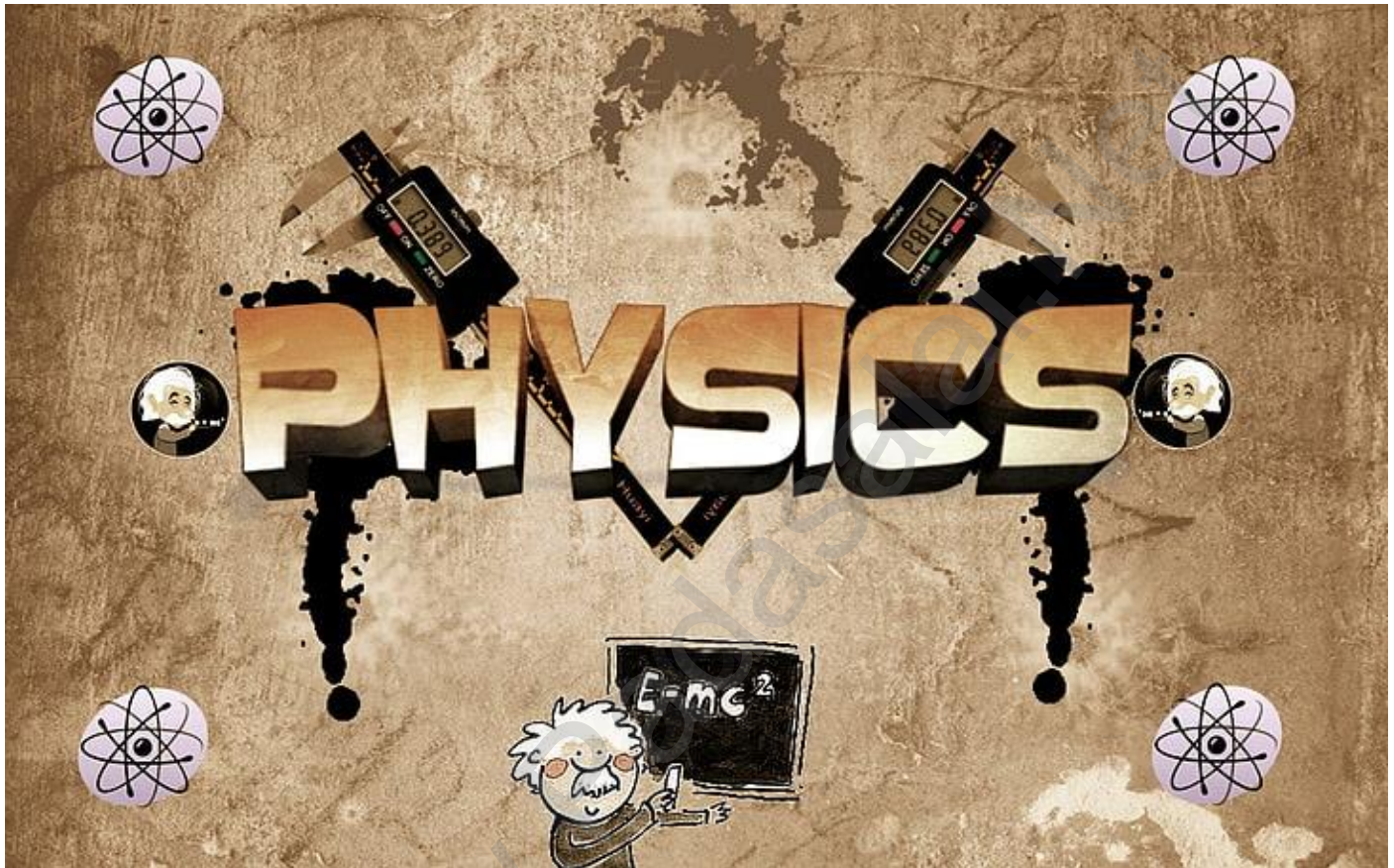


# **XI STANDARD PRIORITISED SYLLABUS BOOK BACK QUESTIONS E.M. - EASY GUIDE**



**BY**

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*XI STANDARD - PHYSICS*

*VERY VERY IMPORTANT QUESTIONS AND ANSWERS ( BOOK QUESTIONS )*

**UNIT: 1 NATURE OF PHYSICAL WORLD AND MEASUREMENT**

**SHORT ANSWER QUESTIONS**

1. What is a physical quantity? Briefly explain the types.

Quantities that can be measured, and in terms of which, laws of physics are described.

Two types. Fundamental quantities      Derived quantities

The quantities which cannot be expressed in terms of any other physical quantities are fundamental quantities. Eg. Length, mass, time, electric current, temperature, luminous intensity and amount of substance.

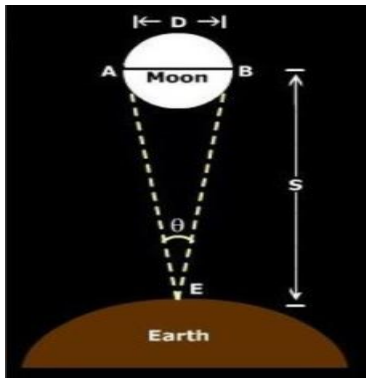
The quantities that can be expressed in terms of fundamental quantities are derived quantities.

Eg. Area, volume, velocity, acceleration etc.,

2. How will you measure the diameter of the Moon using parallax method?

Let  $\theta$  is the angular diameter of the moon.

$S$  – is the distance of the moon from the Earth



From the figure,

$$\theta = \frac{D}{S} \quad \text{So, diameter of moon } D = S \theta$$

By knowing  $\theta$  and  $S$ , the diameter of moon can be calculated.

3. Write the rules for determining significant figures.

1. All non zero digits are significant. Eg. 1342 has *four* significant figures.
2. All zeros between two non zero digits are significant. Eg. 2008 has *four* significant figures.
3. The trailing zeros in a number with a decimal point are significant  
Eg. 40.00 has *four* significant figures
4. The trailing zeros in a number without a decimal point are not significant  
Eg. 30700 has *three* significant figures
5. All zeros are significant if they come from a measurement  
Eg. 30700 m has *five* significant figures
6. The number of significant figures does not depend on the system of units used  
Eg. 1.53 cm      0.0153 m      0.0000153 km all have *three* significant figures

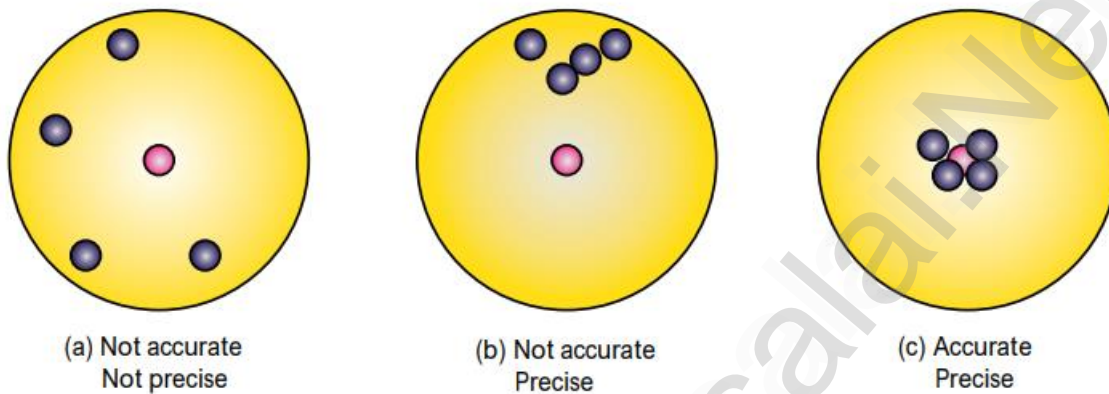
4. What are the limitations of dimensional analysis?

1. It cannot be applied to an equation involving more than three physical quantities.

2. It can only check on whether a physical relation is dimensionally correct but not the correctness of the relation. Eg.  $s = ut + 1/3 at^2$  is dimensionally correct but the correct relation is  $s = ut + 1/2 at^2$
3. It is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.
4. It cannot decide whether the given quantity is a vector or a scalar.
5. It gives no information about the dimensionless constants in the formula like 1,2,...  $\pi$ , e (Euler number) etc.,

5. Define precision and accuracy. Explain with one example.

Accuracy refers to how far we are from the true value and precision refers to how well we measure.



### LONG ANSWER QUESTIONS

1. ( i ) Explain the use of screw gauge and vernier caliper in measuring smaller distances:

Screw gauge:

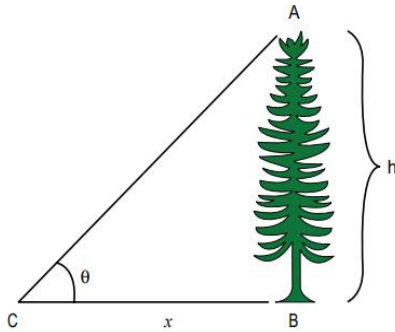
1. Principle: The magnification of linear motion using the circular motion of a screw.
2. It is an instrument used for measuring accurately the dimensions of objects

Vernier Caliper:

1. It is a versatile instrument for measuring the dimensions of an object namely diameter of a hole, or a depth of a hole.
2. The least count is 0.01 cm.

( ii ) ( a ) Triangulation method to measure larger distances:

1. Let  $AB = h$  be the height of the tree or tower to be measured.
2. Let  $C$  be the point of observation and let  $BC = x$
3. From the diagram, the elevation angle  $\angle ACB = \theta$



From right angled triangle ABC,

$$\tan \theta = \frac{AB}{BC} = \frac{h}{x} \quad \text{or height } h = x \tan \theta$$

Knowing  $x$ , the height  $h$  can be determined.

(b) From a point on the ground, the top of a tree is seen to have an angle of elevation  $60^\circ$ . The distance between the tree and a point is 50 m. Calculate the height of the tree.

$$h = x \tan \theta \quad h = 50 x \tan 60^\circ = 50 x 1.732 = 86.6 \text{ m}$$

(iii) Radar method to measure larger distances:

1. RADAR stands for Radio Detection and Ranging
2. To measure accurately the distance of a nearby planet such as Mars.
3. Radio waves are sent from transmitters.
4. After reflection from the planet, they are detected by the receiver.
5. If 't' is the time interval between transmission and reception, then  
Speed = distance travelled / time taken or distance  $d = \text{speed} \times \text{time taken}$   
 $d = v \times t / 2$

The time  $t$  for the distance covered during forward and backward path, it is divided by 2.

2. Explain in detail the various types of errors.

**Error:** The uncertainty in a measurement is called an error.

**Types:** Systematic error, Random error and Gross error.

1. **Systematic errors:** Reproducible inaccuracies that are consistently in the same direction.

**Types:** 1. **Instrumental errors:** Due to an instrument which is not calibrated properly at the Time of manufacture. Measurement with a meter scale whose end is worn out.

2. **Imperfections in procedure:** Due to the limitations in the experimental arrangement.

Using calorimeter with no proper insulation, there will be radiation losses.

3. **Personal errors:** Due to incorrect initial setting up of the experiment or carelessness of the individual

4. **Errors due to external causes :** Due to change in the external conditions during an experiment  
Like changes in temperature, humidity or pressure etc.,

5. **Least count error:** The smallest value that can be measured by the measuring instrument.  
The error due to this measurement is least count error.

2. **Random errors:** Due to random and unpredictable variations in experimental conditions.  
May also be due to personal errors by the observer who performs the experiment.  
They are sometimes called 'chance error'.

3. Gross errors:
1. Reading an instrument without setting it properly.
  2. Without bothering about the sources of errors and the precautions
  3. Recording wrong observations
  4. Using wrong values of the observations in calculations.

To minimise :

1. By choosing the instrument carefully.

2. By applying necessary corrections. 3. By using high precision instrument

4. When an observer is careful and mentally alert.

3. What do you mean by propagation of errors? Explain the propagation of errors in addition and multiplication.

Different types of instruments might have been used for taking readings and a number of Measured quantities may be involved in the final calculation of an experiment.

The error in the final result depends on:

1. The errors in the individual measurements.

2. Nature of the mathematical operations performed.

The various possibilities of the combination of errors:

Error in the sum of two quantities:

Let  $\Delta A$  and  $\Delta B$  be the absolute errors in the two quantities A and B respectively.

Measured value of A =  $A \pm \Delta A$

Measured value of B =  $B \pm \Delta B$

Sum  $Z = A + B$

The error  $\Delta Z$  in Z is given by  $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$$= (A + B) \pm (\Delta A + \Delta B)$$

$$Z \pm \Delta Z = Z \pm (\Delta A + \Delta B)$$

$$\text{or } \Delta Z = \Delta A + \Delta B$$

Maximum possible error in the sum = sum of absolute errors in the individual quantities.

- (ii) Error in the product or multiplication of two quantities:

Let  $\Delta A$  and  $\Delta B$  be the absolute errors in the two quantities A and B respectively.

Consider the product  $Z = A B$

The error  $\Delta Z$  in Z is given by  $Z \pm \Delta Z = (A \pm \Delta A) (B \pm \Delta B)$

$$= (A B) \pm (A \Delta B) \pm (B \Delta A) \pm (\Delta A \cdot \Delta B)$$

Dividing LHS by Z and RHS by AB,  $1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$

$\Delta A / A$ ,  $\Delta B / B$  are both small quantities, their product term  $\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$  can be neglected.

So maximum fractional error in Z is  $\frac{\Delta Z}{Z} = \pm \left( \frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$

The maximum fractional error in the product of two quantities = the sum of the fractional

errors in the individual quantities.

4. Write short notes on the following:

1. **Unit** - An arbitrarily chosen standard of measurement of a quantity which is accepted internationally
2. **Rounding off**: Calculators are widely used now-a-days to do calculations. The result given by a Calculator has too many figures. In no case should the result have more significant figures than The figures involved in the data used for calculation. The result of calculation with numbers containing more than one uncertain digit should be rounded off.

3. **Dimensionless quantities**:

- (i) **Dimensionless variables** : Physical quantities which have no dimensions, but have variable Values. Eg. Specific gravity, strain, refractive index etc.,
- (ii) **Dimensionless constants**: Quantities which have constant values but have no dimensions. Eg.  $\pi$ ,  $e$  ( Euler's number ) , numbers etc.,

5. Explain the principle of homogeneity of dimensions. What are its uses? Give example.

The dimensions of all the terms in a physical expression should be the same.

Uses: 1. Check the dimensional correctness of a given physical equation

Example:  $v = u + at$                        $[L T^{-1}] = [L T^{-1}] + [L T^{-2}] [T]$

$[L T^{-1}] = [L T^{-1}] + [L T^{-1}]$  Dimensions of both sides are the same. Hence correct.

2. To convert a physical quantity from one system of units to another

Example:

If the value of universal gravitational constant in SI is  $6.6 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ , then find its value in CGS System?

$$\begin{aligned} M_1 &= 1 \text{ kg} & L_1 &= 1 \text{ m} & T_1 &= 1 \text{ s} \\ M_2 &= 1 \text{ g} & L_2 &= 1 \text{ cm} & T_2 &= 1 \text{ s} \end{aligned}$$

**Solution**

Let  $G_{\text{SI}}$  be the gravitational constant in the SI system and  $G_{\text{CGS}}$  in the cgs system. Then

The dimensional formula for  $G$  is  $M^{-1} L^3 T^{-2}$

$$G_{\text{SI}} = 6.6 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

$$G_{\text{CGS}} = ?$$

$$n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^a \left[ \frac{L_1}{L_2} \right]^b \left[ \frac{T_1}{T_2} \right]^c$$

$$G_{\text{CGS}} = G_{\text{SI}} \left[ \frac{M_1}{M_2} \right]^a \left[ \frac{L_1}{L_2} \right]^b \left[ \frac{T_1}{T_2} \right]^c$$

$$a = -1 \quad b = 3 \quad \text{and} \quad c = -2$$

$$G_{\text{CGS}} = 6.6 \times 10^{-11} \left[ \frac{1 \text{ kg}}{1 \text{ g}} \right]^{-1} \left[ \frac{1 \text{ m}}{1 \text{ cm}} \right]^3 \left[ \frac{1 \text{ s}}{1 \text{ s}} \right]^{-2}$$

$$= 6.6 \times 10^{-11} \left[ \frac{1 \text{ kg}}{10^{-3} \text{ kg}} \right]^{-1} \left[ \frac{1 \text{ m}}{10^{-2} \text{ m}} \right]^3 \left[ \frac{1 \text{ s}}{1 \text{ s}} \right]^{-2}$$

$$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^6 \times 1$$

$$G_{\text{CGS}} = 6.6 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$$

### 3. Establish relations among various physical quantities

Example:

Obtain an expression for the time period  $T$  of a simple pendulum. The time period  $T$  depends on (i) mass ' $m$ ' of the bob (ii) length ' $l$ ' of the pendulum and (iii) acceleration due to gravity  $g$  at the place where the pendulum is suspended. (Constant  $k = 2\pi$ ) i.e

#### Solution

$$T \propto m^a l^b g^c$$

$$T = k. m^a l^b g^c$$

Here  $k$  is the dimensionless constant. Rewriting the above equation with dimensions

$$[T^1] = [M^a] [L^b] [LT^{-2}]^c$$

$$[M^0 L^0 T^1] = [M^a L^{b+c} T^{-2c}]$$

Comparing the powers of  $M$ ,  $L$  and  $T$  on both sides,  $a=0$ ,  $b+c=0$ ,  $-2c=1$

Solving for  $a, b$  and  $c$   $a = 0$ ,  $b = 1/2$ , and  $c = -1/2$

From the above equation  $T = k. m^0 l^{1/2} g^{-1/2}$

$$T = k \left( \frac{l}{g} \right)^{1/2} = k \sqrt{\frac{l}{g}}$$

Experimentally  $k = 2\pi$ , hence  $T = 2\pi \sqrt{\frac{l}{g}}$

**THINKING SHOULD BECOME YOUR  
CAPITAL ASSET NO MATTER  
WHATEVER UPS AND DOWNS  
YOU COME ACROSS IN YOUR LIFE**

**Dr. A.P.J.KALAM**

**UNIT: 2 KINEMATICS**

**SHORT ANSWER QUESTIONS**

1. What is meant by cartesian coordinate system?

At any given instant of time, the frame of reference with respect to which the position of the object is described in terms of position coordinates ( x, y, z ) is called Cartesian coordinate system.

2. Define a vector. Give examples.

It is a quantity which is described by both magnitude and direction. Geometrically a vector is a Directed line segment. Eg: Force, velocity, displacement, acceleration etc.,

3. Define a scalar. Give examples.

It is a quantity which can be described only by magnitude. Eg: Distance, mass, temperature, speed etc.,

4. Write a short note on the scalar product between two vectors.

The scalar product ( or dot product ) of two vectors is defined the product of the magnitude of both the vectors and the cosine of the angle between them.  $\vec{A} \cdot \vec{B} = AB \cos \theta$

5. Write a short note on vector product between two vectors.

The vector product ( or cross product ) of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them.

$$\vec{C} = \vec{A} \times \vec{B} = ( A B \sin \theta ) \vec{n}$$

6. How do you deduce that two vectors are perpendicular?

If two vectors  $\vec{A}$  and  $\vec{B}$  are perpendicular to each other then their scalar product  $\vec{A} \cdot \vec{B} = 0$ , because  $\cos 90^\circ = 0$ . Then the vectors  $\vec{A}$  and  $\vec{B}$  are said to be mutually orthogonal.

7. Define displacement and distance

**Distance:** The actual path length travelled by an object in the given interval of time during the motion. It is a positive scalar quantity.

**Displacement:** The difference between the final and initial positions of the object in a given interval of time. It can also be defined as the shortest distance between these two positions of the object.

The direction is from the initial to final position of the object. It is a vector quantity.

8. Define velocity and speed.

Velocity at an instant t is defined as the limiting value of the average velocity as  $\Delta t \rightarrow 0$ , evaluated at time t. It is equal to the rate of change of position vector with respect to time. It is a vector.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\Delta t \rightarrow 0$$

**Speed:** The magnitude of velocity v is called speed and is given by  $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$



Speed is always a positive scalar.

9. Define acceleration

Acceleration of a particle at time  $t$  is given by the ratio of change in velocity over  $\Delta t$ , as  $\Delta t \rightarrow 0$

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

The acceleration of the particle at an instant  $t$  is equal to rate of change of velocity. It is a vector.

10. What is the difference between velocity and average velocity.

Velocity at an instant  $t$  is defined as the limiting value of the average velocity as  $\Delta t \rightarrow 0$ , evaluated at time  $t$ . It is equal to the rate of change of position vector with respect to time. It is a vector.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Average velocity is defined as ratio of the displacement vector to the corresponding time interval.

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}. \text{ It is a vector. Its direction is in the direction of the displacement vector } (\Delta \vec{r})$$

11. Define a radian

One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius of the circle.

12. Define angular displacement and angular velocity

The angle described by the particle about the axis of rotation ( or centre O ) in a given time is called angular displacement. Unit is radian.

The rate of change of angular displacement is called angular velocity.

If  $\theta$  is the angular displacement in time  $t$ . then the angular velocity  $\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$

Unit is radian / s

13. What is non uniform circular motion?

If the speed of the object in circular motion is not constant, then it is called non uniform circular motion.

14. Write down the kinematic equations for angular motion.

$$1. \omega = \omega_0 + \alpha t \quad 2. \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad 3. \omega^2 = \omega_0^2 + 2 \alpha \theta \quad 4. \theta = \frac{(\omega_0 + \omega)t}{2}$$

15. Write down the expression for angle made by resultant acceleration and radius vector in the non

uniform circular motion.

$$\tan \theta = \frac{a_t}{\frac{v^2}{r}}$$

## LONG ANSWER QUESTIONS

## 1. Explain in detail the triangle law of addition.

**Statement:** If two vectors  $\vec{A}$  and  $\vec{B}$  are represented by two adjacent sides of a triangle taken in the same order, then the resultant is given by the third side of the triangle taken in the reverse order.

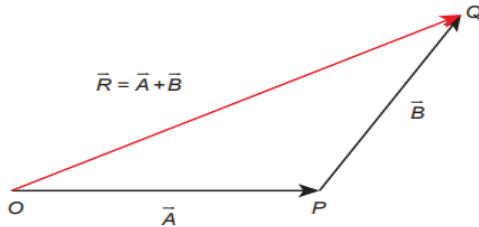


Diagram 1

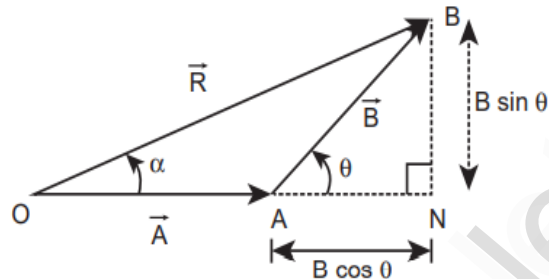


Diagram 2

**Construction:** Head of the first vector  $\vec{A}$  is connected to the tail of the second vector  $\vec{B}$ .

Let  $\theta$  be the angle between  $\vec{A}$  and  $\vec{B}$ . Then  $\vec{R}$  is the resultant vector connecting the tail of the first vector  $\vec{A}$  and head of the second vector  $\vec{B}$ .

From the diagram 1,  $\vec{OQ} = \vec{OP} + \vec{PQ}$  i.e.,  $\vec{R} = \vec{A} + \vec{B}$

**Magnitude of the resultant vector:**

From diagram 2,  $\triangle ABN$  is a right angled triangle. It is obtained by extending the side  $OA$  to  $ON$ .

For  $\triangle OBN$ , we have  $OB^2 = ON^2 + BN^2$

$$\cos \theta = \frac{AN}{B} \therefore AN = B \cos \theta \text{ and}$$

$$\sin \theta = \frac{BN}{B} \therefore BN = B \sin \theta$$

$$\Rightarrow R^2 = (A + B \cos \theta)^2 + (B \sin \theta)^2$$

$$\Rightarrow R^2 = A^2 + B^2 \cos^2 \theta + 2AB \cos \theta + B^2 \sin^2 \theta$$

$$\Rightarrow R^2 = A^2 + B^2 (\cos^2 \theta + \sin^2 \theta) + 2AB \cos \theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

**Direction of the resultant vector:**

If  $\theta$  is the angle between  $\vec{A}$  and  $\vec{B}$ , then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

If  $\vec{R}$  makes an angle  $\alpha$  with  $\vec{A}$ , then in  $\triangle OBN$ ,

$$\tan \alpha = \frac{BN}{ON} = \frac{BN}{OA+AN} = \frac{B \sin \theta}{A+B \cos \theta}$$

$$\alpha = \tan^{-1} \left( \frac{B \sin \theta}{A + B \cos \theta} \right)$$

## 2. Discuss the properties of scalar and vector products.

Sl.No	Scalar product of two vectors ( Dot product )	Vector product of two vectors ( Cross product )
1.	The scalar product of two vectors is defined as the product of the magnitudes of both the vectors and the cosine of the angle between them. $\vec{A} \cdot \vec{B} = AB \cos \theta$	1. The vector product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. $\vec{C} = \vec{A} \times \vec{B} = (AB \sin \theta) \vec{n}$
2.	The scalar product is commutative. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$	2. The vector product is not commutative. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \text{ But } \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
3.	Obey distributive law. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$	3. -----
4.	The angle between the vectors $\theta = \cos^{-1} \left[ \frac{\vec{A} \cdot \vec{B}}{AB} \right]$	4. -----
5.	The scalar product of two vectors will be maximum when $\cos \theta = 1$ , i.e., $\theta = 0^\circ$ , i.e., when the two vectors are parallel. $(\vec{A} \cdot \vec{B})_{\max} = AB$	5. The vector product of two vectors will be maximum when $\sin \theta = 1$ , i.e., $\theta = 90^\circ$ , i.e., when the two vectors are perpendicular to each other. $(\vec{A} \times \vec{B})_{\max} = AB \hat{n}$
6.	The scalar product of two vectors will be minimum when $\cos \theta = -1$ , i.e., $\theta = 180^\circ$ , i.e., when the two vectors are anti parallel. $(\vec{A} \cdot \vec{B})_{\min} = -AB$	6. The vector product of two vectors will be minimum when $ \sin \theta  = 0$ , i.e., $\theta = 0^\circ$ or $180^\circ$ $(\vec{A} \times \vec{B})_{\min} = -A$
7.	In case of a unit vector $\hat{n}$ $\hat{n} \cdot \hat{n} = 1 \times 1 \times \cos 0 = 1$ For example, $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$	7. $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$ The self vector products of unit vectors are thus zero.
8.	In the case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ , $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 1 \cdot 1 \cdot \cos 90^\circ = 0$	8. In the case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ , in accordance with the right hand screw rule: $\hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j}$
9.	The self dot product is given by $(\vec{A})^2 = \vec{A} \cdot \vec{A} = AA \cos \theta = A^2$ Here the angle is $\theta$ . The magnitude of the Vector is $ \vec{A}  = A = \sqrt{\vec{A} \cdot \vec{A}}$	9. The self cross product is the null vector $\vec{A} \times \vec{A} = A A \sin 0^\circ \hat{n} = \vec{0}$

10. The product quantity  $\vec{A} \cdot \vec{B}$  is always a scalar. Positive when the angle between the two vectors is acute and negative when the angle between them is obtuse.
10. The vector product of any two vectors is always another vector whose direction is perpendicular to the plane containing these two vectors.

3. Derive the kinematic equations of motion for constant acceleration.

Consider an object moving in a straight line with uniform or constant acceleration 'a'.

Let  $u$  be the velocity of the object at time  $t = 0$  and  $v$  be the velocity of the body at a later time  $t$ .

1. Velocity - Time relation:

Acceleration is defined as the first derivative of velocity with respect to time.

$$a = \frac{dv}{dt} \text{ or } dv = a dt$$

When time changes from 0 to  $t$ , the velocity changes from  $u$  to  $v$ .

$$\text{Integrating, } \int_u^v dv = \int_0^t a dt = a \int_0^t dt \quad \Rightarrow \quad [v]_u^v = a [t]_0^t$$

$$v - u = a [t - 0] \quad \text{or} \quad v = u + at$$

2. Displacement - Time relation:

Velocity is defined as the first derivative of displacement with respect to time.

$$v = \frac{ds}{dt} \text{ or } ds = v dt \quad \text{since } v = u + at, \text{ we get } ds = (u + at) dt$$

When time changes from 0 to  $t$ , the displacement changes from 0 to  $s$ .

$$\int_0^s ds = \int_0^t u dt + \int_0^t a t dt \quad \text{or} \quad [s]_0^s = u [t]_0^t + a \left[ \frac{t^2}{2} \right]_0^t$$

$$[s - 0] = u [t - 0] + a \left[ \frac{t^2}{2} - \frac{0^2}{2} \right] \quad \Rightarrow \quad s = ut + \frac{1}{2} at^2$$

3. Velocity - Displacement relation:

Acceleration is defined by the first derivative of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{dv}{ds} \frac{ds}{dt} = \frac{dv}{ds} v \quad \text{since } v = \frac{ds}{dt}$$

$$ds = \frac{1}{a} v dv \quad \text{Integrating both sides, } \int_0^s ds = \frac{1}{a} \int_u^v v dv$$

$$[s]_0^s = \frac{1}{a} \left[ \frac{v^2}{2} \right]_u^v \quad \Rightarrow \quad [s - 0] = \frac{1}{a} \left[ \frac{v^2}{2} - \frac{u^2}{2} \right] \quad \Rightarrow \quad s = \frac{1}{2a} [v^2 - u^2]$$

$$2as = v^2 - u^2 \quad \text{or} \quad v^2 = u^2 + 2as$$

4. Derive the equations of motion for a particle (a) falling vertically (b) projected vertically

(a) Equations for a particle

falling vertically

Consider a particle falling downwards from height  $h$ . Air friction is neglected.

The downward direction is chosen as

(b) Equations for a particle projected

vertically

Consider a particle projected upwards.

Air friction is neglected.

The upward direction is chosen as

positive y -axis.

Acceleration  $\vec{a} = g \vec{j}$

Comparing the components,

$$a_x = 0 \quad a_z = 0 \quad a_y = g$$

$$a_y = a = +g \quad s = y$$

The velocity and position of the object

at any time t is given by

$$1) v = u + a t \quad v = u + g t$$

$$2) s = u t + \frac{1}{2} a t^2 \quad y = u t + \frac{1}{2} g t^2$$

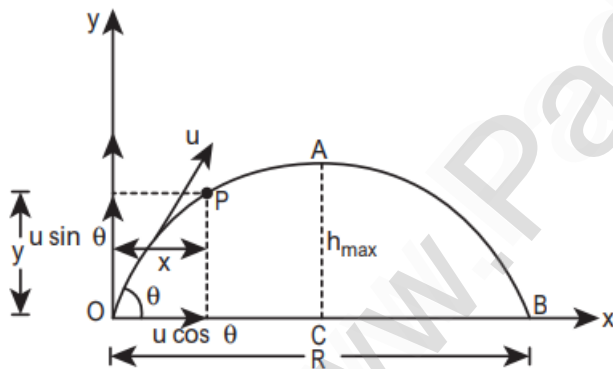
The square of the speed of the particle

when it is at a distance y from the top is

$$3) v^2 = u^2 + 2 a s \quad v^2 = u^2 + 2 g y$$

5. Derive the equation of motion for range and maximum height reached by the particle thrown at an oblique angle  $\theta$  with respect to the horizontal direction.

Maximum height ( $h_{\max}$ )



positive y – axis.

Acceleration  $a = -g$

The velocity and position of the object

at any time t is given by

$$1) v = u + a t \quad v = u - g t$$

$$2) s = u t + \frac{1}{2} a t^2 \quad s = u t - \frac{1}{2} g t^2$$

The velocity of the object at any position

y from the point where the object is thrown is

$$3) v^2 = u^2 + 2 a s \quad v^2 = u^2 - 2 g y$$

The maximum vertical distance travelled by the projectile during its journey is called maximum height.

For the vertical path of the motion,

$$v_y^2 = u_y^2 + 2 a_y s$$

Here,  $u_y = u \sin \theta$ ,  $a_y = -g$ ,  $s = h_{\max}$

At the maximum height,  $v_y = 0$

$$\text{Hence, } 0 = u^2 \sin^2 \theta - 2 g h_{\max} \quad \text{or} \quad 2 g h_{\max} = u^2 \sin^2 \theta$$

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2 g}$$

Horizontal Range ( R )

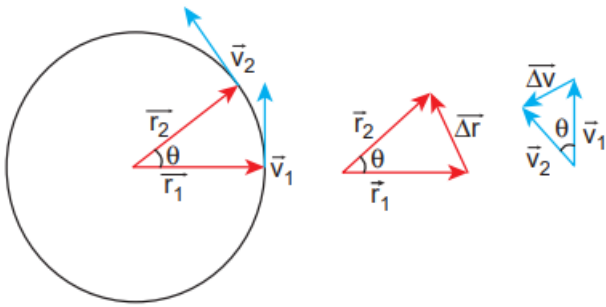
The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground.

Range R = Horizontal component of velocity x time of flight =  $u \cos \theta \times T_f$

$$R = u \cos \theta \times \frac{2 u \sin \theta}{g} = \frac{u^2 2 \sin \theta \cos \theta}{g} = \frac{u^2 \sin 2 \theta}{g}$$

Range is maximum when the angle of projection is maximum i.e.,  $\theta = 45^\circ$   $R_{\max} = \frac{u^2}{g}$

6. Derive the expression for centripetal acceleration.



The acceleration which is acting towards the centre of the circle, perpendicular to velocity and along the radius is called centripetal acceleration.

1. Let the directions of position and velocity vectors shift through the same angle  $\theta$  in a

small interval of time  $\Delta t$ .

2. For uniform circular motion,  $r = |\vec{r}_1| = |\vec{r}_2|$  and  $v = |\vec{v}_1| = |\vec{v}_2|$

3. If the particle moves from position vector  $\vec{r}_1$  to  $\vec{r}_2$ , the displacement is given by

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \text{ and change in velocity from } \vec{v}_1 \text{ to } \vec{v}_2 \text{ is given by } \Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$

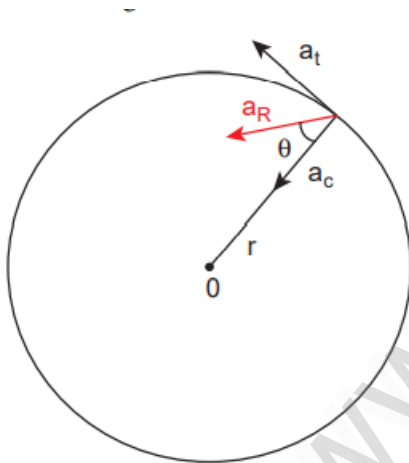
4. The magnitudes of the displacement  $\Delta r$  and  $\Delta v$  satisfy the following relation,

$$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta \quad \text{The -ve sign implies that } \Delta v \text{ points radially inward, towards the centre.}$$

$$5. \Delta v = -v \left( \frac{\Delta r}{r} \right) \quad \text{Then, } a = \frac{\Delta v}{\Delta t} = -\frac{v}{r} \left( \frac{\Delta r}{\Delta t} \right) = -\frac{v^2}{r}$$

6. For uniform circular motion  $v = \omega r$ , where  $\omega$  is the angular velocity of the particle about the Centre. Then the centripetal acceleration can be written as  $a = -\omega^2 r$

7. Derive the expression for total acceleration in the non uniform circular motion.



1. If the speed of the object in circular motion is not constant, then we have non – uniform circular motion.

2. For example, when the bob attached to a string moves in vertical circle, the speed of the bob is not the same at all time.

3. Whenever the speed is not same in circular motion, the particle will have both centripetal and tangential acceleration.

4. The resultant acceleration is obtained by the vector sum of centripetal and tangential acceleration.

5. As centripetal acceleration is  $v^2 / r$ , the magnitude of this

resultant acceleration is given by  $a_R = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$

6. The resultant acceleration makes an angle  $\theta$  with the radius vector. This angle is given by

$$\tan \theta = \frac{a_t}{\frac{v^2}{r}}$$

**IF YOU FAIL, NEVER GIVE UP BECAUSE FAIL MEANS**

**‘ FIRST ATTEMPT IN LEARNING’ - APJ KALAM**

**UNIT: 3 NEWTON'S LAWS OF MOTION EM EASY GUIDE**

**SHORT ANSWER QUESTIONS**

1. Explain the concept of inertia. Write two examples each for inertia of rest, inertia of motion and inertia of direction.

**INERTIA:** The inability of objects to move on its own or change its state of motion is called inertia.

Inertia means resistance to change its state.

**Inertia of rest:** The inability of an object to change its state of rest is called inertia of rest.

**Eg: 1.** When a stationary bus starts to move, the passengers experience a sudden backward push.

**2.** When you beat a carpet, the dust particles come out.

**Inertia of motion:** The inability of an object to change its state of uniform speed ( constant speed ) on its own is called inertia of motion.

**Eg: 1.** When the brake is suddenly applied in a moving bus, passengers move forward and hit against the front seat. **2.** In marathon or in short sprints, the athlete continues to run even after finishing.

**3. Inertia of direction:** The inability of an object to change its direction of motion on its own is called inertia of direction.

**Eg: 1.** When a stone attached to a string is in whirling motion, and if the string is cut suddenly, the Stone will not continue to move in circular path but moves tangentially.

**2.** Protection against rain through umbrella.

2. State Newton's second law.

The force acting on an object is equal to the rate of change of its momentum.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

3. Define one newton.

One newton is defined as the force which acts on 1 kg of mass to give an acceleration  $1 \text{ m s}^{-2}$  in the direction of the force.

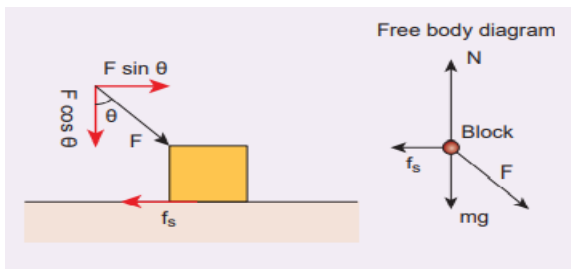
4. Using free body diagram, show that it is easy to pull an object than to push it.

1. When the body is pushed:

1. When a body is pushed at an arbitrary angle  $\theta$  (  $0$  to  $\pi/2$  ), the applied force  $F$  can be resolved into two components as  $F \sin \theta$  parallel to the surface and  $F \cos \theta$  perpendicular to the surface.

2. The total downward force is  $mg + F \cos \theta$ . So normal force acting on the body increases.

As there is no acceleration,  $N_{\text{push}} = mg + F \cos \theta \rightarrow 1$



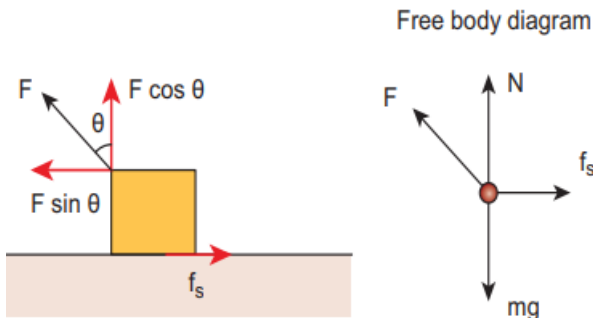
3. As a result the maximal static friction also increases and is equal to

$$f_s^{max} = \mu_s N_{push} = \mu_s (mg + F \cos \theta) \rightarrow 2$$

This equation shows that a greater force needs to be applied to push the object into motion.

2. When the body is pulled:

1. When a body is pulled at an angle  $\theta$ , the applied force is resolved into two components.



2. The total downward force acting on the object is  $N_{pull} = mg - F \cos \theta \rightarrow 3$

3. This equation shows that the normal force is less than  $N_{push}$ .

From 1 and 3 it is easier to pull a body than to

push to make it move.

5. Explain various types of friction. Suggest a few methods to reduce friction.

Friction is form of force. It always opposes the relative motion between an object and the surface where it is placed.

Types: 1. Static friction and 2. Kinetic friction

1. Static friction: It opposes the starting of motion. It is independent of surface area of contact.

$\mu_s$  depends on the nature of materials in mutual contact and magnitude of applied force.

It can take values from zero to  $\mu_s N$ .  $f_s^{max} > f_k$  and  $\mu_s > \mu_k$

2. Kinetic friction: It opposes the relative motion of the object with respect to the surface.

Independent of surface area of contact.

$\mu_k$  depends on nature of materials and temperature of the surface.

Independent of magnitude of applied force. It can never be zero and always equal to  $\mu_k N$

whatever be the speed. ( true  $v < 10 \text{ ms}^{-1}$  )  $f_k < f_s^{max}$  and  $\mu_k < \mu_s$

6. State the empirical laws of static and kinetic friction.

1. Experimentally it is found that the magnitude of static frictional force  $f_s$  satisfies the following

$$\text{empirical relation } 0 \leq f_s \leq \mu_s N$$

where  $\mu_s$  is the coefficient of static friction.

The force of static friction can take any value from zero to  $\mu_s N$

Cases: a)  $f_s$  is zero if the object is at rest and no external force applied on the object.

b)  $f_s$  is less than  $\mu_s N$  when the object is at rest and an external force is applied parallel to surface.



c)  $f_s$  attains its maximum value, when the object begins to slide.

2. Experimentally it is found that the magnitude of kinetic friction satisfies the relation

$$f_k = \mu_k N \quad \text{where } \mu_k \text{ is the coefficient of kinetic friction and } \mu_k < \mu_s$$

7. State Newton's third law.

Newton's third law states that for every action there is an equal and opposite reaction.

8. Under what condition will a car skid on a levelled circular road?

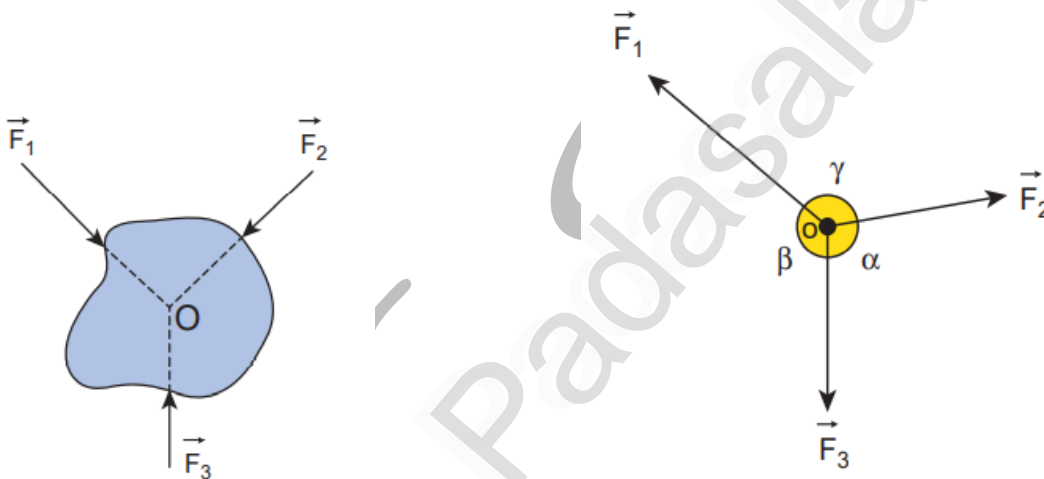
$$\text{If } \frac{mv^2}{r} > \mu_s mg, \text{ or } \mu_s < \frac{v^2}{rg} \text{ (skid)}$$

### LONG ANSWER QUESTIONS

1. What are concurrent forces? State Lami's theorem.

A collection of forces is said to be concurrent, if the lines of forces act at a common point.

If the concurrent forces are in the same plane, they are concurrent as well as coplanar forces.



**Lami's theorem:**

If a system of three concurrent and coplanar forces in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces.

$$\left| \frac{\vec{F}_1}{\sin \alpha} \right| = \left| \frac{\vec{F}_2}{\sin \beta} \right| = \left| \frac{\vec{F}_3}{\sin \gamma} \right|$$

2. Show that in an inclined plane, angle of friction is equal to the angle of repose.

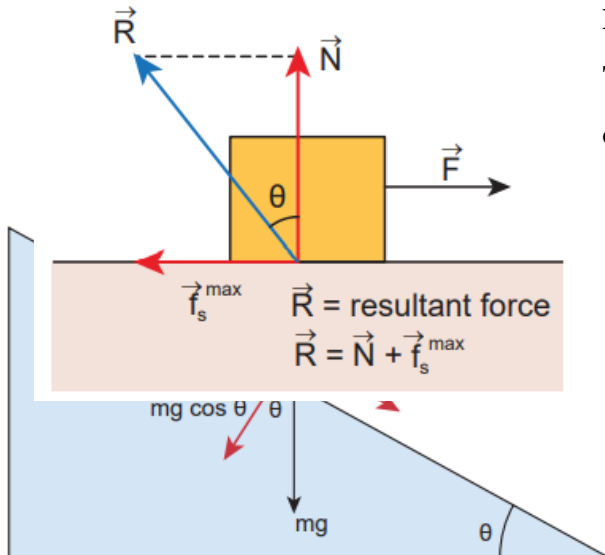
Describe the method of measuring angle of repose.

**Angle of friction:** The angle of friction is defined as the angle between the normal force ( N ) and the resultant force ( R ) of normal force and maximum friction force (  $f_s^{max}$  )

$$R = \sqrt{(f_s^{max})^2 + N^2} \quad \tan \theta = \frac{f_s^{max}}{N} \rightarrow 1$$

From the frictional relation, the object begins to slide when  $f_s^{max} = \mu_s N$

$$\text{or when } \frac{f_s^{max}}{N} = \mu_s \rightarrow 2$$



From equations 1 and 2,  $\mu_s = \tan \theta$

The coefficient of static friction is equal to the tangent of the angle of friction.

**Angle of repose:**

The angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

The gravitational force  $mg$  is resolved into two components.

Parallel component to inclined plane is  $mg \sin \theta$

Perpendicular component to inclined plane is  $mg \cos \theta$

From the diagram,  $N = mg \cos \theta \rightarrow 1$

$$f_s^{max} = mg \sin \theta \rightarrow 2$$

$$2 / 1 \quad \frac{\sin \theta}{\cos \theta} = \frac{f_s^{max}}{N} \quad \tan \theta = \mu_s \quad \text{because} \quad \frac{f_s^{max}}{N} = \mu_s$$

Thus the angle of repose is the same as angle of friction. Angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

### 3. Explain the need for banking of tracks.

In order to avoid skidding of vehicles, usually the outer edge of the road is slightly raised compared to inner edge. This is called banking of roads or tracks. The inclination angle introduced is called angle of banking.

When a vehicle takes a turn, the two forces acting on the vehicle are

1. Downward gravitational force  $mg$
2. Normal force perpendicular to surface ( $N$ )

Resolving the normal force into its components,

$N \cos \theta$  balances the downward gravitational force ( $mg$ )

$N \sin \theta$  provides necessary centripetal force ( $mv^2 / r$ )

$$N \cos \theta = mg \rightarrow 1 \quad N \sin \theta = mv^2 / r \rightarrow 2$$

$$2 / 1 \quad \tan \theta = \frac{v^2}{r g} \quad v^2 = r g \tan \theta \quad v = \sqrt{r g \tan \theta}$$

The banking angle and radius of curvature of the road or track determines the safe speed of the

vehicle at the turning.

**NOTE:**

1. If the speed of the vehicle exceeds this safe speed, then it starts to skid outwards. But the frictional force comes into effect and provides an additional centripetal force to prevent outward skidding.
2. If the speed of the vehicle is little lesser than the safe speed, it starts to skid inwards. But the frictional force reduces centripetal force to prevent inward skidding.

**4. State Newton's three laws and discuss their significance.**

**Newton's first law:** Every object continues to be in the state of rest or of uniform motion ( constant velocity ) unless there is external force acting on it.

We can understand the concept of inertia from first law. Also this law gives the fundamental definition of force.

We can apply first law of Newton in special set of frames called inertial frames of reference.

Newton's first law deals with the motion of objects in the absence of any force.

**Newton's second law:** The force acting on an object is equal to the rate of change of its momentum.

This law is used in inertial frames of reference.

In non-inertial frames Newton's second law cannot be used in this form. It requires some modification.

It gives the dimensional definition of force.

The relation  $F = ma$  is obtained from Newton's second law of motion.

**Newton's third law:** For every action there is an equal and opposite reaction.

This law assures that an isolated force or a single force cannot exist in nature. The forces occur as equal and opposite pairs.

Newton's third law is valid in both inertial and non-inertial frames.

**FAILURE WILL NEVER OVERTAKE ME IF MY**

**DETERMINATION TO SUCCEED IS STRONG ENOUGH - APJ**

**UNIT: 4      WORK ENERGY AND POWER      EM EASY GUIDE**

**SHORT ANSWERS**

**1. Explain how the definition of work in physics is different from general perception.**

The term work refers to both physical as well as mental work. In fact, any activity can generally be called as work. In physics, work is said to be done by the force when the force applied on a body displaces it.

**2. Write the various types of potential energy. Explain the formulae.**

There are three types of potential energies. Each type is associated with a particular force.

**1. The energy possessed by the body due to gravitational force gives rise to gravitational potential energy.**

$$U = mgh$$

m - mass of the object    g- acceleration due to gravity    h- height of the object

**2. The energy due to spring force and other similar forces give rise to elastic potential energy.**

$$U = \frac{1}{2} k (x_f^2 - x_i^2)$$

k – force constant     $x_f$  - final position of the spring     $x_i$  – initial position of the spring

**3. The energy due to electrostatic force on charges gives rise to electrostatic potential energy.**

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$$

$\epsilon_0$  - permittivity of free space     $q_1, q_2$  - point charges

r - distance between the point charges

**3. Write the differences between conservative and non – conservative forces. Give two examples each.**

S.No	Conservative forces	Non-conservative forces
1.	Work done is independent of the path	Work done depends upon the path
2.	Work done in a round trip is zero	Work done in a round trip is not zero
3.	Total energy remains constant	Energy is dissipated as heat energy
4.	Work done is completely recoverable	Work done is not completely recoverable.
5.	Force is the negative gradient of potential energy	No such relation exists.

**Examples for conservative and non-conservative forces:**

**Conservative forces:** Elastic spring force, electrostatic force, magnetic force, gravitational force etc.,

**Non – conservative forces:** Frictional force, force due to air resistance, viscous force etc.,

4. Explain the characteristics of elastic and inelastic collision.

S.No.	Elastic Collision	Inelastic Collision
1.	Total momentum is conserved	Total momentum is conserved
2.	Total kinetic energy is conserved	Total kinetic energy is not conserved
3.	Forces involved are conservative forces	Forces involved are non-conservative forces
4.	Mechanical energy is not dissipated.	Mechanical energy is dissipated into heat, light, sound etc.

5. Define : 1) Power 2) Loss of kinetic energy in inelastic collision

1. Power: It is defined as the rate of work done or energy delivered.

$$\text{Power (P)} = \text{Work done (W)} / \text{time taken (t)}$$

$$P = W / t \quad \text{Unit: watt (W)}$$

2. Loss of kinetic energy in perfect inelastic collision:

In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc.,

Let  $KE_i$  be the total kinetic energy before collision and  $KE_f$  be the total kinetic energy after collision.

Total kinetic energy before and after collision are,

$$KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \quad KE_f = \frac{1}{2} (m_1 + m_2) v^2$$

Then loss of kinetic energy is,  $\Delta Q = KE_i - KE_f$

$$\Delta Q = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \frac{1}{2} (m_1 + m_2) v^2 \quad \text{Here, } v = \frac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)}$$

Using the algebraic equation,  $(a + b)^2 = a^2 + b^2 + 2 a b$

$$\text{Loss of kinetic energy } \Delta Q = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) (u_1 - u_2)^2$$

**LONG ANSWERS**

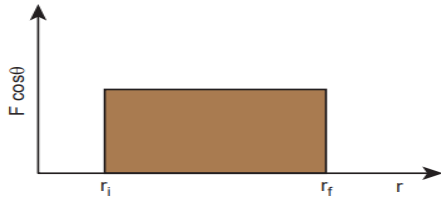
1. Explain with graph the work done by a constant force.

1. When a constant force  $F$  acts on a body, the small work done ( $dW$ ) by the force in producing

A small displacement  $dr$  is given by the relation,  $dW = (F \cos \theta) dr$

2. The total work done in producing a displacement from initial position  $r_i$  to final position  $r_f$  is

$$W = \int_{r_i}^{r_f} dW \quad W = \int_{r_i}^{r_f} (F \cos \theta) dr = (F \cos \theta) \int_{r_i}^{r_f} dr$$



$$3. W = (F \cos \theta) (r_f - r_i)$$

1. The graphical representation of the work done by a constant force is shown.

2. The area under the graph shows the work done by the constant force.

2. State and explain work - energy principle. Mention any three examples for it.

**Work- kinetic energy theorem:** The work done by the force on the body changes the kinetic energy of the the body. This is called work-kinetic energy theorem.

**Explanation:**

1. Let us consider a body of mass  $m$  at rest on a frictionless horizontal surface.

2. The work ( $W$ ) done by the constant force  $F$  for a displacement ( $s$ ) in the same direction is

$$W = F s \quad \text{As constant force } F = ma \quad \text{So, } W = m a s \rightarrow 1$$

$$3. v^2 = u^2 + 2 a s \quad a = \frac{v^2 - u^2}{2 s} \rightarrow 2$$

$$4. \text{Substituting 2 in 1, } W = m \left[ \frac{v^2 - u^2}{2 s} \right] s = m \left[ \frac{v^2 - u^2}{2} \right]$$

$$W = \frac{1}{2} m v^2 - \frac{1}{2} m u^2 \rightarrow 3$$

5. The expression for kinetic energy:

$$\text{In equation 3, } KE = \frac{1}{2} m v^2 \rightarrow 4$$

Kinetic energy of the body is always positive.

$$6. \text{From equations 3 and 4, } \Delta KE = \frac{1}{2} m v^2 - \frac{1}{2} m u^2$$

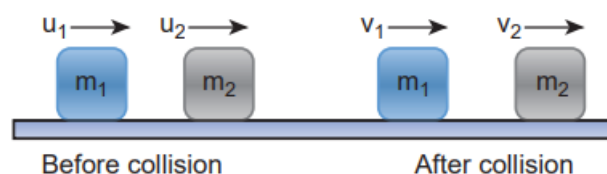
$$\text{So, } W = \Delta KE \quad \text{i.e, Work done} = \text{Change in kinetic energy}$$

**Cases:**

1. If the work done by the force on the body is positive then its kinetic energy increases.
2. If the work done by the force on the body is negative then its kinetic energy decreases.
3. If there is no work done by the force on the body then there is no change in kinetic energy.

3. Arrive at an expression for elastic collision in one dimension and discuss various cases.

1. Consider two elastic bodies of masses  $m_1$  and  $m_2$  moving in a straight line ( along positive X direction ) on a frictionless horizontal surface.



## 2. From the law of conservation of linear momentum,

Total momentum before collision = Total momentum after collision

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{Or } m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \quad \rightarrow 1$$

## 3. For elastic collision,

Total kinetic energy before collision = Total kinetic energy after collision

Using the formula  $a^2 - b^2 = (a + b)(a - b)$ ,  
we can rewrite the above equation as

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 (u_1 + v_1)(u_1 - v_1) = m_2 (v_2 + u_2)(v_2 - u_2) \quad \rightarrow 2$$

## 4. Dividing equation 2 by 1,

$$\frac{m_1 (u_1 + v_1)(u_1 - v_1)}{m_1 (u_1 - v_1)} = \frac{m_2 (v_2 + u_2)(v_2 - u_2)}{m_2 (v_2 - u_2)}$$

Rearranging,  $u_1 - u_2 = v_2 - v_1$

This equation can be rewritten as,

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = -(v_1 - v_2) \quad \rightarrow 3$$

## 5. Finding the values of $v_1$ and $v_2$ from equation 3,

$$v_1 = v_2 + u_2 - u_1$$

Or

$$v_2 = u_1 + v_1 - u_2$$

$$m_1 (u_1 - v_1) = m_2 (u_1 + v_1 - u_2 - u_2)$$

$$m_1 (u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$$

$$m_1 u_1 - m_1 v_1 = m_2 u_1 + m_2 v_1 - 2m_2 u_2$$

$$m_1 u_1 - m_2 u_1 + 2m_2 u_2 - m_1 v_1 + m_2 v_1$$

$$(m_1 - m_2) u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$$

$$\text{or } v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2$$

$$v_2 = \left( \frac{2m_1}{m_1 + m_2} \right) u_1 + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) u_2$$

Special cases:

Masses of first and second bodies	Initial velocities of first and second bodies	Final velocity of the first body	Final velocity of the second body	Inference
Equal. i.e., $m_1 = m_2$	$u_1$ and $u_2$	$v_1 = u_2$	$v_2 = u_1$	Velocities are interchanged.
Equal.	$u_1$ and $0$	$v_1 = 0$	$v_2 = u_1$	Second body moves with the initial velocity of the first body.

i.e.,  $m_1 = m_2$

$m_1 \ll m_2$  i.e.,  
 $m_1 / m_2 = 0$

$u_1$  and  $0$

$v_1 = -u_1$

$v_2 = 0$

The first body which is lighter returns back in the opposite direction with the same initial velocity. The second body which is heavier continues to remain at rest.

$m_2 \ll m_1$  i.e.,  
 $m_2 / m_1 = 0$

$u_1$  and  $0$

$v_1 = u_1$

$v_2 = 2u_1$

The first body which is heavier continues to move with the same initial velocity. The second body which is lighter will move with twice the initial velocity of the first body.

**CREAVITITY IS SEEING  
THE SAME THING BUT  
THINKING DIFFERENTLY.**

**- A.P.J.KALAM**



**UNIT: 5 MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES EM EASY GUIDE**

**SHORT ANSWERS**

1. *Define centre of mass.*

It is defined as a point where the entire mass of the body appears to be concentrated.

2. *Find out the centre of mass for the given geometrical structures.*

- a) Equilateral triangle - At the centre of the medians
- b) Cylinder – On its central axis
- c) Square – Point of intersection of the diagonals.

3. *Define torque and mention its unit.*

The moment of the external applied force about a point or axis of rotation.

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{Unit: N m or J}$$

4. *What are the conditions in which force cannot produce torque?*

- 1) The torque is zero when  $\vec{r}$  and  $\vec{F}$  are parallel or antiparallel.

When parallel  $\theta = 0^\circ$  and when antiparallel  $\theta = 180^\circ$ . In both cases torque is zero.

- 2) If the force acts at the reference point then torque is zero.

5. *Give any two examples of torque in day-to-day life.*

- (i) Opening and closing of a door about the hinges.
- (ii) Turning a nut using a wrench
- (iii) Opening a bottle cap or water tap.

6. *What is the relation between torque and angular momentum?*

Torque is equal to rate of change of angular momentum.  $\tau = \frac{dL}{dt}$

7. *Define couple.*

A pair of forces which are equal in magnitude but opposite in direction and separated by a perpendicular distance so that their lines of action do not coincide that causes a turning effect.

8. *State principle of moments.*

Sum of the clockwise moments is equal to sum of the anticlockwise moments when a body is in

rotational equilibrium.

9. Define centre of gravity.

It is the point at which the entire weight of the body acts irrespective of the position and orientation of the body.

10. Mention any two physical significance of moment of inertia.

- (i) For rotational motion, moment of inertia is a measure of rotational inertia.
- (ii) It is not an invariable quantity. It depends not only on the mass of the body, but also on the way the mass is distributed around the axis of rotation.

11. What is radius of gyration?

It is the perpendicular distance from the axis of rotation to an equivalent point mass, which would have the same mass as well as the same moment of inertia of the object. Unit: m

12. What are the rotational equivalents for the physical quantities

- (i) mass – moment of inertia
- (ii) force – torque

13. What is the condition for pure rolling?

Rolling is the combination of rotational and translational motions.

$$v_{\text{roll}} = 0 = \text{translational velocity} + \text{tangential velocity due to rotation. i.e., } v - r\omega = 0$$

14. What is the difference between sliding and slipping?

**Sliding:** It is the case when  $v_{\text{CM}} > R\omega$  or  $v_{\text{TRANS}} > v_{\text{ROT}}$

The translation is more than the rotation.

**Slipping:** It is the case when  $v_{\text{CM}} < R\omega$  or  $v_{\text{TRANS}} < v_{\text{ROT}}$

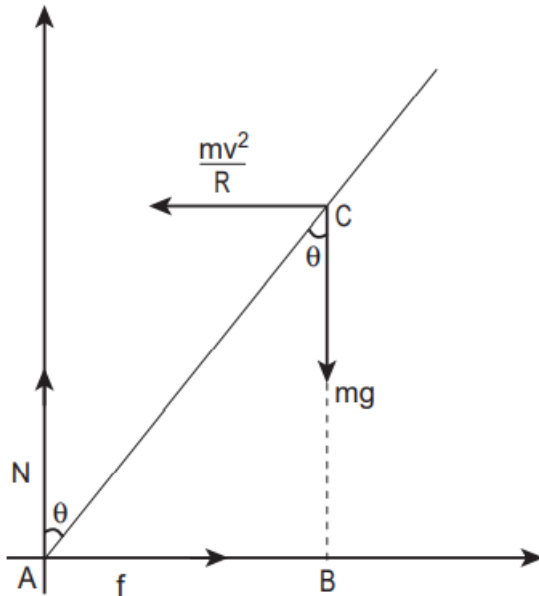
The rotation is more than the translation.

### LONG ANSWER QUESTIONS

1. Explain why a cyclist bends while negotiating a curve road? Arrive at the expression for angle of bending for a given velocity.

1. Let us consider a cyclist negotiating a circular level road ( not banked ) of radius  $r$  with a speed  $v$ .
2. The cycle and the cyclist are considered as one system with mass  $m$ .
3. The system as a frame is rotating about  $Z$ -axis. The system is at rest in this rotating frame.

4. The forces acting on the system are (i) gravitational force (  $mg$  )  
(ii) normal force (  $N$  ) (iii) frictional force (  $f$  ) and (iv) centrifugal force (  $\frac{mv^2}{r}$  )
5. As the system is in equilibrium in the rotational frame of reference, the net external force and net external torque must be zero.



$$mg AC \sin\theta = \frac{mv^2}{r} AC \cos\theta$$

$$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad \theta = \tan^{-1} \left[ \frac{v^2}{rg} \right]$$

For rotational equilibrium,  $\vec{\tau}_{\text{net}} = 0$

6. The torque due to the gravitational force about point A is  $mg AB$  which causes clockwise turn that is taken as negative.

7. The torque due to centrifugal force is  $\frac{mv^2}{r} BC$  Which causes an anticlockwise turn that is taken as Positive.

$$8. -mg AB + \frac{mv^2}{r} BC = 0$$

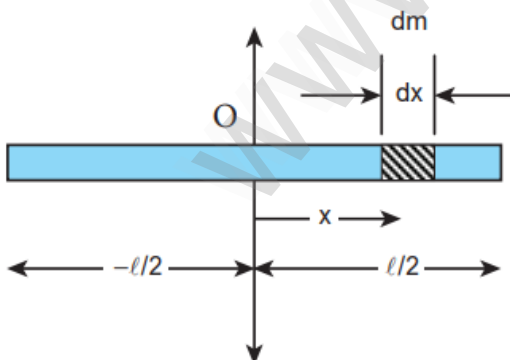
$$mg AB = \frac{mv^2}{r} BC$$

9. From  $\Delta ABC$ ,  $AB = AC \sin\theta$  and  $BC = AC \cos\theta$

10. While negotiating a circular level road of radius  $r$  at velocity  $v$ , a cyclist has to bend by angle  $\theta$  from vertical to stay in equilibrium. ( to avoid a fall )

2. Derive the expression for moment of inertia of a rod about its centre and perpendicular to the rod.

1. Let us consider a uniform rod of mass (  $M$  ) and length (  $l$  ). Moment of inertia of this rod about an axis passing through the centre of mass and perpendicular to the rod is to be determined.



2. The geometric centre is the origin which coincides with the centre of mass.

3. The rod is along the X-axis. An infinitesimal small mass (  $dm$  ) at a distance  $x$  from the origin.

4. The moment of inertia  $dI$  of this mass  $dm$  about the axis is  $dI = (dm) x^2$

5. As the mass is uniformly distributed, the mass per unit

$$\text{length ( } \lambda \text{ ) of the rod is } \lambda = \frac{M}{l}$$

6. The mass of the infinitesimally small length is  $dm = \lambda dx = \frac{M}{l} dx$

7. The moment of inertia  $I$  of the entire rod can be found by integrating  $dI$ ,

$$I = \int dI = \int (dm) x^2 = \int \left(\frac{M}{l} dx\right) x^2$$

$$I = \frac{M}{l} \int x^2 dx$$

8. As the mass is distributed on either side of the origin, the limits of integration are from  $-\frac{l}{2}$  to  $\frac{l}{2}$

9.

$$I = \frac{M}{l} \int_{-\ell/2}^{\ell/2} x^2 dx = \frac{M}{l} \left[ \frac{x^3}{3} \right]_{-\ell/2}^{\ell/2}$$

$$I = \frac{M}{l} \left[ \frac{\ell^3}{24} - \left( -\frac{\ell^3}{24} \right) \right] = \frac{M}{l} \left[ \frac{\ell^3}{24} + \frac{\ell^3}{24} \right]$$

10.

$$I = \frac{M}{l} \left[ 2 \left( \frac{\ell^3}{24} \right) \right]$$

$$I = \frac{1}{12} M \ell^2$$

3. State and prove parallel axis theorem.

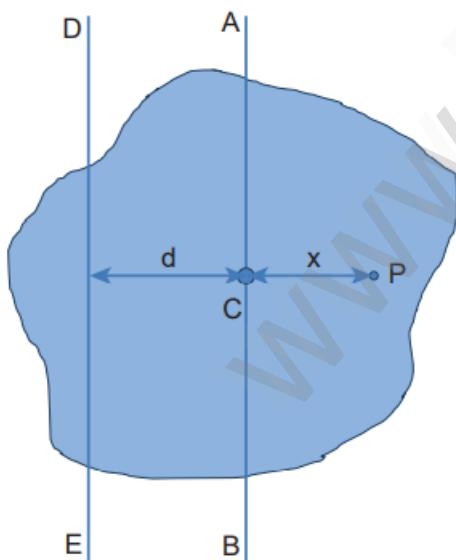
**Theorem:** The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

$$I = I_C + Md^2$$

**Proof:** 1. Consider a rigid body. Its moment of inertia about an axis AB passing through the centre of mass is  $I_C$ .

2. DE is another axis parallel to AB at a perpendicular distance  $d$  from AB. The moment of inertia of the body about DE is  $I$ .

3. Consider a point mass  $m$  on the body at position  $x$  from its centre of mass.



4. The moment of inertia of the point mass about the axis DE is  $m(x+d)^2$

5. The moment of inertia  $I$  of the whole body about DE is the summation of the above expression  $I = \sum m(x+d)^2$

$$6. I = \sum m(x^2 + d^2 + 2xd)$$

$$I = \sum (mx^2 + md^2 + 2d \sum mx)$$

7. Here,  $I_C = \sum mx^2$  is the moment of inertia of the body about the centre of mass.

$\sum mx = 0$  since  $x$  can take positive and negative values with respect to the axis AB.

$$8. \text{ Thus } I = I_C + \sum md^2 = I_C + (\sum m) d^2$$

$$\text{As } \sum m = M$$

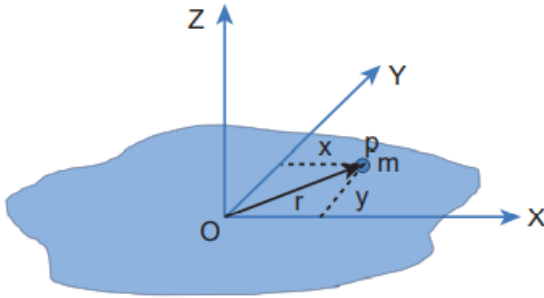
$$\text{So } I = I_C + Md^2 \quad \text{Hence proved.}$$

4. State and prove perpendicular axis theorem.

The moment of inertia of a plane lamina body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.

$$I_Z = I_X + I_Y$$

Proof:



1. Consider a plane lamina object of negligible thickness on which lies the origin O.
2. The X and Y axes lie on the plane and Z axis is perpendicular to it.
3. The lamina is considered to be made up of a large number of particles of mass m.

4. Let us choose one such particle at a point P which has coordinates ( x,y) at a distance r from O.

5. The moment of inertia of the *particle* about Z axis is  $mr^2$

The moment of inertia of the entire body about Z axis is  $I_Z = \sum mr^2$

6. Here,  $r^2 = x^2 + y^2$       So  $I_Z = \sum m (x^2 + y^2)$        $I_Z = \sum mx^2 + \sum my^2$

7. Here  $\sum mx^2$  is the moment of inertia of the body about Y axis and  $\sum my^2$  is the moment of inertia of the body about X axis.

8. Thus,  $I_X = \sum my^2$       and  $I_Y = \sum mx^2$       Substituting in the equation for  $I_Z$ , we get

$$I_Z = I_X + I_Y \quad \text{Hence proved.}$$

5. Discuss rolling on an inclined plane and arrive at the expression for the acceleration.

1. Let us assume a round object of mass m and radius R is rolling down an inclined plane without slipping.

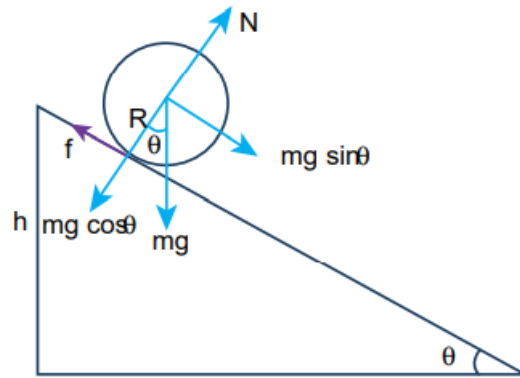
2. The two forces acting on the object along the inclined plane are (i) component of the gravitational Force (  $mg \sin\theta$  ) and the other is the static frictional force (f).

3. Another component of gravitational force  $mg \cos\theta$  is cancelled by the normal force N exerted by the plane.

4. From the freebody diagram,

For translational motion,  $mg \sin\theta$  is the supporting force and f is the opposing force.

$$mg \sin\theta - f = ma \rightarrow 1$$



5. For rotational motion, the torque is set by the frictional force  $f$ .

$$Rf = I\alpha$$

6. WKT,  $a = R\alpha$      $\alpha = a/R$ ,     $I = mK^2$ ,

$$Rf = mK^2 a / R \quad f = ma \left( \frac{K^2}{R^2} \right) \rightarrow 2$$

7. Substitute 2 in 1,     $mg \sin\theta - ma \left( \frac{K^2}{R^2} \right) = ma$

$$mg \sin\theta = ma + ma \left( \frac{K^2}{R^2} \right) = ma \left( 1 + \frac{K^2}{R^2} \right)$$

$$a \left( 1 + \frac{K^2}{R^2} \right) = g \sin\theta$$

8. After rewriting for acceleration, we get     $a = \frac{g \sin\theta}{\left( 1 + \frac{K^2}{R^2} \right)}$

**SUCCESS IS A VEHICLE WHICH  
MOVES ON A WHEEL NAMED HARD WORK  
BUT THE JOURNEY IS IMPOSSIBLE  
WITHOUT THE FUEL NAMED  
SELF CONFIDENCE - Dr. A.P.J. KALAM**

## UNIT: 6 GRAVITATION

## EM SIMPLE GUIDE

## SHORT ANSWERS - TWO MARKS

1. What is meant by superposition of gravitational field?

The total gravitational field at a point due to all the masses is given by the vector sum of the gravitational field due to the individual masses.

2. Define gravitational potential energy.

The W.D. to bring a mass  $m_2$  from infinity to a distance 'r' in the gravitational field of mass  $m_1$ .

3. Is potential energy the property of a single object? Justify.

No. It is a property of a system due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-Object system. The gravitational potential energy depends on relative position, so we need to set a reference level at which the potential energy is zero.

4. Define gravitational potential.

The amount of work required to bring unit mass from infinity to the distance 'r'

5. What is the difference between gravitational potential and gravitational potential energy?

Gravitational potential: W.D. in bringing a body of unit mass from infinity to the point without acceleration.

$$V_{(r)} = - GM / R$$

Gravitational potential energy: The amount of W.D. on the body by the forces acting on it.  $U = - GMm / R$

6. What is meant by escape speed in the case of the Earth?

The minimum speed required by an object to escape from Earth's gravitational field.

7. Why is the energy of a satellite ( or any other planet ) negative?

Energy of a satellite is negative because the satellite is bound to the Earth and it cannot escape from the Earth.

8. What are geostationary and polar satellites?

Geostationary satellites: The satellites appear to be stationary when seen from Earth. Height 36000 km

Polar satellite: The satellites that orbit Earth from North pole to South pole. Height 500 to 800 km

9. Define weight.

The downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the Earth.

10. Why is there no lunar eclipse and solar eclipse every month?

Moon's orbit is tilted  $5^\circ$  with respect to Earth's orbit. Due to this  $5^\circ$  tilt, only during certain periods of the year, the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment.

11. How will you prove that Earth itself is spinning?

The Earth's spinning motion can be proved by observing star's position over a night.

Due to Earth's spinning motion, the stars in sky appear to move in circular motion about the pole star.

12. What is retrograde motion of planets?

The planets move eastwards and reverse their motion for a while and return to eastward motion again.

### 13. Write a note on weightlessness of objects ( astronauts )

The Earth satellite that orbit very close to the Earth experience only gravitational force. The astronauts inside the satellite also experience the same gravitational force. Because of this, they cannot exert any force on the floor of the satellite. Thus, the floor of the satellite also cannot exert any normal force on the astronaut. Therefore, the astronauts inside a satellite are in the state of weightlessness.

#### THREE MARKS

#### 1. Derive the time period of satellite orbiting the Earth.

The distance covered by the satellite during one rotation in its orbit.

$$v = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2\pi(R_E + h)}{T} \quad \sqrt{\frac{GM_E}{R_E + h}} = \frac{2\pi(R_E + h)}{T} \quad T = \frac{2\pi}{\sqrt{GM_E}} (R_E + h)^{\frac{3}{2}}$$

Squaring both sides,  $T^2 = \frac{4\pi^2}{GM_E} (R_E + h)^3$        $h \ll R$       So,  $T^2 = \frac{4\pi^2}{GM_E} R_E^3$

$$T^2 = \frac{4\pi^2}{\frac{GM_E}{R_E^2}} R_E = \frac{4\pi^2}{g} R_E \quad T = 2\pi \sqrt{\frac{R_E}{g}}$$

#### 2. Derive an expression for energy of satellite.

The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy.

Potential energy is  $U = -\frac{GM_s M_E}{(R_E + h)}$       Kinetic energy is  $K.E. = \frac{1}{2} M_s v^2$

But,  $v = \sqrt{\frac{GM_E}{(R_E + h)}}$       On substituting,  $K.E. = \frac{1}{2} \frac{GM_E M_s}{2(R_E + h)}$

Now, the total energy is  $E = \frac{GM_E M_s}{2(R_E + h)} - \frac{GM_s M_E}{(R_E + h)} = -\frac{GM_E M_s}{2(R_E + h)}$

Negative sign indicates that the satellite is bound to the Earth and it cannot escape from the Earth.

#### 3. Explain the variation of g with latitude.

1. As the Earth spins about its own axis, all objects on the surface of the Earth experience centrifugal force. The centrifugal force depends on the latitude of the objects on the Earth.
2. If the Earth were not spinning, the force on the object would have been mg. However, the object experiences an additional centrifugal force due to spinning of the Earth.
3. The centrifugal force is given by  $m\omega^2 R^1$ .       $R^1 = R \cos \lambda$       where  $\lambda$  is the latitude.
4. The component of centrifugal acceleration experienced by the object in the direction opposite to g is

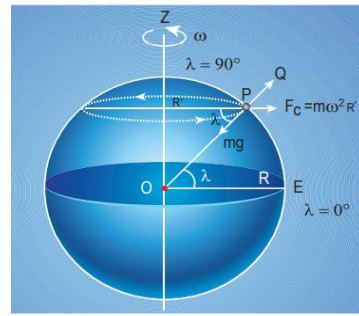
$$a_{PQ} = \omega^2 R^1 \cos \lambda = \omega^2 R \cos^2 \lambda \quad \text{Therefore, } g^1 = g - \omega^2 R \cos^2 \lambda$$

5. At equator,  $\lambda = 0$ ,  $g^1 = g - \omega^2 R$        $g^1$  is minimum.

At poles,  $\lambda = 90^\circ$ ,  $g^1 = g$ , it is maximum.



## Variation of $g$

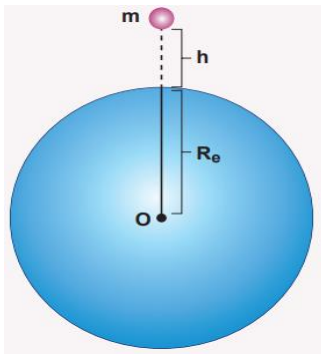


with latitude

### 4. Explain the variation of $g$ with altitude.

1. Consider an object of mass 'm' at a height 'h' from the surface of the Earth.

Acceleration experienced by the object due to Earth is  $g^1 = \frac{GM}{(R_e + h)^2}$



$$2. g^1 = \frac{GM}{R_e^2(1 + \frac{h}{R_e})^2} = \frac{GM}{R_e^2} \left(1 + \frac{h}{R_e}\right)^{-2}$$

$$3. \text{ If } h \ll R_e, \quad \text{By Binomial theorem, } g^1 = \frac{GM}{R_e^2} \left(1 - \frac{2h}{R_e}\right)$$

$$4. \text{ Now, } g^1 = g \left(1 - \frac{2h}{R_e}\right) \quad \text{We find that, } g^1 < g$$

This means that as altitude  $h$  increases,  $g$  decreases.

### LONG ANSWERS – FIVE MARKS

#### 1. Explain in detail the idea of weightlessness using lift as an example.

Let us consider a man inside a lift ( elevator). The two forces acting on him are,

1. Downward gravitational force  $\vec{F}_G = -mg\hat{j}$

2. The normal force exerted by floor on the man acts upward,  $\vec{N} = N\hat{j}$

#### 2. Case (i) Elevator is at rest:

Acceleration of the man is zero. Net force is zero.

Applying Newton's II law on the man,  $\vec{F}_G + \vec{N} = 0 \quad -mg\hat{j} + N\hat{j} = 0$

Comparing the components,  $N - mg = 0$  or  $N = mg \rightarrow 1$

APPARAENT WEIGHT IS EQUAL TO ACTUAL WEIGHT.

#### 3. Case (ii) : When the elevator is moving uniformly in the upward or downward direction:

In uniform motion ( constant velocity ), the net force on the man is still zero.

Hence, APPARAENT WEIGHT IS EQUAL TO ACTUAL WEIGHT.

#### 4. Case (iii): When the elevator is accelerating upwards: $\vec{a} = a\hat{j}$

$\vec{F}_G + \vec{N} = m\vec{a} \quad -mg\hat{j} + N\hat{j} = m a\hat{j} \quad \text{Comparing the components, } N = m(g + a) \rightarrow 2$

APPARENT WEIGHT OF THE MAN IS GREATER THAN HIS ACTUAL WEIGHT.

#### 5. Case (iv): When the elevator is accelerating downwards: $\vec{a} = -a\hat{j}$

$\vec{F}_G + \vec{N} = m\vec{a} \quad -mg\hat{j} + N\hat{j} = -m a\hat{j} \quad \text{Comparing the components, } N = m(g - a) \rightarrow 3$

APPARENT WEIGHT OF THE MAN IS LESSER THAN HIS ACTUAL WEIGHT.

Freely falling objects experience only gravitational force. They are not in contact with any surface. ( neglect air friction ) The normal force acting on the object is zero.

Now the downward acceleration is equal to the acceleration due to the gravity of the Earth.  $a = g$

From 3,  $N = m ( g - g ) = 0$ . This is called the state of weightlessness.

## 2. Derive an expression for escape speed.

1. Consider an object of mass  $M$  on the surface of the Earth. When it is thrown up with an initial speed  $v_i$ , the initial total energy of the object is  $E_i = \frac{1}{2} M v_i^2 - G M M_E / R_E \rightarrow 1$

2. The term  $- G M M_E / R_E$  is the potential energy of the mass  $M$ .

$M_E$  is the mass of the Earth.  $R_E$  is the radius of the Earth.

3. When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero. ( $U(\infty) = 0$ ) and the kinetic energy becomes zero as well.

The final total energy thus becomes zero.  $E_f = 0$

4. According to the law of energy conservation,  $E_i = E_f$

$$\frac{1}{2} M v_i^2 - G M M_E / R_E = 0 \quad \frac{1}{2} M v_i^2 = G M M_E / R_E \rightarrow 2$$

5. ESCAPE SPEED: The minimum speed required by an object to escape Earth's gravitational field.

Now replace  $v_i = v_e$   $\frac{1}{2} M v_e^2 = G M M_E / R_E$   $v_e^2 = 2 G M M_E / R_E M$

$$v_e^2 = 2 G M_E / R_E \rightarrow 3$$

6. Using  $g = G M_E / R_E^2$ ,  $v_e^2 = 2 g R_E$   $v_e = \sqrt{2 g R_E} \rightarrow 4$

From this equation, it is clear that escape speed depends on the value of 'g' and radius of the Earth.

It is completely independent of the mass of the object.

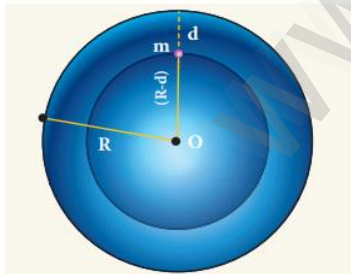
## 3. Explain the variation of g with depth.

1. Consider a particle of mass 'm' which is in a deep mine of the Earth. Assume the depth of the mine as 'd'.

2. The part of the Earth which is above the radius ( $R_e - d$ ) do not contribute to the acceleration.

It is given as  $g^1 = G M^1 / (R_e - d)^2 \rightarrow 1$  Here  $M^1$  is the mass of the Earth of radius ( $R_e - d$ )

3. Density of the earth  $\rho = M / V$  where  $M$  is the Earth and  $V$  its volume.



Now we have,  $\frac{M^1}{V^1} = \frac{M}{V}$  and  $M^1 = \frac{M}{V} V^1$

$$4. M^1 = \left[ \frac{M}{\frac{4}{3} \pi R_e^3} \right] \frac{4}{3} \pi (R_e - d)^3 = \frac{M}{R_e^3} (R_e - d)^3 \rightarrow 2$$

5. Substituting 3 in 1, we get,

$$g^1 = G \frac{M}{R_e^3} (R_e - d)^3 \cdot 1 / (R_e - d)^2 = G M \frac{R_e (1 - \frac{d}{R_e})}{R_e^3}$$

$$g^1 = G M \frac{(1 - \frac{d}{R_e})}{R_e^2}$$

6. Now,  $g^1 = g (1 - d / R_e)$  Here,  $g^1 < g$ . As depth increases,  $g^1$  decreases.

**DON'T READ SUCCESS STORIES, YOU WILL ONLY GET A MESSAGE. READ FAILURE STORIES, YOU WILL GET SOME IDEAS TO TASTE SUCCESS - Dr. A.P.J**

## UNIT: 7      PROPERTIES OF MATTER      EM SIMPLE GUIDE

## SHORT ANSWERS – TWO MARKS

## 1. Define stress and strain

The force per unit area is called stress. Stress  $\sigma = F / A$  Unit:  $N m^{-2}$

The change in size to original size is called strain. Strain  $\epsilon = \Delta L / L$  Dimensionless quantity. No unit.

## 2. State Hooke's law of elasticity.

It states that for a small deformation within the limit, the strain produced in a body is directly proportional to the stress that produces it.

## 3. Define Poisson's ratio.

The ratio of relative contraction ( lateral strain ) to relative expansion ( longitudinal strain ).

## 4. Which one of these is more elastic, steel or rubber? Why?

Steel is more elastic than rubber. If an equal stress is applied to both steel and rubber, steel produces less strain. So the Young's modulus is higher for steel than rubber. The object which has higher Young's modulus is more elastic.

## 5. A spring balance shows wrong readings after using for a long time. Why?

When a spring balance is used for a long time, elastic fatigue is developed in it. Elastic fatigue is the property of a body to lose its elastic property by repeated action of restoring and deforming force.

## 6. What is the effect of temperature on elasticity?

As the temperature of substance increases, its elasticity decreases.

## 7. Write down the expression for the elastic potential energy of a stretched wire.

$$u = \frac{1}{2} \frac{FL}{AL} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

## 8. Define coefficient of viscosity of a liquid.

The coefficient of viscosity is defined as the tangential force per unit area required to maintain unit velocity gradient between the layers of the liquid.

## 9. Distinguish between streamlined flow and turbulent flow.

Streamline flow: 1. The velocity of fluid at a given point is always constant.      2. No intermixing of layers.  
3. Steady, regular and orderly flow      4. The velocity of fluid is always less than critical velocity.

Turbulent flow: 1. The velocity of a fluid at a given point does not remain constant.

2. Always there is intermixing of layers.      3. Unsteady, irregular and disorderly flow.

4. The velocity of a fluid is greater than critical velocity.

## 10. What is Reynold's number? Give its significance.

To find out the nature of flow of fluid, streamlined or turbulent, Reynold formulated an equation

$$R_c = \rho v D / \eta \quad \text{It is a dimensionless number called Reynold's number.}$$

$\rho$  – density of the fluid       $v$  – velocity of the flowing fluid       $D$  – diameter of the pipe where the fluid flows  
and  $\eta$  – the coefficient of viscosity of the fluid

For streamline flow  $Rc < 1000$  For unsteady  $1000 < Rc < 2000$  For turbulent flow  $Rc > 2000$

11. Define terminal velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium.

12. Write down the expression for the Stoke's force and explain the symbols involved in it.

$F = 6 \pi \eta r v$       F- viscous force       $\eta$  – coefficient of viscosity of the liquid      r- radius of the sphere  
and v – velocity of the sphere

13. State Bernoulli's theorem.

The sum of pressure energy, kinetic energy and potential energy per unit mass of an incompressible, non- viscous fluid in a streamlined flow remains a constant.  $\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant}$

14. What are the energies possessed by a liquid? Write down their equations.

A liquid in a steady flow can possess three kinds of energy.

1. Kinetic energy:      K.E. =  $\frac{1}{2} m v^2$       K.E. / m =  $v^2 / 2$

2. Potential energy:      P.E. = mgh      P.E. / m = gh

3. Pressure energy:      Pressure energy = P V      Pressure energy per unit mass =  $P V / m = P / \rho$

15. Two streamlines cannot cross each other? Why?

Two streamlines cannot cross each other because there will be two velocities at the point of intersection which is not possible.

16. Define surface tension of a liquid. Mention its SI unit and dimensions.

The force per unit length of the liquid or the energy per unit area of the surface of a liquid.

$T = F / l$       Unit:  $N m^{-1}$       Dimension:  $[ M T^{-2} ]$

17. How is surface tension related to surface energy?

The surface energy per unit area of a surface is numerically equal to the surface tension.

18. Define angle of contact for a given pair of solid and liquid.

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called angle of contact between the solid and the liquid.

19. What are cohesive and adhesive forces?

The force between the like molecules which holds the liquid together is called cohesive force.

When the liquid is in contact with a solid, the molecules of these solid and liquid will experience an attractive force called adhesive force.

20. What happens to the pressure inside a soap bubble when air is blown into it?

When air is blown into the bubble, the size of the bubble increases. The pressure inside the bubble is inversely proportional to the size of the bubble. As the size increases, pressure decreases proportionally.

21. What do you mean by capillarity? or capillary action?

The rise or fall of a liquid in a narrow tube is called capillarity or capillary action.

22. A drop of oil placed on the surface of water spreads out. But a drop of water placed on oil contracts

to a spherical shape. Why?

Adhesive force between water and oil dominates cohesive force between oil molecules. Here spreading will take place. So oil spreads out.

Cohesive force between oil molecules dominates adhesive force between oil and water. Here contraction will take place. So water contracts to a spherical shape.

23. State the principle and usage of Venturimeter?

Principle: Bernoulli's theorem.

Use: To measure the rate of flow of the incompressible fluid flowing through a pipe.

### THREE MARKS

1. Explain the different types of modulus of elasticity.

Modulus of elasticity = Stress / Strain

Three types: 1. Young's modulus 2. Bulk modulus 3. Rigidity or Shear modulus

1. Young's modulus:  $Y = \text{Tensile or compressive stress} / \text{Tensile or compressive strain}$   $Y = \frac{\sigma_t}{\epsilon_t}$  or  $\frac{\sigma_c}{\epsilon_c}$

2. Bulk modulus:  $K = \text{Normal stress or Pressure} / \text{Volume strain}$   $K = \frac{-\sigma_n}{\epsilon_v} = \frac{-\Delta P}{\frac{\Delta V}{V}}$

3. Rigidity modulus:  $\eta_R = \text{Shearing stress} / \text{Shearing strain}$   $\eta_R = \frac{\sigma_s}{\epsilon_s} = \frac{F_t}{\Delta A \theta}$

2. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.

The volume of the liquid flowing per second through the capillary tube depends on

(i) Coefficient of viscosity of liquid (ii) Radius of the tube (iii) Pressure gradient (P/l)

$V \propto \eta^a r^b (P/l)^c$   $V = k \eta^a r^b (P/l)^c$  Substituting the dimensions on both sides,

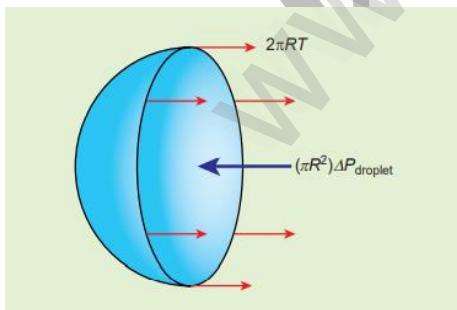
$[L^3 T^{-1}] = [M L^{-1} T^{-1}]^a [L]^b [M L^{-2} T^{-2}]^c$   $[M^0 L^3 T^{-1}] = [M^{a+c} L^{-a+b-2c} T^{-a-2c}]$

Comparing the powers of M, L, T on both sides and solving,  $a = -1$ ,  $b = 4$  and  $c = 1$

$V = k \eta^{-1} r^4 (P/l)^1$  Experimentally  $k = \pi/8$  So,  $V = \frac{\pi P r^4}{8l\eta}$

3. Obtain the expression for the excess of pressure inside a liquid drop

a) Liquid drop: Consider a liquid drop of radius R and the surface tension T.



(i) Force due to surface tension  $F_T = 2\pi RT$  towards right

(ii) Force due to outside pressure  $F_{P1} = P_1 \pi R^2$  towards right

(iii) Force due to inside pressure  $F_{P2} = P_2 \pi R^2$  towards left

As the drop is in equilibrium,  $F_{P2} = F_T + F_{P1}$

$P_2 \pi R^2 = 2\pi RT + P_1 \pi R^2 \Rightarrow (P_2 - P_1) \pi R^2 = 2\pi RT$

Excess pressure  $\Delta P = P_2 - P_1 = 2T/R$

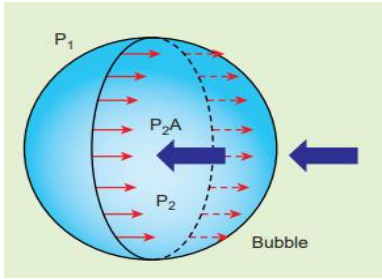
4) Obtain an expression for the excess pressure inside a soap bubble:

Consider a soap bubble of radius R and the surface tension T.

A soap bubble has two liquid surfaces in contact with air, one inside and one outside the bubble.

Therefore, the force on the soap bubble due to surface tension is  $2 \times 2\pi RT$

(i) Force due to surface tension  $F_T = 4 \pi R T$  towards right



(ii) Force due to outside pressure  $F_{P1} = P_1 \pi R^2$  towards right

(iii) Force due to inside pressure  $F_{P2} = P_2 \pi R^2$  towards left

As the bubble is in equilibrium,  $F_{P2} = F_T + F_{P1}$

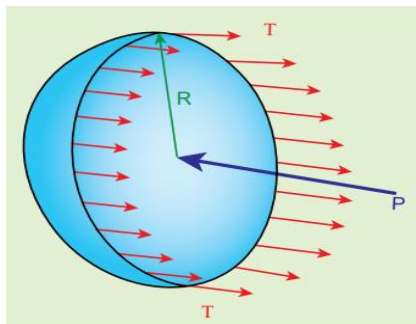
$$P_2 \pi R^2 = 4 \pi R T + P_1 \pi R^2 \Rightarrow (P_2 - P_1) \pi R^2 = 4 \pi R T$$

$$\text{Excess pressure } \Delta P = P_2 - P_1 = 4 T / R$$

5) Obtain an expression for the excess pressure inside an air bubble:

Consider an air bubble of radius  $R$  inside a liquid having surface tension  $T$ .

Let  $P_1$  and  $P_2$  be the pressures outside and inside the air bubble respectively.



(i) Force due to surface tension  $F_T = 2 \pi R T$  towards right

(ii) Force due to outside pressure  $F_{P1} = P_1 \pi R^2$  towards right

(iii) Force due to inside pressure  $F_{P2} = P_2 \pi R^2$  towards left

As the air bubble is in equilibrium,  $F_{P2} = F_T + F_{P1}$

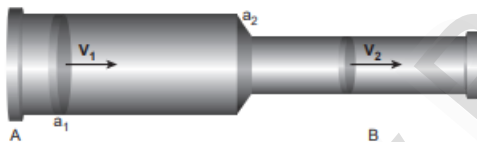
$$P_2 \pi R^2 = 2 \pi R T + P_1 \pi R^2 \Rightarrow (P_2 - P_1) \pi R^2 = 2 \pi R T$$

$$\text{Excess pressure } \Delta P = P_2 - P_1 = 2 T / R$$

6. Obtain an expression for the equation of continuity for a flow of fluid on the basis of conservation of mass.

1. Consider a pipe AB of varying cross sectional area  $a_1$  and  $a_2$  such that  $a_1 > a_2$ .

2. A non-viscous, incompressible liquid flows steadily through the pipe (streamline flow) with velocities  $v_1$  and  $v_2$  in area  $a_1$  and  $a_2$  respectively.



Let  $m_1$  be the mass of fluid flowing through section A in time  $\Delta t$ ,  $m_1 = (a_1 v_1 \Delta t) \rho$

Let  $m_2$  be the mass of fluid flowing through section B in time  $\Delta t$ ,  $m_2 = (a_2 v_2 \Delta t) \rho$

For an incompressible liquid, mass is conserved  $m_1 = m_2$

$$a_1 v_1 \Delta t \rho = a_2 v_2 \Delta t \rho$$

$$a_1 v_1 = a_2 v_2 \Rightarrow a v = \text{constant}$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

#### LONG ANSWERS - FIVE MARKS

1. Derive an expression for the elastic energy stored per unit volume of a wire.

1. When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy.

2. Consider a wire whose un-stretch length is  $L$  and area of cross section is  $A$ . Let a force produce an extension  $l$  and further assume that there is no loss in energy.

3. W.D. by the force  $F = \text{Energy gained by the wire}$ . As  $dW = F dl$ , total W.D.  $W = \int_0^l F dl$

4. From Young's modulus of elasticity  $Y = \frac{F}{A} \times \frac{L}{l}$   $F = \frac{YAl}{L}$  Now,  $W = \int_0^l \frac{YAl}{L} dl$

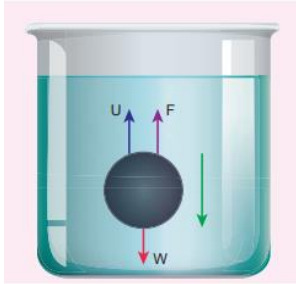
5. On integrating and substituting the limits,  $W = \frac{1}{2} Fl = \text{Elastic potential energy}$

6. ENERGY DENSITY: Energy per unit volume  $u = \frac{1}{2} Fl / AL = \frac{1}{2} \frac{F}{A} \frac{l}{L} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$

2. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using Stokes force.

1. Consider a sphere of radius 'r' which falls freely through a highly viscous liquid of coefficient of viscosity  $\eta$ .

Density of the material of the sphere  $\rho$  and density of the fluid is  $\sigma$ .



2. Gravitational force acting on the sphere  $F_G = mg = \frac{4}{3} \pi r^3 \rho g$  (downward)

Up thrust  $U = \frac{4}{3} \pi r^3 \sigma g$  (upward)

Viscous force  $F = 6\pi \eta r v_t$

3. TERMINAL VELOCITY: The maximum constant velocity acquired by a body while falling freely through a viscous medium.

4. At terminal velocity  $v_t$  The net downward force = upward force  $F_G = U + F$   $F = F_G - U$

$$6\pi \eta r v_t = \frac{4}{3} \pi r^3 \rho g - \frac{4}{3} \pi r^3 \sigma g \quad 6\pi \eta r v_t = \frac{4}{3} \pi r^3 (\rho - \sigma) g \quad v_t = \frac{4 r^2 (\rho - \sigma)}{18 \eta} g$$

5.  $v_t = \frac{2}{9} \times \frac{r^2 (\rho - \sigma)}{\eta} g \Rightarrow v_t \propto r^2$  Terminal velocity of the sphere is proportional to the square of its radius.

3. State and prove Bernoulli's theorem.

Bernoulli's theorem: The sum of pressure energy, kinetic energy and potential energy per unit mass of an incompressible, non viscous fluid in a streamlined flow remains a constant.

Mathematically,  $\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant}$

Proof:



1. Let us consider a flow of liquid through a pipe AB.

$V$  - Volume of the liquid and  $t$  - time taken by the liquid to flow from A to B

2. Let  $a_A$ ,  $v_A$  and  $P_A$  be the area, velocity and pressure exerted by the liquid at A.

Force exerted by the liquid at A is  $F_A = P_A a_A$

3. Distance travelled by the liquid in time  $t$  is  $d = v_A t$

W. D is  $W = F_A d = P_A a_A v_A t$  But  $a_A v_A t = a_A d = V$   $V$  - volume of the liquid entering at A

4. W. D. is the pressure energy (at A)  $W = F_A d = P_A V$

Pressure energy per unit volume at A is  $= P_A V / V = P_A$  Pressure energy per unit mass  $= \frac{P_A V}{m} = \frac{P_A}{\rho}$

Since  $m$  is the mass of the liquid entering at A in a time 't', Pressure energy is  $E_{PA} = m \frac{P_A}{\rho} \rightarrow 1$

5. Potential energy of the liquid at A is  $PE_A = mgh_A \rightarrow 2$  Kinetic energy of the liquid at A is  $\frac{1}{2} m v_A^2 \rightarrow 3$

Total energy at A is  $E_A = 1+2+3 = m \frac{P_A}{\rho} + mgh_A + \frac{1}{2} m v_A^2$

6. Similarly when calculating total energy at B,  $E_B = m \frac{P_B}{\rho} + mgh_B + \frac{1}{2} m v_B^2$

From the law of conservation of energy,  $E_A = E_B$   $m \frac{P_A}{\rho} + mgh_A + \frac{1}{2} m v_A^2 = m \frac{P_B}{\rho} + mgh_B + \frac{1}{2} m v_B^2$

7. Total energy per unit mass  $\frac{P_A}{\rho} + gh_A + \frac{1}{2} v_A^2 = \frac{P_B}{\rho} + gh_B + \frac{1}{2} v_B^2 = \text{constant}$

On dividing the equation by 'g'  $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$

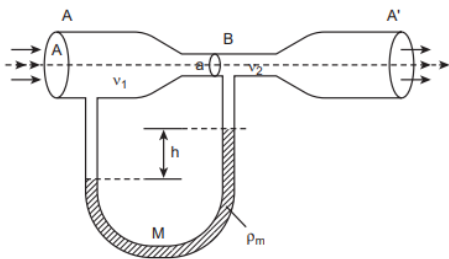
4. Describe the construction and working of Venturimeter.

Use: To measure the rate of flow of the incompressible fluid flowing through a pipe.

Principle: Bernoulli's theorem

Construction: 1. A and A' are two wider tubes of area A connected by a narrow tube B with area 'a'.

2. A manometer in the form of U-tube is also attached between the wide and narrow tubes. The manometer contains a liquid of density  $\rho_m$ .



3. The pressure difference between the tubes A and B is measured by the height difference  $\Delta P = P_1 - P_2$

From the equation of continuity,  $Av_1 = av_2$   $v_2 = \frac{Av_1}{a}$

4. Using Bernoulli's equation,

$$P_1 + \rho \frac{v_1^2}{2} = P_2 + \rho \frac{v_2^2}{2} = P_2 + \frac{\rho}{2} \left[ \frac{A}{a} v_1 \right]^2$$

5. Now, the pressure difference is  $\Delta P = P_1 - P_2 = \frac{\rho}{2} v_1^2 \frac{(A^2 - a^2)}{a^2}$

6. The speed of flow of fluid at the wide end of the tube A is  $v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}$   $v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$

7. The volume of the liquid flowing out per second is  $V = Av_1 = A \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}} = aA \sqrt{\frac{2(\Delta P)}{\rho(A^2 - a^2)}}$

**YOU CANNOT CHANGE YOUR FUTURE. BUT  
YOU CAN CHANGE YOUR HABITS, AND  
SURELY YOUR HABITS WILL CHANGE  
YOUR FUTURE. - ABDUL KALAM**



## UNIT: 8 HEAT AND THERMODYNAMICS E M EASY GUIDE

## SHORT ANSWERS – TWO MARKS

## 1. State Prevost theory of heat exchange.

All bodies emit thermal radiation at all temperatures above absolute zero irrespective of the nature of the surroundings.

## 2. State Stefan - Boltzmann law.

The total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.  $E \propto T^4$  or  $E = \sigma T^4$

## 3. State Wien's displacement law.

The wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body.  $\lambda_m \propto 1/T$  or  $\lambda_m = b/T$  b - Wien's constant

## 4. What are thermodynamic state variables?

The state of a thermodynamic system is represented by a set of variables called thermodynamic variables.

Example: Pressure, Temperature, Volume and Internal energy

## 5. State Zeroth law of thermodynamics.

If two systems A and B are in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.

## 6. State the first law of thermodynamics.

Change in internal energy of the system is equal to the heat supplied to the system minus work done by the System on the surroundings.  $\Delta U = Q - W$

## 7. What is quasi static process?

It is an infinitely slow process in which the system changes its variable ( P,V,T ) so slowly such that it remains In thermal, mechanical and chemical equilibrium with its surroundings throughout.

## 8. What is P V diagram?

P V diagram is a graph between pressure P and volume V of the system. It is used to calculate the amount of work done by the gas during expansion or on the gas during compression.

## 9. Define specific heat capacity at constant pressure.

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K by keeping the pressure constant.

## 10. Define specific heat capacity at constant volume.

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K by keeping the volume constant.

## 11. What are isothermal and adiabatic processes?

A process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change.  $P V = \mu R T$ . A process in which no heat flows into or out of the system (  $Q = 0$  ). So the pressure, volume and temperature of the system may change.

12. What are isobaric and isochoric processes?

A thermodynamic process that occurs at constant pressure. Temperature, volume and internal energy are not constant. A thermodynamic process in which volume of the system is kept constant. Pressure, temperature and internal energy are variables.

13. What is cyclic process?

A thermodynamic process in which the thermodynamic system returns to its initial state after undergoing a series of changes. Change in internal energy is zero. Heat enters into the system as well as leave the system.

14. What are reversible and irreversible processes? Give example.

Reversible process: If it is possible to retrace the path in the opposite direction in such a way that the system and the surroundings pass through the same states as in the initial, direct process.

Slow compression, Expansion of a spring

Irreversible process: All natural processes are irreversible. No unique value of pressure and temperature at Every stage of the process, so it cannot be plotted in a P V diagram.

15. What is a heat engine?

Heat engine is a device which takes heat as input and converts this heat into work by undergoing a cyclic process.

16. Define efficiency of a heat engine.

Efficiency is defined as the ratio of work done by the working substance in one cycle to the amount of heat extracted from the source.

17. State Clausius form of second law of thermodynamics.

Heat always flows from hotter object to colder object spontaneously.

18. State Kelvin- Planck statement.

It is impossible to construct a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work.

19. What is a Carnot engine?

A reversible heat engine operating in a cycle between two temperatures in a particular way is Carnot engine.

20. Define thermal conductivity. State its unit.

The quantity of heat transferred through a unit length of a material in a direction normal to unit surface area due to a unit temperature difference under steady state conditions.

Unit:  $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$  or  $\text{W m}^{-1} \text{K}^{-1}$

### THREE MARKS

1. Define the following: a) one mole b) molar specific heat capacity c) latent heat d) black body

a) One mole: It is the amount of that substance which contains Avogadro number ( $N_A$ ) of particles such as atoms or molecules.

b) Molar specific heat capacity: The heat energy required to increase the temperature of one mole of a substance by 1 K or  $1^\circ \text{C}$ .

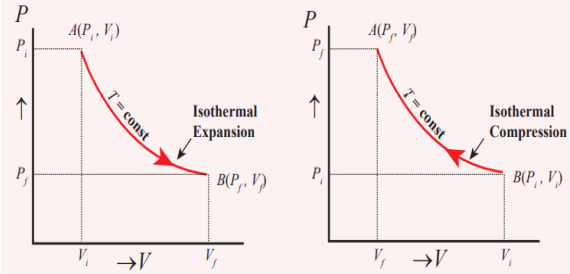
c) Latent heat: The amount of heat energy required to change the state of a unit mass of the material.

$Q = m \times L$        $L = Q / m$       Unit:  $J \text{ kg}^{-1}$

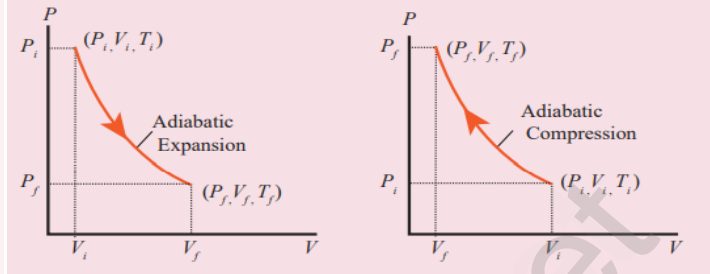
d) **Black body:** An object that absorbs all electromagnetic radiations. It is a perfect absorber and radiator of energy with no reflecting power.

2. Draw the P V diagrams for i) isothermal      ii) adiabatic      iii) isobaric      iv) isochoric processes.

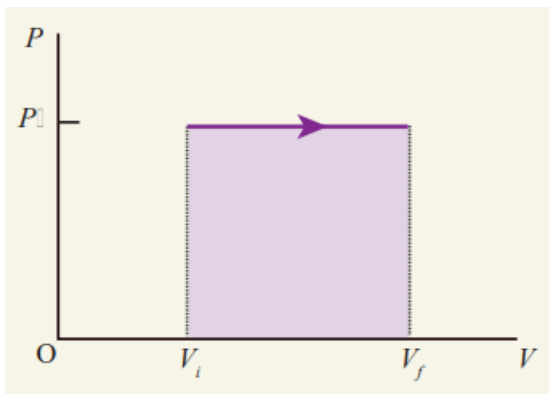
i) **Isothermal process: P V Diagram**



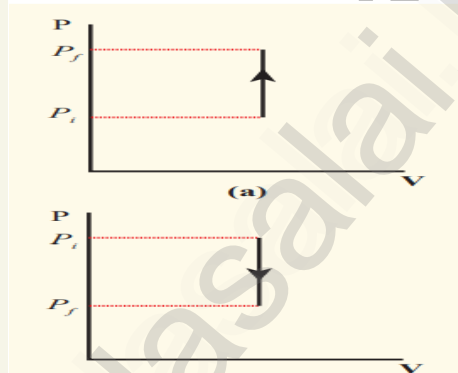
ii) **Adiabatic process: P V Diagram:**



iii) **Isobaric process: P V Diagram**

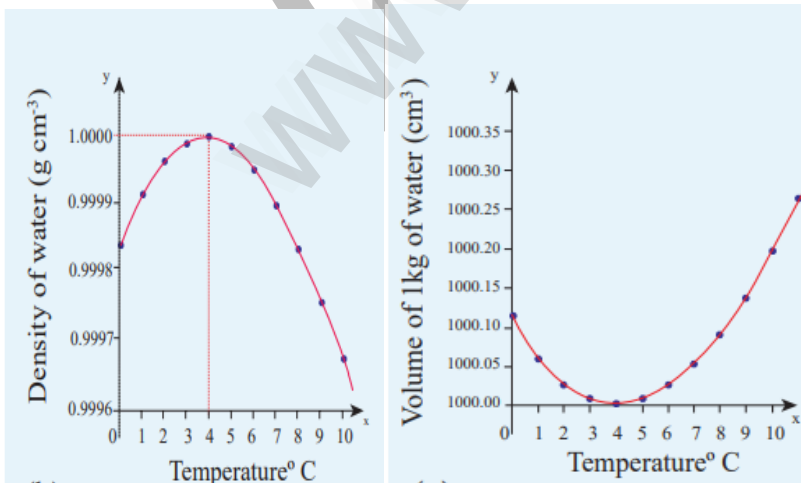


iv) **Isochoric process: P V Diagram:**



3. Describe anomalous expansion of water.

1. At moderate temperatures: Liquids expand on heating and contract on cooling.
2. Water exhibits an anomalous behaviour i.e., it contracts on heating between  $0^{\circ} \text{C}$  and  $4^{\circ} \text{C}$ .
3. The volume of a given amount of water decreases as it is cooled from room temperature until it reaches  $4^{\circ} \text{C}$ .
4. Below  $4^{\circ} \text{C}$ , the volume increases and so its density decreases. This means that water has a maximum density at  $4^{\circ} \text{C}$ . This behaviour of water is called anomalous expansion of water.

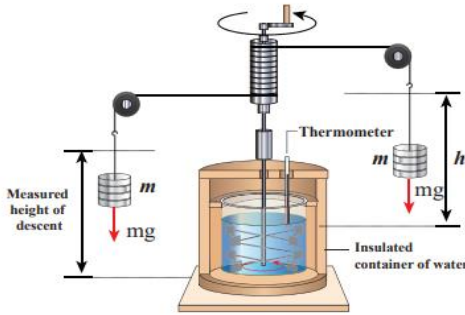


5. Due to the anomalous expansion of water, in lakes and ponds water freezes only at the top and the species living in the lakes and ponds will be safe at the bottom.

4. Explain Joule's mechanical equivalent of heat.

Joule showed that mechanical energy can be converted into internal energy and vice versa.

**Experiment: 1. Two masses are attached with a rope and a paddle wheel.**



2. When the masses fall through a distance 'h' due to gravity, they lose potential energy equal to  $2 mgh$ .
3. When the masses fall, paddle wheel turns. Frictional force comes between water and the wheel due to turning of the wheel.
4. As a result, temperature of water rises. This means gravitational potential energy is converted into internal energy of water.
5. The temperature of water increases due to the work done by the masses. Mechanical work = Heat.

6. Joule found that to raise 1 g of an object by  $1^{\circ}\text{C}$ , 4.186 J of energy is required.  $1 \text{ cal} = 4.186 \text{ J}$ .

It is called Joule's mechanical equivalent of heat.

5. Derive the work done in isothermal process.

1. Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state  $(P_i, V_i)$  to the final state  $(P_f, V_f)$ .

2. W.D. by the gas  $W = \int_{V_i}^{V_f} P dV$  From the ideal gas equation,  $P = \frac{\mu R T}{V}$

3. Now,  $W = \int_{V_i}^{V_f} \frac{\mu R T}{V} dV$   $W = \mu R T \int_{V_i}^{V_f} \frac{dV}{V}$

4. By performing the integration,  $W = \mu R T \ln \left( \frac{V_f}{V_i} \right)$

5. W.D. by the gas during an isothermal expansion is positive. W.D. on the gas during an isothermal compression is negative.

6. Derive the work done in adiabatic process.

1. Consider  $\mu$  mole of an ideal gas is enclosed in a cylinder having perfectly non conducting walls and base. A frictionless insulating piston of cross sectional area  $A$  is fitted in the cylinder.

2. Let  $W$  be the work done when the system goes from the initial state  $(P_i, V_i, T_i)$  to the final state  $(P_f, V_f, T_f)$  adiabatically.  $W = \int_{V_i}^{V_f} P dV$

3.  $P = \text{constant} / V^\gamma$  So,  $W = \int_{V_i}^{V_f} \text{constant} dV / V^\gamma$

4.  $W = \text{constant} \int_{V_i}^{V_f} V^{-\gamma} dV = \text{constant} \left[ \frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_i}^{V_f}$

5.  $\frac{1}{1-\gamma} \left[ \frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right]$

6. Substituting the value of constant as  $P_i V_i^\gamma = P_f V_f^\gamma$   $W = \frac{1}{1-\gamma} [P_f V_f - P_i V_i]$

7. From ideal gas law,  $P_f V_f = \mu R T_f$  and  $P_i V_i = \mu R T_i$  So,  $W_{\text{adia}} = \frac{\mu R}{\gamma-1} [T_i - T_f]$

7. Derive the expression for Carnot engine efficiency.

$\eta = \frac{\text{Work done}}{\text{Heat extracted}} = \frac{W}{Q_H}$  From the I law of thermodynamics,  $W = Q_H - Q_L$

$\eta = Q_H - Q_L / Q_H = 1 - \frac{Q_L}{Q_H}$

After applying isothermal and adiabatic conditions and then divide,  $\left[\frac{V_2}{V_1}\right]^{\gamma-1} = \left[\frac{V_3}{V_4}\right]^{\gamma-1}$

Which implies that  $\frac{V_2}{V_1} = \frac{V_3}{V_4}$  Substituting this in the equation  $\frac{Q_L}{Q_H} = \frac{T_L \ln\left(\frac{V_3}{V_4}\right)}{T_H \ln\left(\frac{V_2}{V_1}\right)}$

$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$  So, the efficiency is  $\eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$

### LONG ANSWERS – FIVE MARKS

#### 1. Explain in detail Newton's law of cooling.

**Statement:** The rate of loss of heat of a body is directly proportional to the difference in the temperature between that body and its surroundings.  $\frac{dQ}{dt} \propto (T - T_s) \rightarrow 1$

Here T – temperature of object and  $T_s$  – temperature of the surroundings

- ve sign indicates that the quantity of heat lost by liquid goes on decreasing with time.

1. Consider an object of mass 'm' and specific heat capacity 's'. If the temperature falls by a small amount dT in time dt, then the amount of heat loss is  $dQ = m s dT \rightarrow 2$

2. Dividing both sides by equation by dt,  $\frac{dQ}{dt} = m s \frac{dT}{dt} \rightarrow 3$

3. From Newton's law of cooling,  $\frac{dQ}{dt} \propto (T - T_s) \Rightarrow \frac{dQ}{dt} = -a (T - T_s) \rightarrow 4$  a - positive constant

4. From 3 and 4,  $-a (T - T_s) = m s \frac{dT}{dt}$  or  $\frac{dT}{T - T_s} = -\frac{a}{m s} dt \rightarrow 5$

5. Integrating equation 5 on both sides,  $\int \frac{dT}{T - T_s} = -\int \frac{a}{m s} dt \Rightarrow \ln(T - T_s) = -\frac{a}{m s} t + b_1$

$b_1$  – constant of integration

6. Taking exponentials both sides,  $T = T_s + b_2 e^{\frac{a t}{m s}}$  Here  $b_2 = e^{b_1} = \text{constant}$ .

#### 2. Derive Meyer's relation for an ideal gas.

1. Consider  $\mu$  mole of an ideal gas in a container with volume V, pressure P and temperature T.

When the gas is heated at constant volume, the temperature increases by dT.

2. No work is done by the gas. The heat flows into the system will increase only the internal energy. Let the change in internal energy be dU.

3. If  $C_v$  is the molar specific heat capacity at constant volume,  $dU = \mu C_v dT \rightarrow 1$

4. If the gas is heated at constant pressure, the temperature increases by dT. If 'Q' is the heat supplied in this process and dV is the change in volume of the gas, then  $Q = \mu C_p dT \rightarrow 2$

5. If W is the work done by the gas, then  $W = P dV \rightarrow 3$  From the I law of thermodynamics,  $Q = dU + W \rightarrow 4$

6. Substituting 1,2, and 3 in 4,  $\mu C_p dT = \mu C_v dT + P dV \rightarrow 5$

The equation of state of ideal gas is  $P V = \mu R T$  Differentiating,  $P dV + V dP = \mu R dT$

7. Since the pressure is constant,  $dP = 0$ ,  $\mu C_p dT = \mu C_v dT + \mu R dT$

$C_p = C_v + R$  or  $C_p - C_v = R$  This is called Meyer's relation.

#### 3. Explain in detail Carnot heat engine.

Carnot engine has four parts. 1. Source 2. Sink 3. Insulating stand 4. Working substance

**Carnot's cycle:** The working substance is subjected to four successive reversible processes.

#### Quasi static isothermal expansion

1. The cylinder is placed on the source.
2. Process is isothermal. Internal energy of the working substance will not change.
3. W.D. by the gas in expanding from volume  $V_1$  to  $V_2$ . Pressure decreases from  $P_1$  to  $P_2$ .
4. This is represented by the path AB.

$$W_{A \rightarrow B} = \int_{v_1}^{v_2} P dV \quad (\text{W.D. by the gas})$$

5. W.D. in the isothermal expansion is

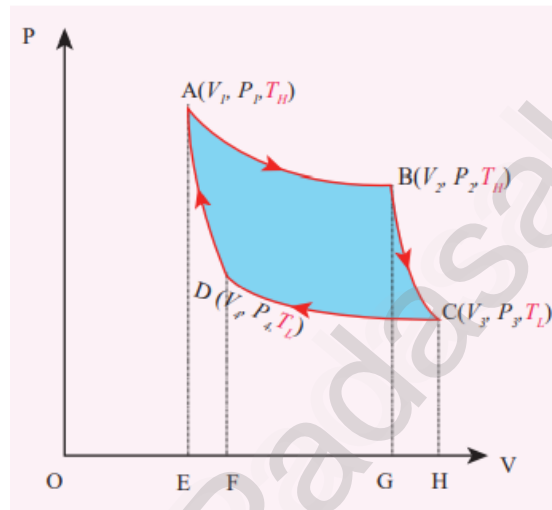
$$W_{A \rightarrow B} = \mu R T_H \ln \left( \frac{v_2}{v_1} \right) = \text{area under curve AB}$$

#### Quasi static adiabatic expansion

1. The cylinder is placed on the insulating stand.
2. Process is adiabatic. Piston is allowed to move out.
3. Gas expands from volume  $V_2$  to  $V_3$ . Pressure falls from  $P_2$  to  $P_3$ .
4. This is represented by the path BC.

$$W_{A \rightarrow B} = \int_{v_2}^{v_3} P dV \quad (\text{W.D. by the gas})$$

5.  $W_{A \rightarrow B} = \frac{\mu R}{\gamma - 1} [T_H - T_L] = \text{Area under curve BC}$



#### Quasi static isothermal compression

1. The cylinder is placed on the sink. The gas is isothermally compressed.
2. Pressure and Volume become  $P_4$  and  $V_4$  respectively.
3. This is represented by the curve CD.

$$W_{C \rightarrow D} = \int_{V_3}^{V_4} P dV = \mu R T_L \ln \left( \frac{v_4}{v_3} \right) = -\mu R T_L \ln \left( \frac{v_3}{v_4} \right)$$

= - area under curve CD. Work is done on the gas.

#### Quasi static adiabatic compression

1. The cylinder is placed on the insulating stand and the gas is compressed adiabatically.
2. Pressure and Volume attains the initial value of  $P_1$  and  $V_1$ .
3. This is represented by the curve DA.

$$W_{D \rightarrow A} = \int_{V_4}^{V_1} P dV = -\frac{\mu R}{\gamma - 1} [T_L - T_H]$$

= Area under the curve DA.

Note: 1. Net work done by the Carnot engine in one cycle =  $W = W_{A \rightarrow B} - W_{C \rightarrow D}$

2. Net work done by the gas in one cycle is equal to the area ABCD of the P-V diagram.

3. The change in internal energy of the working substance after one cycle is zero.

**IT IS VERY EASY TO DEFEAT SOMEONE,**

**BUT IT IS VERY HARD TO WIN SOMEONE – A.P.J.**

## UNIT: 9 KINETIC THEORY OF GASES

## EM EASY GUIDE

## SHORT ANSWERS – TWO MARKS

1. What is microscopic origin of pressure and temperature?

From the kinetic theory of gases, pressure is linked to the velocity of the molecules.

$$P = \frac{1}{3} \frac{N}{V} m \overline{v^2} \quad m - \text{mass of a molecule} \quad N - \text{Avogadro number} \quad V - \text{volume}$$

$\overline{v^2}$  - Avogadro velocity of molecules

$$\text{Average K.E. / molecule} \quad \text{K.E.} = \epsilon = \frac{3}{2} N k T$$

2. Why moon has no atmosphere?

The escape speed of gases on the surface of Moon is much less than the rms speed of gases due to low gravity.

Due to this, all the gases escape from the surface of the Moon.

3. Write the expressions for rms speed, average speed and most probable speed of a gas molecule.

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad V_{\text{ave}} = \sqrt{\frac{8RT}{\pi m}} \quad V_{\text{mp}} = \sqrt{\frac{2RT}{M}}$$

4. What is the relation between the average kinetic energy and pressure?

The internal energy of the gas  $U = \frac{3}{2} NkT$  Also,  $U = \frac{3}{2} PV$  As  $PV = NkT$ ,

$$P = \frac{2}{3} \frac{U}{V} = \frac{2}{3} u \quad \text{Pressure is equal to } 2/3 \text{ of mean K.E. per unit volume.} \quad P = \frac{2}{3} \left( \frac{\rho}{2} \overline{v^2} \right)$$

5. Define the term degrees of freedom.

The minimum number of independent coordinates needed to specify the position and configuration of a thermodynamical system in space .

6. State the law of equipartition of energy.

According to kinetic theory, the average K.E. of system of molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom (x,y or z directions of motion ) so that each degree of freedom will get  $\frac{1}{2} kT$  of energy.

7. Define mean free path and write down its expression.

Average distance travelled by the molecule between collisions is called mean free path.  $\lambda = \frac{kT}{\sqrt{2} \pi d^2 P}$

8. State Boyle's law.

The pressure of a given gas is inversely proportional to its volume provided the temperature remains constant.

$$P V = \frac{2}{3} N \epsilon \quad P V = \text{constant}$$

9. State Charle's law.

For a fixed pressure, the volume of the gas is proportional to internal energy of the gas or average K.E. of the gas and the average K.E. is directly proportional to absolute temperature.  $V \propto T$  or  $\frac{V}{T} = \text{constant}$ .

10. State Avogadro's law.

At constant temperature and pressure, equal volumes of all gases contain the same number of molecules.

At the same temperature, average kinetic energy per molecule is the same for two gases.

### 11. List the factors affecting mean path.

Mean free path increases with increasing temperature.

Mean free path increases with decreasing pressure of the gas and diameter of the gas molecules.

#### THREE MARKS

#### 1. Explain in detail the kinetic interpretation of temperature.

$$1. P = \frac{1}{3} nm \overline{v^2} \quad PV = \frac{1}{3} Nm \overline{v^2} \rightarrow 1$$

$$2. \text{Ideal gas equation, } PV = N k T \rightarrow 2$$

$$3. \text{From 1 and 2, } N k T = \frac{1}{3} Nm \overline{v^2} \quad k T = \frac{1}{3} m \overline{v^2} \rightarrow 3$$

$$4. \text{Multiply both sides of equation 3 by } 3/2, \quad \frac{3}{2} k T = \frac{1}{2} m \overline{v^2}$$

$$5. \frac{1}{2} m \overline{v^2} \text{ is called the average kinetic energy of a single molecule. ( } \overline{KE} \text{ )}$$

$$6. \overline{KE} = \epsilon = \frac{3}{2} k T \quad \text{Temperature of a gas is a measure of the average translational K.E. per molecule of the gas.}$$

$\epsilon \propto T$  and also  $\epsilon$  is independent of mass of the molecule.

$$7. \text{Internal energy } U = N \times \overline{KE} = N \left( \frac{1}{2} m \overline{v^2} \right) = \frac{3}{2} N k T$$

U depends on absolute temperature and is independent of pressure and volume.

#### 2. Obtain the relation between pressure and mean K.E.

$$1. \text{The internal energy of the gas is } U = \frac{3}{2} N k T \quad \text{or } U = \frac{3}{2} P V \rightarrow 1$$

$$2. \text{Internal energy density } u = U / V \text{ (Internal energy per unit volume)}$$

$$3. \text{From equation 1, } P = \frac{2U}{3V} = \frac{2}{3} u \quad \text{Pressure of the gas = two thirds of internal energy density.}$$

$$4. \text{Pressure in terms of mean K.E. is } P = \frac{1}{3} nm \overline{v^2} = \frac{1}{3} \rho \overline{v^2} \rightarrow 2 \quad \text{Here } \rho = nm \text{ is the mass density}$$

$$5. \text{Multiply and divide R.H.S. of equation 2 by } 2, \text{ we get } P = \frac{2}{3} \times \frac{\rho}{2} \overline{v^2} = \frac{2}{3} \times \overline{KE}$$

Pressure is equal to 2/3 of mean kinetic energy per unit volume.

#### LONG ANSWERS – FIVE MARKS

#### 1. Derive an expression for the pressure exerted by a gas.

1. Consider a monoatomic gas of N molecules each having a mass m inside a cubical container of side l.

As the molecules collide, there is no loss of energy but change in momentum occurs.

2. The force experienced per unit area of the walls of the container determines the pressure exerted by the gas.

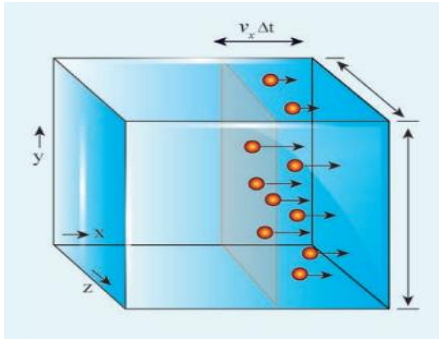
It is essential to determine the total momentum transferred by the molecules in a short interval of time.

3. Of all the molecules inside the container, one half of the n molecules move to the right and other half moves towards left side. Here, n – number density of the molecules.

4. The number of molecules that hit the right side wall in a time interval  $\Delta t$  is  $\frac{n}{2} Av_x \Delta t$

$$\text{The total momentum transferred } \Delta P = \frac{n}{2} Av_x \Delta t \times 2 m v_x \quad \Delta P = A v_x^2 mn \Delta t$$





5. From Newton's II law, change in momentum gives rise to force.

$$F = \frac{\Delta P}{\Delta t} = nm Av_x^2$$

6. Pressure  $P = \frac{F}{A} = nm v_x^2$  Since the molecules move randomly, they do not have the same speed.

Replace  $v_x^2$  by the average  $\overline{v_x^2}$ ,  $P = nm \overline{v_x^2}$

7. As the gas is assumed to move randomly, it has no preferred direction of motion. Effect of gravity is also neglected. All the molecules have same average speed in all the three directions.

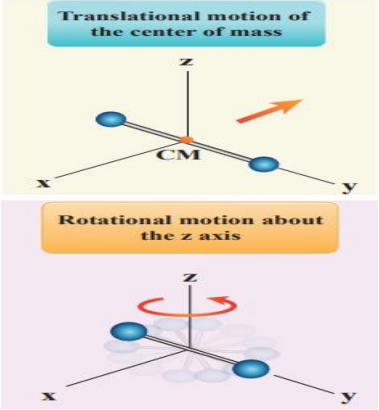
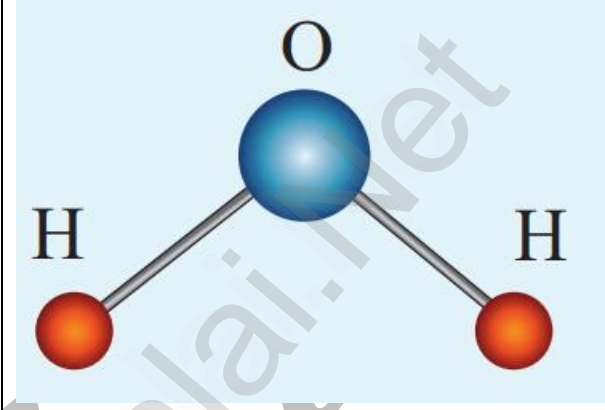
$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

The mean square speed is  $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3 \overline{v_x^2}$   $\overline{v_x^2} = \frac{1}{3} \overline{v^2}$

8. Now the expression for pressure becomes,  $P = \frac{1}{3} nm \overline{v^2} = \frac{1}{3} \frac{N}{V} m \overline{v^2}$  as  $n = \frac{N}{V}$

2. Describe the total degrees of freedom for mono atomic, di atomic and tri atomic molecule.

Mono atomic molecule	Di atomic molecule	Tri atomic molecule
By virtue of its nature, it has 3 translational degrees of freedom. $f = 3$ Example: Helium, Neon, Argon	At normal temperature: It is a system of two point masses fixed at the ends of a massless elastic spring. It has 3 translational degrees. Due to rotation about three mutually perpendicular axis, it has two rotational degrees. Moment of inertia about its own axis of rotation is negligible. $f = 5$	Linear tri atomic molecule At normal temperature, it has three translational degrees and two rotational degrees of freedom. $f = 5$ At high temperature, it has two additional degrees of freedom due to vibration. $f = 7$ Example: CO <sub>2</sub>

	<u>At high temperature</u>	<u>Non linear tri atomic molecule</u>
	At 5000 K, the molecule will have two vibrational degrees. ( one K.E. and one P.E. ) $f = 7$ Example: Hydrogen, Nitrogen and Oxygen	Three atoms at the vertices of a triangle. It has three translational degrees and two rotational degrees about three mutually orthogonal axes. $f = 6$ Example: Water, Sulphur di oxide
	 <p>Translational motion of the center of mass</p> <p>Rotational motion about the z axis</p>	

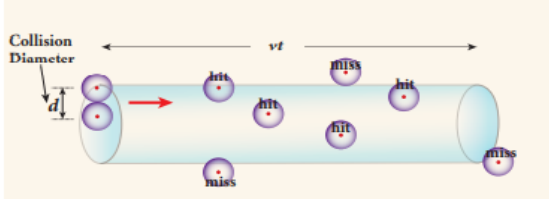
3. Derive the ratio of two specific heat capacities of mono, di, tri atomic molecules.

	Mono atomic molecule	Di atomic molecule	Tri atomic molecule
Average K.E.	$\frac{3}{2} kT$	At low temperature $\frac{5}{2} kT$ At high temperature $\frac{7}{2} kT$	Linear molecule $\frac{7}{2} kT$ Non linear molecule $\frac{6}{2} kT = 3 kT$
Total energy of a mole of a gas U	$\frac{3}{2} kT \times N_A = \frac{3}{2} RT$	At low temperature $\frac{5}{2} kT \times N_A = \frac{5}{2} RT$ At high temperature $\frac{7}{2} kT \times N_A = \frac{7}{2} RT$	Linear molecule $\frac{7}{2} kT \times N_A = \frac{7}{2} RT$ Non linear molecule $3 kT \times N_A = 3 RT$
For one mole Molar specific heat at constant volume $C_V = \frac{dU}{dt}$	$\frac{3}{2} R$	At low temperature $\frac{5}{2} R$ At high temperature $\frac{7}{2} R$	Linear molecule $\frac{7}{2} R$ Non linear molecule $3 R$
$C_P = C_V + R$	$\frac{3}{2} R + R = \frac{5}{2} R$	At low temperature $\frac{5}{2} R + R = \frac{7}{2} R$ At high temperature $\frac{7}{2} R + R = \frac{9}{2} R$	Linear molecule $\frac{7}{2} R + R = \frac{9}{2} R$ Non linear molecule $3 R + R = 4 R$
Ratio of specific heats $\gamma = C_P / C_V$	$\frac{5}{3} = 1.67$	At low temperature $\frac{7}{5} = 1.40$ At high temperature $\frac{9}{7} = 1.28$	Linear molecule $= \frac{9}{7} = 1.28$ Non linear molecule $= \frac{4}{3} = 1.33$

4. Derive an expression for mean free path of the gas.

1. Consider a system of molecules each with diameter 'd'. Let 'n' be the number of molecules per unit volume.

2. Assume that only one molecule is in motion and all others at rest.



3. During a time interval 't' the molecule travels a distance vt in an imaginary cylinder of diameter  $\pi d^2 vt$ .

4. Now, mean free path = distance travelled / No of collisions

$$\lambda = \frac{vt}{n \pi d^2 vt} = \frac{1}{n \pi d^2} \rightarrow 1$$

( No of collisions = No of molecules in the given volume )

5. From the kinetic theory of gases, all the molecules are in random motion. After a detailed calculation, the correct

expression for mean free path is  $\lambda = \frac{1}{\sqrt{2} n \pi d^2} \rightarrow 2$  Mean free path is inversely proportional to number density.

6. In terms of mass of the molecules,  $\lambda = \frac{m}{\sqrt{2} n \pi d^2}$  As  $mn = \rho$ ,  $\lambda = \frac{m}{\sqrt{2} \rho \pi d^2}$

7. In terms of Pressure and absolute temperature,  $n = \frac{P}{kT}$  Substituting 'n' in 2,  $\lambda = \frac{kT}{\sqrt{2} \pi d^2 P}$

**KNOWLEDGE WILL GIVE YOU THE POWER  
BUT CHARACTER WILL  
GIVE YOU RESPECT – Dr. A.P.J.**

## UNIT: 10 OSCILLATIONS

## EM EASY GUIDE

## SHORT ANSWERS – TWO MARKS

1. What are Periodic and non periodic motion? Give example.

Any motion which repeats itself in a fixed time interval is known as periodic motion.

Example: Hands in pendulum clock, revolution of the Earth around the Sun, swing of a cradle, waxing and waning of Moon etc.,

Any motion which does not repeat itself after a regular interval of time is known as non periodic motion.

Example: Occurrence of Earth quake, eruption of volcano etc.,

2. What is oscillatory motion? Give example.

When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory ( or vibratory ) .

Example: Our heart beat, Swinging motion of the wings of an insect, pendulum clock.

3. What is Simple Harmonic Motion?

It is a special type of oscillatory motion in which the acceleration or force on the particle is directly Proportional to its displacement from a fixed point and is always directed towards the fixed point.

4. Define (i) time period (ii) frequency

(i) Time period: The time taken by a particle to complete one oscillation.  $T = 2\pi / \omega$

(ii) Frequency: The number of oscillations produced by the particle per second.  $f = 1 / T$  Unit:  $s^{-1}$  or Hz

5. Define phase and phase difference.

Phase: The phase of a vibrating particle at any instant completely specifies the state of the particle. It expresses the position and direction of motion of the particle at that instant with respect to its mean position.

$y = A \sin (\omega t + \phi_0)$  where  $\phi = \omega t + \phi_0$  is the phase of the vibrating particle.

Phase difference: The equations of two particles executing SHM are

$y_1 = A \sin (\omega t + \phi_1)$  and  $y_2 = A \sin (\omega t + \phi_2)$  .

The phase difference  $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1)$

## THREE MARKS

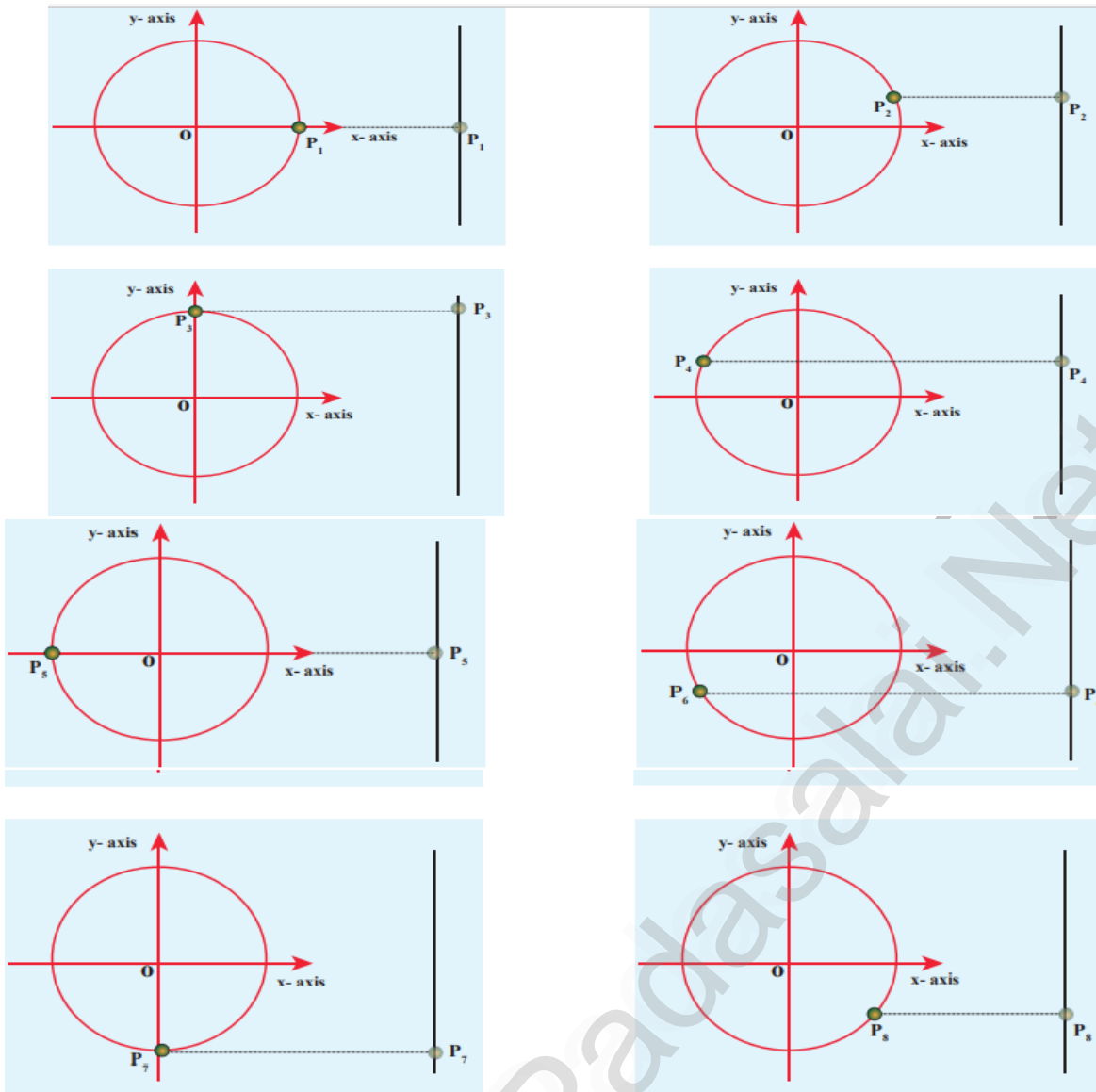
1. Projection of Uniform Circular Motion:

1. Consider a particle of mass  $m$  moving with uniform speed  $v$  along the circumference of a circle of radius  $r$  in anticlockwise direction.

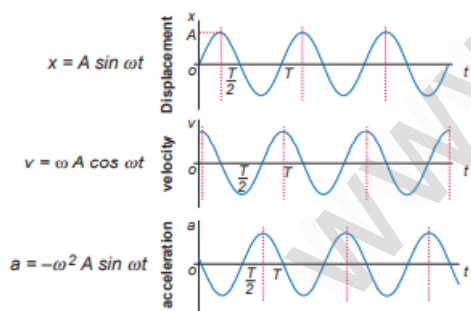
2. The origin of the coordinate system coincides with the centre  $O$  of the circle. If  $\omega$  is the angular velocity of the particle and  $\theta$  is the angular displacement of the particle at any instant  $t$ , then  $\theta = \omega t$ .

3. If a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle ( or a line parallel to the diameter ) traces straight line motion which is SHM in nature.

4. The circle is known as the reference circle of the SHM. SHM is also defined as the motion of the projection of a particle on any diameter of a circle of reference.



2. Discuss the graphical representation of displacement, velocity and acceleration in SHM.



At the mean position,  $y=0$ , the velocity of the particle is maximum.

But the acceleration of the particle is zero.

At the extreme position ( $y = \pm A$ ) the velocity of the particle is zero.

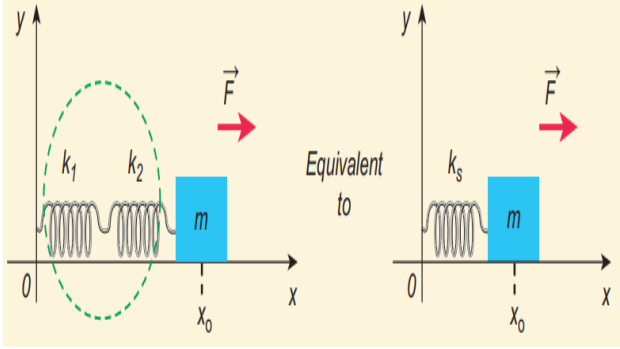
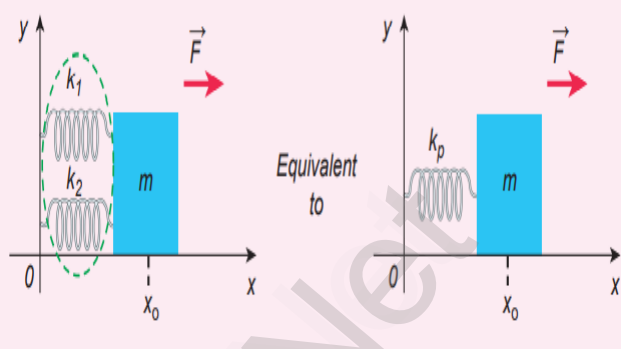
But the acceleration is maximum  $\mp A\omega^2$  acting in the opposite direction.

Velocity is ahead of displacement by  $\frac{\pi}{2}$  and acceleration is ahead of velocity

by  $\frac{\pi}{2}$ . But acceleration is ahead of displacement by a phase angle of  $\pi$ .

Time	0	$\frac{T}{4}$	$\frac{2T}{4}$	$\frac{3T}{4}$	$\frac{4T}{4} = T$
$\omega t$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
Displacement $y = A \sin \omega t$	0	A	0	-A	0
Velocity $v = A \omega \cos \omega t$	$A \omega$	0	$-A \omega$	0	$A \omega$
Acceleration $a = -A \omega^2 \sin \omega t$	0	$-A \omega^2$	0	$A \omega^2$	0

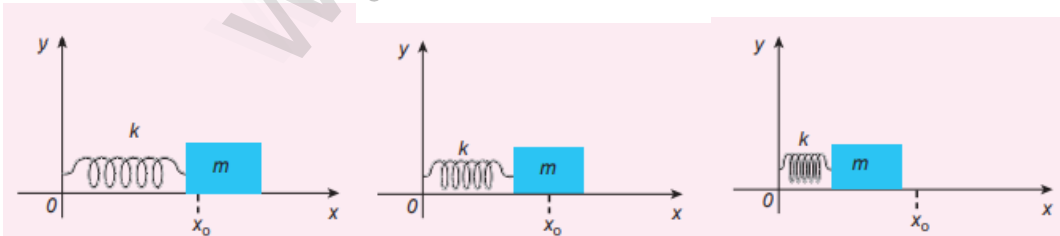
### 3. Derive an expression for effective spring constant of combination of springs in series and parallel.

S.No	Springs in series	Springs in parallel
1.	When two or more springs are connected in series, we can replace all the springs in series with an effective spring whose net effect is the same as if all the springs are in series connection.	1. When two or more springs are connected in parallel, we can replace all the springs in parallel with an effective spring whose net effect is the same as if all the springs are in parallel connection.
2.		
3.	Consider two springs of spring constants $k_1$ and $k_2$ in series combination attached to a mass $m$ .	3. Consider two springs of spring constants $k_1$ and $k_2$ in parallel combination attached to a mass $m$ .
4.	Due to the force $F$ applied towards right, let $x_1$ and $x_2$ be the elongations of springs from equilibrium position. $x = x_1 + x_2$	4. Due to the force $F$ applied towards right, both the springs elongate or compress by the same amount of displacement.
5.	From Hooke's law, net force $F = -k_s(x_1 + x_2)$ or $x_1 + x_2 = -\frac{F}{k_s} \rightarrow 1$ where $k_s$ is called effective spring constant	5. From Hooke's law, net force is $F = -k_p x \rightarrow 1$ where $k_p$ is called effective spring constant.
6.	$x_1 = -F/k_1$ $x_2 = -F/k_2$ Substituting in 1, $-F/k_1 - F/k_2 = -F/k_s$ or $\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$ or $k_s = k_1 k_2 / k_1 + k_2$	6. $F = -k_1 x - k_2 x$ Substituting this in 1, $k_p = k_1 + k_2$
7.	For 'n' springs in series $\frac{1}{k_s} = \sum_{i=1}^n \frac{1}{k_i}$	7. For 'n' springs in parallel, $k_p = \sum_{i=1}^n k_i$

### LONG ANSWERS – FIVE MARKS

#### 1. Explain the horizontal oscillations of a spring. ( Linear Harmonic Oscillator )

1. Consider a system containing a block of mass  $m$  attached to a massless spring with force constant  $k$  placed on a smooth horizontal frictionless surface. Let  $x_0$  be the mean position of mass  $m$  when it is left undisturbed.
2. Suppose the mass is displaced through a small displacement  $x$  towards right from its mean position and then released, it will oscillate back and forth about its mean position  $x_0$ .



3. Let  $F$  be the restoring force which is proportional to the amount of displacement of block. For one dimensional motion,  $F \propto x$      $F = -kx \rightarrow 1$  - ve sign shows that restoring force is opposite to the direction of displacement.  
( By Hooke's law )

4. From Newton's second law, for a particle executing SHM,  $m \frac{d^2x}{dt^2} = -kx$   $\frac{d^2x}{dt^2} = -\frac{k}{m}x \rightarrow 2$

5. We know that in SHM,  $a = \frac{d^2x}{dt^2} = -\omega^2 x \rightarrow 3$

6. On comparing 2 and 3,  $\omega^2 = \frac{k}{m}$   $\omega = \sqrt{\frac{k}{m}}$  rad s<sup>-1</sup> where  $\omega$  – angular frequency of the oscillator.

7. The frequency of the oscillation is  $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$  Hz

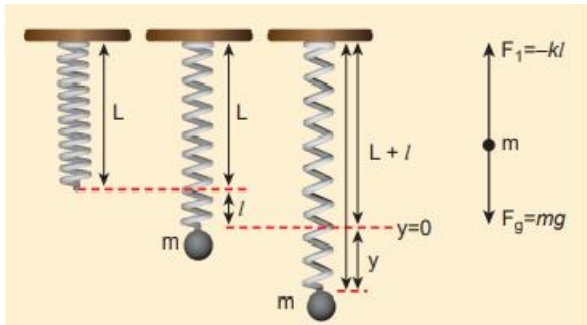
Time period of oscillation is  $T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$  second.

## 2. Describe the vertical oscillations of a spring.

1. Let us consider a massless spring with force constant  $k$  attached to a ceiling. Let the length of the spring before loading mass  $m$  be  $L$ .

2. If a block of mass  $m$  is attached to the other end of the spring, then the spring elongates by a length  $l$ .

3. Let  $F_1$  be the restoring force. Gravitational force acts vertically downward.



When the system is under equilibrium,

$$F_1 + mg = 0 \rightarrow 1 \quad \text{But } F_1 \propto l \quad F_1 = -kl \rightarrow 2$$

$$4. \text{ Substitute 2 in 1, } -kl + mg = 0 \quad mg = kl$$

$$\frac{m}{k} = \frac{l}{g} \rightarrow 3$$

5. When we apply a very small external force on the

mass such that the mass further displaces downward by  $y$

it will oscillate up and down. Now for the total extension  $y+l$ , restoring force  $F_2 \propto (y+l)$

$$F_2 = -k((y+l)) = -ky - kl \rightarrow 4$$

6. From Newton's second law,  $F_2 + mg = m \frac{d^2y}{dt^2} \rightarrow 5$  (from the free body diagram) or

$$\text{The net force acting on the mass due to this stretching is } F = F_2 + mg = -ky - kl + kl = -ky \rightarrow 6$$

7. Substituting 6 in 5,  $m \frac{d^2y}{dt^2} = -ky$   $\frac{d^2y}{dt^2} = -\frac{k}{m}y \rightarrow 7$  It is in the form of simple harmonic differential equation.

We know that in SHM,  $a = \frac{d^2y}{dt^2} = -\omega^2 y \rightarrow 8$

8. From 7 and 8,  $\omega^2 = \frac{k}{m}$   $\omega = \sqrt{\frac{k}{m}}$  rad s<sup>-1</sup> where  $\omega$  – angular frequency of the oscillator.

The frequency of the oscillation is  $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$  Hz

Time period of oscillation is  $T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$  second also from 3,  $T = 2\pi \sqrt{\frac{l}{g}}$

## 3. Discuss the simple pendulum in detail.

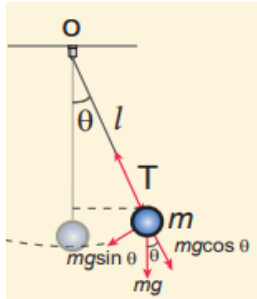
1. A simple pendulum has a bob with mass  $m$  suspended by a long string and the other end is fixed on a stand.

At equilibrium, the pendulum does not oscillate and hangs vertically downward. It is the mean position.

2. When the pendulum is displaced through a small displacement from its mean position and released, the bob

executes to and fro motion. Let  $l$  be the length of the pendulum.

3. The two forces acting on the bob are (i) gravitational force  $\vec{F} = m \vec{g}$  (vertically downwards)  
(ii) tension in the string  $\vec{T}$  (along the string towards the point of suspension)



4. Gravitational force is resolved into

a) Normal component - along the string but opposite to the direction of tension

$$F_{as} = mg \cos \theta$$

b) Tangential component - perpendicular to the string  $F_{ps} = mg \sin \theta$

5. Now,  $T - F_{as} = m \frac{v^2}{l}$        $T - mg \cos \theta = m \frac{v^2}{l} \rightarrow$  This makes the bob to oscillate.

As the tangential force is the restoring force, from Newton's second law,

$$m \frac{d^2 s}{dt^2} + F_{ps} = 0 \quad m \frac{d^2 s}{dt^2} = - F_{ps} \quad m \frac{d^2 s}{dt^2} = - mg \sin \theta \rightarrow 1$$

6. Expressing the arc length 's' in terms of angular displacement ' $\theta$ ',  $s = l \theta$

$$\text{The acceleration is } \frac{d^2 s}{dt^2} = l \frac{d^2 \theta}{dt^2} \rightarrow 2$$

7. Substituting 2 in 1,  $l \frac{d^2 \theta}{dt^2} = - g \sin \theta$        $\frac{d^2 \theta}{dt^2} = - \frac{g}{l} \sin \theta$  If  $\theta$  is small, then  $\sin \theta \approx \theta$

$$\text{So, } \frac{d^2 \theta}{dt^2} = - \frac{g}{l} \theta \rightarrow 3$$

8. We know that in angular SHM,  $a = \frac{d^2 \theta}{dt^2} = - \omega^2 \theta \rightarrow 4$

$$\text{From 3 and 4, } \omega^2 = \frac{g}{l} \quad \omega = \sqrt{\frac{g}{l}} \text{ rad s}^{-1} \text{ where } \omega - \text{angular frequency of the oscillator.}$$

9. The frequency of the oscillation is  $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \text{ Hz}$

10. Time period of oscillation is  $T = \frac{1}{f} = 2\pi \sqrt{\frac{l}{g}}$  second

4. Discuss in detail the energy in simple harmonic motion.

a) EXPRESSION FOR POTENTIAL ENERGY:

For SHM, force and displacement are related by Hooke's law as  $\vec{F} = -k \vec{r}$

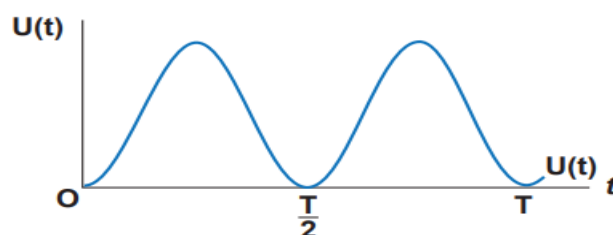
In One dimensional motion,  $F = -k x$       But  $F = -dU/dx$       Now,  $dU = k x dx$

W.D. by the force  $F$  during a small displacement  $dx$  stores as potential energy.

$$U(x) = \int_0^x k x^1 dx = \frac{1}{2} k [(x)^2]_0^x = \frac{1}{2} k x^2 \quad \text{As } k = m \omega^2, \quad U(x) = \frac{1}{2} m \omega^2 x^2 \rightarrow 1$$

For a particle executing SHM,  $x = A \sin \omega t$        $U(t) = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t \rightarrow 2$

The variation of  $U$  with time is





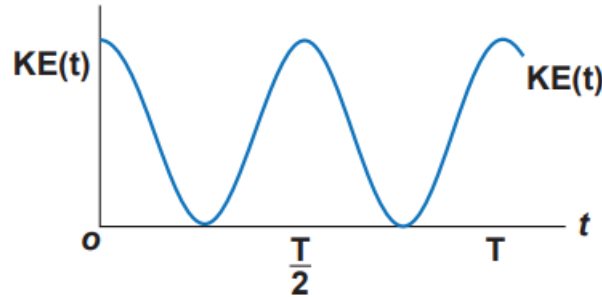
## b) EXPRESSION FOR KINETIC ENERGY:

$$\text{Kinetic energy } KE = \frac{1}{2} m v_x^2 = \frac{1}{2} m \left( \frac{dx}{dt} \right)^2 \quad \text{But } x = A \sin \omega t$$

$$\text{Therefore, velocity is } v_x = \frac{dx}{dt} = A \omega \cos \omega t = A \omega \sqrt{1 - \left( \frac{x}{A} \right)^2} = \omega \sqrt{A^2 - x^2}$$

$$\text{Hence, } KE = \frac{1}{2} m v_x^2 = \frac{1}{2} m \omega^2 (A^2 - x^2) \rightarrow 3 \quad \text{Also } KE = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t \rightarrow 4$$

The variation of KE with time is



## c) EXPRESSION FOR TOTAL ENERGY:

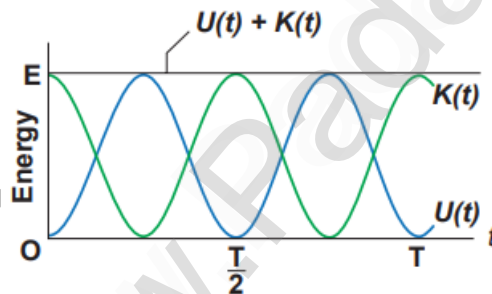
1. Total energy is the sum of kinetic energy and potential energy.  $E = KE + U$

$$E = \frac{1}{2} m \omega^2 (A^2 - x^2) + \frac{1}{2} m \omega^2 x^2 \quad (1 + 3) \quad \text{Now, } E = \frac{1}{2} m \omega^2 A^2 = \text{constant} \rightarrow 5$$

$$\text{Similarly on adding 2 and 4, } E = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t + \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t = \frac{1}{2} m \omega^2 A^2 = \text{constant} \rightarrow 6$$

since  $\sin^2 \omega t + \cos^2 \omega t = 1$

The law of conservation of total energy is proved.



**IF YOU SALUTE YOUR DUTY, YOU NO NEED TO  
SALUTE ANYBODY, BUT IF YOU  
POLLUTE YOUR DUTY, YOU HAVE TO  
SALUTE EVERYBODY - KALAM**

## UNIT: 11 WAVES

## EM EASY GUIDE

## SHORT ANSWERS

1. What is meant by waves?

The disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium.

2. Write down the types of waves.

- a) Mechanical wave – Waves which require a medium for propagation. Eg: Sound waves, ripples on water  
b) Non mechanical wave – Waves which don't require any medium for propagation. Eg: Light

3. What are transverse waves? Give one example.

The constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation of waves. Eg: Light ( Electromagnetic waves )

4. What are longitudinal waves? Give one example.

The constituent of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation of waves. Eg: Sound waves travelling in air.

5. Define wavelength.

For transverse waves, the distance between two neighbouring crests or troughs.

For longitudinal waves, the distance between two neighbouring compressions or rarefactions. S I unit is metre.

6. Write down the relation between frequency, wavelength and velocity of a wave.

Velocity  $v = f\lambda$  where,  $v$  – velocity of the wave  $f$  – frequency  $\lambda$  – wavelength

7. What is meant by interference of waves?

A phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

8. Explain the beat phenomenon.

When two or more waves superimpose each other with slightly different frequencies then a sound of periodically varying amplitude at a point is observed.

The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.  $n = |f_1 - f_2|$  per second.

9. Define intensity of sound and loudness of sound.

Intensity: The sound power transmitted per unit area taken normal to the propagation of the sound wave.

Loudness: The degree of sensation of sound produced in the ear or the perception of sound by the listener.

10. What is meant by end correction in resonance air column apparatus?

The antinodes are not exactly formed at the open end, we have include a correction, called end correction.

11. What is meant by echo? Explain.

A repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at  $20^\circ\text{C}$  is  $344\text{ m s}^{-1}$ . Persistence of hearing is estimated as  $0.1\text{ s}$

Velocity = Distance travelled / time taken =  $2d / t$  Now,  $2d = 344 \times 0.1 = 34.4\text{ m}$  So,  $d = 34.4 / 2 = 17.2\text{ m}$

The minimum distance from a sound reflecting wall to hear an echo at  $20^\circ\text{C}$  is  $17.2\text{ m}$

12. Briefly explain the difference between travelling waves and standing waves.

**Progressive waves:** Crests and troughs are formed in transverse progressive waves and compression and rarefaction are formed in longitudinal progressive waves.

These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.

All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same.

These waves carry energy while propagating.

**Stationary waves:** Crests and troughs are formed in transverse progressive waves and compression and rarefaction are formed in longitudinal stationary waves.

These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.

Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at anti nodes.

These waves do not transport energy.

13. Show that the velocity of a travelling wave produced in a string is  $v = \sqrt{\frac{T}{\mu}}$

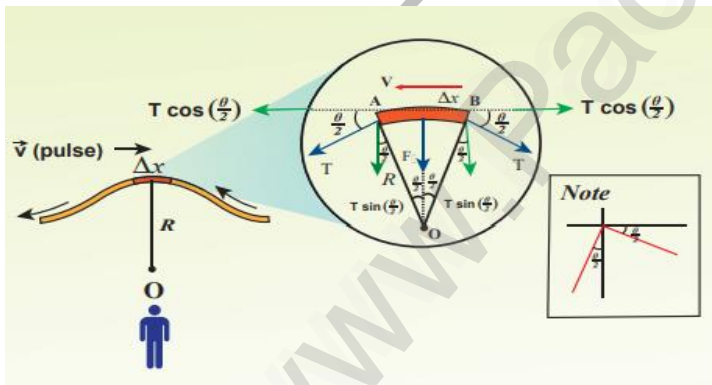
1. Consider an elemental segment in the string. Let A and B be two points on the string at an instant of time.

2. Let  $dl$  and  $dm$  be the length and mass of the elemental string respectively.

$$\text{Linear mass density } \mu = \frac{dm}{dl} \quad dm = \mu dl \rightarrow 1$$

3. From the diagram, angle = arc length / radius  $\theta = \frac{dl}{r}$  and the centripetal acceleration  $a_{CP} = \frac{v^2}{r}$

$$4. \text{ So, the centripetal force } F_{CP} = \frac{dm v^2}{R} = \frac{\mu v^2 dl}{R} \rightarrow 2$$



5. Resolving tension into its components,

The horizontal components  $T \cos \theta / 2$  equal in magnitude and opposite in direction cancel each other.

6. Vertical components  $T \sin \theta / 2$  act towards the centre of the arc get added up.

$$F_T = 2 T \sin \theta / 2 \rightarrow 3$$

$$7. \text{ As the amplitude is very small, } \sin \frac{\theta}{2} \approx \frac{\theta}{2} \quad F_T = 2 T \times \frac{\theta}{2} = T \theta = T \frac{dl}{R} \rightarrow 4 \text{ ( since } \theta = dl/R \text{ )}$$

8. From Newton's second law, radial component of force = centripetal force

$$T \frac{dl}{R} = \frac{\mu v^2 dl}{R} \quad v = \sqrt{\frac{T}{\mu}}$$

3. Describe Newton's formula for velocity of sound waves in air and also discuss Laplace's correction.

**Newton's formula:** Newton assumed sound waves propagate in air isothermally.

As compressions and rarefactions take place very slowly, the temperature of medium remains constant.

From Boyle's law,  $P V = \text{constant}$  Differentiating,  $P dV + V dP = 0$

$$P = - V \frac{dP}{dV} = K_I \quad \text{where } K_I \text{ is the isothermal bulk modulus of air.}$$

Speed of sound in air,  $v_T = \sqrt{\frac{K_I}{\rho}} = \sqrt{\frac{P}{\rho}}$  Here  $\rho$  is the density of air  $1.293 \text{ kg m}^{-3}$

At NTP, P is 76 cm of mercury So,  $P = h \rho g = 0.76 \times 13.6 \times 10^4 \times 9.8 \text{ N m}^{-2}$

On substituting the values in the formula for  $v_T$ , we get the value  $279.80 = 280 \text{ m s}^{-1}$

There is an error in the value as the actual value is  $332 \text{ m s}^{-1}$  at  $0^\circ \text{ C}$ .

Laplace's correction:

Laplace assumed that sound propagates in air adiabatically. As compressions and rarefactions occur very fast, temperature is no longer remains constant.

By Poisson's law,  $P V^\gamma = \text{constant}$  Differentiating,  $V^\gamma dP + P (\gamma V^{\gamma-1} dV) = 0$

$P \gamma = -V \frac{dP}{dV} = K_A$  where  $K_A$  is the adiabatic bulk modulus of air.

Now  $V_A = \sqrt{\frac{K_A}{\rho}} = \sqrt{\frac{P\gamma}{\rho}}$  As  $\sqrt{\frac{P}{\rho}} = 280 \text{ m s}^{-1}$  and  $\gamma = 1.4$

Here  $\gamma = C_P / C_V$  ratio of specific heats of a diatomic gas at constant pressure and constant volume.

So,  $V_A = 280 \times \sqrt{1.4} = 331.30 \text{ m s}^{-1}$  which is very close to experimental data.

#### 4. Explain the formation of stationary waves and write its characteristics.

**Stationary waves:** When a wave hits a rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern formed which are known as stationary waves.

**Explanation:** Consider two harmonic progressive waves which have the same amplitude and same velocity but move in opposite directions.

The displacement of the first wave is  $y_1 = A \sin (kx - \omega t) \rightarrow 1$  ( Incident wave moves towards right )

The displacement of the second wave  $y_2 = A \sin (kx + \omega t) \rightarrow 2$  ( reflected wave moves towards left )

By the principle of superposition,  $y = y_1 + y_2 = A \sin (kx - \omega t) + A \sin (kx + \omega t)$

Using trigonometric identity,  $y (x,t) = 2 A \cos \omega t \sin (kx) \rightarrow 3$

Here,  $2 A \sin (kx) = A^1$  is the amplitude of the wave. Now,  $y (x,t) = A^1 \cos (\omega t)$

Maximum amplitude position is known as antinode. It is given by  $x_m = \left[ \frac{2m+1}{2} \right] \frac{\lambda}{2}$  where  $m = 0,1,2,3,\dots$

Minimum amplitude position is known as node. It is given by  $x_n = n \frac{\lambda}{2}$  where,  $n = 0,1,2,3,\dots$

**Characteristics:** 1. Stationary wave does not advance in a medium, it remains steady at its place.

2. The maximum amplitude position is antinode and minimum amplitude position is node.

3. The distance between two consecutive nodes or antinodes is  $\lambda / 2$

4. The distance between a node and its neighbouring antinode is  $\lambda / 4$ .

5. The transfer of energy along the standing wave is zero.

#### 5. What is a sonometer? Explain its construction and working.

Sonometer implies sound related measurements. A device for demonstrating the relationship between tension, frequency and vibrating length.

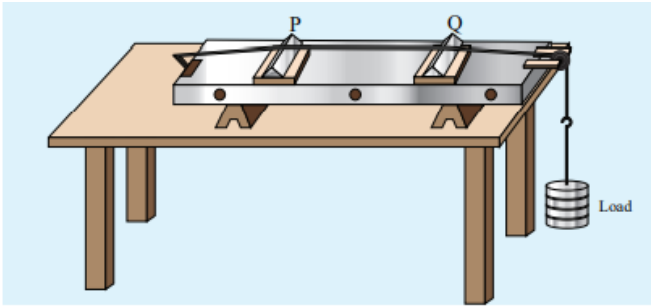
**Construction:** It consists of a hollow box of one metre long with uniform metallic thin string attached to it.

One end of the string is connected to a hook and the other end to a weight hanger through a pulley.

Weights are added to the free end to increase the tension and as one string is used it is called monochord.

By adjusting the positions of the two wooden knife edges, vibrating length can be changed.

Working:



A transverse waves stationary wave is produced in the string.

At the knife edges P and Q, nodes are formed and between the knife edges antinodes are formed.

If  $l$  is the vibrating length,  $l = \frac{\lambda}{2}$  or  $\lambda = 2l$

If  $T$  is the tension in the string and  $\mu$  is the mass per

unit length of the string, the frequency of the vibrating element is  $f = \frac{v}{\lambda} = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$  Hz.

6. Explain how overtones are formed in (i) Closed organ pipe and (ii) Open organ pipe

Sl.No	Closed organ pipe	Open organ pipe
1.	A pipe with one end closed and the other end open.	1. A pipe with both the ends open.
2.	<b>Fundamental mode:</b> The simplest mode of vibration of the air column. Anti -node is formed at the open end and node at closed end.	<b>2. Fundamental mode:</b> The simplest mode of vibration of the air column. Anti -nodes are formed at both open ends.
3.	$L = \frac{\lambda_1}{4}$ $\lambda_1 = 4L$ Frequency $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$ 	$3. L = \frac{\lambda_1}{2}$ $\lambda_1 = 2L$ Frequency $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ 
4.	$L = \frac{3\lambda_2}{4}$ $\lambda_2 = \frac{4L}{3}$ Frequency $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$ 	$4. L = \lambda_2$ $\lambda_2 = L$ Frequency $f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$ 
5.	$L = \frac{5\lambda_3}{4}$ $\lambda_3 = \frac{4L}{5}$ Frequency $f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$ 	$5. L = \frac{3\lambda_3}{2}$ $\lambda_3 = \frac{2L}{3}$ Frequency $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$ 

6.	The frequencies of harmonics are in the ratio 1:3:5	6.	The frequencies of harmonics are in the ratio 1:2:3
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7. How will you determine the velocity of sound using resonance air column apparatus?

**Use:** It is one of the simplest techniques to measure the speed of sound in air at room temperature.

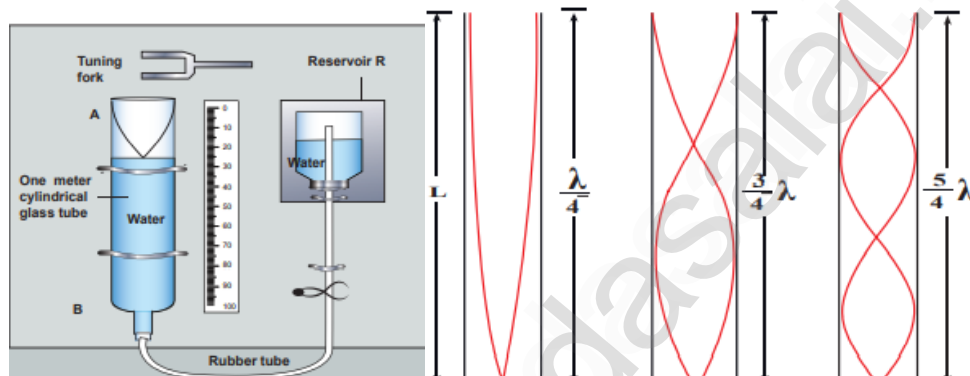
**Description:** 1. It consists of a cylindrical glass tube of one metre length whose one end A is open and other end B is connected to a water reservoir R through a rubber tube.

2. The cylindrical glass tube is mounted on a vertical stand with a scale attached to it.

3. The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir.

4. The surface of the water will act as a closed end and other as the open end. So it behaves like a closed organ pipe.

5. A vibrating tuning fork is held near the open end of the tube. Length of the air column is adjusted so that the air column resonates with the frequency of the tuning fork.



6. For the first resonance,  $\frac{\lambda}{4} = L_1$ . As the antinode is not formed at the open end, an error is included called end correction. So,  $\frac{\lambda}{4} = L_1 + e \rightarrow 1$

7. Now the length of the air column is increased to get second resonance. ,  $\frac{3\lambda}{4} = L_2 + e \rightarrow 2$

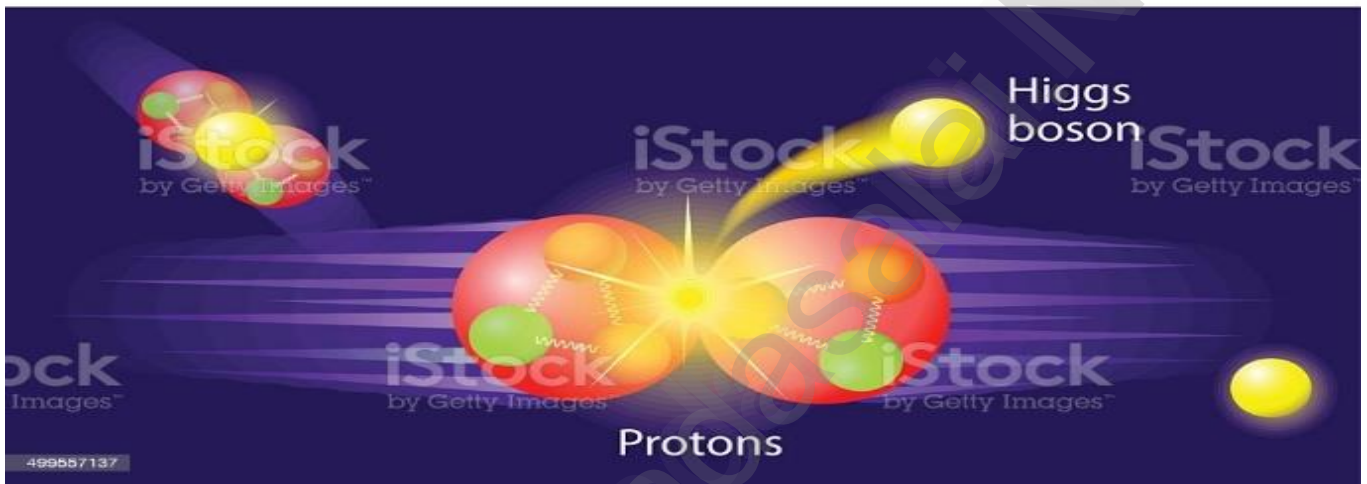
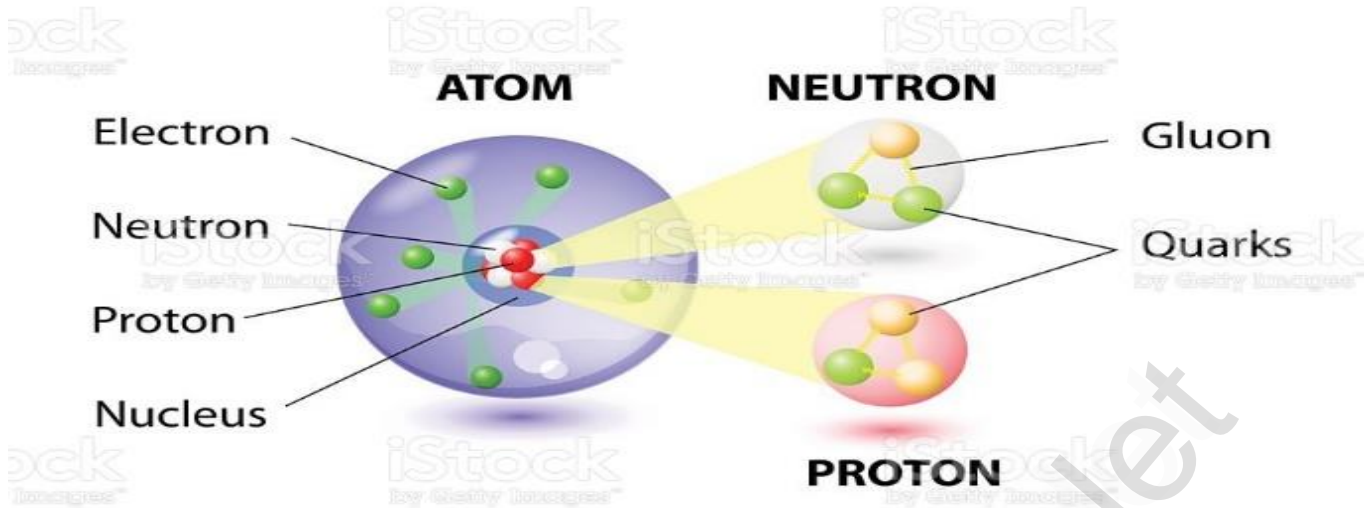
$$2 - 1, \quad \frac{3\lambda}{4} - \frac{\lambda}{4} = L_2 + e - (L_1 + e) \quad \text{Now, } \frac{\lambda}{2} = L_2 - L_1 = \Delta L \quad \text{or } \lambda = 2 \Delta L$$

8. The speed of sound in air at room temperature is  $v = f \lambda = 2 f \Delta L$

The formula to compute end correction is  $e = \frac{L_2 - 3L_1}{2}$

**THE INTELLIGENT BRAIN CANNOT STAND IN FRONT OF  
ANY OBSTACLE - A.P.J. KALAM**

# HIGGS BOSON



*NOTHING IS TOO WONDERFUL TO BE TRUE  
IF IT BE CONSISTENT WITH THE  
LAWS OF NATURE  
- MICHAEL FARADAY*