MODEL COMMON QUATERLY EXAMINATION 2024

XI – STD – MATHEMATICS

Time: 3.00 Hrs **Maximum Marks: 90**

PART – I (Marks: 20)

I. Choose the correct answer: **20 ×1= 20**

1. If A=$\{(x,y) : y = \sin x,\ \ x \in R\}$ and $B = \{(x,y): y = \cos x,\ x \in R\}$ then $A \cap B$ $contains$ **(2)** (1)no element (2) infinitely many elements (3) only one element (4) cannot be determined

2. The number of constant functions from a set containing m elements to a set containing n elements is **(18)** (1) mn (2) m (3) n (4) $m + n$

3. Then function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{(x^2+\cos x)(1+x^4)}{(x-\sin\ \ )(2x-x^3)} + e^{-|x|}$ is **(25)**

   (1) an odd function (2) neither an odd function nor an even function

   (3) an even function (4) both odd function and even function.

4. The value of $\log_{\sqrt{2}} 512$ is **(6)**

   (1) 16 (2) 18 (3) 9 (4) 12

5. If a and b are the real roots of the equation $x^2 - kx + c = 0$, then the distance between the points (a, 0) and (b, 0) is **(16)**

   (1) $\sqrt{k^2 - 4c}$ (2) $\sqrt{4k^2 - c}$ (3) $\sqrt{4c - k^2}$ (4) $\sqrt{k - 8c}$

6. If 3 is the logarithm of 343, then the base is **(10)**

   (1) 5 (2) 7 (3) 6 (4) 9

7. If $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$, then the value of k is **(17)**

   (1) 1 (2) 2 (3) 3 (4) 4

8. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right) =$ **(4)**

   (1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{2}}$

9. $\cos 1° + \cos 2° + \cos 3° + \cdots .. + \cos 179° =$ **(7)**

   (1) 0 (2) 1 (3) -1 (4) 89

10.If a $\Delta\ ABC,\ if$ (1) , if (i) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} > 0$ (ii) $\sin A \sin B \sin C > 0$ $then$ **(20)**(1)both (i) and (ii) are true (2) only (i) is true (2) only (ii) is true (4) neither (i) nor (ii) is true.

11.The sum of the digits at the 10^{th} place of all numbers formed with the help of 2, 4, 5, 7 taken all at a time is **(1)** (1) 432 (2) 108 (3) 36 (4) 18

12.The product of r consecutive positive integers is divisible by **(7)**

(1) $r!$ (2) $(r-1)!$ (3) $(r+1)!$ (4) r^r

13.The number of rectangles that a chessboard has …….. **(19)**

(1) 81 (2) 9^9 (3) 1296 (4) 6561

14.The coefficient of $x^8\ y^{12}$ in the expansion of $(2x+3y)^{20}\ is$ **(3)**

(1)0 (2) $2^8 3^{12}$ (3) $2^8 3^{12} + 2^{12} 3^8$ (4) ${}^{20}C_8\ 2^8 3^{12}$

15.The n^{th} term of the sequence $1, 2, 4, 7, 11, \dots\dots.\ is$ **(12)**

(1) n^3+3n^2+2n (2) n^3-3n^2+3n (3) $\frac{n(n+1)(n+2)}{3}$ (4) $\frac{n^2-n+2}{2}$

16.The coefficient of x^5 in the series $e^{-2x}\ is$ **(18)**

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{-4}{15}$ (4) $\frac{4}{15}$

17.The value of $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots\dots\dots.\ is$ **(19)**

(1) $\frac{e^2+1}{2e}$ (2) $\frac{(e+1)^2}{2e}$ (3) $\frac{(e-1)^2}{2e}$ (4) $\frac{e^2-1}{2e}$

18.The intercept of the perpendicular bisector of the line segment joining $(1,\ 2)\ and\ (3,4)$ with coordinate axes are **(9)**

(1) $5,-5$ (2) $5,5$ (3) $5,3$ (4) $5,-4$

19.If the equation of the base opposite to the vertex $(2,3)$ of an equilateral triangle is $x+y=2$, then the length of a side is **(13)**

(1) $\sqrt{\frac{3}{2}}$ (2) 6 (3) $\sqrt{6}$ (4) $3\sqrt{2}$

20.If the two straight line $x+(2k-7)y+3=0\ and\ 3kx+9y-5=0$ are perpendicular then the value of k is **(19)**

(1) $k=3$ (2) $k=\frac{1}{3}$ (3) $k=\frac{2}{3}$ (4) $k=\frac{3}{2}$

PART – II (Marks: 14)

II. Answer any 7 Questions. Question No. 30 is compulsory. 7×2=14

21.If $\rho(A)$ denotes the power set of A, then find $n(\rho\left(\rho\left(\rho(\phi)\right)\right))$ **(Eg. 1.9)**

22.Solve $(2x+1)^2-(3x+2)^2=0$ $(\boldsymbol{Ex\ 2.6-4})$

23.Solve $\log_4 2^{8x} = 2^{\log_2 8}$ **(Ex. 2.12-4)**

24.Prove that $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right) = -1$ $(\boldsymbol{Ex.3.4-23})$

25.*prove that* $\frac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x} = \tan 4x$ $(\boldsymbol{Ex.3.6-12})$

26.Prove that ${}^{24}C_4 + \sum_{r=0}^{4}({}^{28-r})C_3 = {}^{29}C_4$. $(\boldsymbol{Eg-\ 4.48})$

27.In the binomial expansion of $(a+b)^n$ the coefficients of the 4th and 13th terms are equal then, find n. $(\boldsymbol{Ex.5.1-13})$

28.Find the coefficient of x^4 in the expansion of $\frac{3-4x+x^2}{e^{2x}}$ $(\boldsymbol{Ex.5.4-9})$

29.Find the path traced out by the point $\left(ct,\frac{c}{t}\right)$ *here* $t \neq 0$ is the parameter and c is constant. $(\boldsymbol{Eg.6.2})$

30.Rewrite $\sqrt{3}x+y+4=0$ in to normal form. $(\boldsymbol{Eg.6.20})$

PART – III (Marks: 21)

III. Answer any 7 Questions. Question No. 40 is compulsory. 7×3=21

31.Find the range of the function $\frac{1}{2\cos x-1}$ **(Ex. 1.3-8)**

32.Prove that $ap+q=0$ if $f(x) = x^3-3px+2q$ is divisible by $g(x)=x^2+2ax+a^2$ $(\boldsymbol{Eg.2.18})$

33.if $\theta+\phi = \alpha$ *and* $\tan\theta = k\tan\phi$ *then prove that* $\sin(\theta-\phi) = \frac{k-1}{k+1}\sin\alpha$ **(Ex. 3.4-25)**

34.If $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$, *show that* $\cos 3\theta = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$, $(\boldsymbol{Ex,3.5-3})$

35.Prove that the sum of first n positive odd numbers is n^2. $(\boldsymbol{Eg-\ 4.62})$

36.If $a,\ b,\ c$ are in G.P and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ then prove that x, y, z are in A.P$(\boldsymbol{Ex.5.2-7})$

37.Find the coefficient of x^{15} in $\left(x^2+\frac{1}{x^3}\right)^{10}$ $(\boldsymbol{Ex.5.1-4})$

38.Prove that $c_0^2+c_1^2+c_2^2+\cdots\ldots.+c_n^2=\frac{(2n)!}{(n!)^2}$ $(\boldsymbol{Ex.5.1-16})$

39.Find the equation of the pair of lines through the origin and perpendicular to the pair of lines $ax^2+2hxy+by^2=0$ $(\boldsymbol{Eg.6.35})$

40.The slope of one of the straight lines $ax^2+2hxy+by^2=0$ is three times the other, show that $3h^2=4ab$. $(\boldsymbol{Ex.6.4-9})$

PART – IV (Marks: 35)

IV. Answer all the questions. **7×5=35**

41.on the set of natural numbers let R be the relation defined by aRb if $2a+3b=30$. Write down the relation by listing all the pairs. Check whether it is (a) Reflexive (b) symmetric (c) transitive (d) equivalence $(\boldsymbol{Ex.1.2-5})$ **(OR)**

write the values of f at $-3,\ 5,\ 2,-1,0$ $(\boldsymbol{Ex.1.3-3})$

$$f(x)=\begin{cases} x^2+x-5 & if\ x\in(-\infty,0) \\ x^2+3x-2 & if\ x\in(3,\infty) \\ x^2 & if\ x\in(0,2) \\ x^2-3 & otherwise \end{cases}$$

42.In one root of $k(x-1)^2=5x-7$ is double the other root, show that

$k=2$ or -25. $(\boldsymbol{Ex.2.4-4})$ $(\boldsymbol{OR})$

Resolve into the partial Fractions: $\frac{x^2+x+1}{x^2-5x+6}$ **(Ex. 2.9 – 7)**

43.Prove $\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$ **(Eg. 2.36) (OR)**

Prove that $\frac{\cot(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\ \tan(-\theta)\,\mathrm{cosec}(360°+\theta)}=cos^2\theta\cot\theta$ $(\boldsymbol{Ex.3.3-4})$.

44.In ΔABC, we have (i) $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$ (ii) $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$ (iii) $\tan\frac{C-A}{2}=\frac{c-a}{c+a}\cot\frac{B}{2}$ **(Theorem 3.2) (OR)**

Prove that ${}^{35}C_5+\sum_{r=0}^{4}{}^{(39-r)}C_4={}^{40}C_5$ $(\boldsymbol{Ex.4.3-5})$

45.Using the mathematical induction, show that for any natural number n, $x^{2n} - y^{2n}$ is divisible by $x + y$. **Ex. 4.4 – 10)** **(OR)**

The AM of two numbers exceed their GM by 10 and HM by 16. Find the numbers. (***Ex.* 5.2 – 8**).

46.prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large. **(Ex.5.4- 3)** **(OR)**

if $p - q$ is small compared to either p *or* q , then show that $\sqrt[n]{\frac{p}{q}} \simeq \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$ hence find $\sqrt[8]{\frac{15}{16}}$. (***Ex.* 5.4 – 8**)

47.Show that the straight lines joining the origin to the points of intersection of $3x - 2y + 2 = 0$ *and* $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$ are at right angles. (***Eg.* 6.41**) **(OR)**

Prove that the straight lines joining the origin to the points of intersection of $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and $3x - 2y - 1 = 0$ are at right angles. (***Ex.* 6.4 – 18**).