COMMON QUARTERLY EXAMINATION - 2024	Reg. No.						

XI - MATHEMATICS

Time Allowed : 3.00 Hrs. *Maximum Marks: 90*

Part - I

I. Choose the correct answer: **20 x 1 = 20**

1. Let A and B be subsets of the universal set N, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is
   a) A b) A' c) B d) N
2. The number of relations on a set containing 3 elements is
   a) 9 b) 81 c) 512 d) 1024
3. The range of the function $f(x) = |\lfloor x \rfloor - x|, x \in \mathbb{R}$ is
   a) [0, 1] b) [0, ∞] c) [0, 1) d) (0, 1)
4. If $|x + 2| \le 9$, then x belongs to
   a) $(-\infty, -7)$ b) $[-11, 7]$ c) $(-\infty, -7) \cup [11, \infty)$ d) $(-11, 7)$
5. The number of solutions of $x^2 + |x - 1| = 1$ is
   a) 1 b) 0 c) 2 d) 3
6. If $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$, then the value of k is
   a) 1 b) 2 c) 3 d) 4
7. If $\cos 28^\circ + \sin 28^\circ = k^3$, then $\cos 17^\circ$ is equal to
   a) $\frac{k^3}{\sqrt{2}}$ b) $-\frac{k^3}{\sqrt{2}}$ c) $\pm\frac{k^3}{\sqrt{2}}$ d) $-\frac{k^3}{\sqrt{3}}$
8. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots + \cos 179^\circ =$
   a) 0 b) 1 c) –1 d) 89
9. If $\sin\alpha + \cos\alpha = b$, then $\sin 2\alpha$ is equal to
   a) $b^2 - 1$, if $b \le \sqrt{2}$ b) $b^2 - 1$, if $b > \sqrt{2}$ c) $b^2 - 1$, if $b \ge 1$ d) $b^2 - 1$, if $b \ge \sqrt{2}$
10. The number of 5 digit numbers all digits of which are odd is
    a) 25 b) 5^5 c) 5^6 d) 625
11. The number of ways in which the following prize be given to a class of 30 boys first and second in mathematics, first and second in physics, first in chemistry and first in English is
    a) $30^4 \times 29^2$ b) $30^3 \times 29^3$ c) $30^2 \times 29^4$ d) 30×29^5
12. The number of rectangles that a chessboard has _______.
    a) 81 b) 9^9 c) 1296 d) 6561
13. The value of 2 + 4 + 6 + …, + 2n is
    a) $\frac{n(n-1)}{2}$ b) $\frac{n(n+1)}{2}$ c) $\frac{2n(2n+1)}{2}$ d) $n(n + 1)$
14. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}$ … form an
    a) AP b) GP c) HP d) AGP

15. The sum up to n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+......$ is

a) $\frac{n(n+1)}{2}$ b) $2n(n + 1)$ c) $\frac{n(n+1)}{\sqrt{2}}$ d) 1

16. Which of the following equation is the locus of $(at^2, 2at)$

a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ c) $x^2 + y^2 = a^2$ d) $y^2 = 4ax$

17. The equation of the line with slope 2 and the length of the perpendicular from the origin equal to $\sqrt{5}$ is

a) $x-2y=\sqrt{5}$ b) $2x-y=\sqrt{5}$ c) $2x - y = 5$ d) $x - 2y - 5 = 0$

18. The area of the triangle formed by the lines $x^2 - 4y^2 = 0$ and $x = a$ is

a) $2a^2$ b) $\frac{\sqrt{3}}{2}a^2$ c) $\frac{1}{2}a^2$ d) $\frac{2}{\sqrt{3}}a^2$

19. The function $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x + \cos x$ is

a) an odd function b) neither an odd function nor an even function

c) an even function d) both odd function and even function

20. The product of first n odd natural numbers equals

a) ${}^{2n}C_n \times {}^nP_n$ b) $\left(\frac{1}{2}\right)^n \times {}^{2n}C_n \times {}^nP_n$ c) $\left(\frac{1}{4}\right)^n \times {}^{2n}C_n \times {}^{2n}P_n$ d) ${}^nC_n \times {}^nP_n$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n(P(A \Delta B))$.

22. Solve : $3x^2 + 5x - 2 \leq 0$.

23. Find the square root of $7-4\sqrt{3}$.

24. Show that $\tan(45^\circ + A)=\frac{1+\tan A}{1-\tan A}$

25. If $\frac{1}{7!}+\frac{1}{8!}=\frac{A}{9!}$ then find the value of A.

26. Prove that ${}^{10}C_2 + 2 \times {}^{10}C_3 + {}^{10}C_4 = {}^{12}C_4$

27. Find the coefficient of x^{15} in $\left(x^2+\frac{1}{x^3}\right)^{10}$

28. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

29. Find the domain of $\frac{1}{1-2\sin x}$

30. Find the distinct permutations of the letters of the word MISSISSIPPI?

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. In the set Z of integers, define mRn if m – n is divisible by 7. Prove that R is an equivalence relation.

32. Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$

33. Show that $\cot\left(7\frac{1}{2}^{\circ}\right) = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

34. If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words (i) GARDEN (ii) DANGER.

35. Find the sum of the first 20–terms of the arithmetic progression having the sum of first 10 terms as 52 and the sum of the first 15 terms as 77.

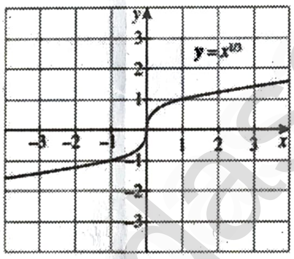
36. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.

37. If one root of $k(x-1)^2 = 5x-7$ is double the other root, show that $k = 2$ or -25.

38. If $\theta+\varphi=\alpha$ and $\tan\theta = k\tan\varphi$, then prove that $\sin(\theta-\phi) = \frac{k-1}{k+1}\sin\alpha$

39. If a, b,c are in geometric progression, and if $a^{1/x} = b^{1/y} = c^{1/z}$, then prove that x,y, z are in arithmetic progression.

40. From the curve $y = x^{(1/3)}$, draw



i) $y = -x^{(1/3)}$ ii) $y = x^{(1/3)} + 1$ iii) $y = x^{(1/3)} - 1$ iv) $y = (x+1)^{(1/3)}$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Write the values of f at – 4, 1,– 2, 7, 0 if

$$f(x) = \begin{cases} -x+4 & if\ -\infty < x \le -3 \\ x+4 & if\ -3 < x < -2 \\ x^2 - x & if\ -2 \le x < 1 \\ x - x^2 & if\ 1 \le x < 7 \\ 0 & otherwise \end{cases}$$

(OR)

b) If $f: R \to R$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.

42. a) Solve : $\frac{x^2-4}{x^2-2x-15} \le 0$

(OR)

b) Simplify : $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

43. a) If $\tan\frac{\theta}{2} = \sqrt{\frac{1-a}{1+a}}\tan\frac{\phi}{2}$, then prove that $\cos\phi = \frac{\cos\theta - a}{1 - a\cos\theta}$

(OR)

b) State and prove Napier's formula

44. a) Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans, so that always Indians will be the majority in the committee.

(OR)

b) By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^2 + 3^2 + 5^2 + \dots\dots\dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

45. a) Compute the sum of first n terms of the series: 6 + 66 + 666 + 6666 + ...

(OR)

b) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

46. a) For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight lines.

(OR)

b) Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.

47. a) Prove: $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$

(OR)

b) How many strings of length 6 can be formed using letters of the word FLOWER if

(i) either starts with F or ends with R?

(ii) neither starts with F nor ends with R?