V.M.G.RAJASEKARAN-RAMANI SRI SARADA SAKTHI MAT.HR.SEC.SCHOOL

CLASS: XI PHYSICS MARKS: 70

I. CHOOSE THE CORRCT ANSWER 15X1=15

1	b	g/2
2	c	13 J
3	b	Torque and energy
4	c	Momentum
5	b	6s
6	b	Zero
7	b	$\sqrt{\frac{10}{7}gh}$
8	b	Always negative
9	a	Increases
10	b	0.02020
11	b	dL/dt = 0
12	c	Inertia of direction
13	a	2:1
14	c	$L\sqrt{2}$
15	d	F

II. ANSWER ANY SIX OF THE FOLLOWING. Q.No 24 is compulsory

6X2=12

16. Define the types of physical quantities.

Physical quantities are classified into two types. They are fundamental and derived quantities.

Fundamental or base quantities are quantities which cannot be expressed in terms of any other physical quantities.

Example: length, mass, time, electric current,

Quantities that can be expressed in terms of fundamental quantities are called derived quantities.

Example, area, volume

17. Define acceleration

The acceleration of the particle at an instant t is equal to rate of change of velocity. Acceleration is a vector quantity. Its SI unit is ms^{-2}.

18. Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).

Solution

Power, P = 75 W

Time of usage, t = 8 hour × 30 days = 240 hours

Electrical energy consumed is the product of power and time of usage.

Electrical energy = power × time of usage = P × t

$$= 75 \text{ watt} \times 240 \text{ hour}$$
$$= 18000 \text{ watt hour}$$
$$= 18 \text{ kilowatt hour} = 18\,\text{kWh}$$
$$1 \text{ electrical unit} = 1\,\text{kWh}$$
$$\text{Electrical energy} = 18 \text{ unit}$$

19. Define torque. Write the unit.

Torque is defined as the moment of the external applied force about a point or axis of rotation. Its SI unit is Nm.

20. Define gravitational potential energy.

Gravitational potential energy of a system of two masses $m_1$and m_2 separated by a distance r as the amount of work done to take the mass m_2 from a distance r to infinity.

21. State Newton's third law of motion. Give one example

Newton's third law states that for every action there is an equal and opposite reaction.

Example: Swimming

22. Define coefficient of restitution.

It is defined as the ratio of velocity of separation (relative velocity) after collision to the velocity of approach (relative velocity) before collision.

23. An iron ball and a feather are both falling from a height of 10 m.

At what time they reach the ground?

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 10}{10}} = \sqrt{2}\,s \approx 1.414\,s$$

24. A mango of mass 400 9 is hanging from a tree. Find the tension on a mango.

Solution: T = m X g = 400 X 10^{-3} X 9.8 = 3.92 N.

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PART – III

ANSWER ANY SIX OF THE FOLLOWING. Q.NO 33 is compulsory

6X3=18

25. Write the limitations of dimensional analysis

1. This method gives no information about the dimensionless constants in the formula like 1, 2 ……..π, e (Euler number), etc.

2. This method cannot decide whether the given quantity is a vector or a scalar.

3. This method is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.

4. It cannot be applied to an equation involving more than three physical quantities.

5. It can only check on whether a physical relation is dimensionally correct but not the correctness of the relation

26. Discuss the properties of scalar product of two vectors

Properties

(i) The product quantity $\vec{A}\cdot\vec{B}$ is always a scalar. It is positive if the angle between the vectors is acute (i.e., θ < 90°) and negative if the angle between them is obtuse (i.e. 90°<θ< 180°).

(ii) The scalar product is commutative, i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) The vectors obey distributive law i.e. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]$

(v) The scalar product of two vectors will be maximum when cos θ = 1, i.e. θ = 0°, i.e., when the vectors are parallel;

$$(\vec{A}\cdot\vec{B})_{max} = AB$$

(vi) The scalar product of two vectors will be minimum, when cos θ = −1, i.e. θ = 180°
$(\vec{A}\cdot\vec{B})_{min} = -AB$, when the vectors are anti-parallel.

(vii) If two vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product $\vec{A}\cdot\vec{B} = 0$, because cos 90°=0. Then the vectors $\vec{A}$ and $\vec{B}$ are said to be mutually orthogonal.

(viii) The scalar product of a vector with itself is termed as self-dot product and is given by $(\vec{A})^2 = \vec{A}\cdot\vec{A} = AA\cos\theta = A^2$. Here angle $\theta = 0°$

27. Which is easier to move an object push or pull. Discuss with relevant diagram.

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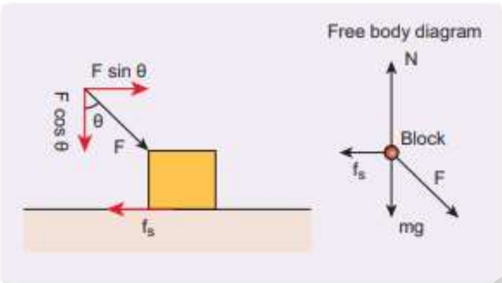
When a body is pushed at an arbitrary angle θ the applied force F can be resolved into two components as F sinθ parallel to the surface and F cosθ perpendicular to the surface as shown in Figure 3.26. The total downward force acting on the body is mg + Fcosθ. It implies that the normal force acting on the body increases. Since there is no acceleration along the vertical direction the normal force N is equal to

$$N_{push} = mg + F\cos\theta \quad (3.29)$$

As a result the maximal static friction also increases and is equal to

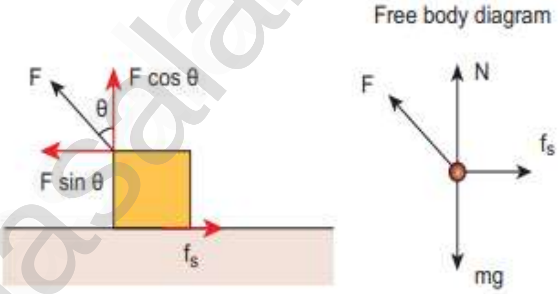
$$f_s^{max} = \mu_s N_{push} = \mu_s\left(mg + F\cos\theta\right) \quad (3.30)$$

Equation (3.30) shows that a greater force needs to be applied to push the object into motion.



When an object is pulled at an angle θ, the applied force is resolved into two components as shown in Figure 3.27. The total downward force acting on the object is

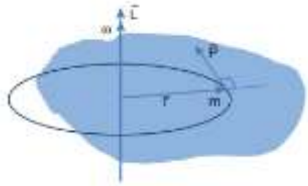
$$N_{pull} = mg - F\cos\theta \quad (3.31)$$



29. Define angular momentum. Deduce the relation between angular momentum and angular velocity.

Angular momentum of a point mass is defined as the moment of its linear momentum.

Let us consider a rigid body rotating about a fixed axis. A point mass m in the body will execute a circular motion about the fixed axis.The point mass m is at a distance r from the axis of rotation. Its linear momentum at any instant is tangential to the circular path. Then the angular momentum ρ L is perpendicular to ρ r and ρ p. Hence, it is directed along the axis of rotation. The angle θ between ρ r and ρ p in this case is 90º. The magnitude of the angular momentum L could be written as,



L= rmv sin90⁰

= rmv

where, v is the linear velocity.

The relation between linear velocity v and angular velocity ω in a circular motion is, v = r ω.

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L= rmrω

= (mr^2) ω

30. State the Kepler's law.

Law of orbits: Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci

Law of area: The radial vector (line joining the Sun to a planet) sweeps equal areas in equal intervals of time.

Law of period: The Square of the time period of revolution of a planet around the Sun in its elliptical orbit is directly proportional to the cube of the semi-major axis of the ellipse

31. Mention the difference between conservative and non-conservative forces.

Table 4.3 Comparison of conservative and non-conservative forces

S.No	Conservative forces	Non-conservative forces
1.	Work done is independent of the path	Work done depends upon the path
2.	Work done in a round trip is zero	Work done in a round trip is not zero
3	Total energy remains constant	Energy is dissipated as heat energy
4	Work done is completely recoverable	Work done is not completely recoverable.
5	Force is the negative gradient of potential energy	No such relation exists.

32. A uniform disc of mass 100g has a diameter of 10cm. Calculate the total energy of the disc when rolling along a horizontal table with a velocity of 20 cms^{-1}.

Solution:

m = 100 x 10^{-3} kg

= 100g

2r = 10 x 10^{-2} m

r = 5 cm

r = 5 x 10^{-2} m

V = 20 cm/s

V = 20 cms^{-1}= 20 x 10^{-2} m/s

E =?

Energy = $\frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(mV^2 + I\omega^2)$, where I = $\frac{1}{2}mr^2$

$= \frac{1}{2}\left[\frac{1}{10} \times 0.2 \times 0.2 + \frac{1}{2} \times \frac{1}{10} \times 25 \times \frac{1}{10^4} \times 16\right]$

$= \frac{1}{2}\left[\frac{4}{1000} + \frac{2}{1000}\right] = \frac{1}{2}\left[\frac{6}{1000}\right]$

Energy = 3×10^{-3} J

33. The ratio of orbital distance of two planets are 1:2, Find the ratio of gravitational field experienced by these two planets.

$E_2 = \frac{GM}{d_2^2}$

The ratio of their orbital distance $\frac{d_1}{d_2} = 2$

$\therefore\ d_1 = 2d_2$

The ratio of gravitational field of two planets.

$\frac{E_1}{E_2} = \frac{GM}{(2d_2)^2} \times \frac{d_2^2}{GM} = \frac{GM}{4d_2^2} \times \frac{d_2^2}{GM}$

$\frac{E_1}{E_2} = \frac{1}{4}$

$E_2 = 4E_1$

PART – IV

ANSWER THE FOLLOWING QUESTIONS IN DETAIL 5X5=25

34. a) Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass 'm' of the bob (ii) length 'l' of the pendulum and (iii) acceleration due to gravity g at the place where the pendulum is suspended. (Constant k = 2π)

$T \propto m^a l^b g^c$

$T = k m^a l^b g^c$

$$[T^1] = [M^a]\,[L^b]\,[LT^{-2}]^c$$

$$[M^0L^0T^1] = [M^a\,L^{b+c}\,T^{-2c}]$$

Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1

Solving for a,b and c a = 0, b = 1/2, and c = −1/2

From the above equation $T = km^0 \ell^{1/2} g^{-1/2}$

$$T = k\left(\frac{\ell}{g}\right)^{1/2} = k\sqrt{\ell/g}$$

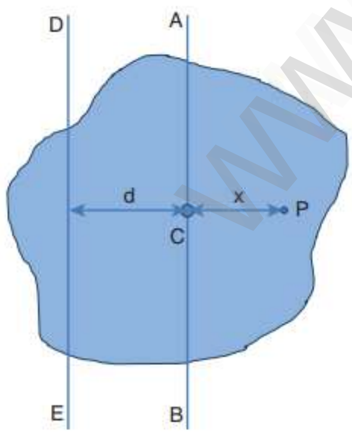
Experimentally k = 2π, hence $T = 2\pi\sqrt{\ell/g}$

b) State and prove parallel axis theorem in moment of inertia.

Parallel axis theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

If IC is the moment of inertia of the body of mass M about an axis passing through the centre of mass, then the moment of inertia I about a parallel axis at a distance d from it is given by the relation,

$$I = I_C + Md^2$$



$$I = \sum m(x+d)^2$$

$$I = \sum m\left(x^2 + d^2 + 2xd\right)$$

$$I = \sum\left(mx^2 + md^2 + 2dmx\right)$$

$$I = \sum mx^2 + \sum md^2 + 2d\sum mx$$

Here, $\sum mx^2$ is the moment of inertia of the body about the centre of mass. Hence, $I_C = \sum mx^2$

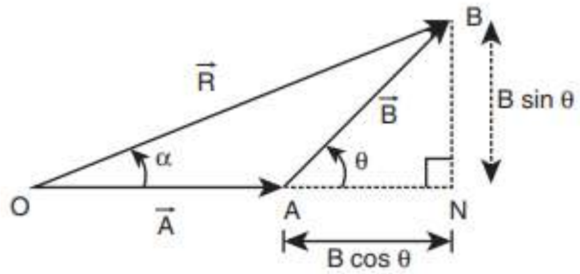
The term, $\sum mx = 0$ because, x can take positive and negative values with respect to the axis AB. The summation $\left(\sum mx\right)$ will be zero.

Thus, $I = I_C + \sum md^2 = I_C + \left(\sum m\right)d^2$

Here, $\sum m$ is the entire mass M of the object $\left(\sum m = M\right)$

$$I = I_C + Md^2$$

35. a) Explain in detail the triangle law of vector addition.



Magnitude of resultant vector:

The magnitude and angle of the resultant vector are determined as follows. Consider the triangle ABN, which is obtained by extending the side OA to ON. ABN is a right angled triangle.

$$\cos\theta = \frac{AN}{B} \therefore AN = B\cos\theta \text{ and}$$

$$\sin\theta = \frac{BN}{B} \quad \therefore BN = B\sin\theta$$

For ΔOBN, we have $OB^2 = ON^2 + BN^2$

$$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

which is the magnitude of the resultant of $\vec{A}$ and $\vec{B}$

(2) Direction of resultant vectors: If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad (2.1)$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

b) Arrive an expression for velocities of two bodies in one dimension elastic collision.

Consider two elastic bodies of masses m_1 and m_2 moving in a straight line on a frictionless horizontal surface. We assume that the mass m_1 moves faster than mass m_2. For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

Mass	Initial velocity	Final velocity
Mass m_1	u_1	v_1
Mass m_2	u_2	v_2

	Momentum of mass m_1	Momentum of mass m_2	Total linear momentum
Before collision	$p_{i1} = m_1u_1$	$p_{i2} = m_2u_2$	$p_i = p_{i1} + p_{i2}$ $p_i = m_1u_1 + m_2u_2$
After collision	$p_{f1} = m_1v_1$	$p_{f2} = m_2v_2$	$p_f = p_{f1} + p_{f2}$ $p_f = m_1v_1 + m_2v_2$

From the law of conservation of linear momentum,

Total momentum before collision (p_i) = Total momentum after collision (p_f)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad (4.46)$$

$$\text{Or } m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad (4.47)$$

	Kinetic energy of mass m_1	Kinetic energy of mass m_2	Total kinetic energy
Before collision	$KE_{i1} = \frac{1}{2}m_1u_1^2$	$KE_{i2} = \frac{1}{2}m_2u_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$
After collision	$KE_{f1} = \frac{1}{2}m_1v_1^2$	$KE_{f2} = \frac{1}{2}m_2v_2^2$	$KE_f = KE_{f1} + KE_{f2}$ $KE_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad (4.48)$$

After simplifying and rearranging the terms,

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

Using the formula $a^2 - b^2 = (a+b)(a-b)$, we can rewrite the above equation as

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad (4.49)$$

Dividing equation (4.49) by (4.47) gives,

$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)} = \frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

Rearranging,

$$u_1 - u_2 = v_2 - v_1 \quad (4.50)$$

Equation (4.50) can be rewritten as

$$u_1 - u_2 = -(v_1 - v_2)$$

$$v_1 = v_2 + u_2 - u_1 \quad (4.51)$$

Or

$$v_2 = u_1 + v_1 - u_2 \quad (4.52)$$

To find the final velocities v_1 and v_2:
Substituting equation (4.52) in equation (4.47) gives the velocity of m_1 as

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - u_2 - u_2)$$

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$$

$$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

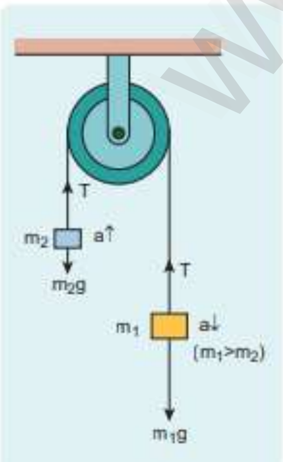
$$\text{or } v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad (4.53)$$

Similarly, by substituting (4.51) in equation (4.47) or substituting equation (4.53) in equation (4.52), we get the final velocity of m_2 as

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \quad (4.54)$$

36. a) Explain the motion of blocks connected by a string in vertical direction.

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley. Two blocks connected by a string over a pulley ($m_1>m_2$) Let the tension in the string be T and acceleration a. When the system is released, both the blocks start moving, m_2 vertically upward and m1 downward with same acceleration a. The gravitational force m_1 g on mass m_1 is used in lifting the mass m_2.



Applying Newton's second law for mass m_2

$$T\hat{j} - m_2 g\hat{j} = m_2 a\hat{j}$$

The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get

$$T - m_2 g = m_2 a \qquad (3.9)$$

Similarly, applying Newton's second law for mass m_1

$$T\hat{j} - m_1 g\hat{j} = -m_1 a\hat{j}$$

As mass m_1 moves downward $(-\hat{j})$, its acceleration is along $(-\hat{j})$

$$T - m_2 g = m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

$$T = m_2 g + m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \qquad (3.13)$$

By comparing the components on both sides, we get

$$T - m_1 g = -m_1 a$$

$$m_1 g - T = m_1 a \qquad (3.10)$$

Adding equations (3.9) and (3.10), we get

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$(m_1 - m_2)g = (m_1 + m_2)a \qquad (3.11)$$

From equation (3.11), the acceleration of both the masses is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \qquad (3.12)$$

If both the masses are equal ($m_1 = m_2$), from equation (3.12)

$$a = 0$$

$$T = m_2 g\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g\left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$$

Equation (3.12) gives only magnitude of acceleration.

For mass m_1, the acceleration vector is given by $\vec{a} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g\hat{j}$

For mass m_2, the acceleration vector is given by $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g\hat{j}$

b) Derive an expression for escape velocity

Escape velocity is defined as the minimum speed required for a body to escape from the earth's gravitational pull".

Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i, the initial total energy of the object is

$$E_i = \frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} \qquad (6.53)$$

where, M_E is the mass of the Earth and R_E- the radius of the Earth. The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

$$E_i = E_f \qquad (6.54)$$

Substituting (6.53) in (6.54) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \qquad (6.55)$$

Consider the escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace v_i with v_e. i.e,

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

Using $g = \frac{GM_E}{R_E^2}$,

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \qquad (6.56)$$

37. a) Derive an expression for centripetal acceleration.

Let the directions of position and velocity vectors shift through the same angle θ in a small interval of time Δt, as shown in Figure 2.52. For uniform circular motion, $r = |\vec{r}_1| = |\vec{r}_2|$ and $v = |\vec{v}_1| = |\vec{v}_2|$. If the particle moves from position vector $\vec{r}_1$ to $\vec{r}_2$, the displacement is given by $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ and the change in velocity from $\vec{v}_1$ to $\vec{v}_2$ is given by $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. The magnitudes of the displacement Δr and of Δv satisfy the following relation

$$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$$

Here the negative sign implies that Δv points radially inward, towards the centre of the circle.

$$\Delta v = -v\left(\frac{\Delta r}{r}\right)$$

Then, $$a = \frac{\Delta v}{\Delta t} = -\frac{v}{r}\left(\frac{\Delta r}{\Delta t}\right) = -\frac{v^2}{r}$$

For uniform circular motion $v = \omega r$, where ω is the angular velocity of the particle

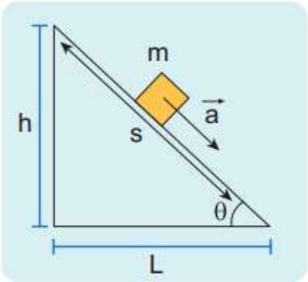
b) Explain the motion of a body on an inclined plane. Obtain an expression for velocity of a body when reaches the ground.

When an object of mass m slides on a frictionless surface inclined at an angle θ, the forces acting on it decides the

a) acceleration of the object

b) speed of the object when it reaches the bottom

The force acting on the object is (i) Downward gravitational force (mg) (ii) Normal force perpendicular to inclined surface (N)



$$-mg\cos\theta\hat{j} + N\hat{j} = 0 (No\ acceleration)$$

By comparing the components on both sides, $N - mg\cos\theta = 0$

$$N = mg\cos\theta$$

The magnitude of normal force (N) exerted by the surface is equivalent to $mg\cos\theta$.

The object slides (with an acceleration) along the x direction. Applying Newton's second law in the x direction

$$mg\sin\theta\hat{i} = ma\hat{i}$$

By comparing the components on both sides, we can equate

$$mg\sin\theta = ma$$

The acceleration of the sliding object is

$$a = g\sin\theta$$

$$v^2 = u^2 + 2as \text{ along the x direction} \quad (3.3)$$

The acceleration a is equal to g sinθ. The initial speed (u) is equal to zero as it starts from rest. Here s is the length of the inclined surface.

The speed (v) when it reaches the bottom is (using equation (3.3))

$$v = \sqrt{2sg\sin\theta} \quad (3.4)$$

38. a) State and prove work energy theorem

Work and energy are equivalents. This is true in the case of kinetic energy also. To prove this, let us consider a body of mass m at rest on a frictionless horizontal surface. The work (W) done by the constant force (F) for a displacement (s) in the same direction is,

W = FXs

The constant force is given by the equation, F = ma

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

Substituting for a in equation (4.8),

$$F = m\left(\frac{v^2 - u^2}{2s}\right) \qquad (4.9)$$

Substituting equation (4.9) in (4.7),

$$W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad (4.10)$$

The expression for kinetic energy:

The term $\left(\frac{1}{2}mv^2\right)$ in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v).

$$KE = \frac{1}{2}mv^2 \qquad (4.11)$$

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad (4.12)$$

Thus, W = ΔKE

b) Derive an expression for moment of inertia of a rod about its centre and perpendicular to the rod.

Let us consider a uniform rod of mass (M) and length (l). Let us find an expression for moment of inertia of this rod about an axis that passes through the centre of mass and perpendicular to the rod. First an origin is to be fixed for the coordinate system so that it coincides with the centre of mass, which is also the geometric centre of the rod. The rod is now along the x axis. We take an infinitesimally small mass (dm) at a distance (x) from the origin. The moment of inertia (dI) of this mass (dm) about the axis is,

$$dI = (dm)\, x^2$$

As the mass is uniformly distributed, the mass per unit length (λ) of the rod is, $\lambda = \frac{M}{\ell}$

The (dm) mass of the infinitesimally small length as, $dm = \lambda\, dx = \frac{M}{\ell} dx$

The moment of inertia (I) of the entire rod can be found by integrating dI,

$$I = \int dI = \int (dm) x^2 = \int \left(\frac{M}{\ell} dx \right) x^2$$

$$I = \frac{M}{\ell} \int x^2 dx$$

As the mass is distributed on either side of the origin, the limits for integration are taken from $-\ell/2$ to $\ell/2$.

$$I = \frac{M}{\ell} \int_{-\ell/2}^{\ell/2} x^2 dx = \frac{M}{\ell} \left[\frac{x^3}{3} \right]_{-\ell/2}^{\ell/2}$$

$$I = \frac{M}{\ell} \left[\frac{\ell^3}{24} - \left(-\frac{\ell^3}{24} \right) \right] = \frac{M}{\ell} \left[\frac{\ell^3}{24} + \frac{\ell^3}{24} \right]$$

$$I = \frac{M}{\ell} \left[2 \left(\frac{\ell^3}{24} \right) \right]$$

$$I = \frac{1}{12} M\ell^2 \qquad (5.41)$$