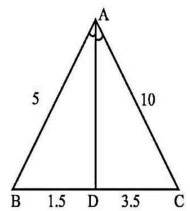
BD = AB – AD = 5.6 – 1.4 = 4.2 cm and EC = AC – AE = 7.2 – 1.8 = 5.4 cm.

$$\frac{AD}{DB}=\frac{AE}{EC} \Rightarrow \frac{1.8}{5.4}=\frac{1.4}{4.2} \Rightarrow \frac{1}{3}=\frac{1}{3}$$

By converse of Basic Proportionality Theorem, DE is parallel to BC.

Hence proved.

Check whether AD is bisector of ∠A of ΔABC if AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm.



Solution : In ΔABC, $\frac{AB}{AC}=\frac{5}{10}=\frac{1}{2}$, $\frac{BD}{DC}=\frac{1.5}{3.5}=\frac{3}{7}$

$$\therefore \frac{AB}{AC}\neq\frac{BD}{DC}$$

By Converse of ABT,

∴ AD is not the bisector of ∠A.

Find the value of 'a' for (2, 3), (4, a) and (6, –3) the given points are collinear.

Solution : Given, (2, 3), (4, a), (6, –3) are collinear. ∴ Area of triangle = 0

$$\frac{1}{2}\begin{bmatrix}2 & 4 & 6 & 2\\ 3 & a & -3 & 3\end{bmatrix}=0$$

$$(2a-12+18)-(12+6a-6)=0$$

$$(2a+6)-(6a+6)=0$$

$$-4a=0$$

$$a=0$$

Show that the points P(–1.5, 3), Q(6, –2), R(–3, 4) are collinear.

Solution : The points are P(– 1.5, 3), Q(6, –2), R(–3, 4)

$$\text{Area of } \Delta PQR = \frac{1}{2}\begin{Bmatrix}x_1 & x_2 & x_3 & x_1\\ y_1 & y_2 & y_3 & y_1\end{Bmatrix} = \frac{1}{2}\begin{Bmatrix}-1.5 & 6 & -3 & -1.5\\ 3 & -2 & 4 & 3\end{Bmatrix}$$

$$=\frac{1}{2}\{(3+24-9)-(18+6-6)\}=\frac{1}{2}\{18-18\}=0$$

The given points are collinear.

Find the equation of a straight line passing through (5, –3) and (7, – 4).

Solution : The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y+3}{-4+3}=\frac{x-5}{7-5}$$

$$2y+6=-x+5$$

$$x+2y+1=0.$$

If the points A(-3, 9) , B (*a, b*) and C(4,-5) are collinear and if *a* + *b* = 1 , then find *a* and *b*.

Solution : Given, A (– 3, 9), B (*a*, *b*), C (4, –5) are collinear **and** $a+b=1$ ∴ $b=1-a$

∴ Area of triangle ABC = 0

$$\therefore \frac{1}{2}\begin{Bmatrix}x_1 & x_2 & x_3 & x_1\\ y_1 & y_2 & y_3 & y_1\end{Bmatrix}=0$$

$$\frac{1}{2}\begin{Bmatrix}-3 & a & 4 & -3\\ 9 & b & -5 & 9\end{Bmatrix}=0$$

$$\frac{1}{2}\begin{Bmatrix}-3 & a & 4 & -3\\ 9 & 1-a & -5 & 9\end{Bmatrix}=0$$

$$\frac{1}{2}\{(-3+3a-5a+36)-(9a+4-4a+15)\}=0$$

$$\{(-2a+33)-(5a+19)\}=0$$

$$-7a+14=0$$

$$-7a=-14$$

$$a=\frac{-14}{-7}$$

$$a=2$$

$a=2$ in $b=1-a$ we get,

$b=1-2=-1$

Find the slope of a line joining the points $\left(-\frac{1}{3},\frac{1}{2}\right)$ and $\left(\frac{2}{7},\frac{3}{7}\right)$

$$\text{slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{\frac{3}{7}-\frac{1}{2}}{\frac{2}{7}+\frac{1}{3}}=\frac{\frac{6-7}{14}}{\frac{6+7}{21}}=-\frac{1}{14}\times\frac{21}{13}=-\frac{3}{26}.$$

Without using Pythagoras theorem, show that the points (1,-4) , (2, –3) and (4, –7) form a right angled triangle.

Solution : Let the given points be A(1, – 4) , B(2, –3) and C(4, –7) . slope $=\frac{y_2-y_1}{x_2-x_1}$

$$\text{slope of AB}=\frac{-3+4}{2-1}=\frac{1}{1}=1$$

$$\text{slope of BC}=\frac{-7+3}{4-2}=\frac{-4}{2}=-2$$

$$\text{slope of AC}=\frac{-7+4}{4-1}=\frac{-3}{3}=-1$$

Slope of AB × slope of AC = (1)(−1) = −1

AB is perpendicular to AC. ∠A = 90°

ΔABC is a right angled triangle.

Show that the given points are collinear (– 3, – 4), (7, 2) and (12, 5).

Solution :

Let points are A (–3, –4), B (7, 2), C (12, 5) , slope $=\frac{y_2-y_1}{x_2-x_1}$

$$\text{Slope of AB}=\frac{2+4}{7+3}=\frac{6}{10}=\frac{3}{5} \qquad \text{Slope of BC}=\frac{5-2}{12-7}=\frac{3}{5}$$

∴ Slope of AB = Slope of BC

∴ AB and BC are parallel. B is the common point.

∴ A, B, C are collinear.

If the three points (3, –1), (*a*, 3) and (1, –3) are collinear, find the value of *a*.

Solution : Let points A (3, –1), B (*a*, 3), C (1, –3) are collinear. slope $=\frac{y_2-y_1}{x_2-x_1}$

∴ Slope of AB = Slope of BC

$$\frac{4}{a-3}=\frac{-6}{1-a}$$

$$4-4a=-6a+18$$

$$2a=14$$

$$a=7$$

Find the slope of a line joining the points (14, 10) and (14, – 6)

Solution :

$$\text{slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{-6-10}{14-14}=\frac{-16}{0}.$$

The slope is undefined.

Find the equation of a straight line whose Inclination is 45° and y intercept is 11

Solution : Given, Slope $m = \tan\theta = \tan 45° = 1$, y intercept, $c = 11$

The equation of a straight line is of the form $y = mx + c$

$$y = x + 11$$

$$x - y + 11 = 0.$$

Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$.

Solution : Equation of the given straight line is $8x - 7y + 6 = 0$

$$\text{slope } m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-8}{-7} = \frac{8}{7} \quad \text{and} \quad \text{y intercept} = \frac{-\text{constant term}}{\text{coefficient of } y} = \frac{-6}{-7} = \frac{6}{7}$$

Find the equation of a straight line passing through **(2, 3) and (–7, –1)**

Solution : The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-3}{-1-3} = \frac{x-2}{-7-2} \qquad 9y - 27 = 4x - 8$$

$$\frac{y-3}{-4} = \frac{x-2}{-9} \qquad 4x - 9y + 19 = 0$$

Find the equation of a straight line passing through $\left(2, \frac{2}{3}\right)$ **and** $\left(\frac{-1}{2}, -2\right)$

Solution :

The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x-2}{\frac{-1}{2} - 2} \Rightarrow \frac{3y-2}{-8} = \frac{x-2}{\frac{-5}{2}} \Rightarrow \frac{3y-2}{-8} = \frac{2x-4}{-5}$$

$$15y - 10 = 16x - 32$$

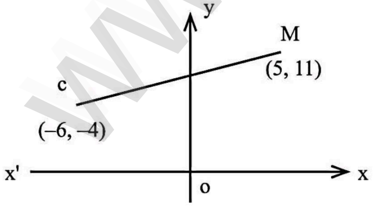
$$16x - 15y - 22 = 0$$

A cat is located at the point(-6,-4) in xy plane. A bottle of milk is kept at (5,11). The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

Solution : Equation of the path CM is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$$\frac{y+4}{11+4} = \frac{x+6}{5+6} \qquad 15x + 90 = 11y + 44$$

$$\frac{y+4}{15} = \frac{x+6}{11} \qquad 15x - 11y + 46 - 0$$



Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1,2).

Solution :

Given slope of the line is $\frac{-5}{4}$ and $(-1, 2)$ is a point on the line.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-5}{4}(x+1)$$

$$4y - 8 = -5x - 5$$

$$5x + 4y - 3 = 0$$

Find the equation of a line whose intercepts on the x and y axes are 4, – 6

Solution :

Given x-intercept = 4 = a

y - intercept = – 6 = b

Equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1 \quad \Rightarrow \frac{x}{4} + \frac{y}{-6} = 1$

$$\frac{x}{4} - \frac{y}{6} = 1 \quad \Rightarrow \frac{3x - 2y}{12} = 1$$

$$3x - 2y - 12 = 0$$

Find the intercepts made by $3x - 2y - 6 = 0$ lines on the coordinate axes.

Solution : $3x - 2y = 6$

Divide by 6 $\Rightarrow \frac{3x}{6} - \frac{2y}{6} = 1 \quad \Rightarrow \frac{x}{2} + \frac{y}{-3} = 1$

Equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ $\qquad \therefore$ x-int = 2, y-int = – 3

Find the slope of the line which is (i) parallel to $3x - 7y = 11$ (ii) perpendicular to $2x - 3y + 8 = 0$.

Solution : (i) $3x - 7y = 11$

$$\text{Slope } m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-3}{-7} = \frac{3}{7}$$

Since parallel lines have same slopes, slope of any line parallel to $3x - 7y = 11$ is $\frac{3}{7}$.

(ii) $2x - 3y + 8 = 0$

$$\text{Slope } m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{-3} = \frac{2}{3}$$

Slope of any line perpendicular to $2x - 3y + 8 = 0$ is $m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution :

Slope of the straight line $x - 2y + 3 = 0$ is Slope $m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-1}{-2} = \frac{1}{2}$

Slope of the straight line $6x + 3y + 8 = 0$ is $m_2 = \frac{-6}{3} = -2$

$$m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$$

Straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Prove that $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=2\operatorname{cosec} A$

Solution : $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=\frac{\sin A(1-\cos A)+\sin A(1+\cos A)}{(1+\cos A)(1-\cos A)}$

$=\frac{\sin A-\sin A\cos A+\sin A+\sin A\cos A}{1-\cos^2 A}$

$=\frac{2\sin A}{1-\cos^2 A}=\frac{2\sin A}{\sin^2 A}=\frac{2}{\sin A}=2\operatorname{cosec} A$

Prove that $\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A+\sin B}=0$

Solution :

$\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A+\sin B}=\frac{(\sin A-\sin B).(\sin A+\sin B)+(\cos A+\cos B).(\cos A-\cos B)}{(\cos A+\cos B).(\sin A+\sin B)}$

$=\frac{(\sin^2 A-\sin^2 B)+(\cos^2 A-\cos^2 B)}{(\cos A+\cos B).(\sin A+\sin B)}$

$=\frac{(\sin^2 A+\cos^2 A)-(\sin^2 B+\cos^2 B)}{(\cos A+\cos B).(\sin A+\sin B)}$

$=\frac{1-1}{(\cos A+\cos B).(\sin A+\sin B)}=0$

Prove that **$\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$**

Solution : $\sec^6\theta=(\sec^2\theta)^3$

$=(1+\tan^2\theta)^3$

$=1+\tan^6\theta+3(1)\tan^2\theta)(1+\tan^2\theta)$ $\therefore (a+b)^3=a^3+b^3+3ab(a+b)$

$=1+\tan^6\theta+3\tan^2\theta.\sec^2\theta$

Show that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2$

Solution : LHS $=\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}=\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}=\tan^2 A$ ...... (1)

RHS $=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2=(-\tan A)^2=\tan^2 A$ ...... (2)

From (1) and (2), $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2$

Prove that $\frac{\cos\theta}{1+\sin\theta}=\sec\theta-\tan\theta$

Solution : $\frac{\cos\theta}{1+\sin\theta}=\frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}=\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}=\frac{1-\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\sec\theta-\tan\theta$

Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\sec\theta+\tan\theta$

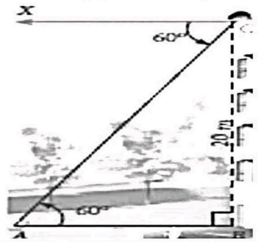
Solution : $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}=\frac{1+\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}=\sec\theta+\tan\theta$

A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60° . Find the distance between the foot of the tower and the ball. ($\sqrt{3}=1.732$)

Solution :

In right triangle ABC, $\tan 60^0=\frac{BC}{AB}\Rightarrow\sqrt{3}=\frac{20}{AB}$

$AB=\frac{20\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{20\times 1.732}{3}=11.54\text{m}$



Hence, the distance between the foot of the tower and the ball is 11.54 m.

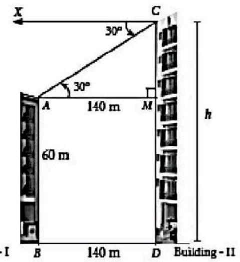
The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30° . If the height of the first building is 60 m, find the height of the second building. ($\sqrt{3}=1.732$)

Solution : In right triangle AMC,

$\tan 30^0=\frac{CM}{AM}$ $\frac{1}{\sqrt{3}}=\frac{CM}{140}$

$CM=\frac{140}{\sqrt{3}}=\frac{140\sqrt{3}}{3}=\frac{140\times 1.732}{3}=80.78$

CD = CM + MD = 80.78 + 60 = 140.78



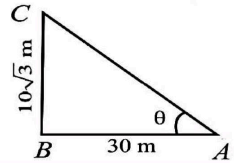
The height of the second building is 140.78 m

Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

Solution : $\tan\theta=\frac{10\sqrt{3}}{30}=\frac{1}{\sqrt{3}}$

$\tan\theta=\tan 30^0$

$\therefore\theta=30^0$



A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.

Solution :