

## XI FIRST TERM ANSWER KEY

### I. Choose the correct answer.

- 1) increase 4 times
- 2) the mass of the satellite
- 3) Conservation of angular momentum
- 4) 5:7
- 5)  $\sqrt{2}v_0$
- 6) force acting on particle
- 7)  $\frac{3}{2}K$
- 8) 1:2
- 9)  $\frac{3}{2}mv^2$
- 10) need not to be zero
- 11) increase
- 12) normal force exerted by the book on the table
- 13) momentum
- 14) -z direction
- 15)  $[ML^{-1}T^0]$

### II. Two Marks.

#### 16) State Kepler's law of period.

Law of period: The square of the time period of revolution of a planet around the Sun in its elliptical orbit is directly proportional to the cube of the semi-major axis of the ellipse.

#### 17) What is the difference between sliding and slipping?

##### Sliding:

- Velocity of center of mass is greater than  $R\omega$  i.e.  $V_{CM} > R\omega$ .
- Velocity of translational motion is greater than velocity of rotational motion.
- Resultant velocity acts in the forward direction.

##### Slipping:

- Velocity of center of mass is lesser than  $R\omega$  i.e.  $V_{CM} < R\omega$
- Velocity of translation motion is lesser than velocity of rotational motion.
- Resultant velocity acts in the backward direction.

#### 18) Define couple.

A pair of forces which are equal in magnitude but opposite in direction and separated by a perpendicular distance so that their lines of action do not coincide that causes a turning effect is called a couple.

#### 19) Coefficient of restitution :

Coefficient of restitution defined as the ratio of velocity of separation (after collision) to velocity of approach (before collision)

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The coefficient of restitution =

#### 20) Power :

Power is a measure of how fast or slow a work is done. Power is defined as the rate of work done or energy delivered. Power  $P = \text{work done } W / \text{time taken } P = W/t$  SI unit Watt, di  $[ML^2T^{-3}]$

#### 21) Define one newton:

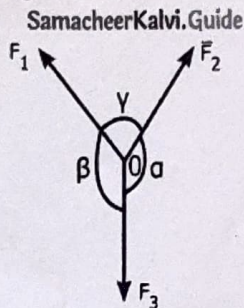
One newton is defined as the force which acts on 1 kg of mass to give an acceleration  $1 \text{ ms}^{-2}$  in the direction of the force.  $1N = kg \text{ m s}^{-2}$

## 22) Lam's a theorem:

If a system of three concurrent and coplanar forces is in equilibrium, then Lamis theorem states that " the magnitude of each force of the system is proportional to sine of the angle between other two forces".

Proof:

Let  $F_1$ ,  $F_2$  and  $F_3$  be three coplanar and concurrent forces act at a common point  $O$  as in figure.



If the point  $O$  is in equilibrium then according to Lamis theorem.

$$|\overline{F_1}| \propto \sin \alpha$$

$$|\overline{F_2}| \propto \sin \beta \quad \text{SamacheerKalvi.Guide}$$

$$|\overline{F_3}| \propto \sin \gamma$$

$$\frac{|\overline{F_1}|}{\sin \alpha} = \frac{|\overline{F_2}|}{\sin \beta} = \frac{|\overline{F_3}|}{\sin \gamma} = \text{Constant}$$

23) A particle moves in a circle of radius 10 m. Its linear speed is given by  $vt^3$  where  $t$  is in second and  $v$  is in  $\text{ms}^{-1}$ .

(a) Find the centripetal and tangential acceleration at  $t = 2$  s.

(b) Calculate the angle between the resultant acceleration and the radius vector.

The linear speed at  $t = 2$  s

$$v = 3t = 6\text{ms}^{-1}$$

The centripetal acceleration at  $t = 2$  s is

$$a_c = \frac{v^2}{r} = \frac{(6)^2}{10} = 3.6\text{ms}^{-2}$$

The tangential acceleration is  $a_t = \frac{dv}{dt} = 3\text{ms}^{-2}$

The angle between the radius vector with resultant acceleration is given by

$$\tan \theta = \frac{a_t}{a_c} = \frac{3}{3.6} = 0.833$$

$$\theta = \tan^{-1}(0.833) = 0.69 \text{ radian}$$

$$\text{In terms of degree } \theta = 0.69 \times 57.17^\circ \approx 40^\circ$$

## 24) Define the gravitational field. Give its unit.

The gravitational field intensity  $E \rightarrow 1$  at a point is defined as the gravitational force experienced by unit mass at that point. It's unit  $\text{N kg}^{-1}$ .

## III. Three Marks

## 25) State Newton's Universal law of gravitation.

The gravitational force between two masses is directly proportional to the product of masses and inversely proportional to square of the distance between the masses.

## 26) How will you measure the diameter of the Moon using parallax method?

Let  $\theta$  is the angular diameter of moon  
 $d$  - is the distance of moon from earth



Diameter of moon  $D = d \cdot \theta$

by knowing  $\theta$ ,  $d$ , diameter of moon can be calculated

**27) Find the dimensions of mass in terms of Energy, length and time.**

Let the dimensions of Energy, Length and Time be  $[E], [L], [T]$  respectively.

We know that Force = mass  $\times$  acceleration

$$\text{Mass} = \frac{\text{Force}}{\text{acceleration}} = \frac{\text{Work done (or) Energy}}{\text{acceleration} \times \text{displacement}}$$

$$[m] = \frac{\text{Energy}}{[\text{acceleration}][\text{displacement}]} = \frac{[E]}{[LT^{-2}][L]}$$

$$= \frac{[E]}{[L^2T^{-2}]} = [EL^{-2}T^2]$$

*W F d*

*$F = ma$   
 $m = \frac{M L T^{-2}}{L T^{-2}} = M$*

*$F = ma$   
 $m = \frac{F}{a} = \frac{F \times L}{L T^{-2}}$*

*$W = F \cdot d$*

*$F = ma$*

*$m = \frac{F}{a} = \frac{W}{\text{dis} \cdot a}$*

*$= \frac{[E]}{[LT^{-2}]L}$   
 $= \frac{[E]}{[L^2T^{-2}]}$*

**28) Define velocity and speed.**

Velocity - Velocity is defined as the rate of change of position vector with respect to time (or) defined as the rate of change of displacement. It is a vector quantity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

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$$\vec{v} = \frac{d\vec{r}}{dt}$$

Unit -  $ms^{-1}$

Dimension -  $[LT^{-1}]$

Speed - Speed is defined as the rate of change of distance. It is a scalar quantity.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time taken}}$$

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Unit -  $ms^{-1}$

Dimension -  $[LT^{-1}]$

**29) Explain the various types of friction suggest a few methods to reduce friction.**

There are two types of Friction:

**(1) Static Friction:**

Static friction is the force which opposes the initiation of motion of an object on the surface.

The magnitude of static frictional force  $f_s$  lies between

$$0 \leq f_s \leq \mu_s N$$

where,  $\mu_s$  - coefficient of static friction

$N$  - Normal force

**(2) Kinetic friction:**

The frictional force exerted by the surface when an object slides is called as kinetic friction. Also called as sliding friction or dynamic friction,

$$f_k = \mu_k N$$

where  $\mu_k$  – the coefficient of kinetic friction

N – Normal force exerted by the surface on the object

**Methods to reduce friction:**

Friction can be reduced

- By using lubricants
- By using Ball bearings
- By polishing
- By streamlining

**30) Under what condition will a car skid on a leveled circular road?**

On a leveled circular road, if the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid  $\mu_s < v^2/rg$

**31) Explain the characteristics of elastic and inelastic collision Elastic collision:**

In a collision, the total K.E of the bodies before collision is equal to the total K.E. of the bodies after collision, then it is an elastic collision.

Total K.E. before collision = Total K.E. after collision

**Inelastic collision:**

In a collision, the total K.E. of the bodies before collision is not equal to total K.E. after collision then it is called as inelastic collision Even though K.E. is not conserved but total energy is conserved. After collision of the two colliding bodies stick together such collision are called as perfectly inelastic a plastic collision.

**32) Define center of gravity.**

The center of gravity of a body is the point at which the entire weight of the body acts, irrespective of the position and orientation of the body.

**33) State conservation of angular momentum?**

When no external torque acts on a body, the net angular momentum of a rotating rigid body remains constant.

**IV. 5 Marks**

34) a) State and explain work energy principle Mention any three examples for it.

**Law:**

Work done by a force on the body changes the kinetic energy of the body, ie change in K.E. = work done. This is called work energy theorem.

**Proof:**

Consider a body of mass m at rest on a frictionless horizontal surface. The work done (W) done by the constant force (F) for displacement (S) in the same direction is  $W = FS \rightarrow (1)$

The constant force is given by  $F = ma \rightarrow (2)$

We know that  $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s}$$

Substituting in equ (2)

$$F = m \left( \frac{v^2 - u^2}{2s} \right) \rightarrow (3)$$

Substituting (3) in (1)

$$W = m \left( \frac{v^2 - u^2}{2s} \right) s$$

$$W = \frac{1}{2} m (v^2 - u^2)$$

$$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

Here the term  $\frac{1}{2} mv^2$  indicates K.E.

$$\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = \Delta K \text{ (change in K.E.)}$$

$$\therefore W = \Delta K$$

Hence proved

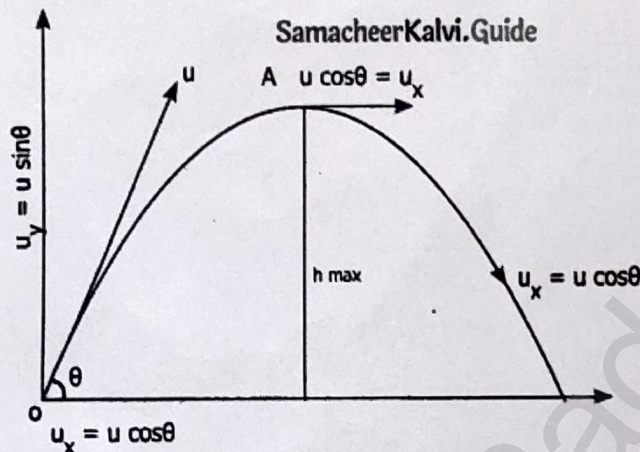
### Examples:

1. A moving hammer drives a nail into the wood. Being in motion, it has K.E. or ability to do work.
2. A fast moving stone can break a windowpane. The stone has K.E. due to its motion and so it can do work.
3. The kinetic energy of air is used to run windmills.

b) Derive the equations of motion, range, and maximum height reached by a particle thrown at an oblique angle  $\theta$  with respect to the horizontal direction.

Answer:

Consider an object thrown with an initial velocity  $u$  at an angle  $\theta$  with horizontal.



Then initial velocity is resolved into two components

$u_x = u \cos \theta$  horizontally and

$u_y = u \sin \theta$  vertically

At maximum height  $u_y = 0$  (since acceleration due to gravity is opposite to the direction of the vertical component).

The Horizontal component of velocity

$u_x = u \cos \theta$  remains constant throughout its motion.

hence after the time  $t$  the velocity along the horizontal motion

$$V_x = U_x + a_x t$$

$$= u_x = u \cos \theta$$

The horizontal distance travelled by the projectile in a time ' $t$ ' is  $S_x = u_x t + \frac{1}{2} a_x t^2$ .

Here  $S_x = x$   $u_x = u \cos \theta$

$$a_x = 0$$

$$\therefore x = u \cos \theta t \quad (1)$$

$$\therefore t = \frac{x}{u \cos \theta} \quad (2)$$

For vertical motion

$$V_y = u_y + a_y t$$

Here  $v_y = 0$

$$u_y = u \sin \theta$$

$$a_y = -g$$

$$v_y = u \sin \theta - gt$$

The vertical distance travelled by the projectile in the same time ' $t$ ' is

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$S_y = y, U_y = u \sin \theta, a_y = -g$$

$$y = u \sin \theta t - \frac{1}{2} g t^2 \quad (4)$$

Substituting the value of t in (4) we get equation:

$$y = u \sin \theta \frac{x}{u \cos \theta} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

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$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

Which indicates the path followed by the projectile is an inverted parabola.

Expression for Maximum height:

The maximum vertical distance travelled by the projectile during its motion is called maximum height.

We know that

$$v_y^2 = u_y^2 + 2a_y s$$

$$\text{Here } u_y = u \sin \theta, a_y = -g, s = h_{\max}$$

$$v_y = 0$$

$$0 = u^2 \sin^2 \theta + 2(-g) h_{\max}$$

$$0 = u^2 \sin^2 \theta - 2gh_{\max}$$

$$2g h_{\max} = u^2 \sin^2 \theta$$

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$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

Expression for horizontal range:

The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground is called horizontal range.

Horizontal range = Horizontal component of velocity x time of flight

$$R = u \cos \theta \times t_f \rightarrow (1)$$

Time of flight ( $t_f$ ) is the time taken by the projectile from point of projection to point the projectile hits the ground again

$$\text{w.k.t } S_y = u_y t_f + \frac{1}{2} a_y t_f^2$$

$$\text{Here } S_y = 0, u_y = u \sin \theta, a_y = -g$$

$$0 = u \sin \theta t_f - \frac{1}{2} g t_f^2$$

$$\frac{1}{2} g t_f^2 = u \sin \theta t_f$$

$$t_f = \frac{2u \sin \theta}{g} \rightarrow (2)$$

Substituting (2) in (1) we get

$$R = \frac{u \cos \theta \times 2u \sin \theta}{g}$$

$$R = 2u^2 \frac{\sin \theta \cos \theta}{g}$$

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$$R = \frac{u^2}{g} \cdot 2 \sin \theta \cos \theta$$

$$R = \frac{u^2}{g} \sin 2\theta \quad [\because 2 \sin \theta \cos \theta = \sin 2\theta]$$

35) a)

Here  $k$  is the dimensionless constant.  
Rewriting the above equation with dimensions

$$[T^1] = [M^a] [L^b] [LT^{-2}]^c$$

$$[M^0 L^0 T^1] = [M^a L^{b+2c} T^{-2c}]$$

Comparing the powers of  $M$ ,  $L$  and  $T$  on both sides,  $a=0$ ,  $b+2c=0$ ,  $-2c=1$

Solving for  $a, b$  and  $c$   $a=0$ ,  $b=1/2$ , and  $c=-1/2$

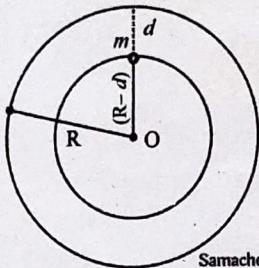
From the above equation  $T = k m^0 \ell^{1/2} g^{-1/2}$

$$T = k \left( \frac{\ell}{g} \right)^{1/2} = k \sqrt{\frac{\ell}{g}}$$

Experimentally  $k = 2\pi$ , hence  $T = 2\pi \sqrt{\frac{\ell}{g}}$

b) Explain the variation of  $g$  with depth from the Earth's surface.

Let us consider a particle of mass  $m$  which is in a deep mine on the Earth. Let the depth of the mine as  $d$ . To calculate  $g'$  at a depth  $d$ , consider the following points.



Particle in a mine

$$g' = g \left( 1 - \frac{d}{R_e} \right) \quad \text{i.e., } g' < g$$

The part of the Earth that is above the radius  $(R_e - d)$  do not contribute to the acceleration.

The result is proved earlier and is given as  $GM$

$$g' = \frac{GM'}{(R_e - d)^2}$$

$M'$  is the mass of the Earth of radius  $(R_e - d)$

Assuming the density of Earth  $\rho$  to be constant,

$$\rho = \frac{M}{V}$$

Where  $M$  is the mass of the Earth and  $V$  its volume, Thus,

$$\rho = \frac{M'}{V'}$$

$$\frac{M'}{V'} = \frac{M}{V} \quad \text{and} \quad M' = \frac{M}{V} V'$$

$$M' = \left( \frac{M}{\frac{4}{3}\pi R_e^3} \right) \left( \frac{4}{3}\pi (R_e - d)^3 \right)$$

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$$M' = \frac{M}{R_e^3} (R_e - d)^3$$

$$g' = G \frac{M}{R_e^3} (R_e - d)^3 \cdot \frac{1}{(R_e - d)^2}$$

$$g' = GM \frac{R_e \left( 1 - \frac{d}{R_e} \right)}{R_e^3}$$

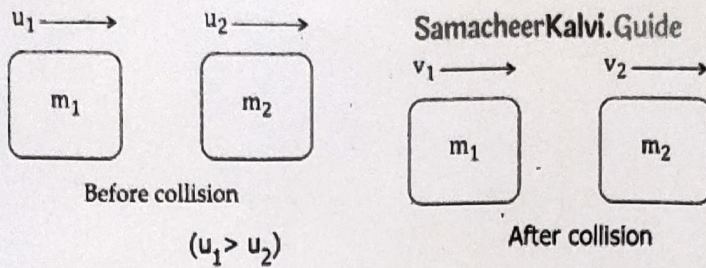
Thus,

$$g' = GM \frac{\left( 1 - \frac{d}{R_e} \right)}{R_e^2}$$

$$g' = g \left( 1 - \frac{d}{R_e} \right)$$

Here also  $g' < g$ . As depth increases,  $g'$  decreases.

36) a) Arrive at an expression for elastic collision in one dimension and discuss various cases.



Consider two elastic bodies of masses  $m_1$  and  $m_2$  moving in a straight line (along positive direction) on a frictionless horizontal surface. In order to have collision assume  $m_1$  moves faster than  $m_2$ .

Let  $U_1$  and  $u_2$  be the initial velocities of  $m_1$  and  $m_2$  respectively. ( $u_1 > u_2$ ). After collision let the masses  $m_1$  and  $m_2$  moves with velocities  $v_1$  and  $v_2$  respectively.

In case of elastic collision both linear momentum and kinetic energies are conserved

$\therefore$  from law of conservation of linear momentum

Total momentum before (Pi)

collision = Total momentum after collision (Pf)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \rightarrow (1)$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \rightarrow (2)$$

Further

Total K.E. before collision ( $KE_i$ ) = Total K.E. after collision ( $KE_f$ )

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(u_2^2 - v_2^2) \rightarrow (3)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(u_2 + v_2)(u_2 - v_2) \rightarrow (4)$$

Dividing  $\frac{4}{2}$  SamacheerKalvi.Guide

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} = \frac{m_2(u_2 + v_2)(u_2 - v_2)}{m_2(u_2 - v_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

rearranging

$$u_1 - u_2 = -(v_1 - v_2) \rightarrow (5)$$

From this it is clear that for any elastic collision, relative speed of two elastic bodies after the collision has the same magnitude as before collision but in opposite direction.

Rewriting the above equation for  $v_1$  &  $v_2$

$$v_1 = v_2 + u_2 - u_1 \rightarrow (6)$$

(or)

$$v_2 = v_1 + u_1 - u_2 \rightarrow (7)$$

To find velocities of  $v_1$  &  $v_2$

Substituting (7) in (2)

$$m_1(u_1 - v_1) = m_2(v_1 + u_1 - u_2 - u_2)$$

$$m_1(u_1 - v_1) = m_2(v_1 + u_1 - 2u_2)$$



$$m_1 u_1 - m_1 v_1 = m_2 v_1 + m_2 u_1 - 2m_2 u_2$$

$$m_1 u_1 - m_2 u_1 + 2m_2 u_2 = m_1 v_1 + m_2 v_1$$

$$u_1(m_1 - m_2) + 2m_2 u_2 = v_1(m_1 + m_2)$$

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + 2 \frac{m_2 u_2}{(m_1 + m_2)}$$

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$$\therefore v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2 \rightarrow v(\beta)$$

Similarly by substituting (6) in (2)

$$v_2 = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) u_2 + \left( \frac{2m_1}{m_1 + m_2} \right) u_1 \rightarrow (9)$$

Case 1:

When bodies have same mass is  $m_1 = m_2 = m$

$$v_1 = (0) u_1 + \left( \frac{2m}{2m} \right) u_2$$

$$v_1 = u_2 \quad \text{SamacheerKalvi.Guide}$$

$$v_2 = \left( \frac{2m}{2m} \right) u_1 + (0) u_2$$

$$v_2 = u_1$$

The velocities get interchanged.

Case 2:

When both bodies have same mass  $m_1 = m_2 = m$ , but second body at rest in  $u_2 = 0$

$$v_1 = 0, v_2 = u_1$$

After collision the first body comes to rest and the second body moves with the velocity of first body.

Case 3 :

The first body very much lighter than the second body in  $m_1 \ll m_2$ ,  $m_1 m_2 \ll 1$ .

the ratio  $m_1 m_2 = 0$  and also second body at rest, ( $u_2 = 0$ )

Dividing numerator and denominator of equation 8 by  $m_2$

$$v_1 = \left( \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} \right) u_1 + \left( \frac{2}{\frac{m_1}{m_2} + 1} \right) (0)$$

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$$v_1 = \left( \frac{0-1}{0+1} \right) u_1$$

$$v_1 = -u_1$$

Similarly dividing numerator and denominator of equation 9 by  $m_1$

$$v_2 = \left( \frac{2 \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \right) u_1 + u_2 + 0$$

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$$v_2 = \left( \frac{2 \times 0}{1+0} \right)$$

$$v_2 = 0$$

From this, the conclusion arrived is the first body which is lighter returns back (rebounds) in opposite direction with the same initial velocity as it has a negative sign. The second body since it has heavier mass continues to remain at rest even after the collision.

Case 4 :

The second body is very much lighter than the first body.

$m_2 \ll m_1$  then the ratio = 0  $m_2 m_1$  and also if the target is at rest ie second body at rest ( $u_2 = 0$ )

$$v_1 = \left( \frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \right) u_1 + \left( \frac{2 \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \right) (0) \quad \text{ie numerator and denominator by } m_1$$

$$v_1 = \left( \frac{1-0}{1+0} \right) u_1$$

$$v_1 = u_1 \quad \text{SamacheerKalvi.Guide}$$

Similarly dividing equ (9) numerator and denominator by  $m_2$

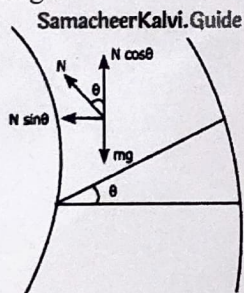
$$v_2 = \left( \frac{2}{1 + \frac{m_2}{m_1}} \right) u_1 + 0$$

$$v_2 = \left( \frac{2}{1} \right) u_1$$

$$v_2 = 2u_1$$

### b) Explain the need for banking of tracks.

In a leveled circular road skidding mainly depends on the co-efficient of static friction  $\mu_s$ . The coefficient of static friction depends on the nature of surface which has a maximum limiting value. To avoid this usually "the outer edge of the road is slightly raised compared to inner edge". This is called banking of roads or tracks. The angle of inclination called banking angle.



Let the surface of the road make angle  $\theta$  with horizontal surface. Then the normal force makes an angle  $\theta$  with vertical. When the car takes a turn, two forces are acting on the car.

- Gravitational force  $mg$  (downwards)
- Normal force  $N$  (Perpendicular to surface).

Normal force ' $N$ ' can be resolved into two components  $N \cos \theta$  and  $N \sin \theta$  and balances downward gravitational force.

$N \sin \theta$  provides necessary centripetal acceleration, According to II law

$$N \cos \theta = mg$$

$$N \sin \theta = mv^2/r$$

Dividing the above equations,

$$\tan \theta = v^2/rg$$

$$v = r \tan \theta \sqrt{g}$$

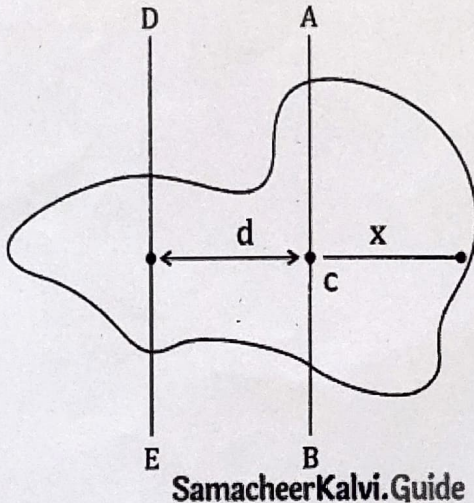
$\therefore$  The banking angle  $\theta$  and radius of curvature of the road or track determines the safe speed of car at the turning.

If the speed exceeds this safe limit, then it starts to skid outward but the frictional force comes into effect and provides an additional centripetal force to prevent outward skidding. But at the same time if the speed is less than the safe limit it starts to skid inward and again frictional force come into effect which reduces centripetal force to prevent inward skidding

However if the speed of the vehicle is sufficiently greater than the correct speed the frictional force cannot stop the car from skidding. So to avoid skidding in circular road or tracks they are banked.

**37) a) State and prove parallel axes theorem.**

Parallel axes theorem states that the moment of Inertia of a body about any axis is equal to the sum of its moment of Inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between two axis.



Let  $I_c$  be the moment of Inertia of the body of mass  $m$  about an axis (AB) passing through the center of mass.

Let  $I$  be the moment of Inertia of a parallel axis (DE) at a distance  $d$  from AB is

$$I = I_c + Md^2$$

Proof:

Consider a rigid body as in figure. Let  $I_c$  be the moment of Inertia of the body about an axis AB passing through center of mass  $c$ .

DE is another axis parallel to AB at a perpendicular distance  $d$  from AB. Let  $I$  be the moment of Inertia about the axis DE.

Consider a point P of mass 'm' on the body at a distance,  $x$  from  $c$ .

The moment of Inertia of the point mass about the axis DE =  $m(x+d)^2$ .

The moment of Inertia  $I$  of the whole body about

$$DE = I = \sum m (x+d)^2$$

$$I = \sum m (x^2 + d^2 + 2xd)$$

$$I = \sum mx^2 + \sum md^2 + 2\sum mx d$$

$$I = \sum mx^2 + \sum md^2 + 2d \sum mx$$

Here  $\sum mx^2 =$  moment of Inertia of the body about the center of mass

$$\therefore I_c = \sum mx^2$$

The term  $\sum mx = 0$ , because  $x$  can take positive and negative values w.r. to the axis AB.

$$I = I_c + \sum md^2$$

$$\text{but } \sum m = M$$

$$\therefore I = I_c + Md^2$$

Hence proved

**b) Prove that at points near the surface of the Earth, the gravitational potential energy of the object is  $U = mgh$ .**

Let us consider the Earth and mass system, with  $r$ , the distance between the mass 'm' and the Earth's centre. Then the gravitational potential energy,

$$U = G M m / r \quad \dots (1)$$

Here  $r = R_e + h$ , where  $R_e$  is the radius of the Earth,  $h$  is the height above the Earth's surface.

$$U = - G M m / (R_e + h) \quad \dots (2)$$

If  $h \ll R_e$ , equation (2) can be modified as

$$U = -G \frac{M_e m}{R_e (1 + h/R_e)}$$

$$U = -G \frac{M_e m}{R_e} (1 + h/R_e)^{-1} \quad \dots (3)$$

38) a)

Table 3.4 Salient Features of Centripetal and Centrifugal Forces	
Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or centre of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the centre of the circular motion
$ F_{cp}  = m\omega^2 r = \frac{mv^2}{r}$	$ F_{cf}  = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects.	Origin of centrifugal force is inertia. It does not arise from interaction.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame.
	In inertial frames there is no centrifugal force.
	In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

Table 5.2 Different types of Equilibrium and their Conditions.

Type of equilibrium	Conditions
Translational equilibrium	<ul style="list-style-type: none"> <li>Linear momentum is constant.</li> <li>Net force is zero.</li> </ul>
Rotational equilibrium	<ul style="list-style-type: none"> <li>Angular momentum is constant.</li> <li>Net torque is zero.</li> </ul>
Static equilibrium	<ul style="list-style-type: none"> <li>Linear momentum and angular momentum are zero.</li> <li>Net force and net torque are zero.</li> </ul>
Dynamic equilibrium	<ul style="list-style-type: none"> <li>Linear momentum and angular momentum are constant.</li> <li>Net force and net torque are zero.</li> </ul>
Stable equilibrium	<ul style="list-style-type: none"> <li>Linear momentum and angular momentum are zero.</li> <li>The body tries to come back to equilibrium if slightly disturbed and released.</li> <li>The centre of mass of the body shifts slightly higher if disturbed from equilibrium.</li> <li>Potential energy of the body is minimum and it increases if disturbed.</li> </ul>
Unstable equilibrium	<ul style="list-style-type: none"> <li>Linear momentum and angular momentum are zero.</li> <li>The body cannot come back to equilibrium if slightly disturbed and released.</li> <li>The centre of mass of the body shifts slightly lower if disturbed from equilibrium.</li> <li>Potential energy of the body is not minimum and it decreases if disturbed.</li> </ul>
Neutral equilibrium	<ul style="list-style-type: none"> <li>Linear momentum and angular momentum are zero.</li> <li>The body remains at the same equilibrium if slightly disturbed and released.</li> <li>The centre of mass of the body does not shift higher or lower if disturbed from equilibrium.</li> <li>Potential energy remains same even if disturbed.</li> </ul>