COMMON QUARTERLY EXAMINATION - 2024

Standard XI

Reg.No.

BUSINESS MATHEMATICS & STATISTICS

ime : 3.00 hrs | Part - I | Marks : 90

Choose the correct answer: 20 x 1 = 20

1. The cofactor of -7 in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ is

a) -18 b) 18 c) -7 d) 7

2. If A is a square matrix of order 3 then $|kA|$ is

a) $k|A|$ b) $-k|A|$ c) $k^3|A|$ d) $-k^3|A|$

3. If A is a square matrix of order 3 and $|A| = 3$, then $|\text{adj } A|$ is equal to

a) 81 b) 27 c) 3 d) 9

4. If $\begin{vmatrix} x & 2 \\ 8 & 5 \end{vmatrix} = 0$ then the value of x is

a) $\frac{-5}{6}$ b) $\frac{5}{6}$ c) $\frac{-16}{5}$ d) $\frac{16}{5}$

5. If $nC_3 = nC_2$ then the value of nC_4 is

a) 2 b) 3 c) 4 d) 5

6. The possible outcomes when a coin is tossed five times

a) 2^5 b) 5^2 c) 10 d) $5/2$

7. There are 10 true or false questions in an examination. Then these questions can be answered in

a) 240 ways b) 120 ways c) 1024 ways d) 100 ways

8. Sum of the binomial coefficients is

a) 2^n b) n^2 c) $2n$ d) $n + 17$

9. If m_1 and m_2 are the slopes of the pair of lines given by $ax^2 + 2hxy + by^2 = 0$ then the value of $m_1 + m_2$ is

a) $\frac{2h}{b}$ b) $\frac{-2h}{b}$ c) $\frac{2h}{a}$ d) $\frac{-2h}{a}$

10. If $kx^2 + 3xy - 2y^2 = 0$ represent a pair of lines which are perpendicular, then k is equal to

a) $\frac{1}{2}$ b) $\frac{-1}{2}$ c) 2 d) -2

11. Combined equation of co-ordinate axes is

a) $x^2 - y^2 = 0$ b) $x^2 + y^2 = 0$ c) $xy = c$ d) $xy = 0$

12. The distance b/w directrix and focus of a parabola $y^2 = 4ax$ is

a) a b) 2a c) 4a d) 3a

13. The value of $\sin(-420^\circ)$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{-\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $\frac{-1}{2}$

14. The value of $1 - 2\sin^2 45^\circ$ is

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) 0

15. The value of $\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

16. $\tan\left(\frac{\pi}{4} - x\right)$ is

a) $\left(\frac{1+\tan x}{1-\tan x}\right)$ b) $\left(\frac{1-\tan x}{1+\tan x}\right)$ c) $1 - \tan x$ d) $1 + \tan x$

17. If $f(x) = \frac{1-x}{1+x}$, $x > 1$ then $f(-x)$ is equal to

a) $-f(x)$ b) $\frac{1}{f(x)}$ c) $\frac{-1}{f(x)}$ d) $f(x)$

18. The graph of $y = 2x^2$ is passing through

a) (0,0) b) (2,1) c) (2,0) d) (0,2)

19. $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} =$

a) 1 b) ∞ c) $-\infty$ d) θ

20. If $y = \log x$, then $y_2 =$

a) $\frac{1}{x}$ b) $\frac{-1}{x^2}$ c) $\frac{-2}{x^2}$ d) e^2

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **$7 \times 2 = 14$**

21. Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

22. Show that $\begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$ is non-singular.

23. In how many ways 5 boys and 3 girls can be seated in a row, so that no two girls are together?

24. In how many ways 8 students can be arranged

i) in a line ii) along a circle

25. Show that perpendicular distances of of the line $x - y + 5 = 0$ from origin and from the point p(2,2) are equal.
26. Find the angle b/w the straight lines $x^2 + 4xy + y^2 = 0$
27. Find the value of the trigonometric ratios $\cos(-105^o)$
28. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$
29. Evaluate : $\lim\limits_{x \to \infty} \frac{\sum n}{n^2}$
30. Differentiate with respect to x : $3x^4 - 2x^3 + x + 8$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Find the inverse of the matrix $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$
32. The technology matrix of an economic system of two Industries is $\begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix}$ Test whether the system is viable as per Hawkins-Simon conditions.
33. If $^nP_r = 1680$ and $^nC_r = 70$, find n and r.
34. Find n, if $\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$
35. Show that the straight lines $x + y - 4 = 0$, $3x + 2 = 0$ and $3x - 3y + 16 = 0$ are concurrent.
36. Find the equation of the tangent to the circle $x^2 + y^2 - 4x + 4y - 8 = 0$ at $(-2,-2)$
37. Prove that $\sin\theta.\cos\theta\left[\sin\left(\frac{\pi}{2}-\theta\right).\operatorname{cosec}\theta + \cos\left(\frac{\pi}{2}-\theta\right).\sec\theta\right] = 1$
38. Prove that $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\sin^2\left(\frac{\alpha-\beta}{2}\right)$
39. If $f(x) = x^3 - \frac{1}{x^3}$, $x \neq 0$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$
40. Evaluate $\lim\limits_{x \to a} \frac{x^{3/5} - a^{3/5}}{x^{1/5} - a^{1/5}}$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ then, show that $(AB)^{-1} = B^{-1}A^{-1}$

(OR)

b) Suppose the inter-industry flow of the product of two sectors X and Y are given under

Production Sector	Consumption Sector		Domestic Demand	Gross Output
	X	Y		
X	15	10	10	35
Y	20	30	15	65

Find the gross output when the domestic demand changes to 12 for X and 18 for Y.

42. a) Show by the principle of Mathematical induction that $2^{3n}-1$ is a divisible by 7 for all $n \in N$. *(OR)*

b) Prove that the terms independent of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is

$$\frac{1.3.5.........(2n-1)2^n}{n!}$$

43. a) Find the equation of the circle passing through the points (0,0), (1,2) and (2,0)

(OR)

b) Show that the pair of straight lines $4x^2+12xy+9y^2-6x-9y+2=0$ represent two parallel straight lines and also find the separate equation of the straight lines.

44. a) If $\tan\alpha=\frac{1}{3}$ and $\tan\beta=\frac{1}{7}$, then prove that $(2\alpha+\beta)=\frac{\pi}{4}$ *(OR)*

b) Prove that $\tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}\left(\frac{m-n}{m+n}\right)=\frac{\pi}{4}$

45. a) If $y=500e^{7x}+600e^{-7x}$ then show that $y_2-49y=0$ *(OR)*

b) Show that $f(x)=\begin{cases}5x-4 & \text{if } 0<x\le 1\\ 4x^3-3x & \text{if } 1<x<2\end{cases}$ is continuous at x = 1

46. a) Find $\frac{dy}{dx}$ of the function $x=a\cos^3\theta$, $y=a\sin^3\theta$

(OR)

b) Prove that $\frac{\sin(180^\circ+A)\cos(90^\circ-A)\tan(270^\circ-A)}{\sec(540^\circ-A)\cos(360^\circ+A)\operatorname{cosec}(270^\circ+A)}=-\sin A\cos^2 A$

47. a) The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹320. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹560. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is ₹380. Find the cost of each item per kg by matrix inversion method.

(OR)

b) Resolve into partial fractions $\frac{3x+7}{x^2-3x+2}$

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THIRUVALLUR DISTRICT

STD: XII COMMON QUARTERLY EXAMINATION - 2024

SUB: BUSINESS MATHEMATICS

M.S. UDHAYA MURUGAN M.Sc., B.Ed
MATHS & BUSINESS MATHS
PHONE NO: 7200142523

I

Part - 1

1. b) 1
2. c) 0.4
3. a) 0
4. b) no solution
5. c) $\frac{2^x}{\log 2} + c$
6. b) $2\sqrt{e^x} + c$
7. b) 2
8. a) $(n-1)!$
9. a) 2
10. a) $MC - MR = 0$
11. c) $\frac{3}{2}$ sq. units
12. d) $e^{\int P\,dy}$
13. a) $\frac{d^2y}{dx^2} = 0$
14. d) $y = \sin x + c$, c is an arbitrary constant
15. c) $f(x+h)$
16. a) $1 + \Delta$
17. a) $y_2 - 2y_1 + y_0$
18. c) 7
19. d) c
20. c) 1

Part - 2

II

21. Let $A = \begin{bmatrix} -5 & -7 \\ 5 & 7 \end{bmatrix}$

order of matrix A is 2×2, $\rho(A) \le 2$

consider second order minor

$$|A| = \begin{vmatrix} -5 & -7 \\ 5 & 7 \end{vmatrix} = -35 + 35 = 0$$

second order minor which is zero.

$$\rho(A) \ne 2$$

Consider first order minor $|-5| \ne 0$ which is not zero

$$\therefore \boxed{\rho(A) = 1}$$

22.

$$A \quad X = B$$

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \ne 0$$

$$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 35 - 27 = 8$$

$$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 18 - 21 = -3$$

by using cramer's rule

$$x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$$

Soln: $\boxed{x = 8,\ y = -3}$

23. If $f'(x) = e^x$ & $f(0) = 2$, then find $f(x)$

WKT $f(x) = \int f'(x) \cdot dx + c$

$= \int e^x \cdot dx + c$

$\boxed{f(x) = e^x + c} \rightarrow ①$

Given $f(0) = 2$; $x = 0$, $f(x) = 2$

$2 = e^0 + c$

$2 = 1 + c$

$2 - 1 = c$

$\boxed{c = 1}$ sub ①

$\boxed{f(x) = e^x + 1}$

24. Evaluate $\int_0^\infty e^{-\frac{x}{2}} \cdot x^5 \, dx$

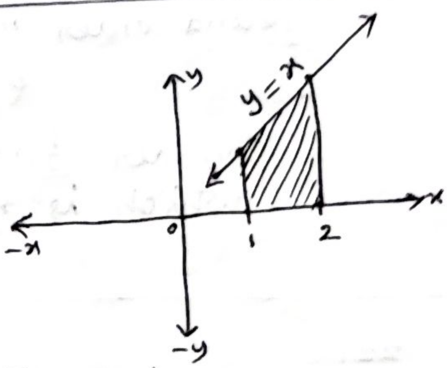
WKT Gamma Integral

$$\int_0^\infty x^n \cdot e^{-ax} \cdot dx = \frac{n!}{a^{n+1}}$$

here $\boxed{n=5}$ $\boxed{a=\frac{1}{2}}$

$$\int_0^\infty e^{-\frac{x}{2}} \cdot x^5 \, dx = \frac{5!}{\left(\frac{1}{2}\right)^{5+1}} = \frac{5!}{\frac{1}{2^6}} = 2^6 \cdot 5!$$

25.



Given Line $y = x$

x-axis at $x = 1$, $x = 2$

$x = 1 \Rightarrow y = 1$, $(1,1)$

$x = 2 \Rightarrow y = 2$, $(2,2)$

Area above x-axis at $x = 1$ to $x = 2$

Area bounded by a line $= \int_a^b y \, dx$

$= \int_1^2 x \, dx = \left[\frac{x^2}{2}\right]_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$ square units

26. If $MR = 20 - 5x + 3x^2$

WKT $R = \int MR \cdot dx + k$

$= \int (20 - 5x + 3x^2)\, dx + k$

$= 20x - 5\frac{x^2}{2} + \cancel{3}\frac{x^3}{\cancel{3}} + k$

$\boxed{R = 20x - \frac{5}{2}x^2 + x^3 + k} \rightarrow (1)$

WKT $R = 0$, $x = 0 \Rightarrow \boxed{K = 0}$ sub (1)

Total revenue $\boxed{R = 20x - \frac{5}{2}x^2 + x^3}$

27. solve $\frac{dy}{dx} + e^x + ye^x = 0$

$\frac{dy}{dx} + e^x \cdot y = -e^x$

It is of the form

$\frac{dy}{dx} + Py = Q$

here $P = e^x$, $Q = -e^x$

$I.F = e^{\int P dx} = e^{\int e^x \cdot dx} = e^{e^x}$

G.S

$y \cdot e^{\int P dx} = \int Q \cdot e^{\int P dx} \cdot dx + c$

$y \cdot e^{e^x} = \int -e^x \cdot e^{e^x} \cdot dx + c$

put $\boxed{t = e^x}$

$\frac{dt}{dx} = e^x$

$\boxed{dt = e^x \cdot dx}$

$= -\int e^t \cdot dt + c$

$= -e^t + c$

$\boxed{y \cdot e^{e^x} = -e^{e^x} + c}$

28. Solve $(D^2-3D-4)y=0$

A.E is $m^2-3m-4=0$

$$(m+1)(m-4)=0$$

$$m_1=-1, \quad m_2=4$$

$-4 = -4 \times 1$, $-4+1=-3$

$m_1 \neq m_2$

$$C.F = Ae^{m_1x}+Be^{m_2x}$$

G.S $\boxed{y = A.e^{-x}+Be^{4x}}$

29. If $f(x)=x^2+3x$ then show that $\Delta f(x)=2x+4$

WKT $\Delta f(x) = f(x+h)-f(x)$, $\boxed{h=1}$

$$\Delta f(x) = \Delta(x^2+3x)$$

$$= (x+1)^2+3(x+1)-x^2-3x$$

$$= \cancel{x^2}+2x+1+\cancel{3x}+3-\cancel{x^2}-\cancel{3x}$$

$$\boxed{\Delta f(x) = 2x+4}$$

30. p.m.f

X	0	1	2	3	4	5
P(X=x)	0	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{4}{20}$	$\frac{5}{20}$

(i) $P(X<3) = P(X=0)+P(X=1)+P(X=2)$

$$= 0+\frac{1}{20}+\frac{2}{20} = \frac{3}{20}$$

(ii) $P(2<X\leq 4) = P(X=3)+P(X=4)$

$$= \frac{3}{20}+\frac{4}{20} = \frac{7}{20}$$

31.

$$A \quad X = B$$

$$\begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

Augmented Matrix

$$[A|B] = \left[\begin{array}{cc:c} 3 & -2 & 6 \\ 6 & -4 & 10 \end{array}\right]$$

$$\sim \left[\begin{array}{cc:c} 3 & -2 & 6 \\ 0 & 0 & -2 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2 - 2R_1 \end{array}$$

$\rho(A)$

$\rho[A, B]$

The last equivalent matrix is an Echelon's form

For A, no. of non-zero row is 1, $\rho(A) = 1$

For [A|B], No. of non-zero row is 2, $\rho[A, B] = 2$

$\therefore \rho(A) \neq \rho[A, B]$

Hence Given equations are inconsistent

Hence proved.

32. Let $I = \int (2\cos x - 3\sin x + 4\sec^2 x - 5\operatorname{cosec}^2 x)\, dx$

$= 2\sin x + 3\cos x + 4\tan x + 5\cot x + c$

33. Since four values of y_x are given

Since the polynomial is of degree is 3

Since fourth order difference is zero

$\therefore \Delta^4 y_0 = 0$

$(E-1)^4 y_0 = 0$

$(E^4 - 4E^3 + 6E^2 - 4E^1 + 1) y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$$81 - 4y_3 + 6(9) - 4(3) + 1 = 0$$

$$81 + 54 - 12 + 1 = 4y_3$$

$$136 - 12 = 4y_3$$

$$\frac{124}{4} = y_3$$

$$\boxed{31 = y_3}$$

missing entry of 3 is 31

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35. Evaluate $\int_{-1}^{1} \frac{2x+3}{x^2+3x+7}\,dx$

let $I = \int_{-1}^{1} \frac{2x+3}{x^2+3x+7}\,dx$

put, $\boxed{t = x^2+3x+7}$

x	-1	1
t	5	11

$$\frac{dt}{dx} = 2x+3$$

$$\boxed{dt = (2x+3).dx}$$

$$I = \int_5^{11} \frac{dt}{t} = \left[\log(t)\right]_5^{11}$$

$$= \log(11) - \log(5)$$

$$= \log\left(\frac{11}{5}\right)$$

36.

p.m.f

X	2	4	6	8
$P(X=x)$	$2k$	$4k$	$6k$	$6k$

WKT $\sum P(X) = 1$

$$P(2) + P(4) + P(6) + P(8) = 1$$

$$2k + 4k + 6k + 6k = 1$$

$$18k = 1$$

$$\boxed{k = \frac{1}{18}}$$

37. Solve $\frac{dy}{dx} - \frac{y}{x} = x$

It is of the form

$$\frac{dy}{dx} + Py = Q$$

here $P = -\frac{1}{x}$, $Q = x$

$$I.F = e^{\int P dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = x^{-1} = \frac{1}{x}$$

G.S $y \cdot e^{\int P dx} = \int Q \cdot e^{\int P dx} \cdot dx + c$

$$y \cdot \frac{1}{x} = \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx + c$$

$$\boxed{\frac{y}{x} = x + c}$$

38. WKT $f(4) - f(3) = \Delta f(3)$

$$= \Delta[f(2) + \Delta f(2)]$$

$$= \Delta f(2) + \Delta^2 f(2)$$

$$= \Delta f(2) + \Delta^2 [f(1) + \Delta f(1)]$$

$$f(4) - f(3) = \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$$

$$\boxed{f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)}$$

$\Delta f(2) = f(3) - f(2)$

$f(2) + \Delta f(2) = f($

$f(1) + \Delta f(1) = f($

hence proved.

39. properties of Mathematical expectation?

(i) $E(a) = a$, where a - constant

(ii) $E(aX+b) = aE(X) + b$

(iii) If $X \geq 0$ if $E(X) \geq 0$

(iv) $V(a) = 0$

(v) $V(aX+b) = a^2 V(X)$

10. Given st. lines $y = mx + c \rightarrow (1)$

diff. 'y' w. r. to x

(ii) $\boxed{\frac{dy}{dx} = m}$ sub (1)

Again diff. 'y' w. r. to 'x'

(iii) $\boxed{\frac{d^2y}{dx^2} = 0}$

(i) $\boxed{y = x\frac{dy}{dx} + c}$

IV

Part-IV

41.) b) Given currently shares $\begin{array}{cc} A & B \\ (0.5 & 0.5) \end{array}$

Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} \end{array}$$

Share after one week

$$(0.5 \quad 0.5)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (0.30+0.10 \quad 0.20+0.40)$$

$$= (0.40 \quad 0.60)$$

$$= \begin{array}{cc} A & B \\ (0.4 & 0.6) \end{array}$$

A's share after one week is 40%.

B's share after one week is 60%.

Shares after two weeks

$$(0.4 \quad 0.6)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (0.24+0.12 \quad 0.16+0.48)$$

$$= \begin{array}{cc} A & B \\ (0.36 & 0.64) \end{array}$$

A's share after two weeks is 36%.

B's share after two weeks is 64%.

Equilibrium reached

$(A \quad B)T = (A \quad B)$ where $A+B=1$

$$(A \quad B)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (A \quad B)$$

$$(0.6A+0.2B \quad 0.4A+0.8B) = (A \quad B)$$

$$0.6A+0.2B=A$$

$$0.6A+0.2(1-A)=A$$

$$0.6A+0.2-0.2A=A$$

$$0.4A+0.2=A$$

$$0.2=A-0.4A$$

$$0.2=A(1-0.4)$$

$$0.2=0.6A$$

$$\frac{0.2}{0.6}\times\frac{10}{10}=A$$

$$\frac{\cancel{2}}{\cancel{6}_3}=A$$

$$\frac{1}{3}=A$$

$$A=0.333$$

$$\boxed{A=33.3\%}$$

$$B=1-A$$

$$=1-0.333$$

$$B=0.667$$

$$\boxed{B=66.7\%}$$

Equilibrium reached A' share is 33.3%

& B' share is 66.7%

42.) b) Evaluate $\int_1^2 \frac{1}{(x+1)(x+2)} \cdot dx$

consider

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \rightarrow (1)$$

$$1 = A(x+2) + B(x+1)$$

put $x=-2$

$1 = B(-1)$

$\boxed{B=-1}$

put $x=-1$

$1 = A(1)+0$

$\boxed{1=A}$ sub (1)

$$\int_1^2 \frac{1}{(x+1)(x+2)} \cdot dx = \int_1^2 \left(\frac{1}{x+1} - \frac{1}{x+2}\right) dx$$

$$= \left[\log|(x+1)| - \log|(x+2)|\right]_1^2$$

$$= \left[\log\left|\frac{x+1}{x+2}\right|\right]_1^2$$

$$= \log\left|\frac{3}{4}\right| - \log\left|\frac{2}{3}\right|$$

$$= \log\left|\frac{3/4}{2/3}\right|$$

$$= \log\left|\frac{9}{8}\right|$$

43. b) WKT Equilibrium

$$P_d = P_s$$

$$25-3x = 5+2x$$

$$25-5 = 3x+2x$$

$$\frac{20}{5} = x$$

$$\boxed{x_0 = 4}$$

$$P_0 = 5+2(4)$$

$$= 5+8$$

$$\boxed{P_0 = 13}$$

$$x_0 P_0 = 4(13) = 52$$

$$C.S = \int_0^{x_0} f(x)\,dx - x_0 P_0 = \int_0^4 (25-3x)\,dx - 52$$

$$= \left[25x - \frac{3x^2}{2}\right]_0^4 - 52 = \left[\left(100 - \frac{3(16)}{2}\right) - (0-0)\right] - 52$$

$$= 100 - 24 - 52$$

$$= 100 - 76$$

$$\boxed{C.S = 24 \text{ units}}$$

$$P.S = x_0 P_0 - \int_0^{x_0} g(x)\,dx = 52 - \int_0^4 (5+2x)\,dx$$

$$= 52 - \left[5x + \frac{2x^2}{2}\right]_0^4$$

$$= 52 - [(20+16) - (0+0)]$$

$$= 52 - 36$$

$$\boxed{P.S = 16 \text{ units}}$$

44) b) Solve $(1-x)\,dy - (1+y)\,dx = 0$

$$(1-x)\,dy = (1+y)\,dx$$

V.S

$$\int \frac{dy}{1+y} = \int \frac{dx}{1-x}$$

$$\log|1+y| = \log|1-x| + \log|c|$$

$$\log(1+y) = \log((1-x)\cdot c)$$

Exponential on both sides

$$(1+y) = (1-x)c$$

$$\frac{1+y}{1-x}$$

45. a) Backward interpolation formula

$$y = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \cdots$$

$x = 2.8$, $n = \frac{x - x_n}{h} = \frac{2.8-3}{1} = -0.2$

$h = 1$

$x_n = 3$

Backward table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		1		
1	2		8	
		9		6
2	11		14	
		23		
3	34			

$$y = 34 + \frac{(-0.2)}{1!}(23) + \frac{(-0.2)(-0.2+1)}{2!}(14) + \frac{(-0.2)(-0.2+1)(-0.2+2)}{3!}(6)$$

$$= 34 - 4.6 + (-0.2)(0.8)(7) + (-0.2)(0.8)(1.8)$$

$$= 29.4 - 1.12 - 0.288$$

$$= 29.4 - 1.408$$

$$\boxed{y = 27.992}$$

29.400
− 1.408
27.992

46) b)

Given $f(x) = k\, e^{-|x|}$, $-\infty < x < \infty$

WKT p.d.f $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\int_{-\infty}^{\infty} k\, e^{-|x|}\, dx = 1$$

$$2k \int_{0}^{\infty} e^{-x}\, dx = 1 \quad (e^{-|x|} \text{ is an even function})$$

$$2k \left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$$

$$-2k\left[e^{-\infty} - e^{0}\right] = 1$$

$$-2k[0 - 1] = 1$$

$$\boxed{k = 1/2}$$

Mean $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \frac{1}{2}\int_{-\infty}^{\infty} x\, e^{-|x|}\, dx$

$\boxed{E(X) = 0}$ $(x \cdot e^{-|x|}$ is an odd function$)$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \frac{1}{2}\int_{-\infty}^{\infty} x^2\, e^{-|x|}\, dx$$

$$= \cancel{2} \cdot \frac{1}{\cancel{2}} \int_0^{\infty} x^2 e^{-x}\, dx \quad (x^2 e^{-|x|} \text{ is an even function})$$

$$= \frac{2!}{1^{2+1}} \quad \left(\because \text{Gamma Integral } \int_0^{\infty} x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}\right)$$

$$\boxed{E(X^2) = 2}$$

$$\text{var}(X) = E(X^2) - [E(X)]^2$$

$$= 2 - 0$$

$\boxed{\text{Var}(X) = 2}$

b)

$$\underset{A}{\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}}$$

Augmented matrix

$$[A|B] = \left[\begin{array}{ccc:c} 2 & 1 & 1 & 5 \\ 1 & 1 & 1 & 4 \\ 1 & -1 & 2 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 5 \\ 1 & -1 & 2 & 1 \end{array}\right] \begin{array}{l} R_1 \leftrightarrow R_2 \\ \\ R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -3 \\ 0 & -2 & 1 & -3 \end{array}\right] \begin{array}{l} R_1 \to 1 \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 3 & 3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \to R_3 - 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & 3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to (-1)R_2 \\ R_3 \end{array}$$

$\rho(A)$

$\rho(A|B)$

The last equivalent matrix is an echelon form

For A, no. of non-zero row is 3, $\rho(A) = 3$

For $[A|B]$, no. of non-zero row is 3, $\rho(A|B) = 3$

$\therefore \rho(A) = \rho([A,B]) = 3 =$ number of unknowns (x, y, z)

consistent & unique solution.

consider

$$x+y+z=4 \rightarrow (1)$$

$$y+z=3 \rightarrow (2)$$

$$3z=3$$

$$\boxed{z=1} \text{ sub (2)}$$

$$y+1=3$$

$$y=3-1$$

$$\boxed{y=2} \text{ sub (1)}$$

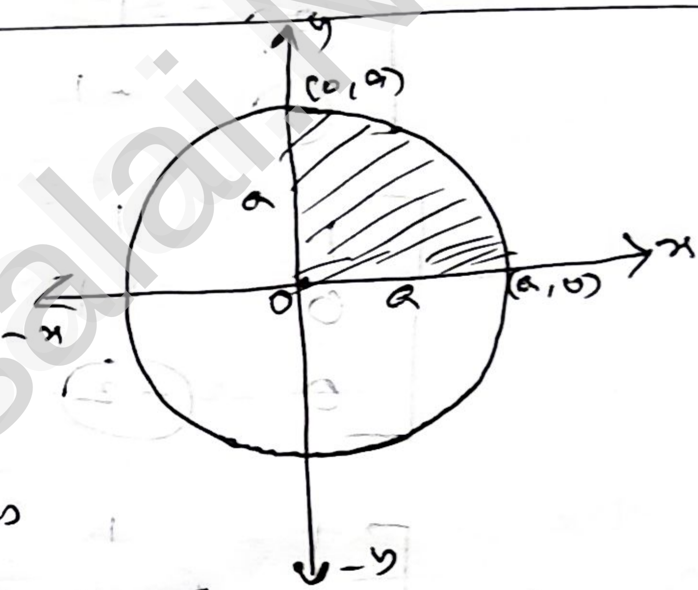
$$x+2+1=4$$

$$x=4-3$$

$$\boxed{x=1}$$

Soln: $\{1,2,1\}=(x,y,z)$

43) a)



WKT eqn of circle with radius 'a' units

$$x^2+y^2=a^2$$

$$\boxed{y=\sqrt{a^2-x^2}}$$

Area about both the axis but take area above x-axis at $x=0$, to $x=a$

Area of circle $= 4 \times$ Area of I quadrant

$$= 4\int_a^b y \, dx = 4\int_0^a \sqrt{a^2-x^2}\, dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= 2\left[\left(0+a^2\sin^{-1}(1)\right)-(0+0)\right]$$

$$= \cancel{2}\, a^2\left(\frac{\pi}{\cancel{2}}\right)$$

$$= \pi a^2 \text{ square units}$$