MODEL COMMON QUATERLY EXAMINATION - 2024

XI – STD – MATHEMATICS

Time: 3.00 Hrs **Maximum Marks: 90**

PART – I (Marks: 20)

I. Choose the correct answer: **20 ×1= 20**

1. Let A and B be subsets of the universal set N, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is **(6)** (1) A (2) A' (3) B (4) $\mathbb{N}$
2. If $n(A) = 2$ and $n(BUC) = 3,$ $then$ $n[(A \times B) \cup (A \times C)]$ is **(9)**
   (1)2^3 (2) 3^2 (3) 6 (4) 5
3. The rule $f(x) = x^2$ is a bijection if the domain and the co-domain are given by**(17)** (1)$\mathbb{R}, \mathbb{R}$ (2) $\mathbb{R}$. $(0, \infty)$ (3) $(0, \infty), \mathbb{R}$ (4) $[0, \infty), [0, \infty)$
4. Given that x, y and b are real numbers $X < Y,\ B > 0,$ $then$ **(2)**
   (1) $xb < yb$ (2) $xb > yb$ (3) $xb \le yb$ (4) $\frac{x}{b} \geq \frac{y}{b}$
5. The number of solutions of $x^2 + |x - 1| = 1$ is **(13)**
   (1) 1 (2) 0 (3) 2 (4) 3
6. The value of $\log_3 11 . \log_{11} 13 . \log_{13} 15 . \log_{15} 27 . \log_{27} 81$ is **(20)**
   (1) 1 (2) 2 (3) 3 (4) 4
7. $\frac{1}{\cos 80°} - \frac{\sqrt{3}}{\sin 80°} =$ **(1)** (1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) 2 (4) 4
8. If $\tan 40° = \lambda,$ $then$ $\frac{\tan 140° - \tan \quad °}{1+tan140°\ \tan 130°} =$ **(6)**
   (1) $\frac{1-\lambda^2}{\lambda}$ (2) $\frac{1+\lambda^2}{\lambda}$ (3) $\frac{1+\lambda^2}{2\lambda}$ (4) $\frac{1-\lambda^2}{2\lambda}$
9. If $\tan \alpha$ and $\tan \beta$ are the $roots$ of $x^2 + ax + b = 0,$
   $then$ $\frac{\sin(\alpha+\beta)}{\sin \alpha \sin \beta}$ is $equal$ to **(13)**
   (1) $\frac{b}{a}$ (2) $\frac{a}{b}$ (3) $-\frac{a}{b}$ (4) $-\frac{b}{a}$
10. If $f(\theta) = [\sin \theta] + [\cos \theta],\ \theta \in R,$ $then$ $f(\theta)$ is in the $interval$ **(15)**
    (1) $[0,2]$ (2) $[1, \sqrt{2}]$ (3) $[1,2]$ (4) $[0,1]$
11. The number of 5 digit numbers all digits of which are odd is **(4)**

(1) 25 (2) 5^6 (3) 625 (4) 5^5

12.Number of sides of a polygon having 44 diagonals is …… **(14)**

(1) 4 (2) 4! (3) 11 (4) 22

13.$1+3+5+7+\cdots..+17$ is equal to **(24)** (1)101 (2) 81 (3) 71 (4) 61

14.The coefficient of x^6 in $(2+2x)^{10}$ is **(2)**

(1) ${}^{10}C_6$ (2) 2^6 (3) ${}^{10}C_6\, 2^6$ (4) ${}^{10}C_6\, 2^{10}$

15.The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}, \ldots\ldots$ form an **(8)**

(1)AP (2) GP (3) HP (4) AGP

16.The value of $1-\frac{1}{2}\left(\frac{2}{3}\right)+\frac{1}{3}\left(\frac{2}{3}\right)^2-\frac{1}{4}\left(\frac{2}{3}\right)^3+\cdots\ldots..is$ **(20)**

(1) $\log\left(\frac{5}{3}\right)$ (2) $\frac{3}{2}\log\left(\frac{5}{3}\right)$ (3) $\frac{5}{3}\log\left(\frac{5}{3}\right)$ (4) $\frac{2}{3}\log\left(\frac{2}{3}\right)$

17.Which of the following point lie on the locus of

$3x^2+3y^2-8x-12y+17=0$ **(3)**

(1) $(0,0)$ (2) $(-2,\ 3)$ (3) $(1,2)$ (4) $(0,-1)$

18.The length of ⊥ from the origin to the line $\frac{x}{3}-\frac{y}{4}=1,\ is$ **(17)**

(1) $\frac{11}{5}$ (2) $\frac{5}{12}$ (3) $\frac{12}{5}$ (4) $-\frac{5}{12}$

19.The area of the triangle formed by the lines $x^2-4y^2=0$ and $x=a$ is **(22)**

(1) $2a^2$ (2) $\frac{\sqrt{3}}{2}\ a^2$ (3) $\frac{1}{2}\ a^2$ (4) $\frac{2}{\sqrt{3}}\ a^2$

20.θ is acute angle between the lines $x^2-xy-6y^2=0$, then $\frac{2\cos\theta+3\sin}{4\sin\theta+5\cos\theta}$ is **(24)**

(1)1 (2) $-\frac{1}{9}$ (3) $\frac{5}{9}$ (4) $\frac{1}{9}$

PART – II (Marks: 14)

II. Answer any 7 Questions. Question No. 30 is compulsory. 7×2=14

21.If $A=\{1,2,3,4\}$ and $B=\{3,4,5,6\}$, find $n\big((A\cup B)\times(A\cap B)\times(A\,\triangle\,B)\big)$**(Eg. 1.8)**

22.If $f:\mathbb{R}\to\mathbb{R}$ is defined as $f(x)=2x^2-1$ find the pre-image of 17,4 and -2. $(\boldsymbol{Eg.1.18})$

23.Solve $\frac{1}{5}\ |10x-2|<1$. **(Ex. 2.2-5)**

24.Prove that $\tan 315° \cot(-405°) + \cot(495°) \tan(-585°) = 2$ $(\boldsymbol{Eg.3.13})$

25.If $a \cos(x + y) = b \cos(x - y)$, show that

$(a + b) \tan x = (a - b) \cot y$. $(\boldsymbol{Ex.3.4 - 10})$

26.If ${}^{n+2}P_4 = 42 \times {}^{n}P_2$, find n. $(\boldsymbol{Eg - 4.26})$

27.Find the last two digits of the number 3^{600} $(\boldsymbol{Ex.5.1 - 8})$

28.Find $\sqrt[3]{65}$ $(\boldsymbol{Eg.5.23})$

29.Find the path traced out by the point $\left(ct, \frac{c}{t}\right)$ *here* $t \neq 0$ is the parameter and c is constant. $(\boldsymbol{Eg.6.2})$

30.Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form. $(\boldsymbol{Eg.6.20})$

PART – III (Marks: 21)

III. Answer any 7 Questions. Question No. 40 is compulsory. 7×3=21

31.P.T $\left((A \cup B' \cup C) \cap (A \cap B' \cap C')\right) \cup \left((A \cup B \cup C') \cap (B' \cap C')\right) = (B' \cap C')$ $(\boldsymbol{Eg.1.3})$

32.prove that $\sqrt{3}$ is an irrational number **(Ex. 2.1-2)**

33.show that $sin^2\frac{\pi}{18} + sin^2\frac{\pi}{9} + sin^2\frac{7\pi}{18} + sin^2\frac{4\pi}{9} = 2.$ $(\boldsymbol{Ex.3.3 - 6})$

34.Prove that $\sin x = 2^{10} \sin\left(\frac{x}{2^{10}}\right) \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{2^2}\right) \ldots\ldots \cos\left(\frac{x}{2^{10}}\right)$ **(Eg. 3.25)**

35.Prove that $32(\sqrt{3}) \sin\frac{\pi}{48} \cos\frac{\pi}{48} \cos\frac{\pi}{24} \cos\frac{\pi}{12} \cos\frac{\pi}{6} = 3.$ $(\boldsymbol{Ex, 3.5 - 11})$

36.If ${}^{10}P_{r-1} = 2 \times {}^{6}P_r$ find r. $(\boldsymbol{Ex.4.2 - 2})$

37.Expand $\left(2x^2 - 3\sqrt{1 - x^2}\right)^4 + \left(2x^2 - 3\sqrt{1 - x^2}\right)^4$ $(\boldsymbol{Ex.5.1 - 1(ii)})$

38.If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ then show that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \cdots$ $(\boldsymbol{Ex.5.4 - 7})$

39.Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form: (i) slope and intercept form (ii) intercept form (iii) Normal form. $(\boldsymbol{Eg.6.19})$

40.The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$. $(\boldsymbol{Ex.6.4 - 8})$

PART – IV (Marks: 35)

IV. Answer all the questions. **7×5=35**

41. If $A \times A$ has 16 elements, $S = \{(a,b) \in A \times A : a < b\}$; $(-1,2)$ and $(0,1)$ are two elements of S, then find the remaining elements of S. $(\boldsymbol{Ex.1.1-10})$ **(OR)**

    Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse. $(\boldsymbol{Eg.1.30})$

42. A manufacturer has 600 liters of a 12 percent solution of acid. How many liters of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent? $(\boldsymbol{Ex.2.3-6})$ **(OR)**

    Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$ $(\mathbf{Ex.2.8-2})$

43. Prove that $\log_{10} 2 + 16 \log_{10} \frac{16}{15} + 12 \log_{10} \frac{25}{24} + 7 \log_{10} \frac{81}{80} = 1$

    $(\mathbf{Ex.2.12-7})$ **(OR)**

    If $x = \sum_{n=0}^{\infty} cos^{2n}\theta,\ y = \sum_{n=0}^{\infty} sin^{2n}\theta$ and $z = \sum_{n=0}^{\infty} cos^{2n}\theta\, sin^{2n}\theta,\ 0 < \theta < \frac{\pi}{2}$

    then show that $xyz = x + y + z$.

    $(\boldsymbol{hint}: use\ the\ formula\ 1 + x + x^2 + \cdots = \frac{1}{1-x}, where\ |x| < 1)$ **(Ex. 3.1-7)**

44. Show that $\cos\frac{\pi}{15} \cos\frac{2\pi}{15} \cos\frac{3\pi}{15} \cos\frac{4\pi}{15} \cos\frac{5\pi}{15} \cos\frac{6\pi}{15} \cos\frac{7\pi}{15} = \frac{1}{128}$ **(Ex. 3.6-4). (OR)**

    Prove that ${}^{24}C_4 + \sum_{r=0}^{4} {}^{(28-r)}C_3 = {}^{29}C_4$ $(\boldsymbol{Eg.4.48})$

45. prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$. $(\boldsymbol{Eg.4.66})$ **(OR)**

    If the binomial coefficients of three consecutive terms in the expansion of $(a + x)^n$ are in the ratio 1: 7: 42, then find n. $(\boldsymbol{Ex.5.1-14})$

46. prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

    $(\boldsymbol{Eg.5.25})$ **(OR)**

    Find the value of

$$\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{1}{9^{n-1}} + \frac{1}{9^{2n-1}}\right). \quad (\boldsymbol{Ex.5.4-10})$$

47. Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form. $(\boldsymbol{Eg.6.20})$ **(OR)**

    Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them. **(Ex. 6.4-14)**