CHENGALPATTU - 603002

COMMON QUARTERLY EXAMINATION - 2024

Standard XI

Reg.No. | | | | | | |

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MATHEMATICS

Time : 3.00 hrs *Marks : 90*

Part - I

I. Choose the correct answer: **20 x 1 = 20**

1. The rule $f(x) = x^2$ is a bijection if the domain and the co-domain are given by
   a) R, R b) R, $(0, \infty)$ c) $(0, \infty)$, R d) $(0, \infty)$, $[0, \infty)$
2. Let R be the universal relation on a set x with more than one element. Then R is
   a) not reflexive b) not symmetric c) transitive d) none of the above
3. If $n(A) = 2$ and $n(B \cup C) = 3$, then $n[(A \times B) \cup (A \times C)]$ is
   a) 2^3 b) 3^2 c) 6 d) 5
4. The solution of $5x - 1 < 24$ and $5x + 1 > -24$ is
   a) (4,5) b) (–5,–4) c) (–5,5) d) (–5,4)
5. If a and b are the real roots of the equation $x^2 - kx + c = 0$, then the distance between the points (a,0) and (b,0) is
   a) $\sqrt{k^2 - 4c}$ b) $\sqrt{4k^2 - c}$ c) $\sqrt{4c - k^2}$ d) $\sqrt{k - 8c}$
6. The value of $\log_a b \log_b c \log_c a$ is
   a) 2 b) 1 c) 3 d) 4
7. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is
   a) $4 + \sqrt{2}$ b) $3 + \sqrt{2}$ c) 9 d) 4
8. Which of the following is not true?
   a) $\sin\theta = -\frac{3}{4}$ b) $\cos\theta = 25$ c) $\tan\theta = \frac{1}{4}$ d) $\sec\theta = -1$
9. If $f(\theta) = |\sin\theta| + |\cos\theta|$, $\theta \in R$, then $f(\theta)$ is in the interval
   a) [0,2] b) $[1, \sqrt{2}]$ c) [1,2] d) [0,1]
10. A wheel is spinning at 2 radian / second. How many seconds will it take to make 10 complete rotations?
    a) 10π seconds b) 20π seconds c) 5π seconds d) 15π seconds
11. In 3 fingers, the number of ways four rings can be worn is _____ ways.
    a) $4^3 - 1$ b) 3^4 c) 68 d) 64
12. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points
    a) 45 b) 40 c) 39 d) 38

13. Number of sides of a polygon having 44 diagonals is
    a) 4 b) 4 ! c) 11 d) 22
14. $1 + 3 + 5 + 7 + \ldots\ldots + 19$ is equal to
    a) 100 b) 81 c) 71 d) 61
15. If a is the arithmetic mean and g is the geometric mean of two numbers, then
    a) $a \leq g$ b) $a \geq g$ c) $a = g$ d) $a > g$
16. The remainder when 38^{15} is divided by 13 is
    a) 12 b) 1 c) 11 d) 5
17. The sum of an infinite G.P is 18. If the first term is 6, the common ratio is
    a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{3}{4}$
18. Which of the following point lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$
    a) (0,0) b) (–2,3) c) (1,2) d) (0,–1)
19. The image of the point (2,3) in the line $y = -x$ is
    a) (–3,–2) b) (–3,2) c) (–2,–3) d) (3,2)
20. If a vertex of a square is at the origin and its one side lies along the line $4x + 3y - 20 = 0$, then the area of the square is
    a) 20 sq.units b) 16 sq.units c) 25 sq.units d) 4 sq.units

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. Let A = {a,b,c}. What is the equivalence relation of smallest cardinality on A? What is the equivalence relation of largest cardinality on A?
22. Solve : $|5x - 12| < -2$
23. Evaluate : $\left(\left((256)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right)^3$
24. Find the value of $\sin(765^\circ)$
25. Show that $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}$
26. If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$, then find the value of A.
27. Evaluate the following : i) $10C_3$ ii) $100C_{99}$
28. Find the middle term in the expansion of $(x + y)^6$
29. Write the first 6 terms of the exponential series e^{-2x}

30. Find the perpendicular distance from the origin to the line $x + y = 1$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Let $f, g : R \to R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. Find fog.

32. Resolve into partial fractions : $\frac{x}{(x+3)(x-4)}$

33. If α and β are the roots of the quadratic equation $x^2 + \sqrt{2}x + 3 = 0$, form a quadratic polynomial with zeros $\frac{1}{\alpha}, \frac{1}{\beta}$.

34 Show that $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{1}{\sqrt{3}}$

35. Find the distinct permutations of the letters of the word MISSISSIPPI.

36. Prove that $10C_2 + 2(10C_3) + 10C_4 = 12C_4$

37. Write the first 6 terms of the sequences whose n^{th} term is $a_n = \begin{cases} 1 & , \text{if } n = 1 \\ 2 & , \text{if } n = 2 \\ a_{n-1} + a_{n-2} & , \text{if } n > 2 \end{cases}$

38. Compute the sum of first n terms of the following series :
$6 + 66 + 666 + 6666 + ...$

39. Show the points $\left(0, \frac{-3}{2}\right)$, $(1, -1)$ and $\left(2, \frac{-1}{2}\right)$ are collinear.

40. Find the value of $\tan^{-1}\sqrt{3} + \cos^{-1}\frac{\sqrt{3}}{2}$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Write the values of f at –4, 1, –2, 7, 0 if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

(OR)

b) If $A + B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$

42. a) If ${}^{(n+2)}C_7 : {}^{(n-1)}P_4 = 13 : 24$, find n.

(OR)

b) Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$

43. a) The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of other, show that $8h^2 = 9ab$.

(OR)

b) By the principle of mathematical induction, prove that, for all integers $n \geq 1$

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

44. a) If A x A has 16 elements $S = \{(a,b) \in A \times A : a < b\}$, (–1,2) and (0,1) are two elements of S, then find the remaining elements of S.

(OR)

b) If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, then find the value of λ and the separate equations of the lines.

45. a) If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.

(OR)

b) Find the equations of parallel line and perpendicular line passing through the point (1,2) to the line 3x + 4y = 7

46. a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

(OR)

b) If A + B + C = 180°, prove that sin2A + sin2B + sin2C = 4 sinA sinB sinC

47. a) Find the co-efficient of x^2 and the co-efficient of x^6 in $\left(x^2 - \frac{1}{x^3}\right)^6$

(OR)

b) Find the largest possible domain for the real valued function given by

$$f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$$

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ST. JOSEPH'S HR. SEC. SCHOOL - CHENGALPATTU

COMMON QUARTERLY EXAMINATION - 2024

STD - 11 MATHEMATICS - KEY MARKS: 90

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23.9.24

PART-I

1	d	$(0,\infty), [0,\infty)$
2	c	transitive
3	c	6
4	c	$(-5,5)$
5	a	$\sqrt{k^2-4c}$
6	b	1
7	a	$4+\sqrt{2}$
8	d	$\sec\theta = -1$
9	b	$[1,\sqrt{2}]$
10	a	10π seconds
11	b	3^4
12	b	40
13	c	11
14	a	100
15	b	$a \geq g$
16	a	12
17	b	$2/3$
18	c	$(1,2)$
19	a	$(-3,-2)$
20.	b	16 sq. units

PART-II

21. $R_1 = \{(a,a)(b,b)(c,c)\}$, $n(R_1)=3$

$R_2 = \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$, $n(R_2)=9$

22. $-2 < 5x-2 < 2$

$0 < 5x < 0$

$0 < x < 0$ no solution

23. $= (256)^{-1/2 \times -1/4 \times 3} = (256)^{3/8} = (2^8)^{3/8} = 2^3 = 8$

24. $\sin(2\times360^\circ+45^\circ) = \sin 45^\circ = 1/\sqrt{2}$

25. $\tan(45^\circ+A) = \frac{\tan45^\circ+\tan A}{1-\tan45^\circ\tan A} = \frac{1+\tan A}{1-\tan A}$

26. $\frac{A}{9\times8\times7!} = \frac{1}{7!} + \frac{1}{8\times7!} \Rightarrow A = 81$

27. $^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 120$, (ii) $^{100}C_{99} = 100$

28. $^6C_3 x^3y^3 = 20x^3y^3$

29. $e^{-2x} = 1 - \frac{2x}{1!} + \frac{(2x)^2}{2!} - \frac{(2x)^3}{3!} + \cdots$

$= 1 - 2x + 2x^2 - \frac{4x^3}{3} + \cdots$

30. $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right| = \left|\frac{0+0-1}{\sqrt{1^2+1^2}}\right| = \frac{1}{\sqrt{2}}$

PART-III

31. $f(x) = \begin{cases} 3x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$, $g(x) = \begin{cases} x & \text{if } x \leq 0 \\ 3x & \text{if } x > 0 \end{cases}$

$x \leq 0$, $fog(x) = 3x$

$x > 0$, $fog(x) = 3x$

32. $= \frac{A}{x+3} + \frac{B}{x-4} \Rightarrow x = A(x-4) + B(x+3)$

$x=4 \Rightarrow B = 4/7$, $x=-3 \Rightarrow A = 3/7$

$= \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$

33. $\alpha+\beta = -\sqrt{2}$, $\alpha\beta = 3$

Sum $= \frac{1}{\alpha}+\frac{1}{\beta} = \frac{-\sqrt{2}}{3}$

Product $= \frac{1}{\alpha}\cdot\frac{1}{\beta} = 1/3$

$x^2 - x\left(\frac{-\sqrt{2}}{3}\right) + 1/3 = 0$

$\Rightarrow 3x^2 + \sqrt{2}x + 1 = 0$

34. $= \frac{2\cos\left(\frac{75+15}{2}\right)\sin\left(\frac{75-15}{2}\right)}{2\cos\left(\frac{75+15}{2}\right)\cos\left(\frac{75-15}{2}\right)} = \frac{2\cos45\sin30}{2\cos45\cos30^\circ}$

$= \tan30^\circ = 1/\sqrt{3}$

35. $= \frac{11!}{4!4!2!} = 34650$

36. $= {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_3 + {}^{10}C_4$

$= ({}^{10}C_2 + {}^{10}C_3) + ({}^{10}C_3 + {}^{10}C_4)$

$= {}^{11}C_3 + {}^{11}C_4$

$= {}^{12}C_4$

37. $a_1 = 1$, $a_2 = 2$

$a_3 = 3$, $a_4 = 5$, $a_5 = 8$, $a_6 = 13$

38. $S = 6(1 + 11 + 111 + \cdots)$ n terms

$= \frac{6}{9}(9 + 99 + 999 + \cdots)$

$= \frac{6}{9}[(10 + 10^2 + 10^3 + \cdots) - (1+1+1+\cdots)]$

$= \frac{6}{9}\left[\frac{10(10^n-1)}{9} - n\right] = \frac{6}{81}[10(10^n-1) - 9n]$

39. slope of AB $= \frac{-1+3/2}{1-0} = 1/2$

slope of BC $= \frac{-1/2+1}{2-1} = 1/2$

Slope AB = Slope BC

A, B, C lying on the same line

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40

$= \tan^{-1}\sqrt{3} + \cos^{-1}\frac{\sqrt{3}}{2}$

$= \pi/3 + \pi/6 = \frac{3\pi}{6} = \pi/2$

PART-IV

41. a.

$f(-4) = 8$, $f(1) = 0$

$f(-2) = 6$ $f(0) = 0$

$f(7) = 0$

b.

$A+B = 45^\circ \Rightarrow \tan(A+B) = \tan 45^\circ$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\tan A + \tan B + \tan A \tan B = 1$

$1 + \tan A + \tan B + \tan A \tan B = 2$

$(1+\tan A)(1+\tan B) = 2$

42 a.

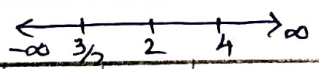
$\frac{(n+2)C_7}{(n-1)P_4} = \frac{13}{24}$

$\frac{(n+2)!}{(n-5)!\times 7!} \times \frac{(n-5)!}{(n-1)!} = \frac{13}{24}$

$(n+2)(n+1)n = \frac{13}{24} \times 7!$

$(n+2)(n+1)n = 13\times 14\times 15 \Rightarrow \boxed{n = 13}$

b.



Intervals	$2x-3$	$x-2$	$x-4$	$\frac{2x-3}{(x-2)(x-4)}$
$(-\infty, 3/2)$	−	−	−	−
$(3/2, 2)$	+	−	−	+
$(2,4)$	+	+	−	−
$(4,\infty)$	+	+	+	+

$\frac{2x-3}{(x-2)(x-4)} < 0$ in $(-\infty, 3/2)$ and $(2,4)$

43 a.

$m_1 + m_2 = \frac{-2h}{b}$, $m_1 m_2 = a/b$

$m + 2m = -2h/b$, $m(2m) = a/b$

$m = -2h/b$, $2m^2 = a/b$

$2\left(\frac{-2h}{3b}\right)^2 = a/b \Rightarrow 2\left(\frac{4h^2}{9b^2}\right) = a/b$

$\boxed{8h^2 = 9ab}$

b

$P(n) = 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$n=1$ LHS = 1, RHS = 1 LHS = RHS

$P(1)$ is true

$n=k$ $P(k) = 1^2+2^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$

$P(k)$ is true.

$n=k+1$, ST. $P(k+1)$ is true.

$P(k+1) = 1^2+2^2+3^2+\cdots+k^2+(k+1)^2$

$P(k+1) = \frac{(k+1)(k+2)(2k+3)}{6}$, $\forall\ n \geq 1$

$P(k+1)$ is true,

44 a.

$n(A\times A) = 16$, $n(A) = 4$, $A = \{-1, 0, 1, 2\}$

$A\times A = \{(-1,-1)(-1,0)(-1,1)(-1,2)(0,-1)(0,0)(0,1)(0,2)(1,-1)(1,0)(1,1)(1,2)(2,-1)(2,0)(2,1)(2,2)\}$

$S = \{(-1,0)(-1,1)(-1,2)(0,1)(0,2)(1,2)\}$

Remaining elements: $(-1,0)(-1,1)(0,2)(1,2)$

b.

$a=\lambda$, $b=12$, $c=-3$, $h=-5$, $g=5/2$, $f=-8$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$-36\lambda + 200 - 64\lambda - 75 + 75 = 0 \Rightarrow \boxed{\lambda = 2}$

$2x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

$2x^2 - 10xy + 12y^2 + 5x - 16y - 3 \equiv (x-2y+c_1)(2x-6y+c_2)$

$c_1 = 3$, $c_2 = -1$

$x - 2y + 3 = 0$, $2x - 6y - 1 = 0$

45 a.

$\frac{1}{\log_x 2} + \frac{1}{\log_x 4} + \frac{1}{\log_x 16} = 7/2$

$\frac{1}{a} + \frac{1}{2a} + \frac{1}{4a} = \frac{7}{2} \Rightarrow a = \log_x 2 \Rightarrow \frac{7}{4a} = \frac{7}{2}$

$a = 1/2 \Rightarrow \log_x 2 = 1/2$, $x^{1/2} = 2 \Rightarrow \boxed{x = 2^2 = 4}$

b.

$3x+4y = 7$, parallel line $3(1)+4(2)+k = 0 \Rightarrow k = -11$

$\boxed{3x+4y = 11}$

perpendicular line $4x - 3y + k = 0 \Rightarrow$

$4(1) - 3(2) + k = 0 \Rightarrow k = 2$

$\boxed{4x - 3y = -2}$

46 a.

$\sqrt[3]{x^3+7} = (x^3+7)^{1/3} = x\left(1+\frac{7}{x^3}\right)^{1/3}$

$= x + \frac{7}{3}\times\frac{1}{x^2} - \frac{49}{9}\times\frac{1}{x^5} + \cdots$

$\sqrt[3]{x^3+4} = (x^3+4)^{1/3} = x\left(1+\frac{4}{x^3}\right)^{1/3}$

$= x + \frac{4}{3}\times\frac{1}{x^2} - \frac{16}{9}\times\frac{1}{x^5} + \cdots$

$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = \left(x + \frac{7}{3}\times\frac{1}{x^2}\right) - \left(x + \frac{4}{3}\times\frac{1}{x^2}\right) = \frac{1}{x^2}$

b.

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cos C$

$= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$

$= 2\sin(180-C)\cos(A-B) + 2\sin C\cos C$

$= 2\sin C[\cos(A-B) - \cos(A+B)]$

$= 4\sin A\sin B\sin C$

47 a.

$T_{r+1} = {}^6C_r (x^2)^{6-r}\left(-\frac{1}{x^3}\right)^r = (-1)^r\, {}^6C_r\, x^{12-5r}$

w.e.g. x^6 $r = 5/6$ not possible.

w.e.g. x^2 $\Rightarrow 12 - 5r = 2 \Rightarrow \boxed{r = 2}$

$T_3 = (-1)^2\, {}^6C_2\, x^{12-10} = 15x^2$,

co-eff. of x^2 is 15

b.

$x < -3$ or $x > 3$, x^2 greater than 9 and hence $9 - x^2$ -ve no square roots in $\mathbb{R}$

x must lie on the interval $[-3, 3]$

$x \geq -1$, $x \leq 1$, $x^2 - 1$, -ve or zero.

no square roots in $\mathbb{R}$. f is not defined.

x must outside $[-1, 1]$, lie on $(-\infty, -1) \cup (1, \infty)$

domain of f $[-3,3] \cap ((-\infty,-1) \cup (1,\infty))$, $\therefore [-3,-1) \cup (1,3]$